Notes and exercises from "An Introduction to Statistical Leaning"" by James, Witten, Hastie & Tibshirani

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Chapter 1 Introduction

The introduction outlines three basic tasks in statistical learning:

1. **Regression** - predicting *continuous numerical* values from other continuous values
2. **Classification** - predicting class membership; has a *categorical* or *qualitative* output.

In both these tasks, we are trying to predict an output.

3. **Clustering** - here we are not trying to predict an output variable, but instead are trying to understand how similar/different a set of observations are, usually based on a number of variables.

   It is clear that I need to revise matrix multiplication, I didn't really understand the example on p. 12.

The book's website is here.

Chapter 2 Statistical Learning

Broadly, the goal of statistical learning is to estimate f in the equation:

$Y = f(X) + \epsilon$

Where:

Y is the *response/output/dependent variable.*
X is one or more *input/independent variables*, or *predictors* or *features.*
f is a function that relates X to Y, and
ϵ is a random error term, that is independent of X and has mean zero.

We estimate f for two ends:

1. To predict Y

Here we are not primarily concerned with the exact nature of the function, rather how accurately it predicts Y. Thus our estimate of Y comes from our estimate of f:

$\hat{Y} = \hat{f}(X)$

The accuracy of Y depends on two quantities:

I. The reducible error

$\hat{f}$ will be a more-a-less imperfect estimate of f. This inaccuracy is termed the *reducible error* as it is reducible by determining a better $\hat{f}$. This reduction is the main point of statistical learning.

II. The irreducible error

Even if our estimate of f was perfect, it would still have error in it, as Y is also a function of ϵ, which by definition cannot be predicted from X - hence *irreducible error*. This error may come from unmeasured (or unmeasurable) variables that might be useful in predicting Y, but since we didn't measure them, they are not included in f. The irreducible error will always provide an upper bound on the accuracy of our prediction of Y.

2. To infer the relationship between Y and $X_1, ..., X_p$

In order to do this we need to know the exact form of $\hat{f}$. We can them aim to answer the following questions: 1. Which predictors are associated with the response? 2. What is the relationship between the response and each predictor? 3. Can the relationship between Y and each predictor be adequately summarised using a linear equation, or is the relationship more complicated?

Some examples will combine both prediction and inference.

How do we estimate f?

We estimate f based on a subset of observations called the *training data*. We then use this function to predict the values of Y for new values of X.

There are two classes of methods for producing $\hat{f}$.

1. Parametric Methods

Parametric methods involve a two-step model-base approach. First, we make an assumption about the form of f; for example, that is it linear:

$f(X) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p$

Where p is the number of variables X. The second step uses the the training data to *fit* or *train* the model. In this case, this means estimating the parameters $\beta_0, \beta_1, ..., \beta_p$ (the coefficients), such that:

$Y \approx \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p$

The most common method for doing this is called *(ordinary) least squares*, but there are many others. Parametric approaches are termed this because they reduce the problem of estimating f to one of estimating a set of *parameters*.

2. Non-parametric Methods