DOCUMENTING THE DYNAMICS, CONNECTEDNESS, AND EFFICIENCY OF THE BITCOIN MARKET

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Introduction

Since its proposal in Satoshi Nakamoto's foundational paper "Bitcoin: A Peer-to-Peer Electronic Cash System," Bitcoin has garnered significant attention for its use as both a cryptocurrency and more recently an investable asset. At its most base level, Bitcoin relies on a public digital ledger technology referred to as blockchain. Likewise, blockchain relies on large groups of decentralized network participants to solve cryptographically and computationally intensive problems in order to create records within the digital ledger (Badev and Chen, 2014). In return for their computational and electrical expenses, users within the network receive newly minted bitcoins as well as any additional transaction fees necessary to incentivize participation within the network. Because of the cryptographic complexity of the process of creating new ledger entries as well as behavioral incentives, Bitcoin is highly secure and circumvents the necessity of a central monetary authority such as the Federal Reserve or the European Central Bank.

From an economic perspective, Bitcoin is unique in that the current algorithm governing its production hard codes its upper limit at 21 million units (Gronwald, 2014). Unless this is changed, the constantly decreasing rate at which bitcoins are generated will converge to zero in the year 2140. As a result of this, it is implied that Bitcoin does not face the inflationary or deflationary pressures experienced by other fiat currencies whose volume fluctuates as per a central regulatory authority. Furthermore, due to the characteristics of blockchain, all of the transactions conducted using Bitcoin are available in the public record. To a certain extent, this allows perfect transparency of transaction information for all users in the market assuming a baseline level of technological proficiency. There has also been significant academic debate as to Bitcoin's classification as either as currency or an investment asset. On one side, it is presented as a fiat currency which can be used in exchanges involving goods and services or be converted to other currency via liquid exchanges (Kroll, Davey, Felten, 2013). On the other hand, recent analysis has suggested that less than 50% of all bitcoins in circulation are used in the transaction process (Badev et al, 2014). Additionally, significant price volatility and the entry of larger institutional investors such as Goldman Sachs highlights the strong possibility of speculative investing within bitcoin exchanges.

In order for the average consumer to obtain and spend bitcoins, they must download and utilize cryptocurrency software such as a digital wallet. From there, several methods for procuring bitcoins exist though the most popular is to purchase the cryptocurrency through an online exchange. At the present moment dozens of exchanges exists, with some handling tens or hundreds of millions of dollars a day worth of transactions. However, significant risk exists within these exchanges as even the largest, such as Mt. Gox in 2013, have experienced collapse (Moore and Christin, 2015). When these collapses occur, bitcoins held or stored on the exchange are oftentimes lost as a result of seizure by the exchange or fraudulent activity. In terms of asset pricing, the failure of these exchanges have also historically had significant impacts on the price of bitcoins. As aforementioned, when Mt. Gox closed in 2013 the value of the cryptocurrency dropped by more than 50% in the span of a few hours (Donier and Bouchaud, 2015).

Back on the topic of bitcoin exchanges, noticeable differences exist in transaction costs, liquidity, and the time it takes to complete a transaction. Foremost, the majority of large exchanges such as itBit or Bitstamp charge between .1% and 1% on every transaction. There can also be fees incurred for the use of credit cards, certain currencies, and transactions in between exchanges in which users must pay fees on both exchanges. Accordingly, the price of a single bitcoin between each exchange can vary by as much as 50 dollars, though this may be a function of other unobserved factors not accounted for purely by transaction costs. Looking at liquidity some exchanges handle thousands of transactions and millions of dollars, as was already mentioned. However, some smaller exchanges handle as few as 25-50 bitcoins a day which translates to between 25,000 to 50,000 dollars a day. For individuals attempting to buy or sell on these exchanges, there may not be enough volume on any given day for a trade to execute which can be qualified as a form of liquidity risk. Finally, depending on administrative and security procedures, it may take up to several days for currency or bitcoins to become available for use on certain exchanges. This can pose significant challenges for anyone involved in arbitrage across exchanges, as exploitable pricing irregularities (in excess of transaction costs) often only exists for extremely short durations of time. Nonetheless, high frequency algorithmic trading continues to be popular in bitcoin markets.

Motivation

On the whole, there is relatively little known about the dynamics and stylized facts of cryptocurrencies and their markets. This can be attributed to the relative immaturity of the cryptocurrencies themselves, the exchanges that trade them, and their accompanying data collection infrastructure. Looking at the existing literature, there are many published articles that debate cryptocurrency from a sociological, behavioral, and micro-economic standpoint. However, there are few articles that detail statistically the price and return movement of cryptocurrencies or their place in the spectrum of the market efficiency hypothesis, and none at all that investigate the connectedness exchanges. This seems illogical, especially given the rising prevalence of cryptocurrency-based investing at both the individual and institutional level.

In terms of choosing a specific digital "coin" to study, the selection criteria was primarily based on longevity, availability of data, and price stability. There are a myriad of cryptocurrencies that seemingly come in and out of existence on a monthly basis, and even though Bitcoin was the first it has thus far maintained dominance since its creation in late 2009. Furthermore, there are multiple reputable data sources such as Quandl and BitcoinCharts that track price, volume, and other financial metrics for Bitcoin on multiple exchanges. Additionally, though there is significantly more volatility than in traditional investable assets, Bitcoin has yet to experience a catastrophic price collapse. Therefore, since Bitcoin is the largest and most well-known of all cryptocurrencies, it makes sense that any formal investigation would use it as a preliminary subject of inquiry.

With respect to the description of stylized facts and dynamics, traditional equity markets have been thoroughly researched and studied using tools derived from mathematics, statistics, physics, and more recently computational finance. With the help of these tools, the fundamental movements and hidden pricing anomalies such as the January Effect have been brought to the attention of the financial community and subsequently leveraged for profit. Furthermore, the computational power necessary to perform these types of analysis has rapidly become available to the average investor. This makes it possible to explore previously unexploited markets with relative ease, which ultimately is the purpose of this paper.

Prior Literature

As aforementioned, the vast majority of Bitcoin-based academic literature neglects it from the perspective of an investable asset. Nonetheless, this paper does build off of two pieces of prior academic literature with relevance to the subject. The first was written in 2014 by William L. Brown, a graduate student at Claremont McKenna College. The paper is titled "An Analysis of Bitcoin Market Efficiency through Measures of Short-Horizon

Return Predictability and Market Liquidity" and effectively attempts to test the market efficiency hypothesis relative to Bitcoin markets. In order to implement this, the author used trade and order data between 2011 and 2013 from the Mt. Gox exchange (which was the largest at the time). Looking at methodology, the author measures market efficiency via the predictability of short-horizon returns and by measurements of liquidity. With respect to short-horizon return predictability, he uses order flow imbalances to predict future price movements and short-horizon past returns to predict future returns within the framework of ordinary least squares (OLS) regression. In turn, the author draws his testing methodology from Tarun Chordia's order flow imbalance ratio referenced in his 2002 paper "Market Liquidity and Trading Activity." Moving to results, Brown's paper comes to the statistically significant conclusion that Bitcoin does not follow the market efficiency hypothesis.

