1. (a) True; each of the first two lines has a direction vector parallel to the direction vector of the third line, so these vectors are each scalar multiples of the third direction vector. Then the first two direction vectors are also scalar multiples of each other, so these vectors, and hence the two lines, are parallel.

   (b) False; for example, the x- and y-axes are both perpendicular to the z-axis, yet the x- and y-axes are not parallel.

   (c) True; each of the first two planes has a normal vector parallel to the normal vector of the third plane, so these two normal vectors are parallel to each other and the planes are parallel.

   (d) False; for example, the xy- and yz-planes are not parallel, yet they are both perpendicular to the xz-plane.

   (e) False; the x- and y-axes are not parallel, yet they are both parallel to the plane $z = 1$.

   (f) True; if each line is perpendicular to a plane, then the lines' direction vectors are both parallel to a normal vector for the plane. Thus, the direction vectors are parallel to each other and the lines are parallel.

   (g) False; the planes $y = 1$ and $z = 1$ are not parallel, yet they are both parallel to the x-axis.

   (h) True; if each plane is perpendicular to a line, then any normal vector for each plane is parallel to a direction vector for the line. Thus, the normal vectors are parallel to each other and the planes are parallel.

   (i) True; see Figure 9 and the accompanying discussion.

   (j) False; they can be skew, as in Example 3.

   (k) True. Consider any normal vector for the plane and any direction vector for the line. If the normal vector is perpendicular to the direction vector, the line and plane are parallel. Otherwise, the vectors meet at an angle θ, $0° \leq \theta < 90°$, and the line will intersect the plane at an angle $90° - \theta$.

2. For this line, we have $\mathbf{r}_0 = 6\,\mathbf{i} - 5\,\mathbf{j} + 2\,\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 3\,\mathbf{j} - \frac{2}{3}\,\mathbf{k}$, so a vector equation is $\mathbf{r} = \mathbf{r}_0 + t\,\mathbf{v} = (6\,\mathbf{i} - 5\,\mathbf{j} + 2\,\mathbf{k}) + t\big(\mathbf{i} + 3\,\mathbf{j} - \frac{2}{3}\,\mathbf{k}\big) = (6 + t)\,\mathbf{i} + (-5 + 3t)\,\mathbf{j} + \big(2 - \frac{2}{3}t\big)\,\mathbf{k}$ and parametric equations are $x = 6 + t$, $y = -5 + 3t$, $z = 2 - \frac{2}{3}t$.

3. For this line, we have $\mathbf{r}_0 = 2\,\mathbf{i} + 2.4\,\mathbf{j} + 3.5\,\mathbf{k}$ and $\mathbf{v} = 3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}$, so a vector equation is $\mathbf{r} = \mathbf{r}_0 + t\,\mathbf{v} = (2\,\mathbf{i} + 2.4\,\mathbf{j} + 3.5\,\mathbf{k}) + t(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}) = (2 + 3t)\,\mathbf{i} + (2.4 + 2t)\,\mathbf{j} + (3.5 - t)\,\mathbf{k}$ and parametric equations are $x = 2 + 3t$, $y = 2.4 + 2t$, $z = 3.5 - t$.

4. This line has the same direction as the given line, $\mathbf{v} = 2\,\mathbf{i} - 3\,\mathbf{j} + 9\,\mathbf{k}$. Here $\mathbf{r}_0 = 14\,\mathbf{j} - 10\,\mathbf{k}$, so a vector equation is $\mathbf{r} = (14\,\mathbf{j} - 10\,\mathbf{k}) + t(2\,\mathbf{i} - 3\,\mathbf{j} + 9\,\mathbf{k}) = 2t\,\mathbf{i} + (14 - 3t)\,\mathbf{j} + (-10 + 9t)\,\mathbf{k}$ and parametric equations are $x = 2t$, $y = 14 - 3t$, $z = -10 + 9t$.

