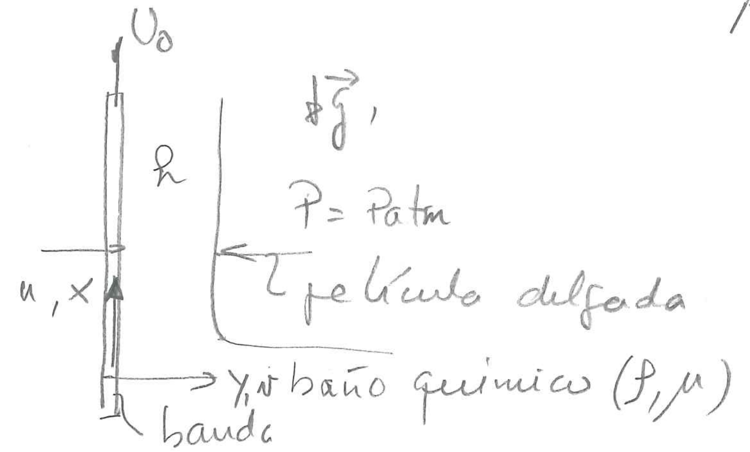
1



Asumiendo: un proceso estacionario, flujo completamente desarrollado, laminar, gradiente de presión nulo, y tensión de corte nula en la superficie externa de la película

Determine una expresión para la velocidad del flujo

Solución:

i) asumiendo flujo incompresible de un fluido Newtoniano, problema bidireccional y bidimensional, las ecuaciones de N-S son

a) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

b) dir-x: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\rho g_x}{\rho}$$

dir-y $$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{\rho g_y}{\rho}$$

2i) De continuidad con flujo desarrollado $v = v(x)$ ó $v = ct$

C.B. en $y = 0$, $v = 0 \Rightarrow ct = 0$

flujo desarrollado, $\frac{\partial u}{\partial x} = 0 \Rightarrow u = u(y)$

3i) de dir-x : $$0 = \mu\frac{d^2 u}{dy^2} - \rho g$$

de dir-y : $$0 = 0$$

4i) De (3i), $$\frac{du}{dy} = \frac{\rho g}{\mu}y + A$$

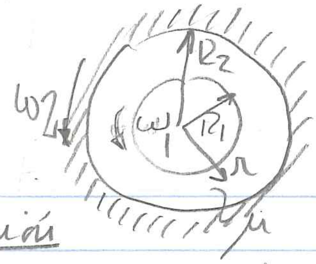
$$u(y) = \frac{\rho g}{\mu}\frac{y^2}{2} + Ay + B$$

5i) C.B. $U(y=0) = U_0 = B$

$$\tau\Big|_{y=h} = \mu\frac{du}{dy}\Big|_{y=h} = 0 = \frac{\rho g h}{\mu} + A \rightarrow A = -\frac{\rho g h}{\mu}$$

6i) Por lo tanto, $$\boxed{u(y) = U_0 + \frac{\rho g}{2\mu}(y^2 - 2hy)}$$

#2



Se pide $\vec{V}=(V_r, V_\theta, V_z)$; fluido - aceite a 20°C

solución

Ecuaciones gobernantes en coordenadas cilíndricas

1) Continuidad

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{\partial V_z}{\partial z}=0$$

Cilindro muy largo $\Rightarrow \frac{\partial (\ )}{\partial z}\equiv 0$, y $V_z=0$

Simetría rotacional $\Rightarrow \frac{\partial (\ )}{\partial \theta}\equiv 0$

Luego en (1) $rV_r = cte \Rightarrow V_r = 0$

$V_\theta = f(r)$

2) Navier-stokes: fuerzas másicas igual a cero

z-direc.: $-\frac{\partial p}{\partial z}=0$ (i)

θ-direc: $0=\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)\right)$ (2i)

r-direc.: $-\frac{\rho V_\theta^2}{r}=-\frac{\partial p}{\partial r}$ (3i)

de (2i): $V_\theta = V_\theta(r) = c_1 r + \frac{c_2}{r}$

Condiciones de borde: $\begin{cases} (a)\ V_\theta(r=R_1)=\omega_1 R_1 \\ (b)\ V_\theta(r=R_2)=\omega_2 R_2 \end{cases}$

$$\Rightarrow c_1 = 2\frac{(\omega_2 r_2^2-\omega_1 r_1^2)}{r_2^2-r_1^2};\quad c_2 = r_1 r_2\frac{(\omega_2 r_2 r_1-\omega_1 r_1 r_2)}{r_1^2-r_2^2}$$

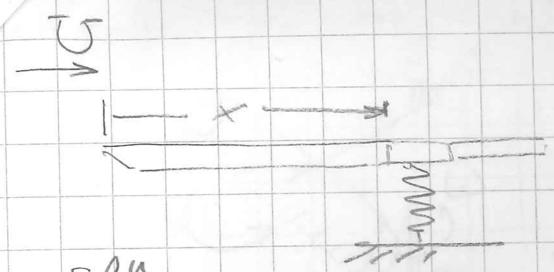
Así, $\vec{V}=(0, V_\theta, 0)$

a) $$V_\theta = \frac{(\omega_2 r_2^2-\omega_1 r_1^2)r}{r_2^2-r_1^2}-\frac{1}{r}\frac{(r_1 r_2)^2(\omega_2-\omega_1)}{(r_2^2-r_1^2)}$$

b) $\omega_2 = -\omega_2$

$$V_\theta = -\frac{(\omega_1 r_1^2+\omega_2 r_2^2)r}{r_2^2-r_1^2}+\frac{1}{r}\frac{(r_1 r_2)^2(\omega_1+\omega_2)}{(r_2^2-r_1^2)}$$ #

Problema 3(a)   Aire a 20°C y $P = 1$ Atm.
$x = 2m$
$\tau_w = 2.1$ Pa
a) Determinar U: veloc. corriente libre
- flujo turbulento
- fluido Newtoniano y flujo incomp.



