

NAVIER - STOKES

$$\partial_t \rho + \partial_\alpha (\rho u_\alpha) = 0 \quad (\text{Continuity})$$

$$\partial_t (\rho u_\alpha) + \partial_\beta (\rho u_\alpha u_\beta) = \partial_\beta \sigma_{\alpha\beta} + \rho f_\alpha + \underbrace{f_{s\alpha}}_{\text{surface tension}} \quad \sigma_{\alpha\beta} = -p \delta_{\alpha\beta} + \tau_{\alpha\beta}$$

$$u_\alpha \left[\partial_t \rho + \partial_\beta (\rho u_\beta) \right] + \left[\rho \partial_t u_\alpha + \rho u_\beta \partial_\beta u_\alpha = \partial_\beta \sigma_{\alpha\beta} + \rho f_\alpha \right] + f_{s\alpha}$$

Continuity eqn. Second form. $\frac{5}{2} \rho u$

$$\sigma_{\alpha\beta} = -p \delta_{\alpha\beta} + \lambda \partial_\gamma u_\gamma \partial_\alpha u_\beta + \mu \left(\partial_\beta u_\alpha + \partial_\alpha u_\beta \right)$$

Stress tensor vs. pressure tensor?

How surface tension force affects mechanical pressure?

Does \bar{p} has to be redefined with \vec{f}_s

mechanical pressure

$$\bar{p} = \sum_\alpha \frac{\sigma_{\alpha\alpha}}{3}$$

Thermodynamic

$$p - \bar{p} = \left(\lambda + \frac{2}{3} \mu \right) \nabla \cdot \mathbf{u}$$

"only Translational energy of molecules"

Monoatomic $K=0$ and no intermolecular?

so $\bar{p} = p$ or incompressible $\nabla \cdot \mathbf{u} = 0$

$$\Rightarrow p = \bar{p}$$

p measures the total energy

relaxation times $\left\{ \begin{array}{l} \text{vibrational} \\ \text{Rotational} \\ \text{Intermolecular attraction} \\ \text{Translational} \end{array} \right\}$

Conversion from one to another

to \leftarrow From $\rightarrow K$ measures conversion

Bulk viscosity K

$$\text{Surface tension force } \vec{f}_\alpha^s = \kappa \rho \nabla \Delta \phi = \kappa \rho \partial_\alpha \partial_\beta \partial_\beta \phi$$

$$= \kappa \partial_\alpha (\rho \partial_\beta \partial_\beta \phi) - \kappa (\partial_\alpha \rho) \partial_\beta \partial_\beta \phi \rightarrow -\kappa \partial_\alpha [\partial_\beta \rho \partial_\beta \phi] + \kappa (\partial_\alpha \rho) (\partial_\beta \partial_\beta \phi)$$

$$= \kappa \partial_\alpha (\rho \partial_\beta \partial_\beta \phi) - \kappa \partial_\alpha (\partial_\beta \rho \cdot \partial_\beta \phi) + \frac{\kappa}{2} \partial_\alpha [\partial_\beta \rho]^2$$

$$= \partial_\alpha \left[\kappa \rho \partial_\beta \partial_\beta \phi + \frac{\kappa}{2} [\partial_\beta \rho]^2 \right] - \partial_\alpha [\kappa \partial_\beta \rho \partial_\beta \phi]$$

$$\Rightarrow \vec{f}_\alpha^s = \partial_\beta \left[\underbrace{\frac{\kappa}{2} (\partial_\alpha \rho)^2 + (\partial_\beta \rho)^2}_{\Delta \rho} + \kappa \rho \partial_\alpha \partial_\beta \phi \right] \delta_{\alpha\beta} - \partial_\beta [\kappa \partial_\alpha \rho \partial_\beta \phi]$$

$$\vec{f}_\alpha^s = \partial_\beta [A \delta_{\alpha\beta} - B_{\alpha\beta}]$$

Kruger says
RHS in NS $-\partial_\beta p_0 + \partial_\beta \mu \partial_\alpha^2 u$

Comparing with Currie
 $-\partial_\beta (p - \lambda \nabla \cdot \mathbf{u}) \delta_{\alpha\beta} + \partial_\beta \mu \partial_\alpha^2 u$

Integrating \vec{f}_s does not solve the problem.