5. A line perpendicular to the given plane has the same direction as a normal vector to the plane, such as $\mathbf{n} = \langle 1, 3, 1 \rangle$. So $\mathbf{r}_0 = \mathbf{i} + 6\,\mathbf{k}$, and we can take $\mathbf{v} = \mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$. Then a vector equation is $\mathbf{r} = (\mathbf{i} + 6\,\mathbf{k}) + t(\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}) = (1 + t)\,\mathbf{i} + 3t\,\mathbf{j} + (6 + t)\,\mathbf{k}$, and parametric equations are $x = 1 + t$, $y = 3t$, $z = 6 + t$.

6. The vector $\mathbf{v} = \langle 1-0, 2-0, 3-0 \rangle = \langle 1, 2, 3 \rangle$ is parallel to the line. Letting $P_0 = (0, 0, 0)$, parametric equations are $x = 0 + 1 \cdot t = t$, $y = 0 + 2 \cdot t = 2t$, $z = 0 + 3 \cdot t = 3t$, while symmetric equations are $x = \dfrac{y}{2} = \dfrac{z}{3}$.

7. The vector $\mathbf{v} = \langle -4-1, 3-3, 0-2 \rangle = \langle -5, 0, -2 \rangle$ is parallel to the line. Letting $P_0 = (1, 3, 2)$, parametric equations are $x = 1 - 5t$, $y = 3 + 0t = 3$, $z = 2 - 2t$, while symmetric equations are $\dfrac{x-1}{-5} = \dfrac{z-2}{-2}$, $y = 3$. Notice here that the direction number $b = 0$, so rather than writing $\dfrac{y-3}{0}$ in the symmetric equation we must write the equation $y = 3$ separately.

8. $\mathbf{v} = \langle 2-6, 4-1, 5-(-3) \rangle = \langle -4, 3, 8 \rangle$, and letting $P_0 = (6, 1, -3)$, parametric equations are $x = 6 - 4t$, $y = 1 + 3t$, $z = -3 + 8t$, while symmetric equations are $\dfrac{x-6}{-4} = \dfrac{y-1}{3} = \dfrac{z+3}{8}$.

9. $\mathbf{v} = \left\langle 2-0, 1-\frac{1}{2}, -3-1 \right\rangle = \left\langle 2, \frac{1}{2}, -4 \right\rangle$, and letting $P_0 = (2, 1, -3)$, parametric equations are $x = 2 + 2t$, $y = 1 + \frac{1}{2}t$, $z = -3 - 4t$, while symmetric equations are $\dfrac{x-2}{2} = \dfrac{y-1}{1/2} = \dfrac{z+3}{-4}$ or $\dfrac{x-2}{2} = 2y - 2 = \dfrac{z+3}{-4}$.

10. $\mathbf{v} = (\mathbf{i} + \mathbf{j}) \times (\mathbf{j} + \mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ is the direction of the line perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.

    With $P_0 = (2, 1, 0)$, parametric equations are $x = 2 + t$, $y = 1 - t$, $z = t$ and symmetric equations are $x - 2 = \dfrac{y-1}{-1} = z$ or $x - 2 = 1 - y = z$.

11. The line has direction $\mathbf{v} = \langle 1, 2, 1 \rangle$. Letting $P_0 = (1, -1, 1)$, parametric equations are $x = 1 + t$, $y = -1 + 2t$, $z = 1 + t$ and symmetric equations are $x - 1 = \dfrac{y+1}{2} = z - 1$.

12. Setting $x = 0$, we see that $(0, 1, 0)$ satisfies the equations of both planes, so they do in fact have a line of intersection. $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 0, 1 \rangle = \langle 1, 0, -1 \rangle$ is the direction of this line. Taking the point $(0, 1, 0)$ as P_0, parametric equations are $x = t$, $y = 1$, $z = -t$, and symmetric equations are $x = -z$, $y = 1$.

13. Direction vectors of the lines are $\mathbf{v}_1 = \langle -2-(-4), 0-(-6), -3-1 \rangle = \langle 2, 6, -4 \rangle$ and $\mathbf{v}_2 = \langle 5-10, 3-18, 14-4 \rangle = \langle -5, -15, 10 \rangle$, and since $\mathbf{v}_2 = -\frac{5}{2}\mathbf{v}_1$, the direction vectors and thus the lines are parallel.