The second paper that motivated this research was written by Jakub Bartos in 2015 and is titled "Does Bitcoin Follow the Hypothesis of Efficient Markets?" Notably, Bartos is an Economics PHD from the University of Economics in Prague. Accordingly, his line of questioning is based off of a traditional supply and demand framework. In terms of data, the paper relies on Google search data as well as pricing information gathered from the Mt. Gox exchange as well as several other smaller exchanges. With respect to methodology, the author uses the number of Google searches for Bitcoin as a proxy for demand and then relates this to Bitcoin returns. In an intermediary step, descriptive statistics for the returns are generated and compared to those of other traditional financial instruments such as gold and stock in Google. Finally, Bartos then builds a regression framework based off of the Error Correction Estimator with the intention of predicting Bitcoin prices. In terms of results, the paper ultimately is unable to forecast Bitcoin prices in a statistically significant manner and therefore concludes that Bitcoin abides by the hypothesis of efficient markets.

Methodology

Broadly speaking, the analysis was conducted using Matlab and is broken into three sections consisting of statistical descriptions of Bitcoin returns, tests designed to uncover the interconnectedness of Bitcoin exchanges, and tests of the market efficiency hypothesis relative to Bitcoin returns. For all of the above sections, certain aspects are replicated for traditional equity indexes for benchmarking purposes. Below, the three sections will be laid out in more detail.

For the first section, descriptive statistics including mean, median, standard deviation, skewness, kurtosis, and minimum/maximum values are generated for the returns of all the Bitcoin exchange, international equity indices, and Bitcoin daily volume for each exchange. This is done for both a daily and monthly frequency for returns of Bitcoin and traditional equity markets. Following this step, a combination of quantile-quantile (QQ) plots and one-sample Kolmogorov-Smirnov (K-S) tests are employed. Particularly, the QQ plot is a graphical technique used to determine whether a set of data comes from a normal distribution. Theoretically, if the population is close to a normal distribution, then the data points included on the QQ plot will follow the 45 degree linear reference line fairly closely. This technique allows for visual inspection of the data's distribution as compared to a normal distribution, but it does not empirically test the hypothesis that the data comes from a normally distributed population. This flaw is compensated for with the K-S test, which tests the null hypothesis that the data comes from a normally distributed population at the 5% significance level. Primarily, the K-S test goes about this process by quantifying the distance between the empirical cumulative distribution function (CDF) calculated from the data and the hypothesized cumulative distribution function (CDF) of the null hypothesis (in this case a normal distribution). For a two sided test, this distance can be expressed as the following equation:

$$(1)\ D^{*} = MAX_{x}\left(\left|\hat{F}(x) - G(x)\right|\right)$$

Notably, D^{*} is the maximum absolute difference between the empirical CDF $\hat{F}(x)$ and the hypothesized CDF $G(x)$. The critical value against which this value is compared is calculated via interpolation from a table.

In the second section of the analysis, the focus shifts to quantitatively measuring the relative connectedness of international and domestic Bitcoin exchanges as well as international equity markets (proxied by large-cap indices). The first test performed is a traditional covariance matrix using all of the exchanges' daily returns as inputs. Mathematically, this test is a measure of the joint variability of two or more random variables with the output potentially indicating similar movements (a larger positive number), no relationship (a number close to 0), or opposite movement (a larger negative number). This can be expressed for a two asset case as shown below:

$$(1)\ cov(A,B) = \frac{1}{N-1}\sum_{i=1}^{N}(A_i - u_A)(B_i - u_B)$$

$$(2)\ C = \left\langle \begin{matrix} cov(A,A) & cov(A,B) \\ cov(B,A) & cov(B,B) \end{matrix} \right\rangle$$

In the above equation, u_A and u_B represent the mean value of A and B, respectively. Furthermore, A and B represent separate assets and the "i" indexing variable represents number of observations. The above calculations are repeated, but with a moving interval of time/observations indexed by 1 at every step, in order to form a "rolling" covariance matrix that is presented concurrently.

Following the covariance matrix calculations, a series of univariate ordinary least square (OLS) regressions and univariate quantile regressions are then performed with the purpose of attempting to measure the ability of returns on one exchange to predict returns on another. This is repeated for every pairwise combination of exchanges. With respect to the OLS regression, it takes the form of the below equation:

$$(1)\ Y_{i,t} = \beta_0 + B_1 exchret_{i,t} + \mu_{i,t}$$

In this equation, the $Y_{i,t}$ is the dependent variable which can be represented by the returns of any Bitcoin exchange that are not the returns of the Bitcoin exchange represented by the independent variable $exchret_{i,t}$. Moreover, the $\mu_{i,t}$ term represents variation/error not captured by the independent variable. In addition to this simple univariate OLS regression, a more complex regression model with the same OLS estimator is used. This model is a quantile regression model, with the difference being that it estimates the relationship between the independent variable $exchret_{i,t}$ and the conditional quantiles of the dependent variable Y. This allows the model to build a more complete picture of the relationship between the two variables, particularly with respect to returns that fall in the upper and lower quantiles. Although there is no native functionality for this form of regression in Matlab, an open-source third party function titled "qregressmatlab" and written by a PHD student at the University of Copenhagen was used.

For the final measure of connectedness, a relatively new test developed to measure the connectedness of international equity markets called the coexceedence measure was used. This methodology was first developed by Kee-Hong Bae from Korea University in his paper "A New

Approach to Measuring Financial Contagion" as an academic response and exposition on the Asian Financial Crisis in the late 1990's. At a high level, this test measures the number of times that markets (proxied by indices) breach a certain threshold of daily return. This threshold is determined by calculating the distribution of each markets' return and calculating the cutoff value for the upper and lower 2.5% of the CDF. Likewise, the number of exceedances for each market is compared to the number of exceedances for every other markets, with intersecting events being termed joint coexceedence events. On the whole, it is a simple yet effective method of measuring joint market responses to macroeconomic and financial shocks.

Moving now to efficiency, the first test utilized is the autocorrelation function. Specifically, the purpose of this is to measure the degree of similarity between a time series of data and lagged variations of itself. The resulting output from the function is bounded between 1 and -1, with interpretations similar to that of the correlation function. With respect to the market efficiency hypothesis, the idea here is that if lagged versions of the market returns are correlated to present returns, than they can potentially be used to efficiently forecast returns which in turn violates the market efficiency hypothesis. In mathematical notation, the autocorrelation function takes the form below:

$$(1)\ r_k = \frac{\sum_{i=1}^{N-k}(Y_i - \bar{Y})(Y_{i+k} - \bar{Y})}{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}$$

In the above formula, the denominator is simply the sample variance of the time series data and the numerator is a lagged variation of the covariance formula shown earlier where "N" is the number of observations and "k" is the number of periods of lag. Following the autocorrelation test, a similar cross correlation test is employed. Particularly, this test measures the degree of correlation between one exchange's returns and lagged variation of another exchange's returns. The idea is fairly similar to that of autocorrelation, though the mathematical notation is somewhat different as shown below:

$$(1)\ c_k = \frac{\sum_{i=1}^{N-k}(X_i - \overline{X})(Y_{i+k} - \overline{Y})}{\sqrt{\sum_{i=1}^{N}(X_i - \overline{X})^2}\sqrt{\sum_{i=1}^{N}(Y_{i+k} - \overline{Y})^2}}$$

In the above formula, the denominator is the standard deviations of both exchanges' returns, with one of the return series being lagged. Likewise, the numerator is identical to that of the autocorrelation function, though with two assets instead of one. For the final measure of the market efficiency hypothesis, a variance ratio test is utilized. Primarily, this test examines the predictability of time series data by comparing variances of differences of the data (returns) calculated over different intervals. Assuming that the data follows the trend of a random walk, the amount of variance of a "q" period difference should be "q" times the variance of a one-period difference. With this information as a benchmark, it is then possible to empirically test whether or not a vector of time series data follows this random walk hypothesis. If it does not, then there is some degree of mean reversion or aversion over time and the market efficiency hypothesis is violated. Given that it was developed fairly recently, there are a variety of different models that fall under the variance ratio test. However, the model chosen for this analysis was developed by Lo and MacKinlay in 1988. Their entire model with estimator and assumptions is too extensive to be shown, though a basic outline is presented below

$$(1)\ Y_t = c + \delta Y_{t-1} + \varepsilon_t$$

$$(2)\ V_k = \frac{Var(Y_t - Y_{t-k})/k}{Var(Y_t - Y_{t-1})}$$

In the above model, the first formula is the random walk hypothesis formula where “c” is an unknown drift parameter and ε_t is an error term that is assumed to be correlated in this analysis. However, it is important to note that in most testing situations this term is assumed to be not independent or identically distributed (IID). The second formula is the actual variance ratio in which the denominator is the variance of a one-period difference while the numerator is the variance of a “k” period difference.

