(1) $F(z) = Uz + \frac{q}{2\pi} \operatorname{Ln} z$

$$F(z) = Ure^{i\theta} + \frac{q}{2\pi} \operatorname{Ln}(re^{i\theta})$$

$$F(z) = Ur(\cos\theta + i\,\mathrm{sen}\,\theta) + \frac{q}{2\pi}(\operatorname{Ln} r + i\theta)$$

$$F(z) = \left(Ur\cos\theta + \frac{q}{2\pi}\operatorname{Ln} r\right) + i\left(Ur\,\mathrm{sen}\,\theta + \frac{q\,\theta}{2\pi}\right)$$

(a) $\Phi = Ur\cos\theta + \frac{q}{2\pi}\operatorname{Ln} r$ Función Potencial

$\Psi = Ur\,\mathrm{sen}\,\theta + \frac{q}{2\pi}\theta$ Función de Corriente

$$u_r = \frac{\partial \Phi}{\partial r} = U\cos\theta + \frac{q}{2\pi r}$$

$$u_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} = -U\,\mathrm{sen}\,\theta$$

(b) Puntos de Estancamiento:

$$\left.\begin{aligned} u_r &= U\cos\theta + \frac{q}{2\pi r} = 0 \\ u_\theta &= -U\,\mathrm{sen}\,\theta = 0 \end{aligned}\right\}$$

$u_\theta = 0$ para $\theta = 0$; $\theta = \pi$

si $\theta = 0 \Rightarrow U = -\frac{q}{2\pi r} \Rightarrow r = \frac{-q}{2\pi U}$ No puede haber $r < 0$

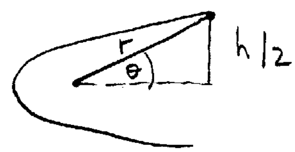
si $\theta = \pi \Rightarrow U = \frac{q}{2\pi r} \Rightarrow$ $\boxed{r = \frac{q}{2\pi U} \ ;\ \theta = \pi}$

reemplazando: $\Psi = \frac{q}{2\pi}(0) + \frac{q}{2\pi}(\pi) = \frac{q}{2}$

Existe un sólo punto de estancamiento, la línea de corriente $\Psi = q/2$ corresponde a la que define la superficie de la embarcación.

(c) Ancho máximo para la línea de corriente $\Psi = q/2$

$$\left.\begin{aligned} U r \operatorname{sen}\theta + \frac{q}{2\pi}\theta &= \frac{q}{2} \\ h &= 2 r \operatorname{sen}\theta \end{aligned}\right\}$$



$$r \operatorname{sen}\theta = \frac{h}{2}$$

$$h + \frac{q\theta}{U\pi} = \frac{q}{U} \Rightarrow h = \frac{q}{U}\left(1 - \frac{\theta}{\pi}\right) \Rightarrow \boxed{h_{max} = \frac{q}{U}}$$

(d) $$\cancelto{0}{\frac{P_\infty}{\gamma}} + \frac{U^2}{2g} = \frac{p}{\gamma} + \frac{V^2}{2g}$$

$$V^2 = \left(U\cos\theta + \frac{q}{2\pi r}\right)^2 + U^2 \operatorname{sen}^2\theta$$

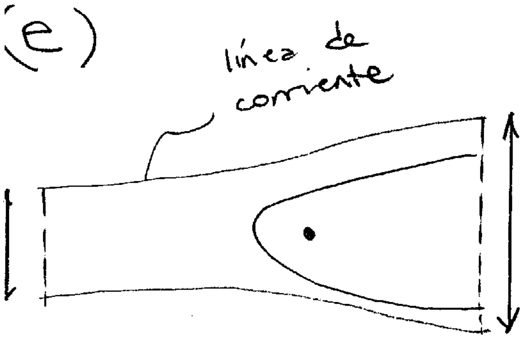
$$= U^2\cos^2\theta + \frac{U\cos\theta\, q}{\pi r} + \frac{q^2}{4\pi^2 r^2} + U^2 \operatorname{sen}^2\theta$$

$$= U^2 + \frac{Uq}{\pi r}\cos\theta + \frac{q^2}{4\pi^2 r^2}$$; reemplazando :

$$\frac{p}{\gamma} = \frac{-1}{2g}\left(\frac{Uq}{\pi r}\cos\theta + \frac{q^2}{4\pi^2 r^2}\right)$$

$$p = \frac{-\rho q}{2\pi r}\left(U\cos\theta + \frac{q}{4\pi r}\right)$$

(e)