14. Direction vectors of the lines are $\mathbf{v}_1 = \langle -2, 4, 4 \rangle$ and $\mathbf{v}_2 = \langle 8, -1, 4 \rangle$. Since $\mathbf{v}_1 \cdot \mathbf{v}_2 = -16 - 4 + 16 \neq 0$, the vectors and thus the lines are not perpendicular.

15. (a) The line passes through the point $(1, -5, 6)$ and a direction vector for the line is $\langle -1, 2, -3 \rangle$, so symmetric equations for the line are $\dfrac{x-1}{-1} = \dfrac{y+5}{2} = \dfrac{z-6}{-3}$.

(b) The line intersects the xy-plane when $z = 0$, so we need $\dfrac{x-1}{-1} = \dfrac{y+5}{2} = \dfrac{0-6}{-3}$ or $\dfrac{x-1}{-1} = 2 \;\Rightarrow\; x = -1$, $\dfrac{y+5}{2} = 2 \;\Rightarrow\; y = -1$. Thus the point of intersection with the xy-plane is $(-1, -1, 0)$. Similarly for the yz-plane, we need $x = 0 \;\Rightarrow\; 1 = \dfrac{y+5}{2} = \dfrac{z-6}{-3} \;\Rightarrow\; y = -3$, $z = 3$. Thus the line intersects the yz-plane at $(0, -3, 3)$. For the xz-plane, we need $y = 0 \;\Rightarrow\; \dfrac{x-1}{-1} = \dfrac{5}{2} = \dfrac{z-6}{-3} \;\Rightarrow\; x = -\frac{3}{2}$, $z = -\frac{3}{2}$. So the line intersects the xz-plane at $\left(-\frac{3}{2}, 0, -\frac{3}{2}\right)$.

16. (a) A vector normal to the plane $x - y + 3z = 7$ is $\mathbf{n} = \langle 1, -1, 3 \rangle$, and since the line is to be perpendicular to the plane, $\mathbf{n}$ is also a direction vector for the line. Thus parametric equations of the line are $x = 2 + t$, $y = 4 - t$, $z = 6 + 3t$.

(b) On the xy-plane, $z = 0$. So $z = 6 + 3t = 0 \;\Rightarrow\; t = -2$ in the parametric equations of the line, and therefore $x = 0$ and $y = 6$, giving the point of intersection $(0, 6, 0)$. For the yz-plane, $x = 0$ so we get the same point of interesection: $(0, 6, 0)$. For the xz-plane, $y = 0$ which implies $t = 4$, so $x = 6$ and $z = 18$ and the point of intersection is $(6, 0, 18)$.

17. From Equation 4, the line segment from $\mathbf{r}_0 = 2\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}$ to $\mathbf{r}_1 = 4\,\mathbf{i} + 6\,\mathbf{j} + \mathbf{k}$ is

$\mathbf{r}(t) = (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)(2\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}) + t(4\,\mathbf{i} + 6\,\mathbf{j} + \mathbf{k}) = (2\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}) + t(2\,\mathbf{i} + 7\,\mathbf{j} - 3\,\mathbf{k}), 0 \le t \le 1.$

18. From Equation 4, the line segment from $\mathbf{r}_0 = 10\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$ to $\mathbf{r}_1 = 5\,\mathbf{i} + 6\,\mathbf{j} - 3\,\mathbf{k}$ is

$$\begin{aligned} \mathbf{r}(t) &= (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)(10\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}) + t(5\,\mathbf{i} + 6\,\mathbf{j} - 3\,\mathbf{k}) \\ &= (10\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}) + t(-5\,\mathbf{i} + 3\,\mathbf{j} - 4\,\mathbf{k}), \quad 0 \le t \le 1. \end{aligned}$$

The corresponding parametric equations are $x = 10 - 5t$, $y = 3 + 3t$, $z = 1 - 4t$, $0 \le t \le 1$.

