1. A function f has an **absolute minimum** at $x = c$ if $f(c)$ is the smallest function value on the entire domain of f, whereas f has a **local minimum** at c if $f(c)$ is the smallest function value when x is near c.

2. (a) The Extreme Value Theorem

   (b) See the Closed Interval Method.

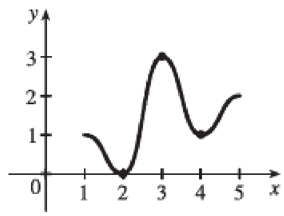
3. Absolute maximum at s, absolute minimum at r, local maximum at c, local minima at b and r, neither a maximum nor a minimum at a and d.

4. Absolute maximum at r; absolute minimum at a; local maxima at b and r; local minimum at d; neither a maximum nor a minimum at c and s.

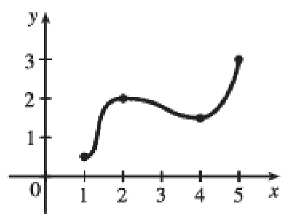
5. Absolute maximum value is $f(4) = 5$; there is no absolute minimum value; local maximum values are $f(4) = 5$ and $f(6) = 4$; local minimum values are $f(2) = 2$ and $f(1) = f(5) = 3$.

6. There is no absolute maximum value; absolute minimum value is $g(4) = 1$; local maximum values are $g(3) = 4$ and $g(6) = 3$; local minimum values are $g(2) = 2$ and $g(4) = 1$.

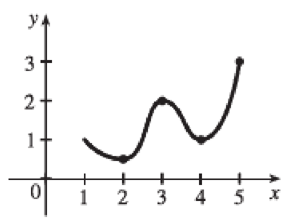
7. Absolute minimum at 2, absolute maximum at 3,
   local minimum at 4



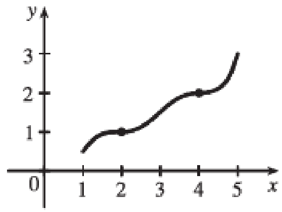
8. Absolute minimum at 1, absolute maximum at 5,
   local maximum at 2, local minimum at 4



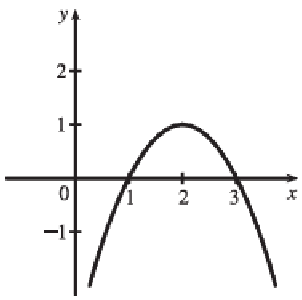
9. Absolute maximum at 5, absolute minimum at 2,
   local maximum at 3, local minima at 2 and 4



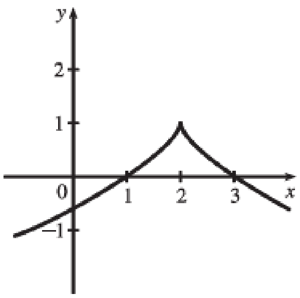
10. f has no local maximum or minimum, but 2 and 4 are critical numbers



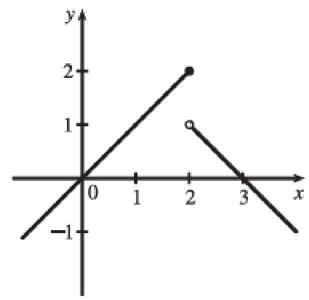
11. (a)



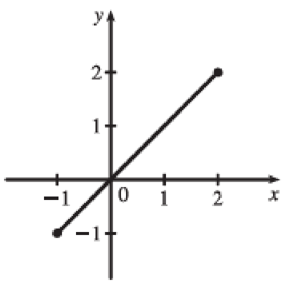
(b)



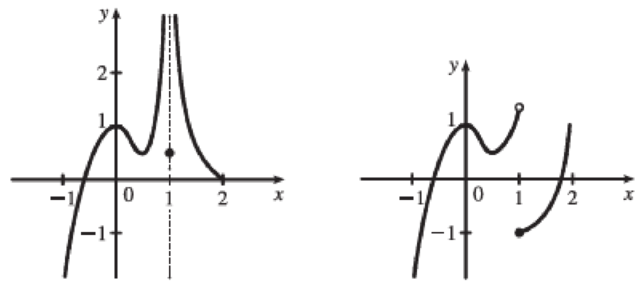
(c)



12. (a) Note that a local maximum cannot occur at an endpoint.