Data Description

Foremost, all data was collected for Bitcoin pricing information from www.bitcoincharts.com from the interval of 12/31/2015 to 4/25/2017. This data is daily in frequency and included the closing price of a bitcoin for each of the 11 international Bitcoin exchanges in the local currency as well as the volume of bitcoins traded that day, measured in bitcoins. The number of pricing observations is 482 due to the fact that the exchanges never cease operation. The 11 exchanges chosen were Coincheck in Japan, Coinbase in the European Union, Bitcoin Deutschland (BTCDE) in the European Union, Bitcoin China (BTCE) in China, Coinbase in the United States, Bitstamp in the United States, Bitbay in Poland, Coinfloor in the United Kingdom, Bitcoin Markets (BTC) in Australia, Mercado Bitcoin in Brazil, and Foxbit in Brazil. In terms of selection criteria these exchanges were chosen due to the fact that they are the largest in terms of volume, they have continuous pricing information over the research interval, and represent distinct geographic regions. Smaller exchanges were omitted due to the possibility that their limited volume of transactions may over-represent idiosyncrasies and market inefficiencies present on those exchanges, thus skewing results.

With respect to the international equity benchmarks, all of the index data was gathered from FactSet between the interval from 12/31/2015 to 4/25/2017. The data is daily in frequency and includes the closing price of 8 indices from each of the geographic regions represented by the Bitcoin exchanges. The number of pricing observations is 350 due to the fact that markets are closed on weekends and holidays. These indices include the S&P 500 from the United States, the Nikkei 225 from Japan, the ASX 200 from Australia, the IBOVESPA from Brazil, the WIG from Poland, the SHCOMP from China, the UKX from the United Kingdom, and the EUROSTOXX 50 from the European Union. These indices were selected to give a broad overview of equity markets in each of the respective regions.

Moving now to statistical description, the tables below presents some of the moments of the distribution of the 11 Bitcoin exchange returns on a daily and monthly basis and volume on a daily basis:

Table 1: Bitcoin Daily Return Statistics

	Mean	Median	StDev	Skewness	Kurtosis	Min	Max
bitbayPLN	0.22%	0.22%	2.94%	-0.96	11.14	-16.00%	13.50%
bitstampUSD	0.23%	0.24%	2.89%	-1.23	10.56	-17.66%	9.98%
btcdeEUR	0.22%	0.30%	3.21%	-1.11	9.84	-19.59%	11.99%
btceCNY	0.20%	0.20%	3.16%	-1.08	14.91	-19.67%	15.36%
btcmarketAUD	0.22%	0.25%	2.95%	-1.23	8.78	-15.61%	9.91%
coinbaseEUR	0.23%	0.32%	2.98%	-1.16	10.89	-18.96%	11.28%
coinbaseUSD	0.23%	0.30%	2.94%	-1.05	10.44	-18.46%	10.91%
coincheckJPY	0.21%	0.20%	3.22%	-1.40	14.54	-21.76%	15.09%
coinfloorGBP	0.25%	0.21%	3.07%	-0.59	10.49	-14.83%	18.80%
foxbitBRL	0.18%	0.19%	2.66%	-0.86	9.99	-16.07%	11.46%
mercadoBRL	0.18%	0.04%	2.66%	-0.82	12.09	-19.21%	10.72%

Examining the above table, it is clear that daily returns on all of the Bitcoin exchanges are exceptionally volatile. Standard deviations of 3% per day are seemingly the norm, with minimum and maximum returns between -20% and 20% on a daily basis. Furthermore, the skewness of returns seems to indicate that large negative returns occur more frequently than large positive returns. Nonetheless, mean and median returns are positive which indicate that a risk tolerant investor with a buy and hold philosophy would expect to make a small amount of money each day over the whole period. Looking at kurtosis, it is clear that the distribution is far from that of a Gaussian distribution, with high kurtosis values indicating “fat” tails on either end of the distribution.

Table 2: Bitcoin Monthly Return Statistics

	Mean	Median	StDev	Skewness	Kurtosis	Min	Max
bitbayPLN	10.44%	15.21%	17.94%	-0.30	1.70	-19.40%	34.69%
bitstampUSD	10.71%	10.47%	16.96%	0.08	1.73	-11.66%	39.08%
btcdeEUR	10.90%	13.88%	16.77%	-0.11	1.63	-11.98%	35.71%
btceCNY	9.93%	11.55%	16.37%	-0.19	1.83	-17.99%	32.51%
btcmarketAUD	10.41%	12.29%	17.95%	-0.16	1.68	-16.95%	37.44%
coinbaseEUR	10.81%	12.90%	17.87%	-0.03	1.76	-15.01%	39.96%
coinbaseUSD	10.82%	10.43%	16.86%	0.13	1.72	-10.07%	39.17%
coincheckJPY	10.46%	16.60%	16.60%	-0.18	1.68	-13.97%	35.38%
coinfloorGBP	12.09%	13.29%	18.48%	0.17	1.82	-14.10%	42.91%
foxbitBRL	8.68%	13.19%	15.79%	-0.22	1.74	-16.12%	31.69%
mercadoBRL	8.76%	11.08%	15.87%	-0.04	1.88	-16.51%	31.95%

With respect to monthly Bitcoin returns, they seem to follow a slightly less volatile trend than daily returns. Mean and median returns are generally positive on a month by month basis, though standard deviations are still larger than either of these metrics. Once again, skewness is slightly negative, but significantly less so than daily returns. The minimum and maximum returns are only slightly greater in magnitude than in the daily case, indicating that volatility seen in daily returns evens out to a certain extent over monthly intervals. Finally, the most notable change is in kurtosis, as the significantly lower value indicates that monthly returns are closer to a normal distribution in that the tails of the return distribution are less saturated.

