Work and Energy (Particles)

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Principle of Work & Energy

$$T_1 + \sum U_{1-2} = T_2$$

$\sum U_{1-2}$ = work done on the system

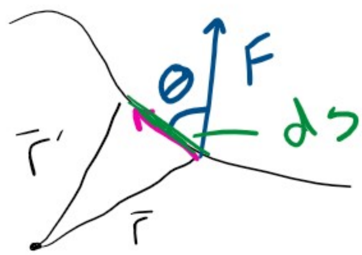
T = Kinetic energy $\frac{1}{2}mv^2$

system of particles

$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

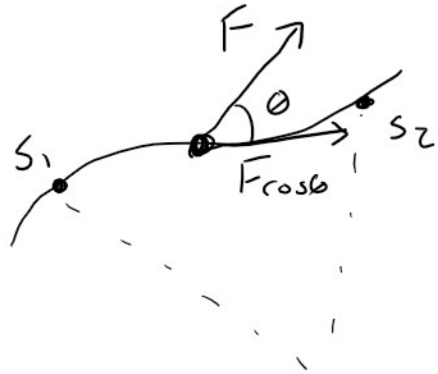
work – if a particle undergoes a displacement in the direction of a force → work done by a force

$$dU = F(ds)\cos\theta$$
$$= \bar{F} \cdot dr$$



Work of a variable force

★ $U_{1-2} = \int_{s_1}^{s_2} F\cos\theta \, ds$ ← not constant



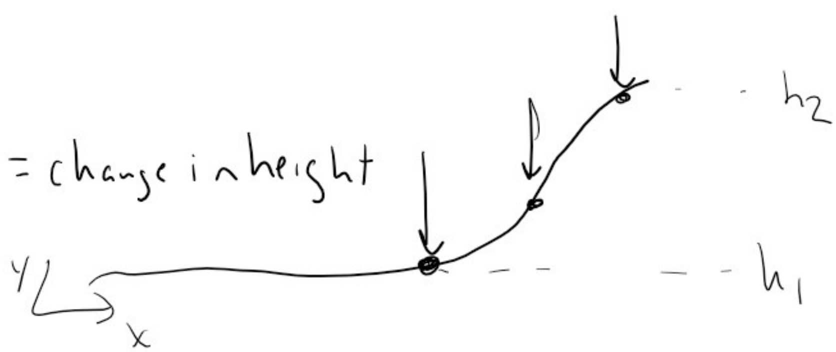
Work of a constant force

$$U_{1-2} = \int_{s_1}^{s_2} F\cos\theta \, ds = F\cos\theta \int_{s_1}^{s_2} ds = F\cos\theta\,(s_2 - s_1)$$

Work of Weight

$$U_{1-2} = -W\Delta h$$

Δh = change in height

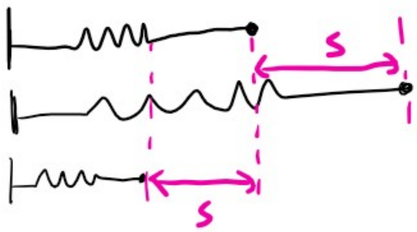


Work of a spring

$\rightarrow x$

Work of a spring

$F_s = Ks$



$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds$$

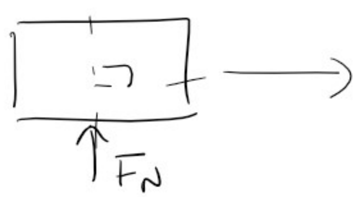
$$= \int_{s_1}^{s_2} Ks \, ds$$

$$= \frac{1}{2}Ks^2 \Big|_{s_1}^{s_2}$$

$$= \frac{1}{2}K{s_2}^2 - \frac{1}{2}K{s_1}^2$$

Work of Normal Force

Always 0



Power & Efficient

[W] [HP]