19. Since the direction vectors are $\mathbf{v}_1 = \langle -6, 9, -3 \rangle$ and $\mathbf{v}_2 = \langle 2, -3, 1 \rangle$, we have $\mathbf{v}_1 = -3\mathbf{v}_2$ so the lines are parallel.

20. The lines aren't parallel since the direction vectors $\langle 2, 3, -1 \rangle$ and $\langle 1, 1, 3 \rangle$ aren't parallel. For the lines to intersect we must be able to find one value of t and one value of s that produce the same point from the respective parametric equations. Thus we need to satisfy the following three equations: $1 + 2t = -1 + s$, $3t = 4 + s$, $2 - t = 1 + 3s$. Solving the first two equations we get $t = 6$, $s = 14$ and checking, we see that these values don't satisfy the third equation. Thus L_1 and L_2 aren't parallel and don't intersect, so they must be skew lines.

21. Since the direction vectors $\langle 1,2,3\rangle$ and $\langle -4,-3,2\rangle$ are not scalar multiples of each other, the lines are not parallel, so we check to see if the lines intersect. The parametric equations of the lines are L_1: $x=t$, $y=1+2t$, $z=2+3t$ and L_2: $x=3-4s$, $y=2-3s$, $z=1+2s$. For the lines to intersect, we must be able to find one value of t and one value of s that produce the same point from the respective parametric equations. Thus we need to satisfy the following three equations: $t=3-4s$, $1+2t=2-3s$, $2+3t=1+2s$. Solving the first two equations we get $t=-1$, $s=1$ and checking, we see that these values don't satisfy the third equation. Thus the lines aren't parallel and don't intersect, so they must be skew lines.

22. Since the direction vectors $\langle 2,2,-1\rangle$ and $\langle 1,-1,3\rangle$ aren't parallel, the lines aren't parallel. Here the parametric equations are L_1: $x=1+2t$, $y=3+2t$, $z=2-t$ and L_2: $x=2+s$, $y=6-s$, $z=-2+3s$. Thus, for the lines to intersect, the three equations $1+2t=2+s$, $3+2t=6-s$, and $2-t=-2+3s$ must be satisfied simultaneously. Solving the first two equations gives $t=1$, $s=1$ and, checking, we see that these values do satisfy the third equation, so the lines intersect when $t=1$ and $s=1$, that is, at the point $(3,5,1)$.

23. Since the plane is perpendicular to the vector $\langle -2,1,5\rangle$, we can take $\langle -2,1,5\rangle$ as a normal vector to the plane. $(6,3,2)$ is a point on the plane, so setting $a=-2$, $b=1$, $c=5$ and $x_0=6$, $y_0=3$, $z_0=2$ in Equation 7 gives $-2(x-6)+1(y-3)+5(z-2)=0$ or $-2x+y+5z=1$ to be an equation of the plane.

24. $\mathbf{j}+2\,\mathbf{k}=\langle 0,1,2\rangle$ is a normal vector to the plane and $(4,0,-3)$ is a point on the plane, so setting $a=0$, $b=1$, $c=2$, $x_0=4$, $y_0=0$, $z_0=-3$ in Equation 7 gives $0(x-4)+1(y-0)+2[z-(-3)]=0$ or $y+2z=-6$ to be an equation of the plane.

25. $\mathbf{i}+\mathbf{j}-\mathbf{k}=\langle 1,1,-1\rangle$ is a normal vector to the plane and $(1,-1,1)$ is a point on the plane, so setting $a=1$, $b=1$, $c=-1$, $x_0=1$, $y_0=-1$, $z_0=1$ in Equation 7 gives $1\,(x-1)+1[y-(-1)]-1(z-1)=0$ or $x+y-z=-1$ to be an equation of the plane.

