This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

### MatrixOpsTF01b 001 10.0 points

When

$$A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_n]$$

and B are matrices such that the product AB is defined, then

$$AB = [B\mathbf{a}_1 \ B\mathbf{a}_2 \ \dots \ B\mathbf{a}_n].$$

True or False?

- 1. FALSE
- 2. TRUE

### InverseProp01a 002 10.0 points

Which of the following shows that if A is an invertible  $n \times n$  matrix and B is any  $n \times n$  matrix such that BA = I, then  $B = A^{-1}$ .

1. Right-multiply each side of the equation I = BA by  $A^{-1}$ . Then

$$A^{-1} = BAA^{-1} = BI = B,$$

so 
$$B = A^{-1}$$
.

**2.** Add  $A^{-1}$  to both sides of the equation I = BA. Then

$$I + A^{-1} = BA + A^{-1},$$

so 
$$A^{-1} = BI = B$$
.

**3.** Left-multiply each side of the equation I = BA by  $A^{-1}$ . Then

$$A^{-1} = A^{-1}BA = BI = B,$$

so 
$$B = A^{-1}$$
.

**4.** Subtract  $A^{-1}$  from both sides of the equation I = BA. Then

$$I - A^{-1} = BA - A^{-1}$$
.

so 
$$A^{-1} = BI = B$$
.

## $\begin{array}{cc} InverseTF02a \\ 003 & 10.0 \ points \end{array}$

If A is an  $n \times n$  invertible matrix, then the same sequence of elementary row operations that row reduces A to the identity  $I_n$  also reduces  $A^{-1}$  to  $I_n$ .

True or False?

- 1. FALSE
- 2. TRUE

- **5.** ii
- **6.** i and ii

## LUDecomp2x3b 005 10.0 points

Determine the Lower Triangular matrix L in an LU-Decomposition of

$$A = \begin{bmatrix} -4 & 0 & 3 \\ -8 & 0 & 11 \end{bmatrix}.$$

$$\mathbf{1.} L = \begin{bmatrix} -4 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\mathbf{2.} L = \begin{bmatrix} -4 & 0 \\ 8 & 5 \end{bmatrix}$$

$$\mathbf{3.}\ L\ =\ \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\mathbf{4.} \ L \ = \ \begin{bmatrix} -4 & 0 \\ -2 & 5 \end{bmatrix}$$

$$\mathbf{5.} \ L \ = \ \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\mathbf{6.} \ L \ = \ \begin{bmatrix} -4 & 0 \\ -8 & 5 \end{bmatrix}$$

# $\begin{array}{cc} Invertible 01 \\ 004 & 10.0 \ points \end{array}$

A is an  $n \times n$  matrix. Which of the following statements are equivalent to A being invertible?

- (i)  $A^T$  is an invertible matrix.
- (ii) The columns of A do not span  $\mathbb{R}^n$ .
- (iii) A is row equivalent to the  $n \times n$  identity matrix.
- 1. None of these.
- 2. ii and iii
- **3.** iii
- 4. i and iii

#### NullSpace01a 006 10.0 points

Find a matrix A so that Nul(A) is the set of all vectors

$$H = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{array}{c} a+3b = 2c, \\ 4a = c-d, \end{array} \right\}$$

in  $\mathbb{R}^4$ .

1. 
$$A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 4 & 0 & -1 & 1 \end{bmatrix}$$

**2.** 
$$A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ 4 & 0 & -1 & 1 \end{bmatrix}$$

**3.** 
$$A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ 4 & 0 & 1 & -1 \end{bmatrix}$$

**4.** 
$$A = \begin{bmatrix} 1 & -3 & -2 & 0 \\ 4 & 0 & 1 & -1 \end{bmatrix}$$

$$\mathbf{5.} \ A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 4 & 0 & -1 & -1 \end{bmatrix}$$

**6.** 
$$A = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 4 & 0 & 1 & -1 \end{bmatrix}$$

# $\begin{array}{c} Rank02b \\ 007 \quad 10.0 \ points \end{array}$

Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -3 & -5 & 7 \\ 1 & 5 & 1 \end{bmatrix}.$$

$$1. \operatorname{rank}(A) = 2$$

$$2. rank(A) = 5$$

3. 
$$\operatorname{rank}(A) = 4$$

**4.** 
$$rank(A) = 1$$

5. 
$$rank(A) = 3$$

#### 009 10.0 points

The matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & -4 \\ 1 & 0 & 2 \end{bmatrix}$$

is invertible.