Table 3: Bitcoin Daily Volume Statistics ($=bitcoins)

	Mean	Median	StDev	Skewness	Kurtosis	Min	Max
bitbayPLN	$5,624.41	$3,123.05	$6,707.00	2.92	13.45	$376.85	$47,497.29
bitstampUSD	$582.52	$477.41	$397.18	2.38	11.12	$124.17	$3,087.99
btcdeEUR	$515.14	$425.05	$340.16	2.09	8.58	$77.82	$2,371.91
btceCNY	$482,378.18	$62,815.76	$898,698.19	2.83	12.25	$1,276.71	$5,915,081.78
btcmarketAUD	$6,579.21	$5,652.20	$3,812.83	2.35	10.21	$683.80	$28,641.67
coinbaseEUR	$6,218.46	$4,815.16	$5,146.75	2.88	14.15	$719.16	$37,326.70
coinbaseUSD	$771.66	$610.38	$553.39	3.00	17.45	$126.24	$5,400.81
coincheckJPY	$407.46	$291.31	$380.80	2.64	13.95	$7.13	$3,309.06
coinfloorGBP	$205.87	$189.61	$114.30	1.03	4.42	$27.09	$725.30
foxbitBRL	$136.94	$124.56	$85.02	1.62	10.42	$11.38	$802.56
mercadoBRL	$263.46	$226.37	$193.79	1.68	7.57	$15.20	$1,433.17

With respect to volume, it is important to note that 1 bitcoin is approximately equal to 1300 USD at the time of writing this paper. Converting the volumes of bitcoin in the above table yields daily volumes on some exchanges of hundreds of millions of dollars. Though this is small compared to traditional equity markets, it is still substantial. Looking at the mean and median values, most exchanges handle less than 1000 bitcoins per day on average. However, the four outliers from this trend are the Polish exchange, the European Union exchange, the Australian exchange, and the Chinese exchange. Particularly, the Chinese exchange is an extreme outlier and handles approximately 500,000 bitcoins on average per day. This could be due to the repressive nature of the Chinese economic system and anonymity offer by Bitcoin, though further research would be necessary to empirically prove this. In terms of volatility, it is clear that trade volumes fluctuate heavily on a day to day basis. Given the high kurtosis, it appears that certain days experience high trade volume while other days experience almost none. This is also substantiated by the minimum and maximum volumes per day.

Table 4: Equity Daily Return Statistics

	Mean	Median	StDev	Skewness	Kurtosis	Min	Max
S&P500	0.04%	0.02%	0.78%	-0.40	5.51	-3.66%	2.45%
Nikkei225	0.02%	0.05%	1.36%	-0.11	6.60	-5.74%	6.18%
ASX200	0.04%	0.13%	1.20%	-0.14	3.67	-4.37%	3.78%
IBOVESPA	0.15%	0.15%	2.24%	0.16	5.52	-7.65%	11.21%
WIG	0.09%	0.06%	1.40%	-0.52	6.62	-8.38%	3.98%
SHCOMP	-0.06%	0.00%	1.32%	-1.85	12.85	-7.84%	4.11%
UKX	0.00%	0.03%	1.36%	-1.57	19.10	-11.51%	5.76%
SX5E	0.03%	0.10%	1.27%	-1.38	15.04	-10.06%	5.16%

Looking at the daily returns of the equity indices, the results are to be expected. Volatility is low and mean returns are close to 0. Minimum and maximum daily returns are fairly low when compared to daily Bitcoin returns, with the exception of the European and United Kingdom indices. For the most part, this can be attributed to the financial shock that occurred as a result of the British referendum to leave the European Union. Looking at kurtosis, values are above those of a normal distribution indicating substantial outliers in the tails of the distributions.

Table 5: Equity Monthly Return Statistics

	Mean	Median	StDev	Skewness	Kurtosis	Min	Max
S&P500	1.36%	1.50%	2.30%	-0.33	1.91	-2.64%	4.45%
Nikkei225	0.78%	0.05%	3.67%	0.93	2.96	-3.39%	8.67%
ASX200	1.45%	2.14%	4.18%	-1.07	3.55	-8.43%	6.25%
IBOVESPA	5.05%	7.93%	8.91%	-0.35	1.64	-8.88%	16.44%
WIG	2.76%	1.75%	6.00%	0.25	1.66	-4.96%	12.02%
SHCOMP	-1.48%	0.89%	5.55%	-0.80	2.24	-12.21%	4.45%
UKX	0.17%	-0.33%	2.39%	0.18	1.65	-3.25%	3.93%
SX5E	1.01%	-1.20%	3.72%	0.51	1.57	-2.69%	7.17%

Looking at monthly returns for the equity markets, there are once again no surprises. Returns are relatively stable and range on average from 5% per month to -1.5% per month.

Kurtosis is more in line with that of a normal distribution, indicating fewer outliers in the tails of the distribution than in the distribution of daily returns. Furthermore, minimum and maximum monthly returns are within reasonable and inferable bounds.

Results

For the most part, the descriptive aspect of returns were covered in the previous section. The results from this portion of the analysis were in line with expectations, particularly the commonly held assertion that Bitcoin exhibits significantly greater volatility and higher rates of return on average than traditional equity markets. This makes sense from the perspective of risk and return, given that assets with greater risk should generally compensate their investors with higher rates of return. However, bitcoins are not a traditional asset, and are subject to additional risks such as the risk of an exchange shutting down and seizing all assets currently held. Therefore, this traditional risk-return viewpoint may not be applicable, and is speculative at best.

On a more empirical note, the results from the QQ plots and accompanying K-S tests are also in line with the intuitive conclusions drawn from the summary statistics of daily returns for Bitcoin exchanges. With consideration to space, one of the QQ plots for the largest Bitcoin exchange (BTCDE) by volume is presented above the QQ plot for the S&P 500 below:

Figure 1: QQ Plot for btceCNY

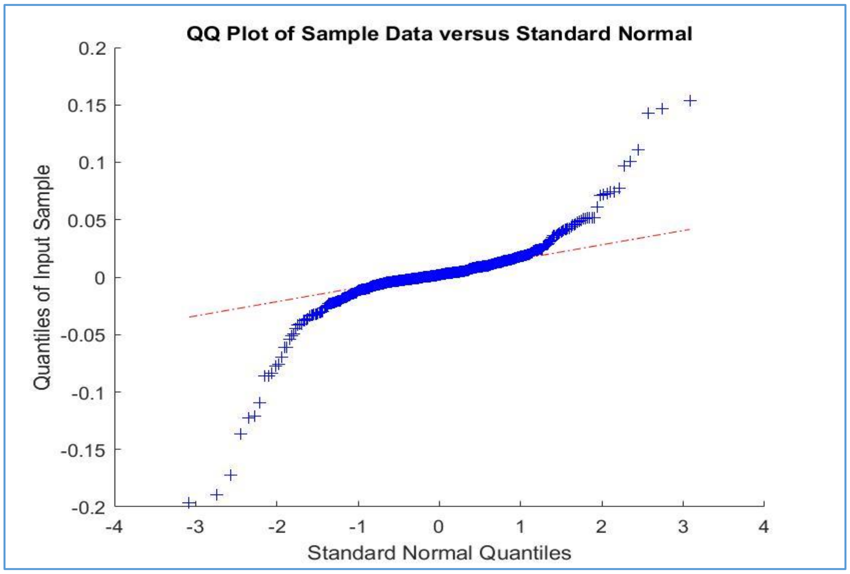
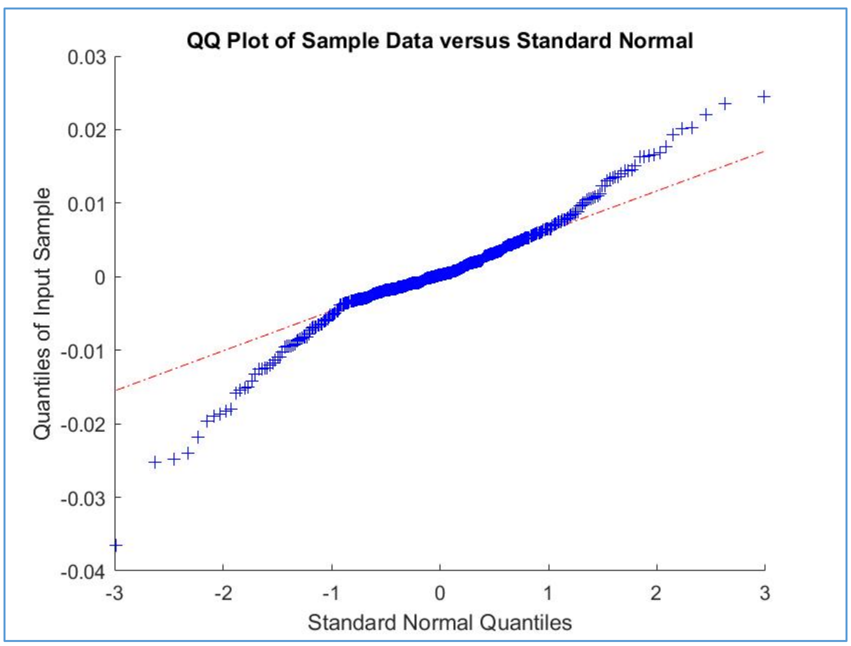


