Astronomy 400A: Homework 2

1. Individual Problems

1. What are the cgs units of the opacity, κ? Explain in words the meaning of this quantity.

2. Show that the Hydrostatic Equilibrium equation can be written as

$$\frac{dP}{d\tau} = \frac{g}{\kappa}. \tag{1}$$

   Here, $g = GM/r^2$ is the local gravitational acceleration.

3. Estimate the average random velocities of Nitrogen particles in our classroom. Use this to calculate the mean free path and mean free time. Do you think that the air in the room is in LTE? You may assume room temperature is $T \approx 300$ K, the radius of a nitrogen molecule is 10^{-8} cm, the mass of a nitrogen molecule is $\approx 28\,m_{\rm H}$, and the density of air is $\rho \approx 0.1$ g cm^{-3}.

2. Group Problems

1. The continuum opacity of the Sun is dominated by the H^- ion. In this problem, we will use the Saha equation to investigate why this is so. Assume the temperature is the effective temperature of the Sun, $T_{\rm eff} \approx 5800$ K, the ionization potential of H^- is 0.75 eV, and the number density of electrons is $n_{\rm e} = 1.9 \times 10^{15}$ cm^{-3}. Assume that $u_{\rm H} = 2$ and $u_{\rm H^-} = 1$.

   (a) Use the Saha equation to calculate the abundance of H^- relative to H.

   (b) Not all H atoms produce emission at optical wavelengths, but those in the $n = 2$ and $n = 3$ states can (these contribute to the Balmer and Paschen series). Determine the abundance of H atoms in the $n = 2$ and $n = 3$ electronic state to the abundance of H atoms in the ground state. Then use your work from part a) to estimate the abundance of H^- relative to these excited H atoms. Assume that $g_z = 2x^2$ and $E_x = -13.6/x^2$ eV.

2. Assuming a uniform value of β (recall that $P_{\rm gas} = \beta P_{\rm tot}$), and defining $U = \int (u_{\rm gas} + u_{\rm rad})dm$, use the Virial theorem to show that for a classical, nonrelativistic gas,

$$E = \frac{\beta}{2}\Omega = -\frac{\beta}{2-\beta}U. \tag{2}$$

Discuss the limits of $\beta \to 0$ and $\beta \to 1$.