Sol^r:
(a) $\tau_w = C_{fx,t} \frac{1}{2} \rho U^2$; donde $C_{fx,t} = \frac{0.0577}{Re_x^{1/5}}$; $Re_x = \frac{U x}{\nu}$
0.1   0.1   0.1

$$\tau_w = \frac{0.0577}{\left(\frac{U x}{\nu}\right)^{1/5}} \cdot \frac{1}{2}\rho U^2 = \frac{0.02885\,\rho\,U^{9/5}}{(x/\nu)^{1/5}}$$

$$\Rightarrow U^{9/5} = \frac{\tau_w \cdot (x/\nu)^{1/5}}{0.02885\,\rho}$$

datos: $\rho = 1.2$ kg/m^3, $\nu = 1.49\times10^{-5}$ m^2/s, $\tau_w = 2.1$ N/m^2, $x = 2m$

$$U^{9/5} = \frac{2.1 \times (2/1.49\times10^{-5})^{1/5}}{0.02885 \times 1.2} = 643.37$$

0.2 $U = 36.32$ m/s

(b) Espesor de capa límite δ @ $x = 2$: $\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}}$ 0.5

$$\delta(x) = \frac{0.37\,x}{(U \cdot x/\nu)^{1/5}} = \frac{0.37 \times 2}{(36.32\times2/1.49\times10^{-5})^{1/5}} = 0.034\,m = 3.4\,cm.$$

(c) Velocidad u @ $Y = 5.5$ cm

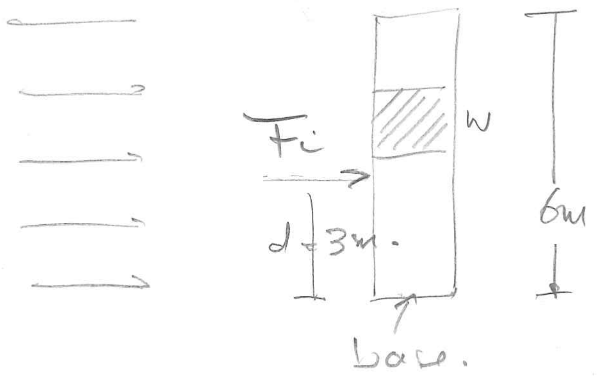
Debido a que $Y = 5.5$ cm $> \delta(x)\big|_{x=2}$, $u(y) = U =$ m/s
0.5

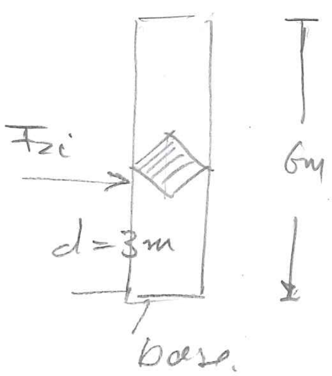
Se pide determinar cual pilar está más solicitado

#35

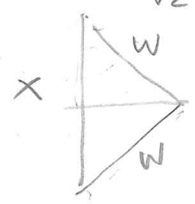
(i) (2i)

$U_0 = 1{,}5\ m/s$





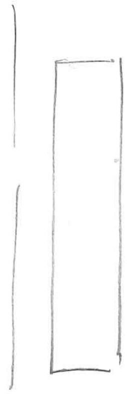
$X = \sqrt{w^2 + w^2} = \sqrt{2}\, w$



$F_i = C_{D_i} \frac{1}{2} \rho U_0^2 A_{t_i}$; $F_{2i} = C_{2i} \frac{1}{2} \rho U_0^2 A_{t_{2i}}$

$A_{t_i} = W \cdot L = 6W,\ m^2$

$C_{D_i} = 2{,}1.$



$A_{t_{2i}} = \sqrt{2}\, WL = 6\sqrt{2}\, W,\ m^2$

$C_{D_{2i}} = 1.6$

$F_i = 2{,}1 \times \frac{1}{2} \rho U_0^2 \times 6W$

$F_i = 6{,}3\ \rho U_0^2 W$

$F_{2i} = 1.6 \times \frac{1}{2} \rho U_0^2 \sqrt{2}\, 6W$

$F_{2i} = 6{,}788\ \rho U_0^2 W.$

$$\frac{F_{2i}}{F_i} = \frac{6{,}788}{6.3} = 1{,}077$$

$\Rightarrow$ El pilar (2i) está más solicitado ya que $F_{2i} = 1{,}077 F_i$ $(F_{2i} > F_i)$

$M_{F_i} = F_i \times d$

$M_{F_{2i}} = F_{2i} \times d. > M_{F_i}$