

Dynamics - branch of mechanics that focuses on the motion of bodies under the action of forces

- statics (bodies at rest)
 - dynamics (bodies in motion)
 - strength of materials (deformable bodies)
 - fluids
- } rigid bodies

Particles - an object of negligible dimension

rigid body - do not deform but they have dimensions

Newton's 2nd Law of motion $\vec{F} = m\vec{a}$

	SI	English
time	s	s
mass	kg	slug
length	m	ft
force	N	lb
gravity	9.81 $\frac{m}{s^2}$	32.2 $\frac{ft}{s^2}$

memorize

if given kg multiply by gravity to get force

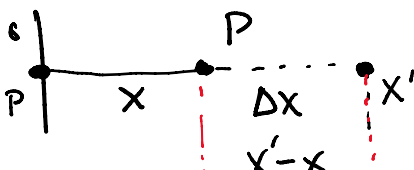
if given lb you need to divide by gravity to get mass

$$F = ma$$

$$[N] = [kg] \left[\frac{m}{s^2} \right]$$

Rectilinear Motion (Motion in 1D)

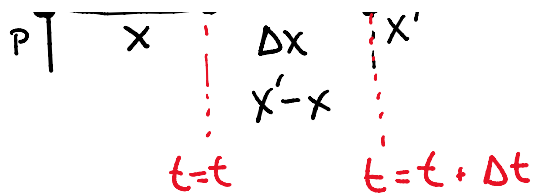
Velocity



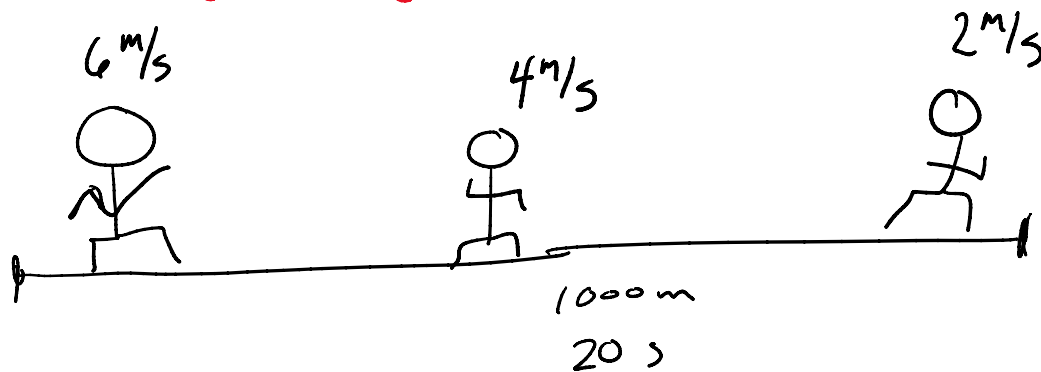
$$V_{AVG} = \frac{\Delta x}{\Delta t}$$

$$V \lim_{\Delta t \rightarrow 0} = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous Velocity



$$V \lim_{\Delta t \rightarrow 0} = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



$$V_{\text{AVG}} = \frac{1000 \text{ m}}{20 \text{ s}} = 5 \text{ m/s}$$

Acceleration

$$a_{\text{AVG}} = \frac{\Delta v}{\Delta t}$$

$$a \lim_{\Delta t \rightarrow 0} = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Instantaneous
Acceleration

