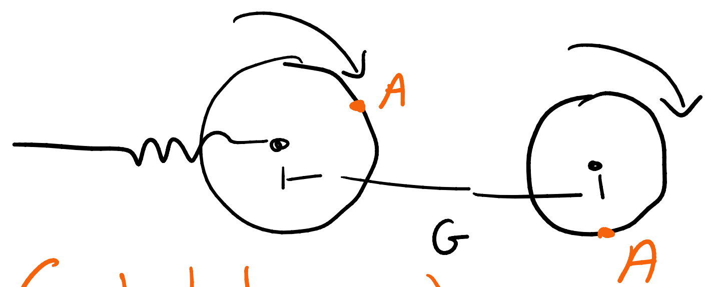
Conservation of energy (Rigid Bodies)

Tuesday, November 29, 2022 3:33 PM

Rigid Bodies

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

(Kinetic)



(potential energy)

$$V_{TOTAL} = V_G + V_E$$

gravity   spring

$$V_G = W \Delta y_G$$

$$V_E = \frac{1}{2} k s^2$$

s = distance of unstretched length

Conservation of Energy

$$T_1 + V_1 = T_2 + V_2$$

+ potential

Datum line
(0 potential energy)

− potential

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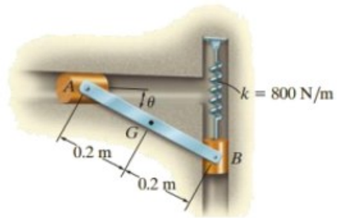
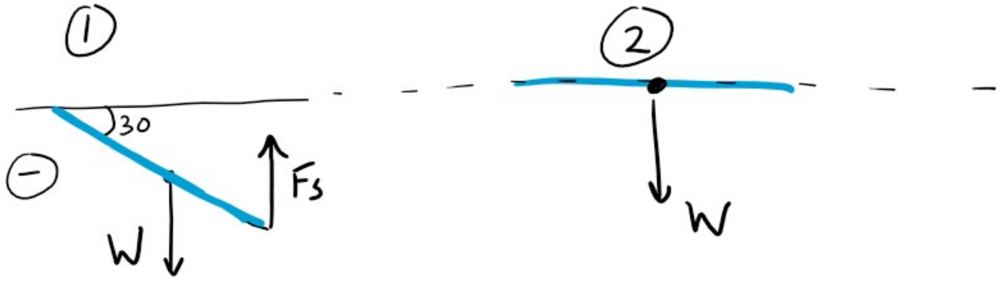
Example 1: The 10kg rod AB is confined so that its ends move in the horizontal and vertical slots. The spring has a stiffness of k = 800N/m and is unstretched when θ= 0°. Determine the angular velocity of AB when θ=0° if the rod is released from rest when θ = 30°. Neglect the mass of the slider blocks





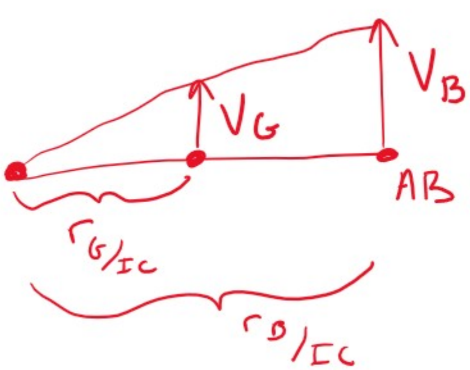
$$\cancel{T_1}^{0} + V_1 = T_2 + \cancel{V_2}^{0}$$

$$V_G + V_s = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$mgh + \frac{1}{2} k s^2$$

$$-(10)(9.81)(0.2 \sin 30) + \frac{1}{2}(800)(0.4 \sin 30)^2 = \underbrace{\frac{1}{2}(10)(V_G)^2}_{\frac{1}{2}(10)\left[(\omega)(0.2)\right]^2} + \frac{1}{2}\left(\frac{1}{12}(10)(0.4)^2\right)\omega^2$$