Watts

$$550 \frac{ft \cdot lb}{s} = 1 \, HP$$

$$Power = \frac{du}{dt}$$ work over a given time

$$P = F \cdot V = F v \cos\theta$$

$$\varepsilon = \frac{\text{output}}{\text{input}}$$

$$= \frac{\text{energy out}}{\text{energy in}} \quad \text{or} \quad \frac{\text{power out}}{\text{power in}}$$

e is always less than 1

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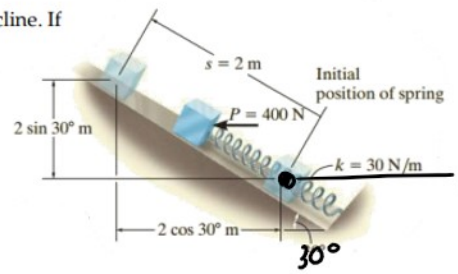
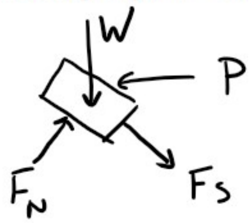
Example 1: The 10 kg block rests on the smooth incline. If the spring is originally stretched 0.5m, determine the total work done by all forces acting on the block when a horizontal force P = 400N pushes the block up the plane s = 2m



(x=0, y=0)



$$\sum U_{T1-2} = U_{CF} + U_W + \cancel{U_{FN}} + U_{FS}$$
$$= 692.8 + (-98.1) + 0 + (-90) = 505\,J$$

$$U_{FN} = 0$$

$$U_{CF} = \overset{[N]}{F}\cos\theta\,\overset{[M]}{(s_2 - s_1)}$$
$$= 400\cos\theta(2-0)$$
$$= 692.8\,J$$

Unit of Work is a Joule (J)

[N·m]

$$U_W = -W\Delta h$$
$$= -(9.81)(10)(2\sin 30)$$
$$= -98.1\,J$$

$$U_{FS} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$
$$= -\left(\frac{1}{2}(30)(2.5)^2 - \frac{1}{2}(30)(0.5)^2\right) = -90\,J$$

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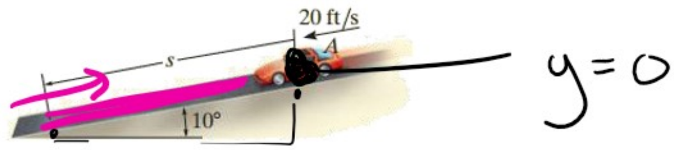
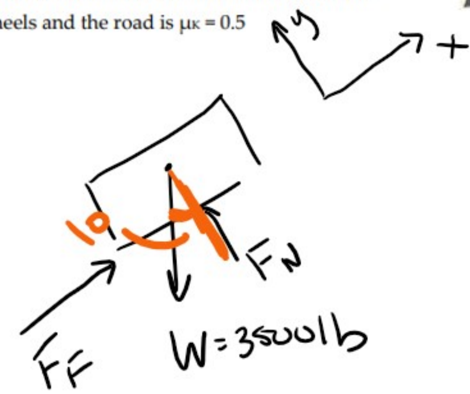
3500

Example 2: The [redacted] lb automobile travels down the 10 ° inclined road at a speed of 20ft/s. If the drivers jams on the brakes causing his wheels to lock, determine how far the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_K = 0.5$





$F_F = \mu_K F_N = (0.5)(3446.8)$

$= 1723.4 \text{ lb}$

$\Sigma F_y = 0$

$F_N - 3500\cos 10$

$F_N = 3446.8 \text{ lb}$

$\frac{1}{2}mv^2$

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}\left(\frac{3500}{32.2}\right)(20)^2 + 607s - 1723.4s = 0$

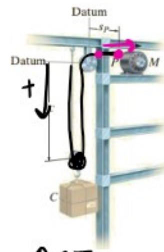
$s = 19.5 \text{ ft}$

$\Sigma U_{12} = U_W + U_{FF}$ model as constant force

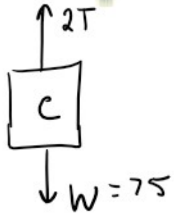
$U_W = -3500(-s \cdot \sin 10) = 607.8\,s$

$U_{FF} = -1723.4\cos(0)(s) = -1723.4s$

Example 4: The motor M of the hoist shown lifts the 75 lb crate C so that the acceleration of point P is 4 ft/s². Determine the power that must be supplied to the motor at the instant P has a velocity of 2 ft/s. Neglect the mass of the pulley and the cable and take ε = 0.85



$$l = S_C + S_C + S_P$$
$$= 2S_C + S_P$$
$$0 = 2V_C + V_P$$
$$0 = 2A_C + A_P$$
$$A_C = \frac{-A_P}{2}$$
$$= \frac{-4}{2} = -2\ \text{ft/s}^2$$

