This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

ParallelFace05c
001 10.0 points

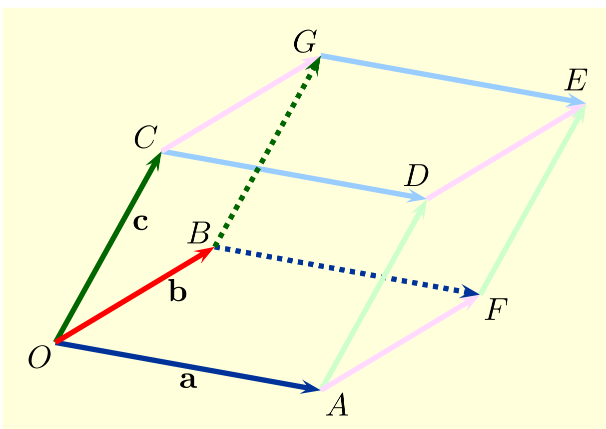
The vectors

$$\mathbf{a} = \langle 4, -4, -1\rangle, \quad \mathbf{b} = \langle 3, -2, -1\rangle,$$

and

$$\mathbf{c} = \langle 2, 2, 4\rangle,$$

shown below form adjacent edges of the parallelepiped



Describe the face $CDEG$ in vector form.

1.

$$\langle 3s + 2t,\ -2s + 2t,\ -s + 4t\rangle,$$

for $-1 \le s,\ t \le 1.$

2.

$$\langle 2s + 3t,\ 2s - 2t,\ 4s - t\rangle,$$

for $0 \le s,\ t \le 1.$

3.

$$\langle 2 + 4s + 3t,\ 2 - 4s - 2t,\ 4 - s - t\rangle,$$

for $0 \le s,\ t \le 1.$

correct

4.

$$\langle 2 + 4s + 3t,\ 2 - 4s - 2t,\ 4 - s - t\rangle,$$

for $-1 \le s,\ t \le 1.$

5.

$$\langle 4 + 3s + 2t,\ -4 - 2s + 2t,\ -1 - s + 4t\rangle,$$

for, $-1 \le s,\ t \le 1.$

6.

$$\langle 3 + 4s + 2t,\ -2 - 4s + 2t,\ -1 - s + 4t\rangle,$$

for, $0 \le s,\ t \le 1.$

Explanation:

The face $CDEG$ of the parallelepiped lies in the unique plane in which the vertices C, D, and G lie. Now in vector form this plane is

$$\mathbf{x} = \mathbf{c} + s\mathbf{a} + t\mathbf{b}, \quad -\infty \le s,\ t \le \infty.$$

But the points in the parallelogram $CDEG$ lying in this plane correspond to $0 \le s \le 1$ and $0 \le t \le 1$, *i.e.*, to

$$\mathbf{c} + s\mathbf{a} + t\mathbf{b}\,, \quad 0 \le s,\ t \le 1.$$

Consequently, the face $CDEG$ is given in vector form by

$$\boxed{\langle 2 + 4s + 3t,\ 2 - 4s - 2t,\ 4 - s - t\rangle,}\,.$$

for $0 \le s,\ t \le 1.$

CalC13a30aNC
002 10.0 points

Find an equation for the set of all points in 3-space equidistant from the points

$$A(1, -1, -2)\,, \qquad B(2, 1, 2)\,.$$

1. $2x + 4y + 8z - 3 = 0$ **correct**

2. $8x + 4y + 2z + 3 = 0$

3. $2x + 4y + 8z + 3 = 0$

4. $4x - 8y + 2z + 3 = 0$

5. $4x + 2y - 8z - 3 = 0$

6. $8x - 2y - 4z - 3 = 0$

Explanation:

