

#### Matrices and Matrix Calculations - Spring 2017

Midterm Exam I, February 16, 2017

In all multiple choice problems you don't have to show your work. In all non-multiple choice problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

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This print-out should have 17 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

#### FinM4a24 001 10.0 points

Pandit is an aging dog who has to be kept on a strict diet containing, among other things, 12.1 grams of protein and 7.4 grams of fat. Two dog foods are available to Pandit's owner.

Food A has 20% protein and 10% fat, while Food B has 7% protein and 8% fat.

How many grams of food A should Pandit's owner use in his diet?

- 1. # grams food A = 51
- 2. # grams food A = 53
- 3. # grams food A = 52
- 4. # grams food A = 49
- 5. # grams food A = 50

is the unique parabola passing through the points

$$(1, 14), (-1, 2), (-3, 6),$$

determine b.

- 1. b = 6
- **2.** b = 9
- **3.** b = 5
- **4.** b = 8
- **5.** b = 7

#### FitParabola01b 002 10.0 points

When the graph of the function

$$y = ax^2 + bx + c$$

- 1. TRUE
- 2. FALSE

### $\begin{array}{cc} Linear System T/F01a \\ 003 & 10.0 \ points \end{array}$

Elementary row operations on an augmented matrix can change the the solution set of the associated linear system.

True or False?

- 1. FALSE
- 2. TRUE

# $\begin{array}{cc} RowReduceMan 02a \\ 005 & 10.0 \ points \end{array}$

The augmented matrix of a linear system of equations has been reduced by row operations to

$$\begin{bmatrix} 1 & 2 & -3 & 7 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

- (a) Continue row operations to write the matrix in reduced row echelon form.
- (b) Then determine the solution set of the original system.

## $\begin{array}{cc} LinSysUniqueTF02\\004&10.0\ points\end{array}$

If a system of linear equations has no free variables, then it has a unique solution.

True or False?

### $\begin{array}{ccc} AxisIntersect01a \\ 006 & 10.0 \ points \end{array}$

When P is the plane in  $\mathbb{R}^3$  given in vector form by

$$\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + t \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix},$$

determine where P intersects the z-axis.

1. 
$$z = 2$$

**2.** 
$$z = 3$$

#### 3. z = 4

**4.** 
$$z = 1$$

5. 
$$z = 5$$

## $\begin{array}{cc} M340LSpanM02\\007&10.0\ points \end{array}$

Given

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \ \mathbf{v_2} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$

determine all values of  $\lambda$  for which

$$\mathbf{w} = \begin{bmatrix} 3 \\ 2 \\ \lambda \end{bmatrix}$$

is a vector in  $Span\{v_1, v_2, v_3\}$ ?

- 1.  $\lambda = -1, 5$
- **2.**  $\lambda = 1$
- **3.**  $\lambda = 1, 5$
- 4.  $\lambda = 5$
- **5.**  $\lambda = -1$
- **6.**  $\lambda = -1, 1$

If  $\mathbf{u}, \mathbf{v}$  are vectors in  $\mathbb{R}^3$ , when can  $\mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$  be visualized as a plane through the origin in  $\mathbb{R}^3$ .

True or False?

- 1. NEVER
- 2. SOMETIMES
- **3.** ALWAYS

### $\begin{array}{cc} Consistent 01d \\ 009 & 10.0 \ points \end{array}$

Describe geometrically the conditions on a vector  ${\bf b}$  in  $\mathbb{R}^2$  under which the equation

$$\begin{bmatrix} -2 & -1 \\ 8 & 4 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

has a solution in  $\mathbb{R}^2$ .

- **1. b** lies on line y 4x = 0
- **2.** any **b** not on line y 4x = 0
- **3.** arbitrary **b** in  $\mathbb{R}^2$
- **4. b** lies on line y + 4x = 0
- 5. any b not on line y + 4x = 0

## $\begin{array}{cc} SolSetsLinSysTF03 \\ 011 & 10.0 \ points \end{array}$

If the equation  $A\mathbf{x} = \mathbf{b}$  has more than one solution, then so does the homogeneous equation  $A\mathbf{x} = \mathbf{0}$ .

True or False?

- 1. TRUE
- 2. FALSE

# MatEquTF02b 010 10.0 points

If the matrix equation  $A\mathbf{x} = \mathbf{b}$  is consistent, then  $\mathbf{b}$  is in the set spanned by the columns of A.

True or False?

- 1. FALSE
- 2. TRUE

## $\begin{array}{cc} Three Points 01a \\ 012 & 10.0 \ points \end{array}$

Determine the linear equation of the unique plane in  $\mathbb{R}^3$  containing the points

P(1, 2, 3), Q(3, 7, 6),

and

R(3, 3, 19).

1. 
$$77x + 26y - 8z = 1$$

**2.** 
$$77x + 26y + 8z = 1$$

3. 
$$77x - 26y - 8z = 1$$

**4.** 
$$77x - 26y + 8z + 1 = 0$$

**5.** 
$$77x - 26y - 8z + 1 = 0$$

**6.** 
$$77x + 26y + 8z + 1 = 0$$

#### BalChemEqt01a 013 10.0 points

When butane  $C_4H_{10}$  burns in the presence of oxygen  $O_2$  it produces carbon dioxide  $CO_2$ and water  $H_2O$ , represented chemically by

$$C_4H_{10} + O_2 \longrightarrow CO_2 + H_2O$$
.

If 35 molecules of water were produced in one particular reaction, how many molecules of butane were burned in that reaction?

- 1. # molecules = 8
- 2. # molecules = 7
- 3. # molecules = 4
- 4. # molecules = 5
- 5. # molecules = 6

## $\begin{array}{cc} LinIndependMan01a \\ 014 & 10.0 \ points \end{array}$

Find all values h for which the vectors

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$$

are linearly independent.

#### LinIndepTF01c 015 10.0 points

The columns of any  $4 \times 5$  matrix are linearly dependent.

True or False?

- 1. FALSE
- 2. TRUE

#### LinTransform01e 016 10.0 points

A transformation  $T:\mathbb{R}^n\to\mathbb{R}^m$  is linear if and only if

$$T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1T(\mathbf{v}_1) + c_2T(\mathbf{v}_2)$$

for all vectors  $\mathbf{v}_1$ ,  $\mathbf{v}_2$  in  $\mathbb{R}^n$  and all scalars  $c_1$ ,  $c_2$ .

True or False?

- 1. FALSE
- 2. TRUE

1. 
$$A = \frac{1}{2} \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$$

$$\mathbf{2.} \ A = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$$

**3.** 
$$A = \frac{1}{2} \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$

**4.** 
$$A = \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$$

**5.** 
$$A = \frac{1}{2} \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$$

**6.** 
$$A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$$

#### MatrixTrans01a 017 10.0 points

Determine the Standard Matrix for the transformation rotating the plane counter-clockwise about the origin through  $60^{\circ}$ .