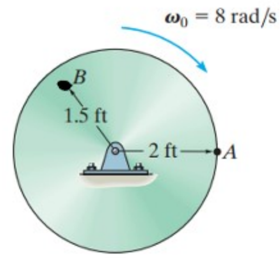
Example 2:

The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6$ rad/s^2, determine the magnitude of the velocity and the n and t components of acceleration of point B just after the wheel undergoes 2 revolutions.



$\omega_0 = 8$ rad/s

$\alpha = 6$ rad/s^2

$V_B = ?$

$A_N = ?$

$A_T = ?$

$\theta_0 = 0$

$\theta_f = 2$ revs

$(2\text{rev})\dfrac{2\pi \text{rad}}{1\text{rev}} = 12.56$ rad/s

$$\omega_f^2 = \omega_0^2 + 2\alpha(\theta_f - \theta_0)$$

$$\omega_f^2 = 8^2 + 2(6)(12.56 - 0)$$

$$\omega_f = 14.65 \text{ rad/s}$$

$$V = r\omega$$
$$= (1.5)(14.65) = 22 \text{ ft/s}$$

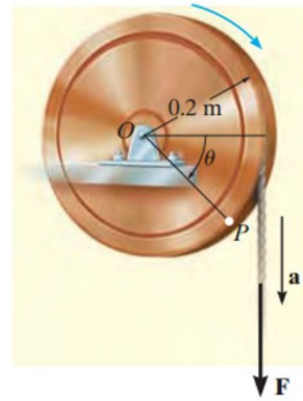
$$a_t = r\alpha$$
$$= (1.5)(6) = 9 \text{ ft/s}^2$$

$$a_N = \omega^2 r$$
$$= (14.65)^2(1.5) = 322 \text{ ft/s}^2$$

Thursday, September 29, 2022 1:06 PM

Example 3.

A cord is wrapped around a wheel initially at rest when θ = 0. If a force is applied to the cord and gives it an acceleration a = 4t m/s² where t is in seconds, determine as a function of time, the angular velocity of the wheel, and the angular position of the line OP in radians.



$\omega_0 = 0$

$\theta_0 = 0$

$a_t = 4t$

$a_t = \alpha r$

$\alpha = \frac{a_t}{r} = \frac{4t}{.2} = 20t \ \text{rad/s}^2$

$\alpha = \frac{d\omega}{dt}$

$20t = \frac{d\omega}{dt}$

$\int_0^t 20t\, dt = \int_0^\omega d\omega$

$10t^2 \Big|_0^t = \omega \Big|_0^\omega$

$\omega = 10t^2$

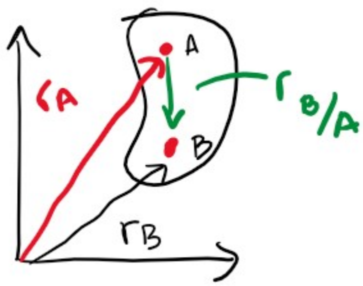
$\omega = \frac{d\theta}{dt}$

$10t^2 = \frac{d\theta}{dt}$

$\int_0^t 10t^2\, dt = \int_0^\theta d\theta$

$\frac{10t^3}{3} = \theta$

General Planar Motion → combination of translation and fixed axis rotation



$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

$$\underbrace{d\bar{r}_B}_{\text{translation + rotation}} = \underbrace{d\bar{r}_A}_{\text{translation}} + \underbrace{d r_{B/A}}_{\text{rot}}$$

$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$$

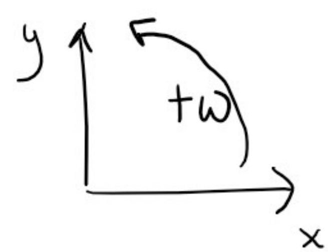
$$\bar{V}_B = \bar{V}_A + \bar{\omega} \times \bar{r}_{B/A} \qquad V_A = V_B + \omega \times \bar{r}_{A/B}$$

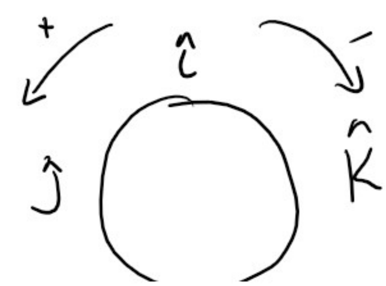
V_B = Velocity of point "B"

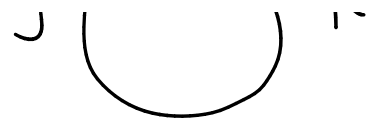
V_A = Velocity of point "A"