True or False?

- 1. TRUE
- 2. FALSE

#### DetElemOps01TF 008 10.0 points

When the matrix

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is obtained from

$$A = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

by interchanging rows, then

$$det[B] = det[A]$$
.

True or False?

- 1. FALSE
- 2. TRUE

# $\begin{array}{cc} {\bf Subspace TF01} \\ {\bf 010} & {\bf 10.0 \ points} \end{array}$

Let H be the set of points inside and on the unit circle in the xy-plane. That is, let  $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \le 1 \right\}$ .

H is a subspace of  $\mathbb{R}^2$ . True or false?

- 1. FALSE
- 2. TRUE

#### BasisNull02a 011 10.0 points

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 3 & 6 & 24 & -12 \\ -3 & -7 & -27 & 13 \\ 1 & -1 & -1 & -1 \end{bmatrix}.$$

$$\mathbf{1.} \left\{ \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$2. \left\{ \begin{bmatrix} -2\\-1\\0\\1 \end{bmatrix} \right\}$$

$$\mathbf{3.} \left\{ \begin{bmatrix} 2\\3\\1\\0 \end{bmatrix}, \begin{bmatrix} -2\\-1\\0\\1 \end{bmatrix} \right\}$$

$$4. \left\{ \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$\mathbf{5.} \left\{ \begin{bmatrix} -2\\3\\1\\0 \end{bmatrix}, \begin{bmatrix} 2\\-1\\0\\1 \end{bmatrix} \right\}$$

$$6. \left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

### $\begin{array}{cc} LinIndSetsTF01b \\ 012 & 10.0 \ points \end{array}$

When  $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p$  are vectors in  $\mathbb{R}^n$  and

$$H = \operatorname{Span}\{\mathbf{b}_1, \, \mathbf{b}_2, \, \dots, \, \mathbf{b}_p\},\,$$

then  $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}$  is a basis for H. True or False?

- 1. TRUE
- 2. FALSE

#### CoordVec01a 013 10.0 points

Find the vector  $\mathbf{x}$  in  $\mathbb{R}^2$  having coordinate vector

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 4\\5 \end{bmatrix}, \begin{bmatrix} 6\\7 \end{bmatrix} \right\}$$

for  $\mathbb{R}^2$ .

1. 
$$\mathbf{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

**2.** 
$$\mathbf{x} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$$

3. 
$$\mathbf{x} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

4. no such x exists

$$\mathbf{5.} \ \mathbf{x} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

Determine the dimension of the subspace

$$\operatorname{Span}\left\{ \begin{bmatrix} 1\\0\\2 \end{bmatrix}, \begin{bmatrix} 3\\1\\1 \end{bmatrix}, \begin{bmatrix} 9\\4\\-2 \end{bmatrix}, \begin{bmatrix} -7\\-3\\1 \end{bmatrix} \right\}$$

of  $\mathbb{R}^3$ .

- 1.  $\dim = 5$
- **2.** dim = 2
- **3.**  $\dim = 3$
- **4.**  $\dim = 4$
- **5.**  $\dim = 1$

# $\begin{array}{cc} RankTF06b \\ 015 & 10.0 \ points \end{array}$

If B is an echelon form of an an  $m \times n$  matrix A, and if B has three nonzero rows, then the first three row of A form a basis for Row(A).

True or False?

- 1. FALSE
- 2. TRUE