26. Since the line is perpendicular to the plane, its direction vector $\langle 1,2,-3\rangle$ is a normal vector to the plane. An equation of the plane, then, is $1[x-(-2)]+2(y-8)-3(z-10)=0$ or $x+2y-3z=-16$.

27. Since the two planes are parallel, they will have the same normal vectors. So we can take $\mathbf{n}=\langle 2,-1,3\rangle$, and an equation of the plane is $2(x-0)-1(y-0)+3(z-0)=0$ or $2x-y+3z=0$.

28. Since the two planes are parallel, they will have the same normal vectors. So we can take $\mathbf{n}=\langle 1,1,1\rangle$, and an equation of the plane is $1[x-(-1)]+1(y-6)+1[z-(-5)]=0$ or $x+y+z=0$.

29. Since the two planes are parallel, they will have the same normal vectors. So we can take $\mathbf{n}=\langle 3,0,-7\rangle$, and an equation of the plane is $3(x-4)+0[y-(-2)]-7(z-3)=0$ or $3x-7z=-9$.

30. First, a normal vector for the plane $2x + 4y + 8z = 17$ is $\mathbf{n} = \langle 2, 4, 8 \rangle$. A direction vector for the line is $\mathbf{v} = \langle 2, 1, -1 \rangle$, and since $\mathbf{n} \cdot \mathbf{v} = 0$ we know the line is perpendicular to $\mathbf{n}$ and hence parallel to the plane. Thus, there is a parallel plane which contains the line. By putting $t = 0$, we know the point $(3, 0, 8)$ is on the line and hence the new plane. We can use the same normal vector $\mathbf{n} = \langle 2, 4, 8 \rangle$, so an equation of the plane is $2(x - 3) + 4(y - 0) + 8(z - 8) = 0$ or $x + 2y + 4z = 35$.

31. Here the vectors $\mathbf{a} = \langle 1 - 0, 0 - 1, 1 - 1 \rangle = \langle 1, -1, 0 \rangle$ and $\mathbf{b} = \langle 1 - 0, 1 - 1, 0 - 1 \rangle = \langle 1, 0, -1 \rangle$ lie in the plane, so $\mathbf{a} \times \mathbf{b}$ is a normal vector to the plane. Thus, we can take $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 1 - 0, 0 + 1, 0 + 1 \rangle = \langle 1, 1, 1 \rangle$. If P_0 is the point $(0, 1, 1)$, an equation of the plane is $1(x - 0) + 1(y - 1) + 1(z - 1) = 0$ or $x + y + z = 2$.

32. Here the vectors $\mathbf{a} = \langle 2, -4, 6 \rangle$ and $\mathbf{b} = \langle 5, 1, 3 \rangle$ lie in the plane, so $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -12 - 6, 30 - 6, 2 + 20 \rangle = \langle -18, 24, 22 \rangle$ is a normal vector to the plane and an equation of the plane is $-18(x - 0) + 24(y - 0) + 22(z - 0) = 0$ or $-18x + 24y + 22z = 0$.

33. Here the vectors $\mathbf{a} = \langle 8 - 3, 2 - (-1), 4 - 2 \rangle = \langle 5, 3, 2 \rangle$ and $\mathbf{b} = \langle -1 - 3, -2 - (-1), -3 - 2 \rangle = \langle -4, -1, -5 \rangle$ lie in the plane, so a normal vector to the plane is $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -15 + 2, -8 + 25, -5 + 12 \rangle = \langle -13, 17, 7 \rangle$ and an equation of the plane is $-13(x - 3) + 17[y - (-1)] + 7(z - 2) = 0$ or $-13x + 17y + 7z = -42$.

34. If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\mathbf{a} = \langle 3, 1, -1 \rangle$ is one vector in the plane. We can verify that the given point $(1, 2, 3)$ does not lie on this line, so to find another nonparallel vector $\mathbf{b}$ which lies in the plane, we can pick any point on the line and find a vector connecting the points. If we put $t = 0$, we see that $(0, 1, 2)$ is on the line, so $\mathbf{b} = \langle 1 - 0, 2 - 1, 3 - 2 \rangle = \langle 1, 1, 1 \rangle$ and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 1 + 1, -1 - 3, 3 - 1 \rangle = \langle 2, -4, 2 \rangle$. Thus, an equation of the plane is $2(x - 1) - 4(y - 2) + 2(z - 3) = 0$ or $2x - 4y + 2z = 0$. (Equivalently, we can write $x - 2y + z = 0$.)