ω = angular velocity

$r_{B/A}$ = position of "B" w.r.t "A"





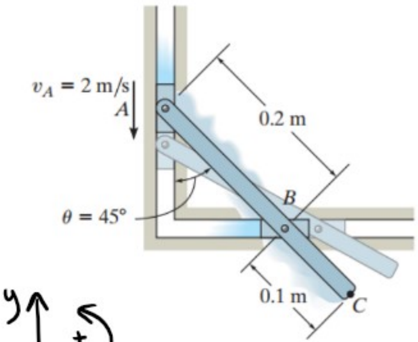


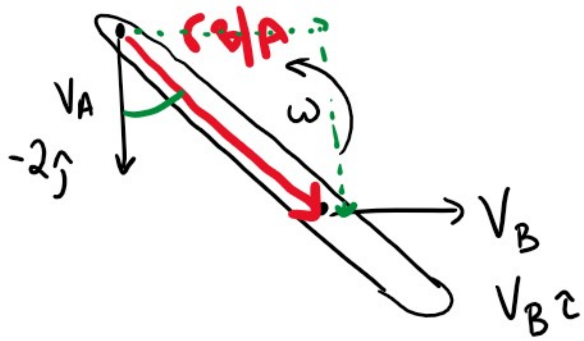
$\hat{i} \times \hat{j} = +k$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{i} = +\hat{j}$

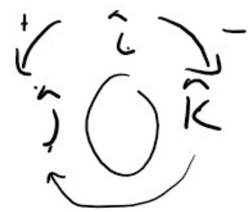
Example 1: The link is guided by two blocks at A and B, which move in fixed slots. If the velocity of A is 2 m/s downward, determine the velocity of B at the instant θ = 45°.





$$r_{B/A} = .2\sin 45\,\hat{i} - .2\cos 45\,\hat{j}$$

$$\bar{V}_B = \bar{V}_A + \bar{\omega} \times \bar{r}_{B/A}$$



$$V_B\hat{i} = -2\hat{j} + \omega\hat{K} \times (.2\sin 45\,\hat{i} - .2\cos 45\,\hat{j})$$

$$V_B\hat{i} = -2\hat{j} + .2\sin 45\,\omega\,\hat{j} + .2\cos 45\,\omega\,\hat{i}$$

collect terms

$\hat{i}$:

$$V_B = 0.2\cos 45\,\omega$$

$$V_B = 0.2\cos 45(14.1)$$

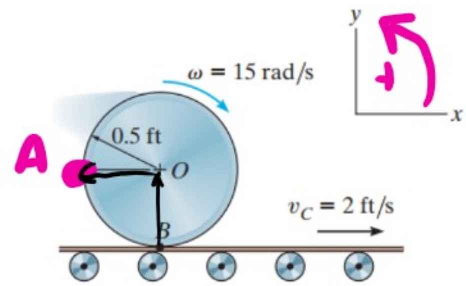
$$= 2\ \text{m/s} \longrightarrow$$

$\hat{j}$:

$$0 = -2 + .2\sin 45\,\omega$$

$$\omega = 14.1\ \text{rad/s}$$

Example 2: The cylinder rolls without slipping on the surface of a conveyor belt moving at 2 ft/s. Determine the velocity of point A. The cylinder has a clockwise angular velocity $\omega = 15$ rad/s



point of contact takes on same velocity as moving surface

$V_A = ?\; V_A\hat{i} + V_A\hat{j}$

$V_B = 2\hat{i}$

$\omega = -15\hat{k}$

$r_{A/B} = 0.5\hat{j} - 0.5\hat{i}$
$= -0.5\hat{i} + 0.5\hat{j}$

$$\bar{V}_A = \bar{V}_B + \bar{\omega} \times \bar{r}_{A/B}$$

$$V_A\hat{i} + V_A\hat{j} = 2\hat{i} + (-15)\hat{k} \times (-0.5\hat{i} + 0.5\hat{j})$$

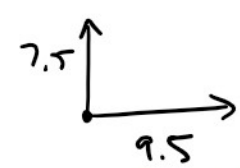
$$V_A\hat{i} + V_A\hat{j} = 2\hat{i} + 7.5\hat{j} + 7.5\hat{i}$$

i:

$V_A = 2 + 7.5 = 9.5$

j:

$V_A = 7.5$



$$V_A = \sqrt{9.5^2 + 7.5^2} = 12.1 \text{ ft/s}$$

$$\theta = \tan^{-1}\left(\frac{7.5}{9.5}\right) = 38.3^\circ$$

Example 3 : The collar in C is moving download with a velocity of 2 m/s. Determine the angular velocity of CB **at this instant**. CB start completely vertically.