This print-out should have 12 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixVecProd04 001 10.0 points

Determine $\mathbf{v}\mathbf{u}^T$ when

$$\mathbf{u} = \begin{bmatrix} -3\\2\\-5 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} a\\b\\c \end{bmatrix}.$$

1.
$$\mathbf{v}\mathbf{u}^T = \begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$$

2.
$$\mathbf{v}\mathbf{u}^T = \begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$$

3.
$$\mathbf{v}\mathbf{u}^T = -5a + 2b - 3c$$

4.
$$\mathbf{v}\mathbf{u}^T = -3a + 2b - 5c$$

InverseMatrix01a 002 10.0 points

Solve for X when AX + B = C,

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}.$$

Find L in an LU decomposition of

$$A = \begin{bmatrix} 5 & 0 & 0 & 1 \\ 20 & 0 & 5 & 2 \\ 20 & 0 & 10 & -2 \end{bmatrix}.$$

1.
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -4 & -2 & 1 \end{bmatrix}$$

2.
$$L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}$$

$$3. \ L = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

4.
$$L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$\mathbf{5.} \ \ L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

6.
$$L = \begin{bmatrix} -1 & 0 & 0 \\ 4 & -1 & 0 \\ 4 & 2 & -1 \end{bmatrix}$$

Let H be the set of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

where a and b are real. Determine if H is a subspace of \mathbb{R}^3 , and then check the correct answer below.

- 1. H is not a subspace of \mathbb{R}^3 because it does not contain 0.
- **2.** *H* is a subspace of \mathbb{R}^3 because it can be written as Nul(A) for some matrix *A*.
- **3.** H is not a subspace of \mathbb{R}^3 because it is not closed under vector addition.
- **4.** *H* is a subspace of \mathbb{R}^3 because it can be written as $Span\{\mathbf{v}_1, \mathbf{v}_2\}$ with $\mathbf{v}_1, \mathbf{v}_2$ in \mathbb{R}^3 .

$\begin{array}{cc} Invertible 01/02 \\ 005 & 10.0 \ points \end{array}$

A is an $n \times n$ matrix. Which of the following statements are equivalent to A being invertible?

- (i) The linear transformation $\mathbf{x} \to A\mathbf{x}$ is not one-to-one.
- (ii) A is not row equivalent to the $n \times n$ identity matrix.
- (iii) The columns of A do not form a basis of \mathbb{R}^n .
- 1. ii and iii
- 2. i and iii
- **3.** i
- 4. None of these
- **5.** All of these
- **6.** i and ii

1.
$$volume = 12$$

2. volume =
$$10$$

3.
$$volume = 9$$

4. volume =
$$11$$

5.
$$volume = 8$$

Rank02c 006 10.0 points

Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 6 & 1 \\ -3 & 6 & 0 \end{bmatrix}.$$

1.
$$rank(A) = 4$$

2.
$$rank(A) = 3$$

3.
$$rank(A) = 1$$

4.
$$rank(A) = 5$$

5.
$$\operatorname{rank}(A) = 2$$

$\begin{array}{cc} Basis Nul 01a \\ 008 & 10.0 \ points \end{array}$

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 3 & 6 & -3 & -6 \\ 1 & 2 & -4 & 4 \\ 1 & 2 & 0 & -6 \end{bmatrix}.$$

DetVolume01a 007 10.0 points

Compute the volume of the parallelepiped with adjacent edges \overline{OP} , \overline{OQ} , and \overline{OR} determined by vertices

$$P(4, -4, -4)$$
, $Q(2, -4, -3)$, $R(2, 2, 1)$,

where O is the origin in 3-space.

In the vector space V of all real-valued functions, find a basis for the subspace

 $H = \operatorname{Span}\{\sin t, \sin 2t, \sin t \cos t\}.$

- 1. $\{\cos t, \sin 2t\}$
- **2.** $\{ \sin t, \sin 2t \}$
- 3. $\{\sin 2t, \sin t \cos t\}$
- 4. $\{\sin t, \sin 2t, \sin t \cos t\}$
- 5. $\{\cos t, \sin 2t, \sin t \cos t\}$

PolyCoordVec01a 010 10.0 points

Find the coordinate vector $[\mathbf{p}]_{\mathcal{B}}$ in \mathbb{R}^3 for the polynomial

$$\mathbf{p}(t) = 1 + 4t + 7t^2$$

with respect to the basis

$$\mathcal{B} = \left\{1 + t^2, \ t + t^2, \ 1 + 2t + t^2\right\}$$

for \mathbb{P}_2 .

$$\mathbf{1.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$$

$$\mathbf{2.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -2\\ -6\\ 1 \end{bmatrix}$$

$$\mathbf{3.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix}$$

4.
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix}$$

5.
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -1\\2\\6 \end{bmatrix}$$

6.
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -6\\1\\2 \end{bmatrix}$$

ChangeBasis01b 011 10.0 points

Determine the change of coordinates matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$ from basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ to $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ of a vector space V when

$$\mathbf{b}_1 = 6\mathbf{c}_1 - 2\mathbf{c}_2, \quad \mathbf{b}_2 = 9\mathbf{c}_1 - 4c_2.$$