Astronomy 400A: Homework 3

You may collaborate on all problems this week, but please write your own code and generate your own solutions for the numerical problem.

1. Solve the Lane-Emden equation numerically for $n = 1.5$ and $n = 3$. Write your own numerical integration routine for this, using a higher-order midpoint technique. Your solution should include your code, and plots or tables showing the behavior of θ versus ξ. You should also give ξ_1, $(d\theta/d\xi)_{\xi_1}$, and $M(\xi_1) = M(R_*)$ for each case. You can compare your values to those in Table 5.1 of the book to make sure you have done this correctly.

2. Recall the four basic equations of stellar structure:

$$\frac{dM(R)}{dr} = 4\pi r^2 \rho, \tag{1}$$

$$\frac{dP}{dr} = -g\rho, \tag{2}$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon, \tag{3}$$

$$\frac{dT}{dr} = \frac{-3\kappa\rho L}{16\sigma T^3 4\pi r^2}. \tag{4}$$

Derive the behavior of $M(r)$, $P(r)$, $L(r)$, and $T(r)$ near the center of the star by using a Taylor expansion. Your answers should be expressed in terms of r, and ρ_c, P_c, ϵ_c, κ_c, and T_c. Keep terms up to third order in r in your solutions.