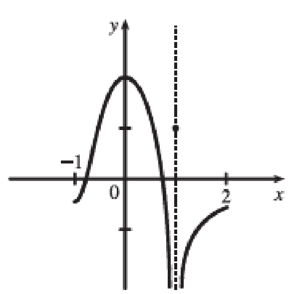


(b)

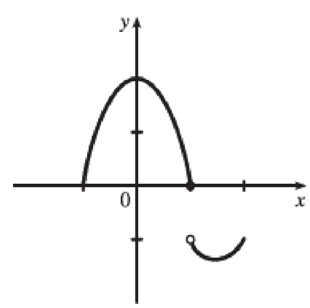


Note: By the Extreme Value Theorem, f must *not* be continuous.

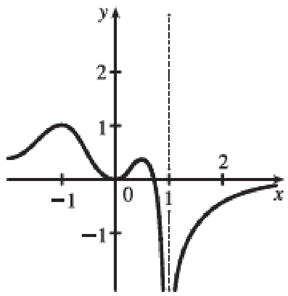
13. (a) *Note:* By the Extreme Value Theorem, f must *not* be continuous; because if it were, it would attain an absolute minimum.



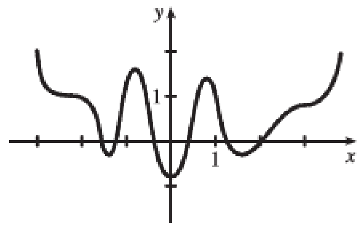
(b)



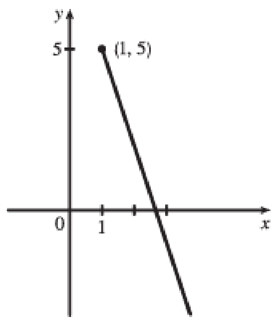
14. (a)



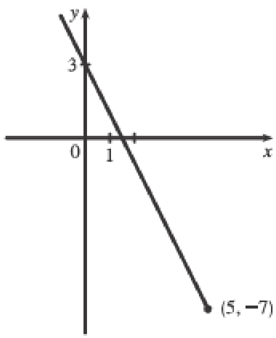
(b)



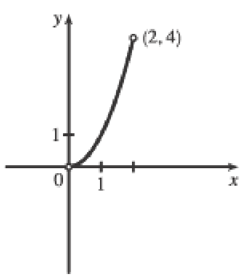
15. $f(x) = 8 - 3x$, $x \geq 1$. Absolute maximum $f(1) = 5$; no local maximum. No absolute or local minimum.



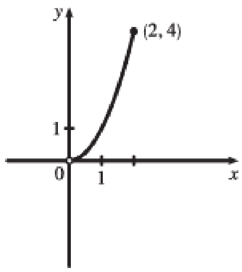
16. $f(x) = 3 - 2x$, $x \leq 5$. Absolute minimum $f(5) = -7$; no local minimum. No absolute or local maximum.



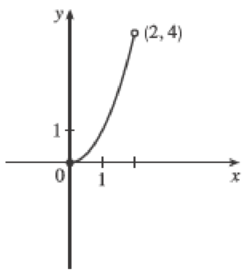
17. $f(x) = x^2$, $0 < x < 2$. No absolute or local maximum or minimum value.