$$\omega = 4.82 \text{ rad/s}$$

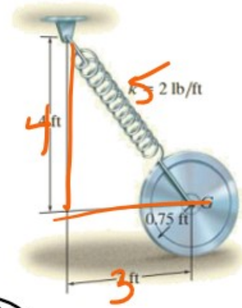


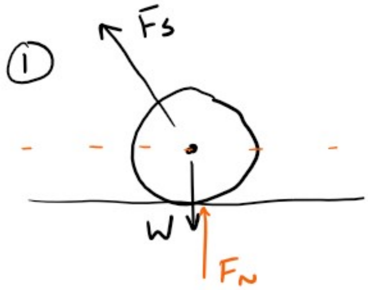
$$V = \omega r$$

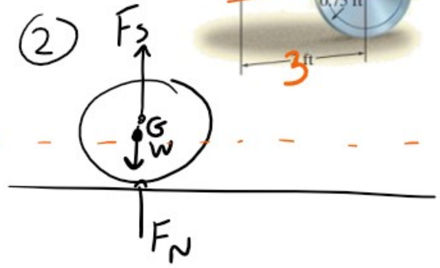
$$V_G = \omega r_{G/IC}$$

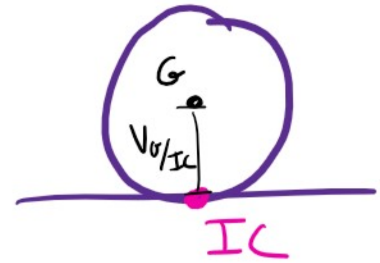
$$V_G = \omega (0.2)$$

Example 2: The wheel has a weight of 30lb and a radius of gyration of $k_G = 0.6$ ft. It is attached to a spring which has a stiffness $k = 2$ lb/ft and an unstretched length of 1 ft. If the disk is released from rest in the position shown and rolls without slipping, determine its angular velocity at the instant G moves 3 ft to the left.









$$V_G = \omega\,(V_{G/IC}) = (0.75)\,\omega$$

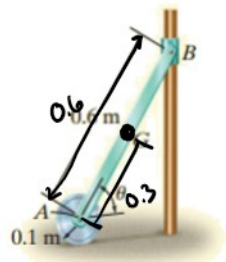
$$\overset{0}{\cancel{T_1}} + V_1 = T_2 + V_2$$

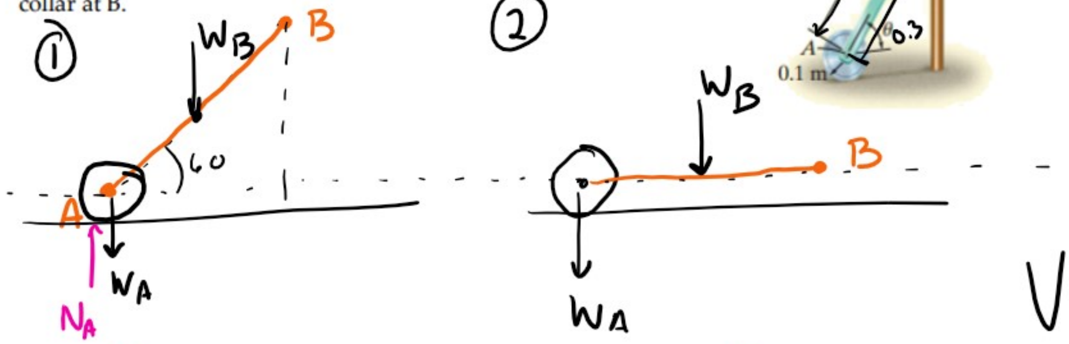
$$\frac{1}{2}k s_1^2 = \underbrace{\frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2}_{T_2} + \frac{1}{2} k s_2^2$$

$$\frac{1}{2}(2)(4)^2 = \underbrace{\frac{1}{2}\left(\frac{30}{32.2}\right) V_G^2}_{\frac{1}{2}\left(\frac{30}{32.2}\right)(0.75\omega)^2} + \frac{1}{2}\Big[\underbrace{\frac{1}{2}\left(\frac{30}{32.2}\right)(0.6)^2}_{I_G}\Big]\omega^2 + \frac{1}{2}(2)(3)^2$$

$$\omega = 4.03\ \text{rad/s}$$

Example 3: The 10 kg homogenous disk is attached to a uniform 5 kg rod AB. If the assembly is released from rest when θ = 60°, determine the angular velocity of the rod when θ =0°. Assume that the disk rolls without slipping. Neglect friction along the guide and the mass of the collar at B.





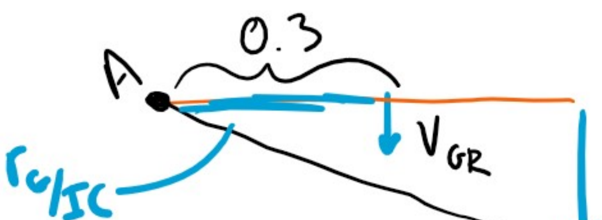
$$V = V_G + \cancel{V_s}$$

$$\sum \cancel{T_1}^{\,0} + \sum V_1 = \sum T_2 + \cancel{\sum V_2}^{\,0}$$

$$mgh = \frac{1}{2} m_w \cancel{V_{Gw}}^{2}_{\,0} + \frac{1}{2} \cancel{J_{Gw}\omega_w}^{2}_{\,0} + \frac{1}{2} m_R \underline{V_{G_R}}^2 + \frac{1}{2} I_{G_r}\omega^2$$

$$(5)(9.81)(0.3\sin 60) = \frac{1}{2}(5)(0.3\omega)^2 + \frac{1}{2}\left[\frac{1}{12}(5)(0.6)^2\right]\omega^2$$

$$\omega = 6.52 \ \text{rad/s}$$



$$V_{GR} = 0.3\omega$$

At this moment since the rod is fully extended the wheel is momentarily at rest (IC)