Discrepancy in $\lambda \nabla \cdot \mathbf{u}$

Is the model assuming incompressibility?

modified stress tensor embedding f_α^s

$$\sigma'_{\alpha\beta} = -p \delta_{\alpha\beta} + \lambda \delta_{\alpha\beta} \partial_r u_r + \mu (\partial_\beta u_\alpha + \partial_\alpha u_\beta) + A \delta_{\alpha\beta} - B_{\alpha\beta}$$

Defining a pressure tensor $P_{\alpha\beta}$, in which μ term is excluded

$$P_{\alpha\beta} = \left(p - \frac{\kappa}{2} (\partial_r \rho)^2 - \kappa \rho \partial_r \partial_r \rho - \lambda \partial_r u_r \right) \delta_{\alpha\beta} + \kappa (\partial_\alpha \rho)(\partial_\beta \rho)$$

Pressure tensor including the thermodynamic pressure p , the surface tension force and the bulk viscosity term that vanishes only for incompressible fluids or low speeds
(?)
Even for simple molecular interactions $\lambda = -2/3$ to make $\bar{p} = p$.

Gibbs - Duhem $U = TS - PV + \sum \mu_k N_k \rightarrow dU = TdS - PdV + \sum \mu_k dN_k$

$$\Rightarrow SdT - Vdp + \sum N_k d\mu_k = 0 \quad @ T$$

$$dp = \bar{p} \sum z_k d\mu_k$$

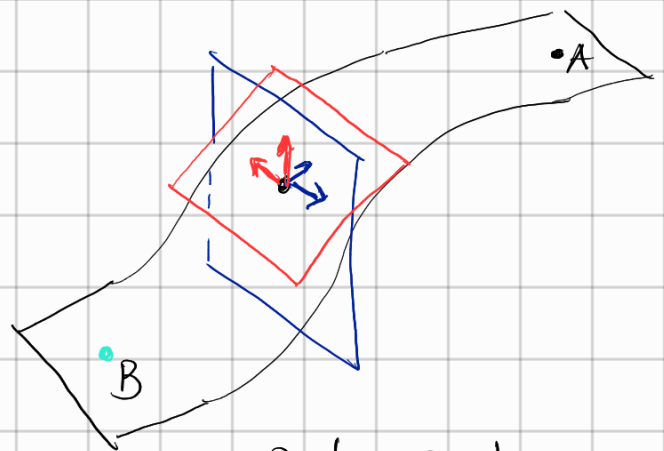
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$$P_{\alpha\beta}$$

$$dP_{\alpha\beta} = \sum \bar{p}_k d\mu_{\alpha\beta}^k$$



$$P_{\alpha\beta}|_B = P_{\alpha\beta}|_A = p = p_b$$

In a system that is globally in non-equilibrium, but locally in equilibrium, the pressure becomes a tensor to account for directionality.

$$P_{\alpha\beta} = (p_b - A) \delta_{\alpha\beta} + B_{\alpha\beta}$$

Along the interface, the pressure must be corrected by tension

Q: Cubic EoS contains unstable zones where $\frac{\partial V}{\partial P} > 0$

In a continuum model, properties have to be defined for thermodynamics to apply. Is this enough argument to accept that we can not allow pressure regions where $\partial V / \partial P > 0$? If not, how can we still define those variables? (local equilibrium)

Now, if it was enough argument, the pressure has to be corrected and contains information about neighbors (directionality). Under which argument can we say that apart from $P_{\alpha\beta}$, all other thermodynamic variables are scalar variables, as assumed by Gibbs - Duhem?

Under what argument, the local pressure is interchangeable by the pressure tensor (scalar vs tensor)
Still something is missing.

CHECK PAPER PROPOSING EXPRESSIONS FOR TENSOR QUANTITIES:
 $\sigma_{\alpha\beta}^k, H_{\alpha\beta}^k, F_{\alpha\beta}^k, U_{\alpha\beta}^k \Rightarrow$ POTENTIALLY $M_{\alpha\beta}^k$

"On tensorial forms of thermodynamic potentials in mixtures theory"
Wyrwał, Marynowicz, Świąska (2009)

Unless, GD is applied to blk first and then to the interface part, using the force. \rightarrow Do this tomorrow.