We have to find the set of points $P(x, y, z)$ such that

$$\|\overline{AP}\| = \|\overline{BP}\|.$$

Now by the distance formula in 3-space,

$$\|\overline{AP}\|^2 = (x-1)^2 + (y+1)^2 + (z+2)^2,$$

while

$$\|\overline{BP}\|^2 = (x-2)^2 + (y-1)^2 + (z-2)^2.$$

After expansion therefore,

$$\|\overline{AP}\|^2 = x^2 - 2x + y^2 + 2y + z^2 + 4z + 6,$$

while

$$\|\overline{BP}\|^2 = x^2 - 4x + y^2 - 2y + z^2 - 4z + 9.$$

Thus $\|\overline{AP}\| = \|\overline{BP}\|$ when

$$x^2 - 2x + y^2 + 2y + z^2 + 4z + 6 = x^2 - 4x + y^2 - 2y + z^2 - 4z + 9.$$

Consequently, the set of all points equidistant from A and B satisfies the equation

$$\boxed{2x + 4y + 8z - 3 = 0}.$$

Notice that this is a plane perpendicular to the line segment joining A and B (since it must contain the perpendicular bisector of the line segment $\overline{AB}$).

keywords: plane, locus points, equidistant two points

CalC13d12s
003 10.0 points

If $\mathbf{a}$ is a vector parallel to the xy-plane and $\mathbf{b}$ is a vector parallel to $\mathbf{k}$, determine $\|\mathbf{a} \times \mathbf{b}\|$ when $\|\mathbf{a}\| = 2$ and $\|\mathbf{b}\| = 4$.

1. $\|\mathbf{a} \times \mathbf{b}\| = -4\sqrt{2}$

2. $\|\mathbf{a} \times \mathbf{b}\| = 4\sqrt{2}$

3. $\|\mathbf{a} \times \mathbf{b}\| = 8$ **correct**

4. $\|\mathbf{a} \times \mathbf{b}\| = 4$

5. $\|\mathbf{a} \times \mathbf{b}\| = -8$

6. $\|\mathbf{a} \times \mathbf{b}\| = -4$

7. $\|\mathbf{a} \times \mathbf{b}\| = 0$

Explanation:

For vectors $\mathbf{a}$ and $\mathbf{b}$,

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta$$

when the angle between them is θ, $0 \le \theta < \pi$.

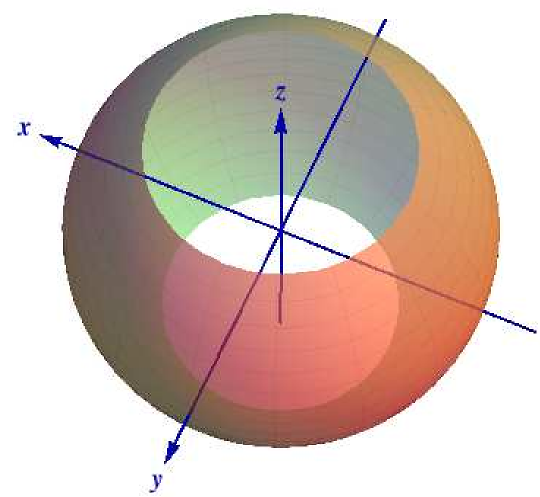
But $\theta = \pi/2$ in the case when $\mathbf{a}$ is parallel to the xy-plane and $\mathbf{b}$ is parallel to $\mathbf{k}$ because $\mathbf{k}$ is then perpendicular to the xy-plane. Consequently, for the given vectors,

$$\boxed{\|\mathbf{a} \times \mathbf{b}\| = 8}.$$

keywords: cross product, length, angle,

SphericalCoords04click
004 10.0 points

The surface S shown in



consists of the portion of the sphere

$$x^2 + y^2 + z^2 = 4$$

where

$$x^2 + y^2 \geq 1 .$$

Use spherical polar coordinates (ρ, θ, ϕ) to describe S.

