Astronomy 400A: Homework 5

You may collaborate on all problems this week.

1. Assume a 1 $M_\odot$ core of a 10 $M_\odot$ star collapses, and the remaining 9 $M_\odot$ of material is ejected to infinity, with velocity= 0 at infinity. 100% of the energy resulting from the collapse of the core went into neutrinos, and 1% of this was absorbed by the envelope to drive its ejection. Calculate the final radius of the collapsed core.

2. Consider a white dwarf with $M = 1$ $M_\odot$, $R = 0.01$ $R_\odot$, that has accreted 10^{-4} $M_\odot$ of Hydrogen onto its surface. Assume that this material is completely fused into He, and that this energy is radiated at the Eddington Luminosity. You may assume the opacity is due to electron scattering. Compute the timescale over which this energy is radiated away.

3. The Crab supernova remnant is $4' \times 2'$ on the sky at a distance of 2000 pc, and its measured expansion velocity is 1.5×10^8 cm s^{-1}.

   (a) Estimate the linear dimensions of the Crab nebula.

   (b) Assuming constant velocity over the course of its expansion, estimate the age of the Crab nebula. Do you think the actual age is shorter or longer than this simple estimate?

4. The Crab pulsar occassionally undergoes "glitches", where the period suddenly changes by a small amount. In one glitch, $|\Delta P| = 10^{-8} P$. Assuming this glitch was caused by contraction of the neutron star, compute the fraction of the radius that the star contracted. You may assume that the neutron star is a sphere with moment of inertia $I = 2/5 M R^2$, $M = 1.5$ $M_\odot$ and $R = 20$ km.

5. Gamma ray bursts are observed approximately once per day. There are approximately 100,000 neutron stars in the Milky Way Galaxy. Assuming GRBs arise from these Galactic neutron stars, how often would a given neutron star need to undergo a burst? How does this timescale compare to the age of a neutron star?

6. Determine the minimum rotation period a 1.5 $M_\odot$ neutron star, with $R = 20$ km, could have while remaining gravitationally bound.