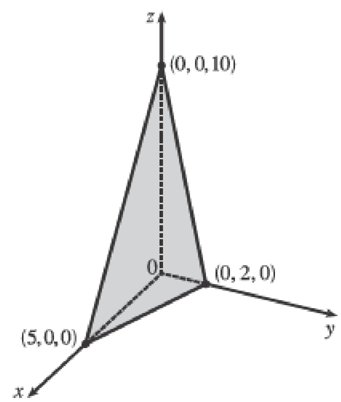
35. If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\mathbf{a} = \langle -2, 5, 4 \rangle$ is one vector in the plane. We can verify that the given point $(6, 0, -2)$ does not lie on this line, so to find another nonparallel vector $\mathbf{b}$ which lies in the plane, we can pick any point on the line and find a vector connecting the points. If we put $t = 0$, we see that $(4, 3, 7)$ is on the line, so $\mathbf{b} = \langle 6 - 4, 0 - 3, -2 - 7 \rangle = \langle 2, -3, -9 \rangle$ and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -45 + 12, 8 - 18, 6 - 10 \rangle = \langle -33, -10, -4 \rangle$. Thus, an equation of the plane is $-33(x - 6) - 10(y - 0) - 4[z - (-2)] = 0$ or $33x + 10y + 4z = 190$.

36. Since the line $x = 2y = 3z$, or $x = \dfrac{y}{1/2} = \dfrac{z}{1/3}$, lies in the plane, its direction vector $\mathbf{a} = \left\langle 1, \frac{1}{2}, \frac{1}{3} \right\rangle$ is parallel to the plane. The point $(0, 0, 0)$ is on the line (put $t = 0$), and we can verify that the given point $(1, -1, 1)$ in the plane is not on the line. The vector connecting these two points, $\mathbf{b} = \langle 1, -1, 1 \rangle$, is therefore parallel to the plane, but not parallel to $\langle 1, 2, 3 \rangle$. Then $\mathbf{a} \times \mathbf{b} = \left\langle \frac{1}{2} + \frac{1}{3}, \frac{1}{3} - 1, -1 - \frac{1}{2} \right\rangle = \left\langle \frac{5}{6}, -\frac{2}{3}, -\frac{3}{2} \right\rangle$ is a normal vector to the plane, and an equation of the plane is $\frac{5}{6}(x - 0) - \frac{2}{3}(y - 0) - \frac{3}{2}(z - 0) = 0$ or $5x - 4y - 9z = 0$.

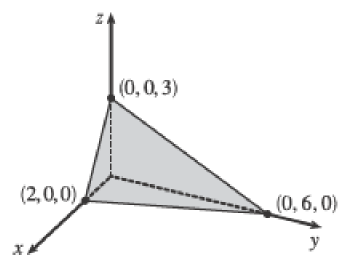
37. A direction vector for the line of intersection is $\mathbf{a} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, -1 \rangle \times \langle 2, -1, 3 \rangle = \langle 2, -5, -3 \rangle$, and $\mathbf{a}$ is parallel to the desired plane. Another vector parallel to the plane is the vector connecting any point on the line of intersection to the given point $(-1, 2, 1)$ in the plane. Setting $x = 0$, the equations of the planes reduce to $y - z = 2$ and $-y + 3z = 1$ with simultaneous solution $y = \frac{7}{2}$ and $z = \frac{3}{2}$. So a point on the line is $\left(0, \frac{7}{2}, \frac{3}{2}\right)$ and another vector parallel to the plane is $\left\langle -1, -\frac{3}{2}, -\frac{1}{2} \right\rangle$. Then a normal vector to the plane is $\mathbf{n} = \langle 2, -5, -3 \rangle \times \left\langle -1, -\frac{3}{2}, -\frac{1}{2} \right\rangle = \langle -2, 4, -8 \rangle$ and an equation of the plane is $-2(x + 1) + 4(y - 2) - 8(z - 1) = 0$ or $x - 2y + 4z = -1$.