1. $S =$ all points $P(\rho, \theta, \phi)\}$ with

$$\rho = 2, \ \ 0 \leq \theta \leq 2\pi, \ \ \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3} .$$

2. $S =$ all points $P(\rho, \theta, \phi)\}$ with

$$\rho = 1, \ \ 0 \leq \theta \leq 2\pi, \ \ \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3} .$$

3. $S =$ all points $P(\rho, \theta, \phi)$ with

$$\rho = 2, \ \ 0 \leq \theta \leq 2\pi, \ \ \frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6} .$$

correct

4. $S =$ all points $P(\rho, \theta, \phi)\}$ with

$$\rho = 1, \ \ 0 \leq \theta \leq 2\pi, \ \ \frac{\pi}{6} \leq \frac{5\pi}{6} .$$

5. $S =$ all points $P(\rho, \theta, \phi)\}$ with

$$\rho = 2, \ \ 0 \leq \theta \leq 2\pi, \ \ \frac{\pi}{6} \leq \phi \leq \pi .$$

6. $S =$ all points $P(\rho, \theta, \phi)\}$ with

$$\rho = 1, \ \ 0 \leq \theta \leq 2\pi, \ \ \frac{\pi}{3} \leq \phi \leq \pi .$$

Explanation:

In spherical polar coordinates (ρ, θ, ϕ),

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta ,$$

and

$$z = \rho \cos\phi ,$$

with $0 \leq \theta \leq 2\pi$ and $0 \leq \psi \leq \pi$. We need to find further restrictions on ρ, θ, and ϕ so that

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 \geq 1 .$$

Now

$$\rho^2 = x^2 + y^2 + z^2 = 4 ,$$

i.e., $\rho = 2$. But then,

$$z^2 = 4\cos^2\phi = 4 - x^2 - y^2 \leq 3 .$$

Consequently, S consists of all points P with $\rho = 2$ and

$$\boxed{0 \leq \theta \leq 2\pi, \ \ \frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}} .$$

FinM4e05
005 10.0 points

Solve for X when $AX + B = C$,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 2 \\ -1 & 4 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}.$$

1. $X = \begin{bmatrix} 3 & -4 \\ -2 & 6 \end{bmatrix}$

2. $X = \begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}$ **correct**

3. $X = \begin{bmatrix} 4 & -4 \\ -1 & 3 \end{bmatrix}$

4. $X = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

5. $X = \begin{bmatrix} 4 & -4 \\ 2 & 6 \end{bmatrix}$

Explanation:

By the algbra of matrices,

$$X = A^{-1}(C - B).$$

But the inverse of any 2×2 matrix

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

is given by

$$D^{-1} = \begin{bmatrix} \dfrac{d_{22}}{\Delta} & -\dfrac{d_{12}}{\Delta} \\ -\dfrac{d_{21}}{\Delta} & \dfrac{d_{11}}{\Delta} \end{bmatrix}$$

with $\Delta = d_{11}d_{22} - d_{12}d_{21}$, so

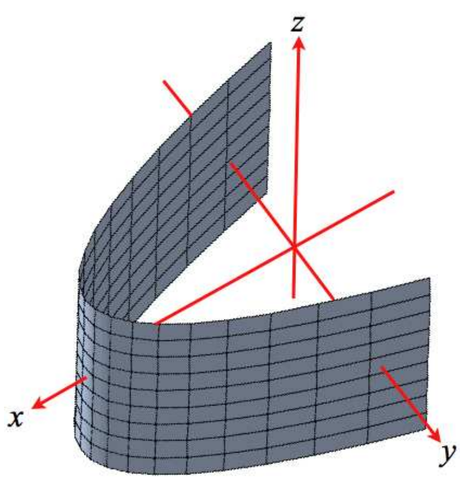
$$X = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \left(\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ -1 & 4 \end{bmatrix} \right)$$
$$= \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix}.$$