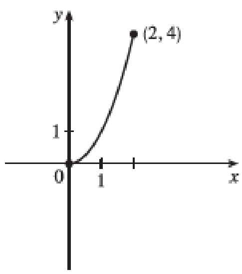
18. $f(x) = x^2$, $0 < x \le 2$. Absolute maximum $f(2) = 4$; no local maximum. No absolute or local minimum.



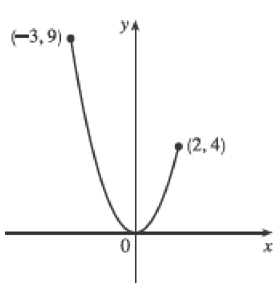
19. $f(x) = x^2$, $0 \le x < 2$. Absolute minimum $f(0) = 0$; no local minimum. No absolute or local maximum.



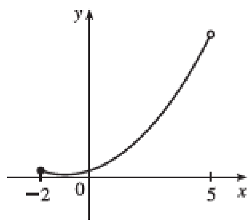
20. $f(x) = x^2$, $0 \le x \le 2$. Absolute maximum $f(2) = 4$. Absolute minimum $f(0) = 0$. No local maximum or minimum.



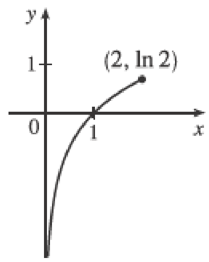
21. $f(x) = x^2$, $-3 \le x \le 2$. Absolute maximum $f(-3) = 9$. No local maximum. Absolute and local minimum $f(0) = 0$.



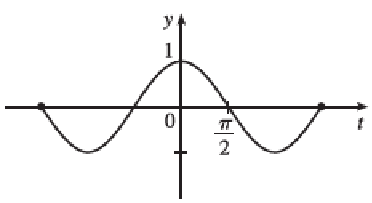
22. $f(x) = 1 + (x+1)^2$, $-2 \le x < 5$. No absolute or local maximum. Absolute and local minimum $f(-1) = 1$.



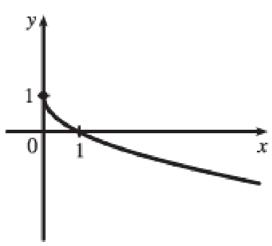
23. $f(x) = \ln x$, $0 < x \le 2$. Absolute maximum $f(2) = \ln 2 \approx 0.69$; no local maximum. No absolute or local minimum.



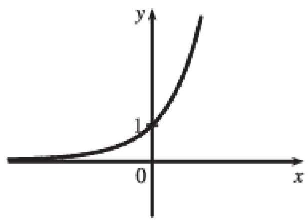
24. $f(t) = \cos t$, $-\frac{3\pi}{2} \le t \le \frac{3\pi}{2}$. Absolute and local maximum $f(0) = 1$; absolute and local minima $f(\pm\pi, -1)$.



25. $f(x) = 1 - \sqrt{x}$. Absolute maximum $f(0) = 1$; no local maximum. No absolute or local minimum.

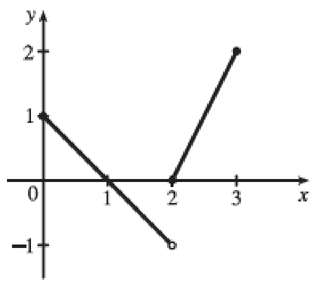


26. $f(x) = e^x$. No absolute or local maximum or minimum value.



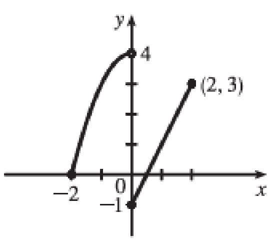
27. $f(x) = \begin{cases} 1 - x & \text{if } 0 \le x < 2 \\ 2x - 4 & \text{if } 2 \le x \le 3 \end{cases}$

Absolute maximum $f(3) = 2$; no local maximum. No absolute or local minimum.