38. $\mathbf{n}_1 = \langle 1, 0, -1 \rangle$ and $\mathbf{n}_2 = \langle 0, 1, 2 \rangle$. Setting $z = 0$, it is easy to see that $(1, 3, 0)$ is a point on the line of intersection of $x - z = 1$ and $y + 2z = 3$. The direction of this line is $\mathbf{v}_1 = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, -2, 1 \rangle$. A second vector parallel to the desired plane is $\mathbf{v}_2 = \langle 1, 1, -2 \rangle$, since it is perpendicular to $x + y - 2z = 1$. Therefore, a normal of the plane in question is $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 4 - 1, 1 + 2, 1 + 2 \rangle = \langle 3, 3, 3 \rangle$, or we can use $\langle 1, 1, 1 \rangle$. Taking $(x_0, y_0, z_0) = (1, 3, 0)$, the equation we are looking for is $(x - 1) + (y - 3) + z = 0 \quad \Leftrightarrow \quad x + y + z = 4$.

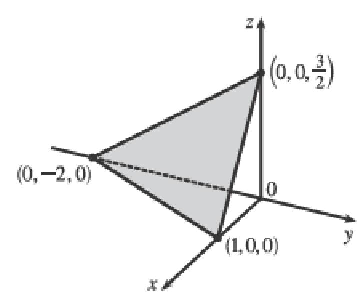
39. To find the x-intercept we set $y = z = 0$ in the equation $2x + 5y + z = 10$ and obtain $2x = 10 \quad \Rightarrow \quad x = 5$ so the x-intercept is $(5, 0, 0)$. When $x = z = 0$ we get $5y = 10 \quad \Rightarrow \quad y = 2$, so the y-intercept is $(0, 2, 0)$. Setting $x = y = 0$ gives $z = 10$, so the z-intercept is $(0, 0, 10)$ and we graph the portion of the plane that lies in the first octant.



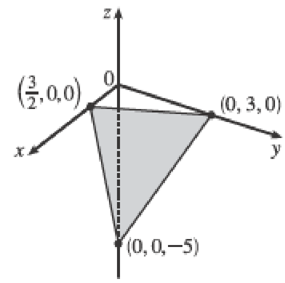
40. To find the x-intercept we set $y = z = 0$ in the equation $3x + y + 2z = 6$ and obtain $3x = 6 \quad \Rightarrow \quad x = 2$ so the x-intercept is $(2, 0, 0)$. When $x = z = 0$ we get $y = 6$ so the y-intercept is $(0, 6, 0)$. Setting $x = y = 0$ gives $2z = 6 \quad \Rightarrow \quad z = 3$, so the z-intercept is $(0, 0, 3)$. The figure shows the portion of the plane that lies in the first octant.



41. Setting $y = z = 0$ in the equation $6x - 3y + 4z = 6$ gives $6x = 6 \quad \Rightarrow \quad x = 1$, when $x = z = 0$ we have $-3y = 6 \quad \Rightarrow \quad y = -2$, and $x = y = 0$ implies $4z = 6 \quad \Rightarrow \quad z = \frac{3}{2}$, so the intercepts are $(1, 0, 0)$, $(0, -2, 0)$, and $\left(0, 0, \frac{3}{2}\right)$. The figure shows the portion of the plane cut off by the coordinate planes.



42. Setting $y = z = 0$ in the equation $6x + 5y - 3z = 15$ gives $6x = 15 \quad \Rightarrow \quad x = \frac{5}{2}$, when $x = z = 0$ we have $5y = 15 \quad \Rightarrow \quad y = 3$, and $x = y = 0$ implies $-3z = 15 \quad \Rightarrow \quad z = -5$, so the intercepts are $\left(\frac{5}{2}, 0, 0\right)$, $(0, 3, 0)$, and $(0, 0, -5)$. The figure shows the portion of the plane cut off by the coordinate planes.