Figure 2: QQ Plot for S&P 500



Looking at the QQ plots, it is fairly clear that neither the BTCE Bitcoin exchange nor the S&P 500 fall entirely on the 45 degree line that would be expected of normally distributed data. However, after inspecting the Y-axis, it becomes apparent that the degree of variation is significantly higher in the sample Bitcoin market than in the sample traditional equity market. Furthermore, the plot for Bitcoin shows significantly larger tails that extend to significantly higher and lower rates of return than in the traditional equity market, hence the "twisting" effect seen on the plot. This confirms and re-emphasizes the high standard deviation and extreme minimum and maximum values for daily returns in Bitcoin markets seen Table 1. Moreover, the same trends persist for all of the Bitcoin markets and all of the equity indices, respectively. More than just visual inspection, the K-S provide additional support that neither of the asset groups follow a normal distribution at a daily frequency, as shown below:

Table 6: Bitcoin K-S Test Results

	K-S Result	P-Value
bitbayPLN	1	1.63E-88
bitstampUSD	1	1.01E-89
btcdeEUR	1	5.91E-89
btceCNY	1	2.28E-88
btcmarketAUD	1	9.34E-92
coinbaseEUR	1	5.14E-88
coinbaseUSD	1	8.09E-89
coincheckJPY	1	3.84E-87
coinfloorGBP	1	1.04E-89
foxbitBRL	1	4.15E-91
mercadoBRL	1	6.70E-90

Table 7: Equity K-S Test Results

	K-S Result	P-Value
S&P500	1	4.47E-74
Nikkei225	1	1.44E-70
ASX200	1	1.72E-72
IBOVESPA	1	2.27E-68
WIG	1	2.97E-72
SHCOMP	1	4.27E-72
UKX	1	4.18E-71
SX5E	1	2.43E-71

With respect to interpretation, a result of "1" indicates that the null hypothesis that the data follows a normal distribution has been rejected. This should be fairly intuitive, given the near 0 p-values generated by all of the tests. Therefore, over the past year, it can be definitively stated that Bitcoin market returns do not follow a normal distribution and exhibit significantly higher degrees of volatility than the benchmark of traditional international equities.

Moving into the second part of analysis, the focus of the paper shifts to documenting the connectedness of Bitcoin exchanges. The first set of tests performed in this section are a standard covariance matrix as well as more descriptive moving covariance matrix utilizing a 10 day "rolling" window of daily returns between each Bitcoin exchange. The static correlation matrix is also replicated for traditional equity markets for benchmarking purposes. The outputs from this process are shown below:

Table 8: Bitcoin Static Covariance Matrix

	bitbayPLN	bitstampUSD	btcdeEUR	btceCNY	btcmarketAUD	coinbaseEUR	coinbaseUSD	coincheckJPY	coinfloorGBP	foxbitBRL	mercadoBRL
bitbayPLN	1.000	0.911	0.819	0.887	0.836	0.918	0.901	0.899	0.870	0.818	0.708
bitstampUSD		1.000	0.871	0.924	0.889	0.975	0.980	0.939	0.911	0.866	0.730
btcdeEUR			1.000	0.808	0.820	0.886	0.866	0.855	0.817	0.797	0.663
btceCNY				1.000	0.822	0.916	0.916	0.922	0.855	0.824	0.696
btcmarketAUD					1.000	0.882	0.876	0.849	0.856	0.812	0.680
coinbaseEUR						1.000	0.982	0.936	0.920	0.866	0.722
coinbaseUSD							1.000	0.929	0.910	0.860	0.714
coincheckJPY								1.000	0.858	0.847	0.708
coinfloorGBP									1.000	0.826	0.703
foxbitBRL										1.000	0.800
mercadoBRL											1.000

All of the exchanges exhibit a high degree of correlation across the distribution of daily returns. Notably, all of the pricing data from which the returns are derived are based off the local currency for each exchange. Therefore, it is inferable that at least some of the differences between return correlations are due to idiosyncratic events affecting the valuation of the local currency. If all of the exchanges were normalized to a single currency, it is expected that the correlations would be higher. Nonetheless, in theory all of the exchanges should be perfectly correlated given they are all trading a homogenous asset, assuming no transaction costs or inefficiencies. In actuality though, there are numerous inefficiencies including transactions costs for buying Bitcoin, fees for converting to other currencies, fees for transfers across exchanges, and delays up to several days before funds/bitcoins become available to trade. However, as was noted in the introduction, algorithmic trading does exist between markets with suggests that occasionally arbitrage opportunities do exists for brief periods of time. Likewise, a covariance matrix for traditional equity markets is presented below for reference purposes:

Table 9: Equity Static Covariance Matrix

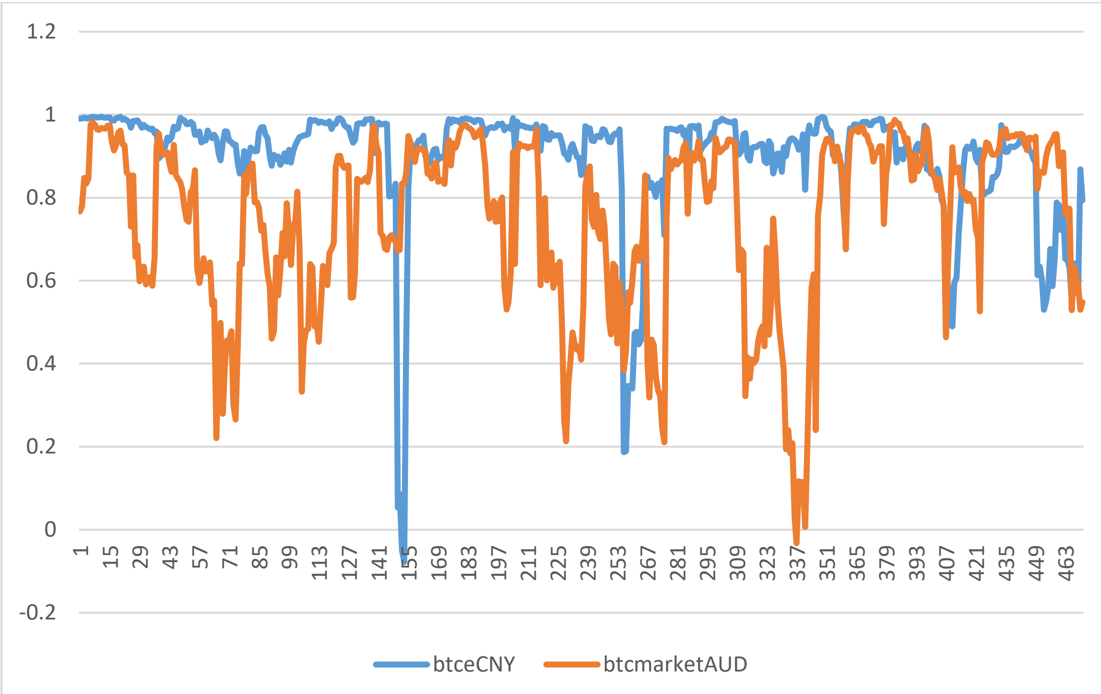
	S&P500	Nikkei225	ASX200	IBOVESPA	WIG	SHCOMP	UKX	SX5E
S&P500	1	-0.01953	0.303497	0.447906	0.37307	0.164144	0.586197	0.598751
Nikkei225		1	0.516929	0.174692	0.276907	0.25276	0.180778	0.186724
ASX200			1	0.38478	0.55509	0.25858	0.534227	0.511107
IBOVESPA				1	0.418437	0.159196	0.483575	0.47347
WIG					1	0.177093	0.657085	0.688867
SHCOMP						1	0.146293	0.141371
UKX							1	0.904594
SX5E								1