28. $f(x) = \begin{cases} 4 - x^2 & \text{if } -2 \le x < 0 \\ 2x - 1 & \text{if } 0 \le x \le 2 \end{cases}$

Abolute minimum $f(0) = -1$; no local minimim. No absolute or local maximum.



29. $f(x) = 5x^2 + 4x \quad\Rightarrow\quad f'(x) = 10x + 4.\quad f'(x) = 0 \quad\Rightarrow\quad x = -\frac{2}{5}$, so $-\frac{2}{5}$ is the only critical number.

30. $f(x) = x^3 + x^2 - x \quad\Rightarrow\quad f'(x) = 3x^2 + 2x - 1$.

$f'(x) = 0 \quad\Rightarrow\quad (x + 1)(3x - 1) = 0 \quad\Rightarrow\quad x = -1, \frac{1}{3}$. These are the only critical numbers.

31. $f(x) = x^3 + 3x^2 - 24x \quad\Rightarrow\quad f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8)$.

$f'(x) = 0 \quad\Rightarrow\quad 3(x + 4)(x - 2) = 0 \quad\Rightarrow\quad x = -4, 2$. These are the only critical numbers.

32. $f(x) = x^3 + x^2 + x \quad \Rightarrow \quad f'(x) = 3x^2 + 2x + 1.\quad f'(x) = 0 \quad \Rightarrow \quad 3x^2 + 2x + 1 = 0 \quad \Rightarrow \quad x = \dfrac{-2 \pm \sqrt{4 - 12}}{6}$.

Neither of these is a real number. Thus, there are no critical numbers.

33. $s(t) = 3t^4 + 4t^3 - 6t^2 \quad \Rightarrow \quad s'(t) = 12t^3 + 12t^2 - 12t.\quad s'(t) = 0 \quad \Rightarrow \quad 12t(t^2 + t - 1) \quad \Rightarrow$

$t = 0$ or $t^2 + t - 1 = 0$. Using the quadratic formula to solve the latter equation gives us

$t = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \dfrac{-1 \pm \sqrt{5}}{2} \approx 0.618, -1.618$. The three critical numbers are 0, $\dfrac{-1 \pm \sqrt{5}}{2}$.

34. $g(t) = |3t - 4| = \begin{cases} 3t - 4 & \text{if } 3t - 4 \geq 0 \\ -(3t - 4) & \text{if } 3t - 4 < 0 \end{cases} = \begin{cases} 3t - 4 & \text{if } t \geq \frac{4}{3} \\ 4 - 3t & \text{if } t < \frac{4}{3} \end{cases}$

$g'(t) = \begin{cases} 3 & \text{if } t > \frac{4}{3} \\ -3 & \text{if } t < \frac{4}{3} \end{cases}$ and $g'(t)$ does not exist at $t = \frac{4}{3}$, so $t = \frac{4}{3}$ is a critical number.

35. $g(y) = \dfrac{y - 1}{y^2 - y + 1} \quad \Rightarrow$

$$g'(y) = \frac{(y^2 - y + 1)(1) - (y - 1)(2y - 1)}{(y^2 - y + 1)^2} = \frac{y^2 - y + 1 - (2y^2 - 3y + 1)}{(y^2 - y + 1)^2} = \frac{-y^2 + 2y}{(y^2 - y + 1)^2} = \frac{y(2 - y)}{(y^2 - y + 1)^2}.$$

$g'(y) = 0 \quad \Rightarrow \quad y = 0, 2$. The expression $y^2 - y + 1$ is never equal to 0, so $g'(y)$ exists for all real numbers.

The critical numbers are 0 and 2.

36. $h(p) = \dfrac{p - 1}{p^2 + 4} \quad \Rightarrow \quad h'(p) = \dfrac{(p^2 + 4)(1) - (p - 1)(2p)}{(p^2 + 4)^2} = \dfrac{p^2 + 4 - 2p^2 + 2p}{(p^2 + 4)^2} = \dfrac{-p^2 + 2p + 4}{(p^2 + 4)^2}$.