43. Substitute the parametric equations of the line into the equation of the plane: $(3 - t) - (2 + t) + 2(5t) = 9 \quad \Rightarrow \quad 8t = 8 \quad \Rightarrow \quad t = 1$. Therefore, the point of intersection of the line and the plane is given by $x = 3 - 1 = 2$, $y = 2 + 1 = 3$, and $z = 5(1) = 5$, that is, the point $(2, 3, 5)$.

44. Substitute the parametric equations of the line into the equation of the plane: $(1 + 2t) + 2(4t) - (2 - 3t) + 1 = 0 \quad \Rightarrow \quad 13t = 0 \quad \Rightarrow \quad t = 0$. Therefore, the point of intersection of the line and the plane is given by $x = 1 + 2(0) = 1$, $y = 4(0) = 0$, and $z = 2 - 3(0) = 2$, that is, the point $(1, 0, 2)$.

45. Parametric equations for the line are $x = t$, $y = 1 + t$, $z = \frac{1}{2}t$ and substituting into the equation of the plane gives $4(t) - (1 + t) + 3\left(\frac{1}{2}t\right) = 8 \quad \Rightarrow \quad \frac{9}{2}t = 9 \quad \Rightarrow \quad t = 2$. Thus $x = 2$, $y = 1 + 2 = 3$, $z = \frac{1}{2}(2) = 1$ and the point of intersection is $(2, 3, 1)$.

46. A direction vector for the line through $(1, 0, 1)$ and $(4, -2, 2)$ is $\mathbf{v} = \langle 3, -2, 1 \rangle$ and, taking $P_0 = (1, 0, 1)$, parametric equations for the line are $x = 1 + 3t$, $y = -2t$, $z = 1 + t$. Substitution of the parametric equations into the equation of the plane gives $1 + 3t - 2t + 1 + t = 6 \quad \Rightarrow \quad t = 2$. Then $x = 1 + 3(2) = 7$, $y = -2(2) = -4$, and $z = 1 + 2 = 3$ so the point of intersection is $(7, -4, 3)$.

47. Setting $x = 0$, we see that $(0, 1, 0)$ satisfies the equations of both planes, so that they do in fact have a line of intersection. $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 0, 1 \rangle = \langle 1, 0, -1 \rangle$ is the direction of this line. Therefore, direction numbers of the intersecting line are $1, 0, -1$.

48. The angle between the two planes is the same as the angle between their normal vectors. The normal vectors of the two planes are $\langle 1, 1, 1 \rangle$ and $\langle 1, 2, 3 \rangle$. The cosine of the angle θ between these two planes is

$$\cos\theta = \frac{\langle 1, 1, 1 \rangle \cdot \langle 1, 2, 3 \rangle}{|\langle 1, 1, 1 \rangle|\,|\langle 1, 2, 3 \rangle|} = \frac{1 + 2 + 3}{\sqrt{1 + 1 + 1}\,\sqrt{1 + 4 + 9}} = \frac{6}{\sqrt{42}} = \sqrt{\frac{6}{7}}.$$

49. Normal vectors for the planes are $\mathbf{n}_1 = \langle 1, 4, -3 \rangle$ and $\mathbf{n}_2 = \langle -3, 6, 7 \rangle$, so the normals (and thus the planes) aren't parallel. But $\mathbf{n}_1 \cdot \mathbf{n}_2 = -3 + 24 - 21 = 0$, so the normals (and thus the planes) are perpendicular.

50. Normal vectors for the planes are $\mathbf{n}_1 = \langle -1, 4, -2 \rangle$ and $\mathbf{n}_2 = \langle 3, -12, 6 \rangle$. Since $\mathbf{n}_2 = -3\mathbf{n}_1$, the normals (and thus the planes) are parallel.