Moment of Inertia (mass)

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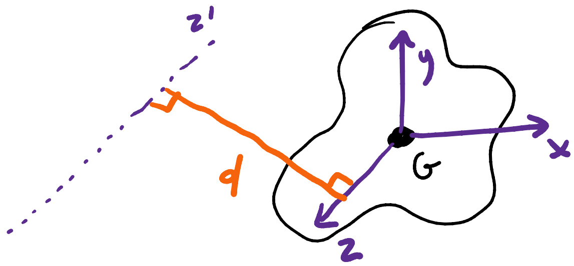
$\Sigma \bar{F} = m\bar{a}$

$\Sigma \bar{M} = I\bar{\alpha}$

We can figure out MoI about any point

Parallel-axis theorem



Table

$I_{z'} = I_G + md^2$

mass

distance

$\rho = \dfrac{\text{mass}}{\text{Volume}} = \dfrac{[Kg]}{[m^3]}$

density

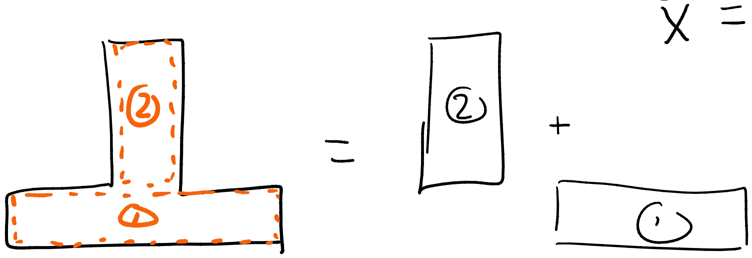
K

radius of gyration

$I = mK^2$

$K = \sqrt{\dfrac{I}{m}}$

$\bar{X} = \dfrac{\Sigma \tilde{X} m}{\Sigma m}$ ← coordinate of each particle



$I = \Sigma(I_G + md^2)$

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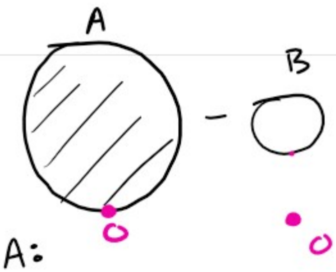
$\pi r^2 h$



A:

$I_{O_A} = I_G + md^2$

$= \frac{1}{2} mr^2 + md^2$

$= \frac{1}{2}(15.71)(0.25)^2 + (15.71)(0.25)^2$

$= 1.473 \text{ Kg m}^2$

B: $I_{O_B} = I_G + md^2$

$= \frac{1}{2} mr^2 + md^2$

$= \frac{1}{2}(3.93)(0.125)^2 + (3.93)(0.25)^2$

$= 0.276 \text{ Kg m}^2$

$I_O = I_G + md^2$

$\rho = \frac{m}{V}$

$m_A = \left(8000 \frac{Kg}{m^3}\right)\left(\pi (0.25m)^2 (0.01m)\right)$

$= 15.71 \text{ Kg}$

$m_B = (8000)\left(\pi (0.125)^2 (0.01)\right)$

$= 3.93 \text{ Kg}$

$I_O = I_{OA} - I_{OB}$

$= 1.473 - 0.276 = 1.197 \text{ Kgm}^2$

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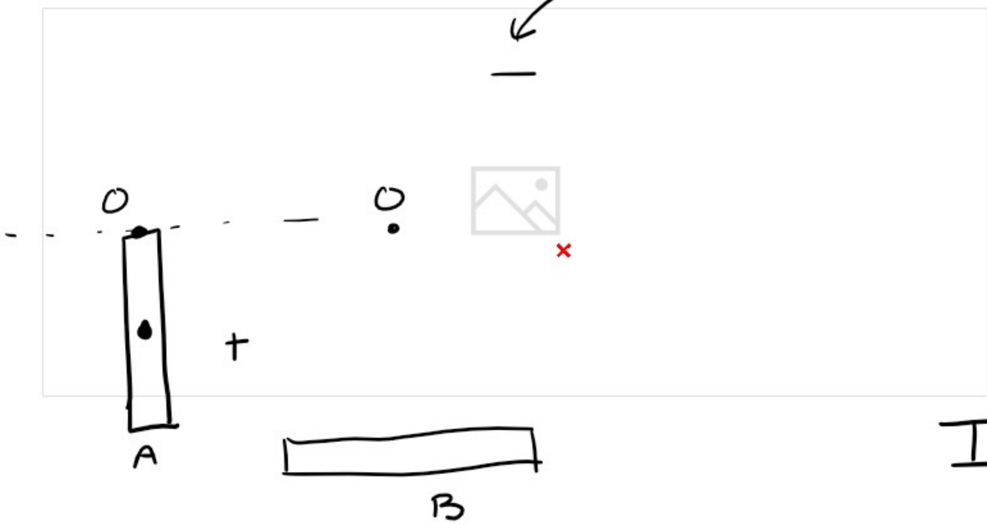
thickness is negligible



$$I_G = \frac{1}{12} m l^2$$

$$I_{OA} = \frac{1}{12} m L^2 + m d^2$$

$$\frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2 + \left(\frac{10}{32.2}\right)(1)^2 = 0.414 \text{ slugft}^2$$

$$I_{OB} = \frac{1}{12} m L^2 + m d^2$$

$$= \frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2 + \left(\frac{10}{32.2}\right)(2)^2 = 1.346 \text{ slugft}^2$$

$$I_O = I_{OA} + I_{OB} = 0.414 + 1.346 = 1.76 \text{ slug}\cdot\text{ft}^2$$

$$\bar{Y} = \frac{\sum \tilde{y} m}{\sum m} = \frac{(1)\left(\frac{10}{32.2}\right) + (2)\left(\frac{10}{32.2}\right)}{\frac{10}{32.2} + \frac{10}{32.2}} = 1.5 \text{ ft}$$

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$$K=\sqrt{\frac{I_0}{m}}$$

$$I_0 = I_A + I_B + I_C$$

$$I_A:\ I_G + md^2$$
$$\frac{1}{2}mr^2 + md^2$$
$$\frac{1}{2}(8)(0.2)^2 + (8)(1.2)^2 = 11.68$$

$$I_B:\ \frac{1}{2}mr^2 + md^2$$
$$\frac{1}{2}(2)(0.1)^2 + (2)(0.6)^2 = 0.73$$

$$I_C:\ I_G + md^2$$
$$\frac{1}{12}ml^2 + md^2$$
$$\frac{1}{12}(4)(1.5) + (4)(0.25)^2 = 1$$

$$I_0 = 11.68 + 0.73 + 1$$
$$= 13.41\,Kgm^2$$

$$K = \sqrt{\frac{13.41\,\cancel{Kg}m^{\cancel{2}}}{14\,\cancel{Kg}}} \leftarrow 2+4+8$$

$$\boxed{K = 0.979m}$$

$$I = mK^2$$
$$=$$