$h'(p) = 0 \quad \Rightarrow \quad p = \dfrac{-2 \pm \sqrt{4 + 16}}{-2} = 1 \pm \sqrt{5}$. The critical numbers are $1 \pm \sqrt{5}$. [$h'(p)$ exists for all real numbers.]

37. $h(t) = t^{3/4} - 2t^{1/4} \quad \Rightarrow \quad h'(t) = \frac{3}{4}t^{-1/4} - \frac{2}{4}t^{-3/4} = \frac{1}{4}t^{-3/4}(3t^{1/2} - 2) = \dfrac{3\sqrt{t} - 2}{4\sqrt[4]{t^3}}$.

$h'(t) = 0 \quad \Rightarrow \quad 3\sqrt{t} = 2 \quad \Rightarrow \quad \sqrt{t} = \frac{2}{3} \quad \Rightarrow \quad t = \frac{4}{9}$. $h'(t)$ does not exist at $t = 0$, so the critical numbers are 0 and $\frac{4}{9}$.

38. $g(x) = \sqrt{1 - x^2} = (1 - x^2)^{1/2} \quad \Rightarrow \quad g'(x) = \frac{1}{2}(1 - x^2)^{-1/2}(-2x) = \dfrac{-x}{\sqrt{1 - x^2}}.\quad g'(x) = 0 \quad \Rightarrow \quad x = 0$.

$g'(x)$ does not exist $\quad \Rightarrow \quad 1 - x^2 = 0 \quad \Rightarrow \quad x = \pm 1$. The critical numbers are -1, 0, and 1.

39. $F(x) = x^{4/5}(x - 4)^2 \quad \Rightarrow$

$$F'(x) = x^{4/5} \cdot 2(x - 4) + (x - 4)^2 \cdot \tfrac{4}{5}x^{-1/5} = \tfrac{1}{5}x^{-1/5}(x - 4)[5 \cdot x \cdot 2 + (x - 4) \cdot 4]$$
$$= \frac{(x - 4)(14x - 16)}{5x^{1/5}} = \frac{2(x - 4)(7x - 8)}{5x^{1/5}}$$

$F'(x) = 0 \quad \Rightarrow \quad x = 4, \frac{8}{7}$. $F'(0)$ does not exist. Thus, the three critical numbers are 0, $\frac{8}{7}$, and 4.

40. $g(x) = x^{1/3} - x^{-2/3} \quad \Rightarrow \quad g'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3} = \frac{1}{3}x^{-5/3}(x+2) = \dfrac{x+2}{3x^{5/3}}$.

$g'(-2) = 0$ and $g'(0)$ does not exist, but 0 is not in the domain of g, so the only critical number is -2.

41. $f(\theta) = 2\cos\theta + \sin^2\theta \quad \Rightarrow \quad f'(\theta) = -2\sin\theta + 2\sin\theta\,\cos\theta$. $f'(\theta) = 0 \quad \Rightarrow \quad 2\sin\theta\,(\cos\theta - 1) = 0 \quad \Rightarrow \quad \sin\theta = 0$ or $\cos\theta = 1 \quad \Rightarrow \quad \theta = n\pi$ [n an integer] or $\theta = 2n\pi$. The solutions $\theta = n\pi$ include the solutions $\theta = 2n\pi$, so the critical numbers are $\theta = n\pi$.

42. $g(\theta) = 4\theta - \tan\theta \quad \Rightarrow \quad g'(\theta) = 4 - \sec^2\theta$. $g'(\theta) = 0 \quad \Rightarrow \quad \sec^2\theta = 4 \quad \Rightarrow \quad \sec\theta = \pm 2 \quad \Rightarrow \quad \cos\theta = \pm\frac{1}{2} \quad \Rightarrow$ $\theta = \frac{\pi}{3} + 2n\pi$, $\frac{5\pi}{3} + 2n\pi$, $\frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + 2n\pi$ are critical numbers.