Although a one-to-one comparison is not possible due to the fact that the equity markets are not trading a homogenous asset, the above table does provide insight into the baseline interconnectedness of equities. On a more descriptive note, a sample of 11 of the 471 values generated by the moving correlation function and a graph of the moving correlation between several assets is presented below:

Table 10: Bitcoin Moving Covariance Matrix

bitbayPLN	bitstampUSD	btcdeEUR	btceCNY	btcmarketAUD	coinbaseEUR	coincheckJPY	coinfloorGBP	foxbitBRL	mercadoBRL
0.840	0.977	0.415	0.969	0.730	0.919	0.863	0.803	0.219	0.326
0.936	0.987	0.714	0.985	0.854	0.953	0.906	0.904	0.719	0.384
0.917	0.988	0.714	0.985	0.657	0.953	0.906	0.902	0.717	0.348
0.921	0.988	0.722	0.986	0.686	0.956	0.911	0.906	0.685	0.269
0.905	0.980	0.540	0.979	0.599	0.944	0.873	0.877	0.718	0.160
0.907	0.977	0.497	0.969	0.614	0.963	0.966	0.865	0.736	0.122
0.908	0.982	0.502	0.975	0.634	0.962	0.969	0.893	0.745	0.134
0.902	0.988	0.594	0.970	0.591	0.957	0.966	0.918	0.786	0.057
0.918	0.986	0.701	0.968	0.603	0.958	0.935	0.928	0.802	0.083
0.938	0.985	0.694	0.965	0.609	0.953	0.907	0.930	0.806	0.060
0.935	0.984	0.686	0.966	0.588	0.987	0.909	0.953	0.877	0.007

Figure 3: CoinbaseUSD Moving Correlation with btceCNY and btcmarketsAUD



In the above table and graph, the Coinbase exchange in the United States is used as the benchmark index and is subsequently compared against other exchanges in order to generate a correlation coefficient for each time period. In contrast to the static correlation matrix, it becomes evident that return correlations fluctuate on a frequent basis and sometimes become almost entirely uncorrelated. During these times, it may be possible to exploit pricing discrepancies, though further inquiry would be needed in order to validate this claim. On the whole though, even the largest Bitcoin exchanges in the United States, China, and Europe experience periods of return discontinuity and even times when returns move in opposite directions for brief periods of time. For additional detail, the “moving_corr” table in the Matlab file contains the moving correlation coefficients for the entire period.

In the next subsection of the connectedness analysis, OLS regressions as well as quantile regressions are performed for each Bitcoin exchange. Below are the coefficients from the OLS regression:

Table 11: OLS Regression Coefficients

	bitbayPLN	bitstampUSD	btcdeEUR	btceCNY	btcmarketAUD	coinbaseEUR	coinbaseUSD	coincheckJPY	coinfloorGBP	foxbitBRL	mercadoBRL
bitbayPLN	1.000	0.896	0.893	0.953	0.840	0.931	0.902	0.983	0.909	0.740	0.640
bitstampUSD	0.928	1.000	0.966	1.011	0.909	1.006	0.997	1.044	0.968	0.797	0.672
btcdeEUR	0.753	0.786	1.000	0.797	0.757	0.825	0.795	0.858	0.783	0.662	0.551
btceCNY	0.826	0.846	0.820	1.000	0.769	0.865	0.853	0.938	0.831	0.694	0.586
btcmarketAUD	0.833	0.871	0.891	0.880	1.000	0.891	0.873	0.925	0.890	0.732	0.613
coinbaseEUR	0.905	0.945	0.952	0.971	0.874	1.000	0.968	1.009	0.947	0.773	0.644
coinbaseUSD	0.902	0.963	0.944	0.984	0.881	0.995	1.000	1.016	0.950	0.778	0.646
coincheckJPY	0.824	0.845	0.853	0.906	0.781	0.869	0.851	1.000	0.820	0.701	0.586
coinfloorGBP	0.835	0.859	0.854	0.881	0.825	0.895	0.873	0.899	1.000	0.717	0.610
foxbitBRL	0.906	0.942	0.962	0.980	0.903	0.972	0.952	1.024	0.954	1.000	0.800
mercadoBRL	0.786	0.796	0.802	0.830	0.758	0.813	0.792	0.858	0.814	0.802	1.000

For all of the above coefficients, the P-values were statistically significant. Thus, it can be stated that returns on one exchange are significant drivers of returns in the same period on another exchange. This makes intuitive sense given they are all trading the same asset, though there are discontinuities especially when considering the Brazilian exchanges’ predictive power with respect to other exchanges. This could be due to volatility in the Brazilian Real or other idiosyncratic factors with the country. To further analyze the predictive power of exchange returns across deciles, quantile regressions were performed and the coefficients for the Coinbase USD exchange are presented below:

Table 12: Quantile Regression Coefficients for CoinbaseUSD

Decile	bitbayPLN	bitstampUSD	btcdeEUR	btceCNY	btcmarketAUD	coinbaseEUR	coincheckJPY	coinfloorGBP	foxbitBRL	mercadoBRL
<.1	0.787	0.980	0.673	0.836	0.745	0.957	0.814	0.823	0.861	0.542
0.1	0.790	0.984	0.702	0.847	0.771	0.963	0.815	0.829	0.875	0.586
0.2	0.798	0.986	0.728	0.853	0.784	0.964	0.819	0.833	0.890	0.614
0.3	0.800	0.998	0.730	0.853	0.785	0.968	0.826	0.862	0.890	0.630
0.4	0.821	1.002	0.750	0.858	0.792	0.970	0.834	0.869	0.894	0.633
0.5	0.845	1.002	0.769	0.868	0.824	0.974	0.846	0.873	0.899	0.669
0.6	0.864	1.003	0.773	0.868	0.838	0.974	0.849	0.884	0.911	0.681
0.7	0.880	1.018	0.779	0.877	0.838	0.977	0.849	0.897	0.945	0.681
0.8	0.885	1.024	0.781	0.879	0.861	0.978	0.851	0.897	0.951	0.681
0.9	0.903	1.024	0.797	0.883	0.869	0.980	0.855	0.904	0.955	0.740
>.9	0.910	1.029	0.807	0.889	0.878	0.981	0.893	0.922	0.956	0.755

The above table is the output from the "qregressmatlab" function, with CoinbaseUSD used as the dependent variable and all other exchanges as the independent variable. All results were statistically significant, and the entirety of results are available in the Matlab code. As opposed to the OLS regressions, this analysis reveals more interesting insight into how exchange returns drive other exchange returns at different points on the CDF. Particularly, a clear trend emerges in that explanatory power diminishes slightly during extreme declines in return (at the <.1, .1, and .2 deciles) while explanatory power increases during periods of extreme increases in return (>.9, .9, and .8). This trend suggests that the exchanges move together during periods of high returns, though experience more disconnected movement during times of negative returns. This could potentially be due changes in local currency valuation, though that alone does not account for the differences above.

