Astronomy 400A: Homework 4

You may collaborate on all problems this week.

1. The age of the Universe is $\sim$ 13.7 Gyr. The main-sequence lifetime of the Sun is $\sim$ 10 Gyr.

   (a) Estimate the main-sequence lifetime of a 0.8 $M_\odot$ star. How does this compare to the age of the Universe? Are there any star clusters in the Galaxy where you would expect to see 0.8 $M_\odot$ stars that have left the main-sequence and headed toward the red giant branch?

   (b) Estimate the main-sequence lifetime of a 10 $M_\odot$ star. How does this compare to the age of the Universe? Are there any star clusters in the Galaxy where you would expect to see 10 $M_\odot$ stars that have left the main-sequence and headed toward the red giant branch?

   (c) Sketch an H-R diagram for a star cluster with an age of 1 Gyr. In particular, indicate the stellar mass where stars are no longer found on the main-sequence.

   (d) White dwarfs are known to exist with masses $\lesssim 0.4$ $M_\odot$. Explain how these came to be.

2. Using the Eddington luminosity limit, derive an upper limit for the mass and luminosity of main-sequence stars. Estimate the effective temperatures for such stars. You may assume that $\kappa = 0.4$ cm^2 g^{-1}, as appropriate for electron scattering.

3. For stars that burn hydrogen via the CNO cycle, the cores and envelopes are typically fully-ionized, and so one can assume $\kappa \approx$ constant. However, for lower-mass stars ($M \lesssim 1.5$ $M_\odot$), partial ionization is important in some zones, and $\kappa = \kappa_0 \rho T^{-7/2}$ is a better approximation. Use this opacity law and assume $n = 4$, and derive the relation between L and M. *Hint: use the same sorts of homology relations as used in the constant κ case.*

4. If hydrogen is deposited onto the surface of a white dwarf, it can undergo fusion. Assume a thin layer of material with solar composition ($X = 0.7, Y = 0.3$) has been deposited onto a white dwarf with $M = M_\odot$ and $R = 0.01 R_\odot$.

   (a) Calculate the fraction of the thin layers's mass (f) that must be fused into He to supply sufficient energy to eject the entire layer. Express your answer for f in terms of physical constants and M, R.

   (b) Derive the dependence of f on M for $M < M_{\mathrm{ch}}$.