Note: The values of θ that make $g'(\theta)$ undefined are not in the domain of g.

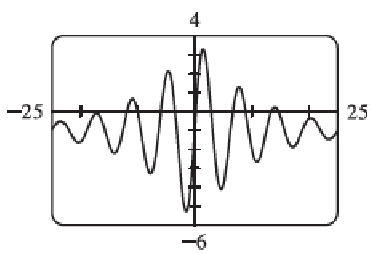
43. $f(x) = x^2 e^{-3x} \quad \Rightarrow \quad f'(x) = x^2(-3e^{-3x}) + e^{-3x}(2x) = xe^{-3x}(-3x+2)$. $f'(x) = 0 \quad \Rightarrow \quad x = 0, \frac{2}{3}$

[e^{-3x} is never equal to 0]. $f'(x)$ always exists, so the critical numbers are 0 and $\frac{2}{3}$.

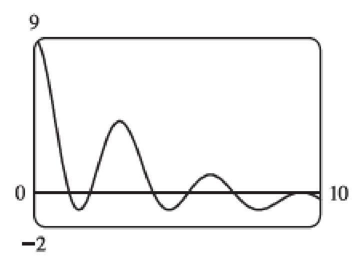
44. $f(x) = x^{-2}\ln x \quad \Rightarrow \quad f'(x) = x^{-2}(1/x) + (\ln x)(-2x^{-3}) = x^{-3} - 2x^{-3}\ln x = x^{-3}(1 - 2\ln x) = \dfrac{1 - 2\ln x}{x^3}$.

$f'(x) = 0 \quad \Rightarrow \quad 1 - 2\ln x = 0 \quad \Rightarrow \quad \ln x = \frac{1}{2} \quad \Rightarrow \quad x = e^{1/2} \approx 1.65$. $f'(0)$ does not exist, but 0 is not in the domain of f, so the only critical number is $\sqrt{e}$.

45. The graph of $f'(x) = 5e^{-0.1|x|}\sin x - 1$ has 10 zeros and exists everywhere, so f has 10 critical numbers.



46. A graph of $f'(x) = \dfrac{100\cos^2 x}{10 + x^2} - 1$ is shown. There are 7 zeros between 0 and 10, and 7 more zeros since f' is an even function. f' exists everywhere, so f has 14 critical numbers.



47. $f(x) = 3x^2 - 12x + 5$, $[0, 3]$. $f'(x) = 6x - 12 = 0 \quad \Leftrightarrow \quad x = 2$. Applying the Closed Interval Method, we find that $f(0) = 5$, $f(2) = -7$, and $f(3) = -4$. So $f(0) = 5$ is the absolute maximum value and $f(2) = -7$ is the absolute minimum value.

48. $f(x) = x^3 - 3x + 1$, $[0, 3]$. $f'(x) = 3x^2 - 3 = 0 \quad \Leftrightarrow \quad x = \pm 1$, but -1 is not in $[0, 3]$. $f(0) = 1$, $f(1) = -1$, and $f(3) = 19$. So $f(3) = 19$ is the absolute maximum value and $f(1) = -1$ is the absolute minimum value.

49. $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$. $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1) = 0 \quad \Leftrightarrow$ $x = 2, -1$. $f(-2) = -3$, $f(-1) = 8$, $f(2) = -19$, and $f(3) = -8$. So $f(-1) = 8$ is the absolute maximum value and $f(2) = -19$ is the absolute minimum value.

50. $f(x) = x^3 - 6x^2 + 9x + 2$, $[-1, 4]$. $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3) = 0 \quad \Leftrightarrow \quad x = 1, 3$. $f(-1) = -14$, $f(1) = 6$, $f(3) = 2$, and $f(4) = 6$. So $f(1) = f(4) = 6$ is the absolute maximum value and $f(-1) = -14$ is the absolute minimum value.