For the final portion of the connectedness section, a coexceedence measure based on Kee-Hong Bae's methodology was employed. Below are the total number of joint coexceedence events for both Bitcoin and equity markets over the period:

Table 13: Bitcoin Joint Coexceedence Events

One Event	Two Events	Three Events	Four Events	Five Events	Six Events	Seven Events	Eight Events	Nine Events	Ten Events	Eleven Events
19	8	1	3	3	0	2	5	4	1	9

Table 14: Equity Joint Coexceedence Events

One Event	Two Events	Three Events	Four Events	Five Events	Six Events	Seven Events	Eight Events
42	17	12	4	2	1	0	0

Notably, "One Event" corresponds to only one exchange experiencing a coexceedence event while "Eleven Events" corresponds to all of the exchanges experiencing a coexceedence event. The same reasoning applies to the table for equities, given there were only 8 indices studied. On the whole, there are two distinct categories in the above table for Bitcoin in that either most, if not all, exchanges experience extreme returns or only a handful or singular exchange experiences extreme returns. This continues to reaffirm the notion developed by the quantile regression framework, particularly that exchanges have a high degree of idiosyncrasy in their returns. On the other hand, coexceedence events are primarily concentrated in only a singular or few markets for equity markets.

In the final section of the analysis, the purpose of the tests conducted is to explore the applicability of the efficient market hypothesis within the context of Bitcoin exchanges. The first test performed is a test of autocorrelation, with the idea being to determine if prior returns have any statistically significant impact on future returns for each exchange. The results of this analysis are shown in graphical format below for Bitcoin with a daily and weekly return frequency:

Figure 4: Daily Bitcoin Autocorrelation

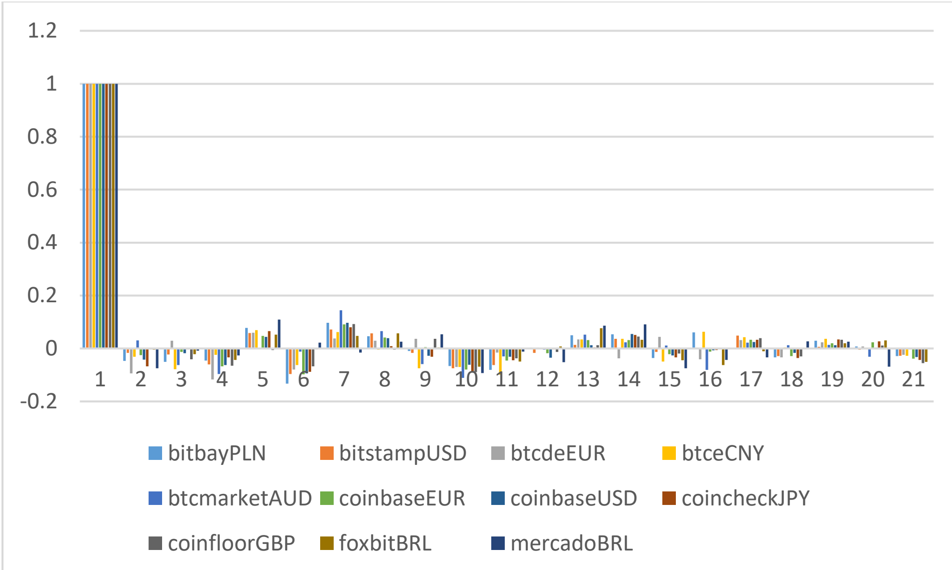
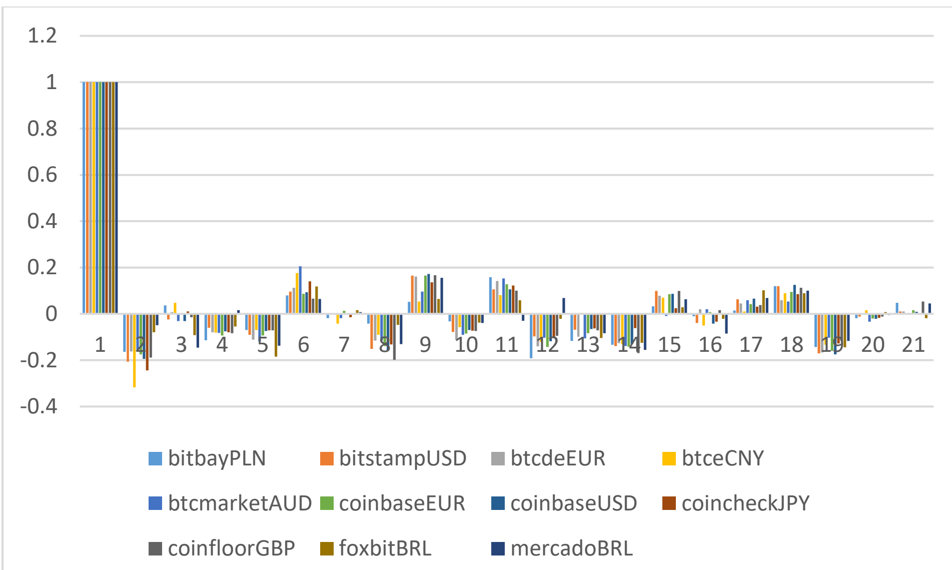


Figure 5: Weekly Bitcoin Autocorrelation



For the autocorrelation analysis, it does not appear there is any definitive evidence of autocorrelation. The only trend that is visually evident is a negative correlation with a time lag of between 2 and 4 periods on both weekly and daily return frequencies, though this is not statistically significant. On the whole, it does not appear that market efficiency is violated from the standpoint of autocorrelation.

For the second part of the analysis, a cross correlation function was utilized with the intention of comparing the relationship of returns on one exchange those of another exchange. The test was performed on a daily and weekly frequency, with Coinbase USD used as the baseline exchange against which all the other exchanges were compared. The graphical results of this are shown below:

