3. Probability Problems

You must show work supporting your answer to receive credit.

Place your answers in a file `probability.pdf`.

(1) 1. A professor has 15 students and during lecture will (uniformly) at random choose a student to answer a question. The professor asks 8 questions during the lecture. What is the probability no student will have to answer more than one question?

15^8 = total outcomes →

no student would have to answer more than 1 question

↳ means 8 / 15 students ask 1 question each

$15P_8 = \frac{15!}{(15-8)!} = \frac{15!}{7!}$

$\therefore \frac{\frac{15!}{7!}}{15^8}$ = no student answer more than 1 question

$= 0.101236 = \boxed{10.124\%}$

$\left(\frac{14}{15}\right)\left(\frac{13}{15}\right)\left(\frac{12}{15}\right)\left(\frac{11}{15}\right)\left(\frac{10}{15}\right)\left(\frac{9}{15}\right)\left(\frac{8}{15}\right) = 10.124\%$

(2) 2. An integer from the range 00000 - 99999 is generated uniformly at random. We are interested only in even integers that start with 2 odd digits where all digits are unique. If we randomly generate 8 of these numbers in succession, what is the probability we get exactly 5 numbers that meet our criteria?

random #s $(100{,}000)^8$ → 8 random #s

1st digit = 5 ways to choose

2nd digit = 4 ways to choose

3rd digit = 7 ways to choose

4th digit = 6 ways to choose

5th digit = 5 ways to choose

(5)(4)(7)(6)(5) → desired #

$\left(\frac{5\cdot4\cdot7\cdot6\cdot5}{100{,}000}\right)^5$ = meets criteria

$\left(\frac{5\cdot4\cdot7\cdot6\cdot5}{100{,}000}\right)^5\left(1-\left(\frac{5\cdot4\cdot7\cdot6\cdot5}{100{,}000}\right)\right)^3\binom{8}{5}$

$= \boxed{6.345\times10^{-6}}$

↑ want 5 exact #s

$\frac{5}{8}$ → selecting 5/8 to meet criteria =

(3) 3. You roll 3 six-sided, fair dice. Let A be the event that at least 2 dice show 4 or above. Let B be the event that all 3 dice show the same value. Are A and B independent?

six sided fair dice

A = 2 dice 4 & above

B = 3 dice = same value

$P(A_1) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3}{8}$

$P(A_2) = \binom{3}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = \frac{1}{8}$

$P(A_1)+P(A_2) = \frac{4}{8} = \frac{1}{2}$ $P(A) = \frac{1}{2}$

$P(B) = \frac{6}{6}\cdot\underbrace{\frac{1}{6}\cdot\frac{1}{6}}_{\text{other 2 dice (same)}} = \frac{1}{36}$

$P(A)\cdot P(B) = \frac{1}{2}\left(\frac{1}{36}\right) = \frac{1}{72}$

$P(A\cap B)$ = all 3 dices show same values $= \frac{1}{6^3}+\frac{1}{6^3}+\frac{1}{6^3} = \frac{3}{6^3} = \frac{3}{6\cdot6\cdot6} = \frac{1}{2(36)} = \frac{1}{72}$

$P(A)\cdot P(B) = P(A\cap B)$ ∴ A and B are independent

④

1. In poker, a flush is any 5-card hand where all the cards of the same suit. For this problem we will not distinguish between an ordinary flush and special flushes (like straight and royal flushes), meaning we will call any hand that has all 5 cards from the same suit a flush. Poker-player Paul loves a flush. What is the expected number of hands of poker he has to play to get a flush. (We assume each hand is dealt from a new deck containing of randomly ordered cards).

Suit: 4 ways → 4 suits

to construct a flush from given suit $= \binom{13}{5} \to 13$ in each suit

to deal a 5 card hand $\binom{52}{5} \to 52$ cards in deck
total outcome

calculating flush in any situation

$$P(\text{flush}) = \frac{4\binom{13}{5}}{\binom{52}{5}} = 0.001981 = p$$

so geometric

geometric seria $(\frac{1}{p})$ / distribution =

$p = 0.001981$

$\frac{1}{p}$ = expected value of a geometric variable (random)

expected # of hands to get a flush $= \frac{1}{p} = \frac{1}{0.001981} = \boxed{504.85}$

⑤

2. A basketball team has a superstar. When their superstar plays, they win 70% of the time. When their superstar does not play they win 50% of the time. Entering a 5 game stretch, the superstar had been recovering from an injury and said the chance they would play the next 5 games was 75%. You go on a trip to the jungle (no internet access). When you return you find out the team won 4 of the 5 games. What is the probability the superstar played those 5 games? You may assume the superstar doesn't get injured during those games (either they play all or none of the 5).

S = superstar plays
W = wins
C = team won 4/5 games

$P(S) = 75\% = 0.75$
$P(W|S) = 70\% = 0.7$, superstar plays
$P(W|\bar{S}) = 50\% = 0.5$, superstar doesn't play

① superstar plays
$P(C|S) = \binom{5}{1} 0.70^4 \times (1-0.7) = 5 \times 0.7^4 \times 0.3 = 0.36015$

② superstar doesn't play
$P(C|\bar{S}) = \binom{5}{1} 0.5^4 \times (1-0.5) = 5 \times 0.5^5 = 0.15625$

total probability: $P(S) \cdot P(C|S) + P(\bar{S}) \cdot P(C|\bar{S})$
$= 0.75(0.36015) + (0.25) \cdot (0.15625)$
$= 0.2701125 + 0.0390625$
$= 0.309175$

BAYES theorem: $P(S|C) = \frac{P(C|S) \cdot P(S)}{P(C)}$

$$= \frac{0.36015 \cdot 0.75}{0.309175} = \frac{0.2701125}{0.309175} = 0.873655$$

$\boxed{87.37\%}$