Optimization: Timetable Scheduler Modelling

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December 2022

1 Problem

N classes: 1, 2, ..., N needs to be assigned.
Each class i has t(i): number of periods needed, g(i): teacher, s(i): number of students.
M rooms: 1, 2, ..., M available. Each room i has c(i) number of seats.
There are 5 scholar days a week, from Monday to Friday, each day is divided into 12 periods. (Morning: 6 periods/ Afternoon: 6 periods).

Create a timetable for those classes satisfying the following constraints:

1. Classes with the same teacher can not be on the same period.
2. Number of studens needs to be smaller than the number of seats.

2 Solution

I will divide 5 days - 60 periods into 10 blocks, each block contains 6 periods.

1. I define a boolean variable Assign(i, m, k, b) for whether "assign class i in room b in period b of block k" or not
2. A class is assigned to a room if and only if there are enough seats for students of the class in that room.
3. All of classes can not be assigned in 2 separated blocks.
4. Teacher can not be at the different rooms or classes simultaneously, so sum of Assign() for him/her ¡= 1 at each period. Also, the total period taught by him/her in each block cannot exceed max periods (6).
5. A class i cannot change room during its section, then the total periods taken by it can only be 0 (not assigned) or t(i) (is assigned)

6. A class i has to be assigned once in a whole week then total periods taken by it for whole week = t(i)
7. One room cannot contain 2 class at the same period, then when a class i is assigned to room j, the next t(i) - 1 periods of room j need to be blocked.

With supports of Google OR-Tools, we can solve this problem !!!

3 Mathematical Modelling

3.1 Decision variable

Let $X(i, m, k, b) \in \{0, 1\}$ a boolean decision variable for the option: Assign class (i) to room (m) in period (b) of block (k)

$$1 \leq i \leq N$$
$$1 \leq m \leq M$$
$$1 \leq k \leq 10$$
$$1 \leq b \leq 6$$

3.2 Constraints

We will formulate the mathematical equality or inequality based on the constraints mentioned above.

3.2.1 The chosen room has enough seats for students

$$X(i, m, k, b) = 0 \Leftarrow s(i) \geq c(m) \ \forall i \leq N \ \forall m \leq M \ \forall k \leq 10 \ \forall b \leq 6$$

Where:

- s(i): Number of students in class i
- c(m): Number of seats in room m

3.2.2 All of the classes can not be assigned in 2 separated blocks

On each block, if a class (i) is assigned, then it is assigned into at least t(i) periods. Then what we need to do is limit the periods a class can take in a block to t(i) when it is assigned and 0 when not being assigned.

$$\sum_{m=1}^{M} \sum_{b=1}^{6} X(i, m, k, b) \in \{t(i), 0\} \ \forall i \in [\ 1, N\ ] \ \forall k \leq 10$$

Where: t(i) is the periods needed of chosen class (i).

3.2.3 Teacher can not be at the different rooms or classes simultaneously

Let G set of all given teachers. The above constraint can be divided into two following conditions:

1. Each teacher can only teach one class in each period

$$\sum_{m=1}^{M} \sum_{\substack{i=1 \\ g(i)=g}}^{N} X(i,m,k,b) \leq 1 \ \forall g \in G \ \forall b \leq 6 \ \forall k \leq 10$$

   Where: g(i) is the teacher of class i

2. Total periods taught by him/her in each block cannot exceed max periods (6 in this problem)

$$\sum_{m=1}^{M} \sum_{\substack{i=1 \\ g(i)=g}}^{N} \sum_{b=1}^{6} X(i,m,k,b) \leq 6 \ \forall g \in G \ \forall k \leq 10$$

3.2.4 A class i cannot change room during its section

When a class is assigned to a room in some periods of block, it is necessary to ensure that they will attached each other in exactly t(i) periods. With the constraint *All classes can not be assigned in 2 separated blocks*, the total periods of a class in a block and in a room assigned equals to exactly t(i), which leads to impossibility for the class to appear in 2 different rooms or blocks simultaneously.

$$\sum_{k=1}^{10} \sum_{b=1}^{6} X(i,m,k,b) \in \{t(i), 0\} \ \forall m \leq M \ \forall i \leq N$$

Where: t(i) is the periods needed of chosen class (i).

3.2.5 Each class has to be assigned once in whole week

In order to satisfy this condition, the total periods assigned for the given class should be exactly equal to the number of periods it needs.

$$\sum_{k=1}^{10} \sum_{b=1}^{6} \sum_{m=1}^{M} X(i,m,k,b) = t(i) \ \forall i \leq N$$

Where: t(i) is the periods needed of chosen class (i).

3.2.6 One room contains only one class at each period

It could be specified by two following sub-constraints:

1. At every period, there must be less than 1 class assigned to the given room

$$\sum_{i=1}^{N} X(i,m,k,b) \leq 1 \ \forall k \leq 10 \ \forall m \leq M \ \forall b \leq 6$$

2. When assigning class (i) in the given room to a period, the next t(i)-1 periods need to be blocked.

$$\sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{\substack{b'=b \\ X(i,m,k,b)=1}}^{b+t(i)-1} X(i,m,k,b') = X(i,m,k,b) \times t(i) \ \forall k \leq 10 \ \forall b \leq 6$$

THE END