Figure 6: CoinbaseUSD Daily Cross Correlation

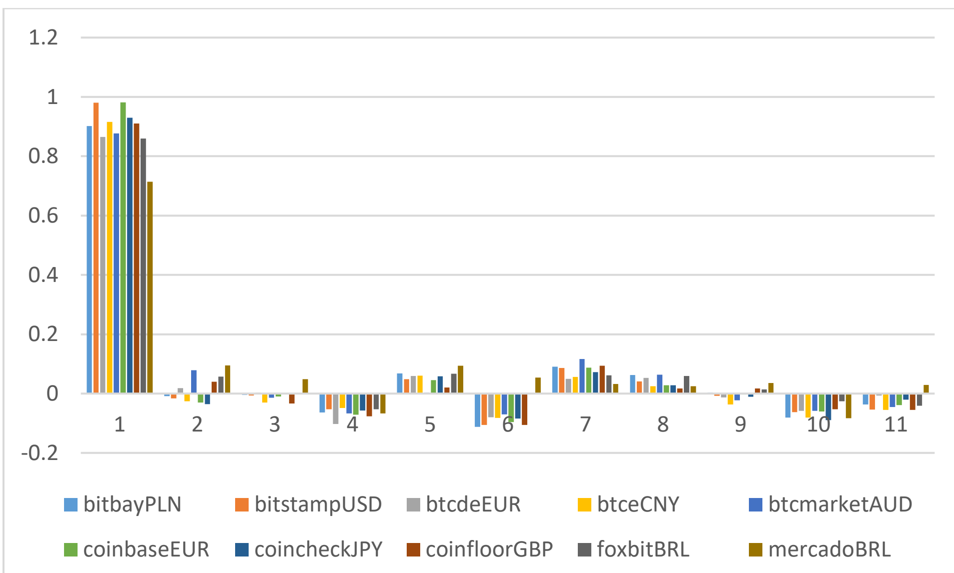
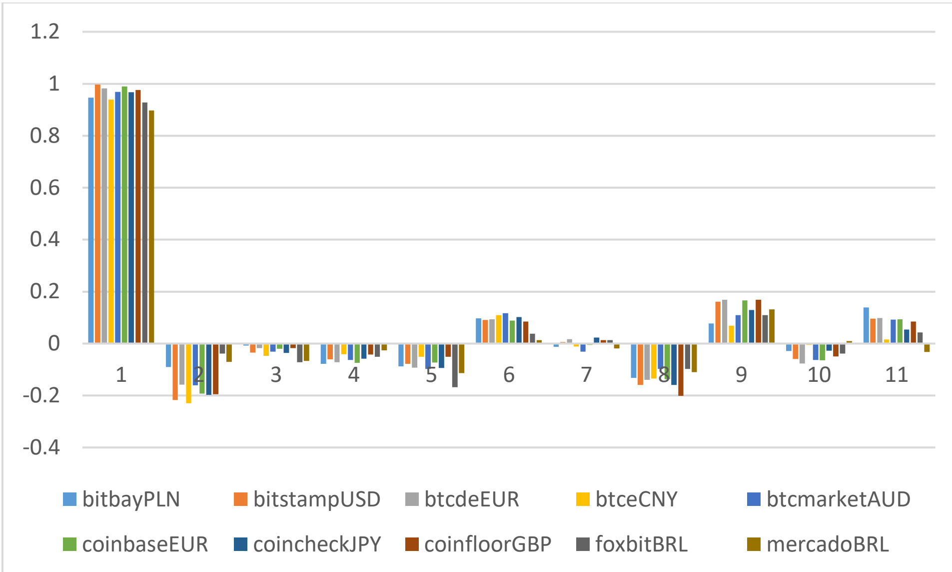


Figure 7: CoinbaseUSD Weekly Cross Correlation



As with autocorrelation, there does not appear to be any statistically significant evidence of cross correlation between exchanges. For the most part, correlation movements appear to be random at both the daily and weekly frequency, and thus the market efficiency hypothesis does not appear to be violated.

For the final portion of the market efficiency analysis, a variance ratio test developed Lo and MacKinlay in 1988 is utilized. Below are the hypothesis tests results as well as the actual variance ratio for both Bitcoin and equity markets:

Table 15: Bitcoin V-Ratio Test Summary

Periods	bitbayPLN	bitstampUSD	btcdeEUR	btceCNY	btcmarketAUD	coinbaseEUR	coinbaseUSD	coincheckJPY	coinfloorGBP	foxbitBRL	mercadoBRL
2	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0

Table 16: Bitcoin V-Ratio

Periods	bitbayPLN	bitstampUSD	btcdeEUR	btceCNY	btcmarketAUD	coinbaseEUR	coinbaseUSD	coincheckJPY	coinfloorGBP	foxbitBRL	mercadoBRL
2	0.957	0.987	0.908	0.972	1.035	0.979	0.962	0.937	1.001	0.997	0.917
4	0.866	0.933	0.839	0.873	0.947	0.928	0.899	0.891	0.934	0.959	0.860
6	0.833	0.906	0.795	0.861	0.886	0.892	0.859	0.884	0.867	0.970	0.922
8	0.847	0.922	0.782	0.873	0.946	0.909	0.877	0.903	0.864	1.014	0.959
10	0.851	0.923	0.783	0.837	0.950	0.916	0.875	0.889	0.861	1.041	0.991

Table 17: Equity V-Ratio Test Summary

Periods	S&P500	Nikkei225	ASX200	IBOVESPA	WIG	SHCOMP	UKX	SX5E
2	0	1	0	0	0	1	0	0
4	0	1	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0
8	0	1	0	0	0	0	0	0
10	0	1	0	0	0	0	0	0

Table 18: Equity V-Ratio

Periods	S&P500	Nikkei225	ASX200	IBOVESPA	WIG	SHCOMP	UKX	SX5E
2	0.913	0.701	1.082	1.069	1.110	0.851	1.115	1.053
4	0.862	0.618	1.157	1.055	1.093	0.929	0.985	0.971
6	0.852	0.544	1.081	0.988	0.966	0.974	0.698	0.759
8	0.836	0.501	0.996	0.982	0.938	0.970	0.556	0.694
10	0.830	0.469	0.956	0.996	0.946	1.007	0.526	0.689

In the V-Ratio test summary tables, a value of “0” indicates that the null hypothesis that the price series is a random walk failed to reject while a value of “1” indicates a rejection of the null hypothesis. Looking into the results for Bitcoin, it can be definitely stated that the price series is more similar to a random walk rather than mean averting or mean reverting. Once again, it seems that the market efficiency hypothesis holds within Bitcoin markets. Surprisingly though, the variance ratio analysis for equity markets indicates that the Nikkei 225 does not follow a random walk and is heavily mean reverting. This indicates that opportunity exists within the market for momentum trading, and the market efficiency hypothesis is potentially violated.

Conclusion and Extensions

On the whole, this exposition of Bitcoin returns did not uncover any groundbreaking results. The results showed that Bitcoin is a highly volatile asset in comparison to traditional equities, and experiences extreme shifts in volume traded per day. Likewise, even with the idiosyncrasies in local currency stability, the exchanges were found to be highly connected in terms of daily and monthly returns. The findings with respect to the market efficiency hypothesis were in line with the research performed by Jakub Bartos and in opposition to those of William Brown. Notably, both of these individuals conducted their research during a period where Bitcoin was still a fairly immature cryptocurrency, hence the data they used may have had different underlying trends that the data used for this study. Of all of the results documented by this paper, likely the most promising in terms of arbitrage opportunities are the discontinuities between upper and lower quantile returns made evident by the quantile regressions and the moving correlation matrix. Nonetheless, transaction costs and frictions would still have to be accounted for in order to determine if a viable arbitrage opportunity exists.

In terms of extensions, it would be interesting to focus some of the connectedness research on particular geographic areas. Particularly, it would be intriguing to group exchanges in Southeast Asia, North America, Europe, and the rest of the world and compare via these groupings rather than individual exchanges. Furthermore, it would also be highly interesting to form portfolios using Bitcoin and traditional asset classes and document the effect it has on the Markowitz frontier. Finally, on a non-Bitcoin related note, the fact that the Nikkei 225 apparently violates the market efficiency hypothesis was a highly surprising result. Additional study and hypothesis testing is warranted by this result, as it would be a major finding if a major equity market was found to be predictable.

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