Thus

$$\boxed{X = \begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}.}$$

CalC13f04c
006 10.0 points

Which one of the following equations has graph



1. $y - x^2 + 4 = 0$

2. $x + y^2 - 4 = 0$ **correct**

3. $z - y^2 + 4 = 0$

4. $z + x^2 - 4 = 0$

5. $x - z^2 + 4 = 0$

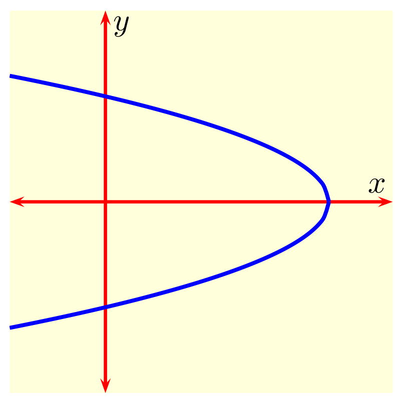
6. $y + z^2 - 4 = 0$

Explanation:

The graph is a parabolic cylinder that has constant value on any line parallel to the z-axis, so it will be the graph of an equation containing no z-term. This already eliminates the equations

$$y + z^2 - 4 = 0, \quad z - y^2 + 4 = 0,$$
$$z + x^2 - 4 = 0, \quad x - z^2 + 4 = 0.$$

On the other hand, the intersection of the graph with the xy-plane, *i.e.* the $z = 0$ plane, is a parabola opening to the left on the x-axis as shown in



Consequently, the graph is that of the equation

$$\boxed{x + y^2 - 4 = 0}.$$

keywords: quadric surface, graph of equation, cylinder, 3D graph, parabolic cylinder, trace

CalC15b16s
007 10.0 points

Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{7xy^4}{x^2 + y^8}$, if it exists.

1. 14

2. 7

3. 3.5

4. The limit does not exist. **correct**

5. 0

Explanation:

CalC15d11s
008 10.0 points

Find the linearization, $L(x, y)$, of

$$f(x, y) = y\sqrt{x}$$

at the point $(9, 1)$.

1. $L(x, y) = 3 - \frac{1}{6}x - \frac{3}{2}y$

2. $L(x, y) = \frac{3}{2} + 3x - \frac{1}{6}y$

3. $L(x, y) = 3 + \frac{3}{2}x - \frac{1}{6}y$

4. $L(x, y) = -\frac{3}{2} + \frac{1}{6}x + 3y$ **correct**

5. $L(x, y) = -\frac{3}{2} + 3x + \frac{1}{6}y$

6. $L(x, y) = \frac{3}{2} - \frac{1}{6}x + 3y$

Explanation:
The linearization of $f = f(x, y)$ at a point (a, b) is given by

$$L(x, y) = f(a, b)+(x-a)\frac{\partial f}{\partial x}\Big|_{(a,b)}+(y-b)\frac{\partial f}{\partial y}\Big|_{(a,b)}.$$

But when $f(x, y) = y\sqrt{x}$,

$$\frac{\partial f}{\partial x} = \frac{y}{2\sqrt{x}}, \qquad \frac{\partial f}{\partial y} = \sqrt{x};$$

thus when $(a, b) = (9, 1)$,

$$\frac{\partial f}{\partial x}\Big|_{(a,b)} = \frac{1}{6}, \qquad \frac{\partial f}{\partial y}\Big|_{(a,b)} = 3,$$

while $f(a, b) = 3$. Consequently,

$$\boxed{L(x, y) = -\frac{3}{2} + \frac{1}{6}x + 3y}.$$

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keywords:

Tangent01a
009 10.0 points

If $\mathbf{r}(x)$ is the vector function whose graph is trace of the surface

$$z = f(x, y) = 2x^2 - y^2 - 3x + 2y$$

on the plane $y = 2x$, determine the tangent vector to $\mathbf{r}(x)$ at $x = 1$.