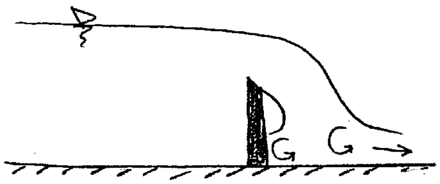


$$F + P_\infty a + P_\infty (a + h_{max}) = \rho U^2 (a + h_{max}) - \rho U^2 a$$

si $P_\infty = 0$ y el ancho es igual a 1

$$F = \rho U^2 h_{max}$$

② (2A)



(a) – Régimen permanente
– Fluido ideal e incompresible
– Análisis a lo largo de una línea de corriente

$$\cancel{z_1} + h_1 + \frac{V_1^2}{2g} = z_2 + h_2 + \frac{V_2^2}{2g} \quad ; \quad Q = V_1 h_1 = V_2 h_2$$

$$h_1 + \frac{V_1^2}{2g} = a + h_2 + \frac{V_1^2}{2g}\frac{h_1^2}{h_2^2}$$

$$Q^2\left(\frac{1}{2gh_1^2} - \frac{1}{2gh_2^2}\right) = a + h_2 - h_1$$

(b) Si $V_1 \ll V_2$ $\qquad h_1 = a + h_2 + \frac{V_2^2}{2g}$

$$h_1 - a - h_2 = \frac{Q^2}{2gh_2^2}$$

$$Q = \sqrt{2gh_2^2(h_1 - a - h_2)}$$

(c) Las suposiciones entregarán un valor de Q distinto al real, debido a la distribución de velocidad que no es uniforme, y a la posible contracción del flujo

$$Q = C_v\, C_c \sqrt{2gh_2^2(h_1 - a - h_2)}$$

coeficientes de velocidad y contracción pueden corregir esta predicción.

(2B) $$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$z_2 - z_1 = L\,\mathrm{sen}\,\theta$$

$$V_1 A_1 = V_2 A_2$$

Del manómetro $P_1 + \gamma(h+a) = P_2 + \gamma(h - L\,\mathrm{sen}\,\theta) + \gamma_m a$

$$\frac{P_1}{\gamma} + \cancel{h} + a = \frac{P_2}{\gamma} + \cancel{h} - L\,\mathrm{sen}\,\theta + \frac{\gamma_m}{\gamma} a$$

reemplazando:

$$\cancel{\frac{P_2}{\gamma}} - L\,\mathrm{sen}\,\theta + a\left(\frac{\gamma_m}{\gamma} - 1\right) = L\,\mathrm{sen}\,\theta + \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$a\left(\frac{\gamma_m}{\gamma} - 1\right) = 2L\,\mathrm{sen}\,\theta + \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

$$a = \frac{1}{\left(\frac{\gamma_m}{\gamma} - 1\right)}\left[2L\,\mathrm{sen}\,\theta + \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)\right]$$

③ $V = \dfrac{V_0 (R+h)}{r}$

(3A) Ecuación de Euler en la dirección $\hat{r}$:

$$-\frac{V^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - g\cos\theta$$

reemplazando V, e integrando:

$$+\frac{V_0^2 (R+h)^2}{r^3} - g\cos\theta = \frac{1}{\rho}\frac{\partial P}{\partial r}$$

$$\rho V_0^2 (R+h)^2 \int_r^{R+h} \frac{dr}{r^3} - \rho g\cos\theta\,(R+h-r) = \int_P^0 dP$$

$$-\rho V_0^2 (R+h)^2 \left.\frac{1}{2r^2}\right|_r^{R+h} - \rho g\cos\theta\,(R+h-r) = 0 - P$$

$$P = \frac{\rho V_0^2}{2}\left[1 - \frac{(R+h)^2}{r^2}\right] + \rho g\cos\theta\,(R+h-r)$$

(3B) Cierre de la válvula:

$$-\frac{\partial u}{\partial t} = \frac{\partial}{\partial s}\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) \qquad \text{Integramos}$$

$$-\frac{L}{g}\frac{\partial u}{\partial t} = \cancel{(z_2 - z_1)}^{\,0} + \frac{\cancel{P_2} - P_1}{\gamma}^{\,0} + \frac{V_2^2 - \cancel{V_1^2}}{2g}$$

$$-\frac{L}{g}\frac{\partial u}{\partial t} = \frac{V^2}{2g} - H$$

$$\frac{\partial u}{\partial t} = -\frac{V^2}{2L} + \frac{gH}{L}$$

$$\int_{V_0}^{V} du = -\frac{V_0^2}{2L}\int_0^t (1-t)^2\,dt + \frac{gH}{L}t$$

$$V - V_0 = \frac{V_0^2}{6L}\left[1-(1-t)^3\right] + \frac{gH}{L}t$$