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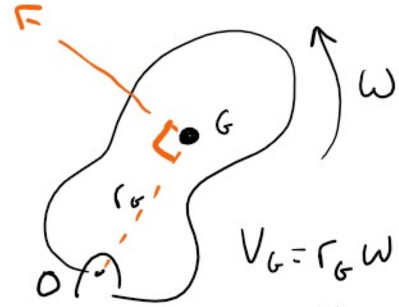
Everything is with respect to center of gravity

Translation - Energy due to rotation is $\emptyset$

$$T = \frac{1}{2} m V_G^2$$



$V_G = r_G \omega$

$\frac{1}{2}(\underbrace{I_G + m r_G^2}_{I_o})$

Rotation about a fixed axis

$$T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$$
$$= \frac{1}{2} m (r_G \omega^2) + \frac{1}{2} I_G \omega^2$$
$$= \frac{1}{2} m r_G^2 \omega^2 + \frac{1}{2} I_G \omega^2$$
$$= \frac{1}{2} I_o \omega^2$$

General Planar Motion

$$T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$$

Translation Rotational

need to relate V_G to ω

Forces that do work

Variable Force $= \int_{s_1}^{s_2} F \cos\theta \, ds$

Constant Force $= F \cos\theta (s_2 - s_1)$

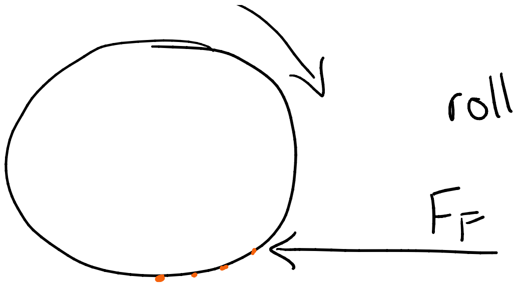
Work of Weight $= -W \Delta Y$

Work of a spring $= U_s = -\left(\frac{1}{2} K s_2^2 - \frac{1}{2} K s_1^2\right)$

Work of moment $= U_m = \int_{\theta_1}^{\theta_2} M \, d\theta$ (if constant) $U_m = M(\theta_2 - \theta_1)$

* work is negative when displacement of body is opposite direction of force

rolls without slipping



rolls without slipping

(Does no work)

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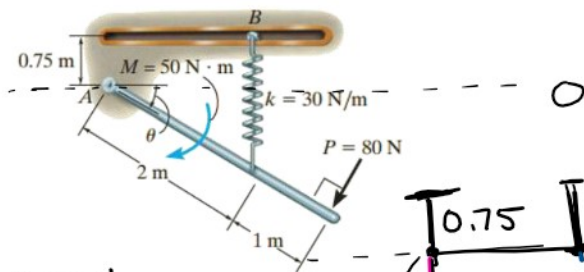
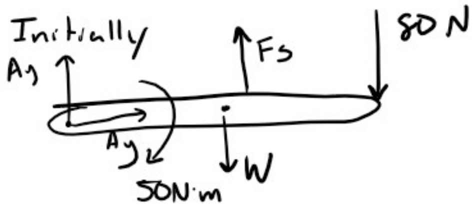
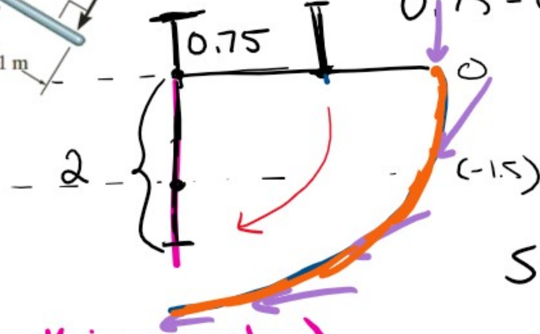
Example 1: The bar shown has a mass of 10kg and is subjected to a couple moment M = 50 Nm and a force of P = 80N which is always applied perpendicular to the end of the bar. Also, the spring has an unstretched length of 0.5m and remains in the vertical position due to the roller guide at B. Determine the total work done by all the forces acting on the bar when it has rotated downward for θ = 0° to θ = 90°.







$0.75 - 0.5 = 0.25$

$S = r\theta$
$= 3(\frac{\pi}{2}) = 4.712$

$U_W = -W\Delta Y$ (If we go down work is positive)

$-\underbrace{(10kg)(9.81 m/s^2)}_{W}(-1.5-0) = 147.2J$

$2.75 - 0.5 = 2.25$

$U_M = M(\theta_2 - \theta_1)$ ☆radians

$= 50(\pi/2 - 0) = 78.5J$

$U_s = -\left[\frac{1}{2}Ks_2^2 - \frac{1}{2}Ks_1^2\right] = -\left[\frac{1}{2}(30)(2.25)^2 - \frac{1}{2}(30)(0.25)^2\right] = -75J$

$U_P = 80(4.712) = 377J$

$U_T = U_W + U_M + U_S + U_P = \boxed{528J}$

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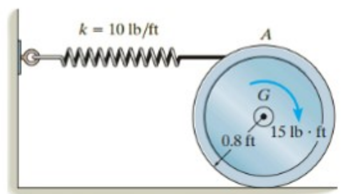
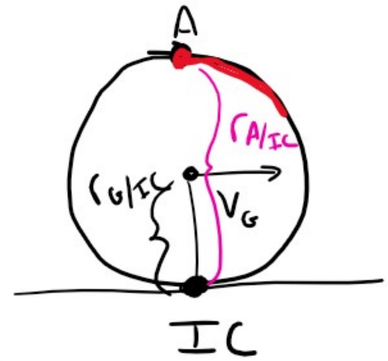
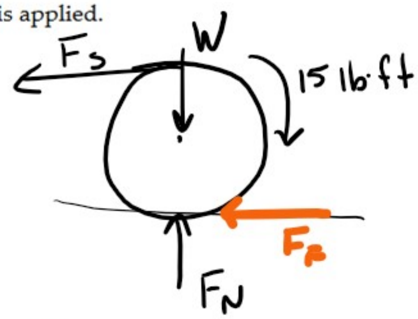
Example 2: The wheel shown weighs 40 lbs and has a radius of gyration $k_G = 0.6$ ft about its mass enter G. If it is subjected to a clockwise couple moment of 15 lb ft and rolls from rest without slipping, determine its angular velocity after its center G moves 0.5 ft. The spring has a stiffness k = 10 lb/ft and is initially unstretched when the couple moment is applied.







No slip

$$\boxed{s = r\theta} \quad v = r\omega \quad a = r\alpha$$

$$T_2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$$

$$T_2 = \frac{1}{2}\left(\frac{40}{32.2}\right) V_G^2 + \frac{1}{2}\underbrace{\left[\left(\frac{40}{32.2}\right)(0.6)^2\right]}_{k_g^2}\omega^2$$

$$\cancel{T_1} + \Sigma U_{1-2} = T_2$$

$$0.5 = 0.8\theta$$

$$\theta = 0.625 \text{ rad}$$

$$S_A = r_{A/IC}\,\theta$$

$$(1.6)(0.625) = 1 \text{ ft}$$

$$T_2 = \frac{1}{2}\left(\frac{40}{32.2}\right)\left[(0.8)\omega\right]^2 + \frac{1}{2}\left[\frac{40}{32.2}(0.6)^2\right]\left[\omega^2\right]$$

$$= \frac{1}{2}\left(\frac{40}{32.2}\right)\left[0.8^2 + 0.6^2\right]\omega^2$$

$$= 0.6211\,\omega^2$$

$$\Sigma U_{1-2} = U_M + U_s$$

$$= (15)(0.625) - \left[\frac{1}{2}(10)(1)^2\right]$$

$$= 9.375 - 5 = 4.375$$

$$U_M = M(\theta_2 - \theta_1)$$

$$U_s = -\left[\frac{1}{2} k s_2^2 - \cancel{\frac{1}{2} k s_1^2}\right]$$

$$4.375 = 0.6211\,\omega^2$$

$$\boxed{\omega = 2.65 \text{ rad/s}}$$