All
valid

$$a = \frac{d^2x}{dt^2} \quad a = v \frac{dv}{dx}$$

If acceleration is constant

$$(1) V_f = V_0 + at$$

$$(2) X_f = X_0 + V_0 t + \frac{1}{2} at^2$$

$$(3) V_f^2 = V_0^2 + 2a(X_f - X_0)$$

Kinematic equations
of motion
only valid when a is
constant

Memorize

Dynamics

Sunday, May 8, 2022 11:39 AM

Example 1

The motion of a particle is given by the equation $x = t^3 - 9t^2 + 24t - 8$ in

A) Determine when $v = 0$

B) The Position and total distance traveled when $a = 0$

$$v = \frac{dx}{dt} = \frac{d(t^3 - 9t^2 + 24t - 8)}{dt} = 3t^2 - 18t + 24$$

$$0 = 3t^2 - 18t + 24$$

$$0 = t^2 - 6t + 8$$

$$t = 2, 4 \text{ s}$$

$$a = \frac{dv}{dt} = 6t - 18$$

$$0 = 6t - 18$$

$$t = 3 \text{ s}$$

plug 3s into x equation

$$x(3) = 3^3 - 9(3)^2 + 24(3) - 8 = 10 \text{ in}$$

Total Distance

$$x @ t = 0$$

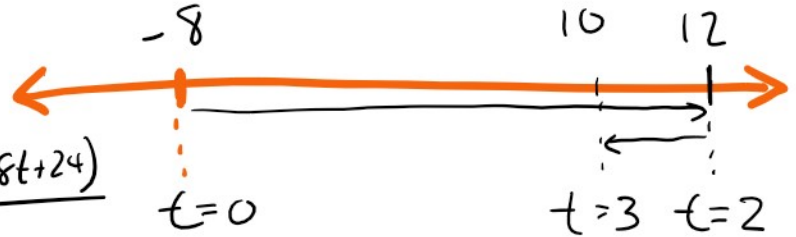
$$t = 2$$

$$t = 3$$

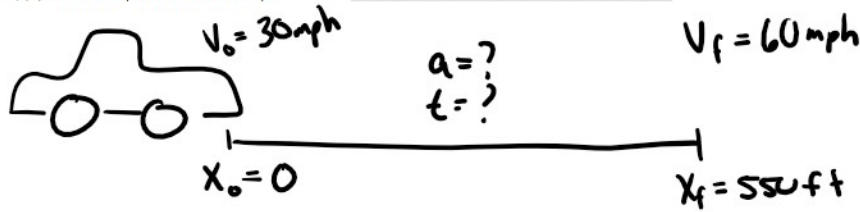
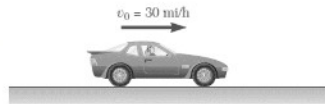
$$~~t = 4~~ \text{ past end}$$

$$\left. \begin{array}{l} x = -8 \text{ in} \\ x = 12 \text{ in} \\ x = 10 \text{ in} \end{array} \right\} \text{ plug into } x \text{ eq}$$

$$\text{total distance} = 20 + 2 = 22 \text{ in}$$



A motorist enters a freeway at 30 mph and accelerates uniformly to 60 mph. From the odometer in the car, the motorist knows that she travelled 550 ft while accelerating. Determine (a) the acceleration of the car, (b) the time required to reach 60 mph



$$\left(\frac{30 \text{ mi}}{\text{hr}}\right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) \left(\frac{1 \text{ hr}}{3600 \text{ s}}\right) = 44 \text{ ft/s} \leftarrow V_0$$

$$\frac{60 \text{ mi}}{\text{hr}} = 88 \text{ ft/s} \quad V_f$$

$$a) V_f^2 = V_0^2 + 2a(x_f - x_0)$$

$$88^2 = 44^2 + 2a(550 - 0)$$

$$a = 5.28 \text{ ft/s}^2$$

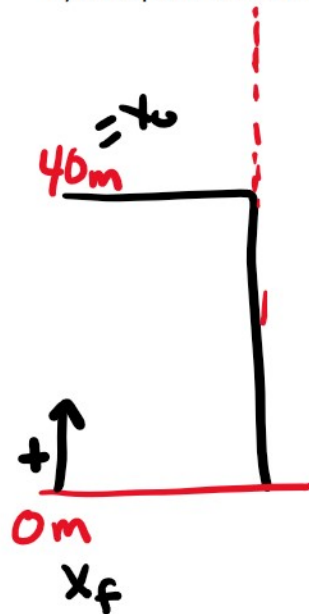
$$b) V_f = V_0 + at$$

$$88 = 44 + 5.28t$$

$$t = 8.3 \text{ s}$$

Example 3

A stone is thrown vertically upward from a point on a bridge located 40m above the water. Knowing that it strikes the water 4s after release, determine, a) the speed with which the stone was thrown upward. $V_0 = ?$
 B) The speed with which the stone strikes the water. $V_f = ?$



$$t = 4s$$

$$a = -g = -9.81 \text{ m/s}^2$$

$$a) \quad x = x_0 + V_0 t + \frac{1}{2} a t^2$$

$$0 = 40 + V_0(4) + \frac{1}{2}(-9.81)(4)^2$$

$$V_0 = 9.62 \text{ m/s}$$

$$b) \quad V_f = V_0 + a t$$

$$= 9.62 + (-9.81)(4)$$

$$= -29.6 \text{ m/s}$$

$$= 29.6 \text{ m/s} \downarrow$$