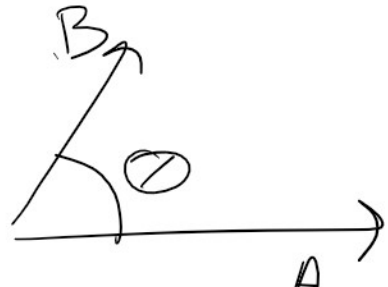


$$+\downarrow \Sigma F_y = ma_y$$
$$W - 2T = \left(\frac{75}{32.2}\right)(a_c)$$
$$75 - 2T = \left(\frac{75}{32.2}\right)(-2)$$
$$T = 39.83\ \text{lb}$$

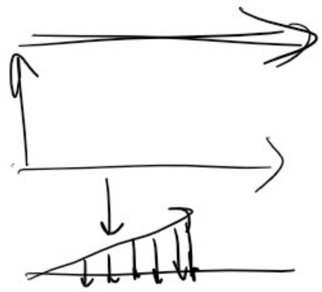
$$\varepsilon = \frac{\text{power out}}{\text{power in}}$$

$$P = T \cdot V$$
$$= (39.83)(2) = \left(76.66\ \frac{\text{lbft}}{\text{s}}\right)\left(\frac{1\ \text{HP}}{550\ \frac{\text{lbft}}{\text{s}}}\right)$$
$$= .1448\ \text{H}$$

$$\frac{0.1448}{.85} = .170\ \text{HP}$$



$$A_x B_x + A_y B_y + A_z B_z = A \cdot B$$
$$A \cdot B = |A||B| \cos \theta$$



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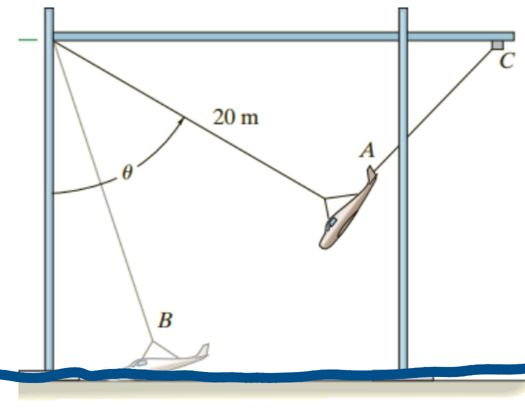
Example 1: The gantry structure in the photo is used to test the response of an airplane during a crash. As shown, the plane having a mass of 8 Mg is hoisted back until θ= 60° and then the pull back cable AC is released when the plane is at rest. Determine the speed of the plane just before it crashes into the ground θ= 15°. Also, what is the maximum tension developed in the supporting cable during the motion. Treat the airplane as a particle and neglect the effect of lift by the wings.



0

$$T_1 + V_1 = T_2 + V_2 \; : \; V = V_G + V_E$$

Gravitational

mgh

Elastic

$\frac{1}{2}ks^2$

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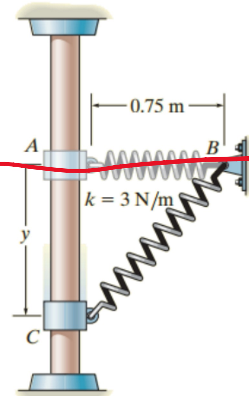
Example 2: A smooth 2kg color fits loosely on the vertical shaft. If the spring is unstretched when the collar is in position A, determine the speed at which the collar is moving when y=1m if a) it is released from rest at A, and b) it is released at A with an upward velocity $V_A = 2$ m/s



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a) $\cancel{T_A}^{0} + \cancel{V_A}^{0} = T_c + V_c$

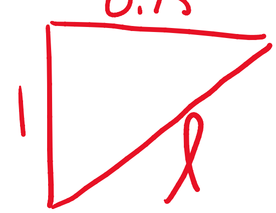
$$0 = \frac{1}{2}(2)v^2 + (-mgh) + \frac{1}{2}ks^2$$

$$0 = \frac{1}{2}(2)v^2 + (-2)(9.81)(1) + \frac{1}{2}(3)(\underset{1.25-0.75}{0.5})^2$$

$V_A = V_G + V_s$

$v = 4.39\ m/s$

$V_c = V_G + V_s$



$l = \sqrt{1^2 + 0.75^2} = 1.25$

$$\frac{1}{2}m\underset{2}{V_A}^2 + 0 = \frac{1}{2}mV_c^2 - mgh + \frac{1}{2}ks^2$$

$V_c = 4.82\ m/s$