1. tangent vector $= \langle 2, 0, 1 \rangle$

2. tangent vector $= \langle 2, 2, -3 \rangle$

3. tangent vector $= \langle 2, 1, 1 \rangle$

4. tangent vector $= \langle 1, 2, 1 \rangle$

5. tangent vector $= \langle 1, 0, -3 \rangle$

6. tangent vector $= \langle 1, 2, -3 \rangle$ **correct**

Explanation:
The graph of

$$z = f(x, y) = 2x^2 - y^2 - 3x + 2y$$

is the set of all points

$$(x, y, f(x, y))$$

as x, y vary in 3-space. So the intersection of the surface with the plane $y = 2x$ is the set of all points

$$(x, 2x, f(x, 2x)), \quad -\infty < x < \infty.$$

But

$$f(x, 2x) = -2x^2 + x.$$

Thus the surface and the plane $y = 2x$ intersect in the graph of

$$\mathbf{r}(x) = \langle x, 2x, -2x^2 + x \rangle.$$

Now the tangent vector to the graph of $\mathbf{r}(x)$ is the derivative

$$\mathbf{r}'(x) = \langle 1, 2, -4x+1 \rangle .$$

Consequently, at $x = 1$ the graph of $\mathbf{r}(x)$ has

$$\boxed{\text{tangent vector} = \langle 1, 2, -3 \rangle} .$$

keywords:

CalC15e07s
010 10.0 points

Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ when

$$z = x^2 + 3xy + y^2 ,$$

and

$$x = 2s + 4t , \qquad y = st .$$

1. $\dfrac{\partial z}{\partial s} = 4x + 12y + 3xt + 2yt$

2. $\dfrac{\partial z}{\partial s} = 8x + 12y + 3xt + 2yt$

3. $\dfrac{\partial z}{\partial s} = 8x + 12y + 3xs + 2ys$

4. $\dfrac{\partial z}{\partial s} = 8x + 6y + 3xs + 2ys$

5. $\dfrac{\partial z}{\partial s} = 4x + 6y + 3xt + 2yt$ **correct**

6. $\dfrac{\partial z}{\partial s} = 4x + 6y + 3xs + 2ys$

Explanation:

By the Chain Rule for Partial Differentiation,

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} .$$

Now

$$\frac{\partial z}{\partial x} = 2x + 3y, \quad \frac{\partial x}{\partial s} = 2$$

while

$$\frac{\partial z}{\partial y} = 3x + 2y, \quad \frac{\partial y}{\partial s} = t .$$

Thus

$$\frac{\partial z}{\partial s} = 2(2x + 3y) + t(+3x + 2y) .$$

Consequently,

$$\boxed{\frac{\partial z}{\partial s} = 4x + 6y + 3xt + 2yt} .$$

CalC15f11s
011 10.0 points

Find the directional derivative, $f_{\mathbf{v}}$, of the function

$$f(x, y) = 7 + 2x\sqrt{y}$$

at the point $P(1, 9)$ in the direction of the vector

$$\mathbf{v} = \langle 3, 4 \rangle .$$

1. $f_{\mathbf{v}} = \dfrac{58}{15}$ **correct**

2. $f_{\mathbf{v}} = 4$

3. $f_{\mathbf{v}} = \dfrac{61}{15}$

4. $f_{\mathbf{v}} = \dfrac{59}{15}$

5. $f_{\mathbf{v}} = \dfrac{62}{15}$

Explanation:

Now for an arbitrary vector $\mathbf{v}$,

$$f_{\mathbf{v}} = \nabla f \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) ,$$

where we have normalized so that the direction vector has unit length. But when

$$f(x, y) = 7 + 2x\sqrt{y} ,$$

then

$$\nabla f = (2\sqrt{y})\,\mathbf{i} + \left(\frac{x}{\sqrt{y}}\right)\mathbf{j} .$$

At $P(1, 9)$, therefore,

$$\nabla f\Big|_P = 6\,\mathbf{i} + \frac{1}{3}\mathbf{j} .$$

Consequently, when $\mathbf{v} = \langle 3, 4 \rangle$,

$$\boxed{f_{\mathbf{v}}(1, 9) = \left\langle 6, \frac{1}{3} \right\rangle \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = \frac{58}{15}} .$$

keywords: