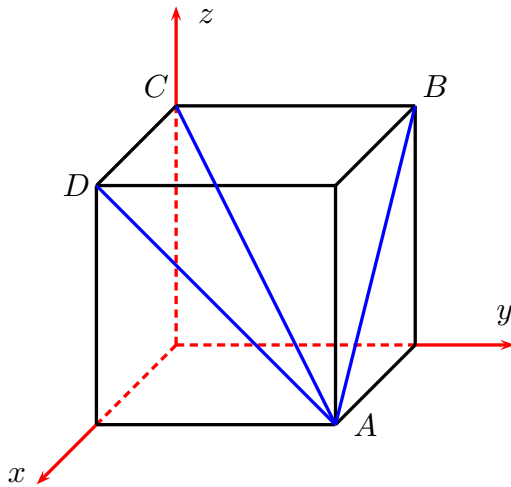


This print-out should have 30 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

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**CalC13c52a**  
**001 10.0 points**

The box shown in



is the unit cube having one corner at the origin and the coordinate planes for three of its adjacent faces.

Determine the projection of  $\overrightarrow{AD}$  on  $\overrightarrow{AC}$ .

1. projection =  $-\frac{1}{2}(\mathbf{i} - \mathbf{k})$
2. projection =  $-\frac{2}{3}(\mathbf{i} + \mathbf{j} - \mathbf{k})$  **correct**
3. projection =  $-\frac{1}{2}(\mathbf{j} - \mathbf{k})$
4. projection =  $\frac{1}{2}(\mathbf{i} - \mathbf{k})$
5. projection =  $\frac{2}{3}(\mathbf{i} + \mathbf{j} - \mathbf{k})$
6. projection =  $\frac{1}{2}(\mathbf{j} - \mathbf{k})$

**Explanation:**

The projection of a vector  $\mathbf{b}$  onto a vector  $\mathbf{a}$  is given in terms of the dot product by

$$\text{proj}_{\mathbf{a}} \mathbf{b} = \left( \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right) \mathbf{a}.$$

On the other hand, since the unit cube has side-length 1,

$$\mathbf{A} = (1, 1, 0), \quad \mathbf{C} = (0, 0, 1),$$

while  $\mathbf{D} = (1, 0, 1)$ . In this case  $\overrightarrow{AC}$  is a directed line segment determining the vector

$$\mathbf{a} = \langle -1, -1, 1 \rangle = -\mathbf{i} - \mathbf{j} + \mathbf{k},$$

while  $\overrightarrow{AD}$  determines the vector

$$\mathbf{b} = \langle 0, -1, 1 \rangle = -\mathbf{j} + \mathbf{k}.$$

For these choices of  $\mathbf{a}$  and  $\mathbf{b}$ ,

$$\mathbf{a} \cdot \mathbf{b} = 2, \quad \|\mathbf{a}\|^2 = 3.$$

Consequently, the projection of  $\overrightarrow{AD}$  onto  $\overrightarrow{AC}$  is given by

$$\text{proj}_{\mathbf{a}} \mathbf{b} = -\frac{2}{3}(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

---

keywords: projection, dot product, unit cube, component,

---

**CalC13d04a**  
**002 10.0 points**

Which of the following expressions are well-defined for all vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ , and  $\mathbf{d}$ ?

- I  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ ,
- II  $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$ ,
- III  $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$ .

1. III only
2. I and III only **correct**
3. none of them
4. II and III only
5. II only

6. I and II only

7. all of them

8. I only

**Explanation:**

The cross product is defined only for two vectors, and its value is a vector; on the other hand, the dot product is defined only for two vectors, and its value is a scalar.

For the three given expressions, therefore, we see that

I is well-defined because it is the cross product of two vectors.

II is not well-defined because the second term in the cross product is a dot product, hence not a vector.

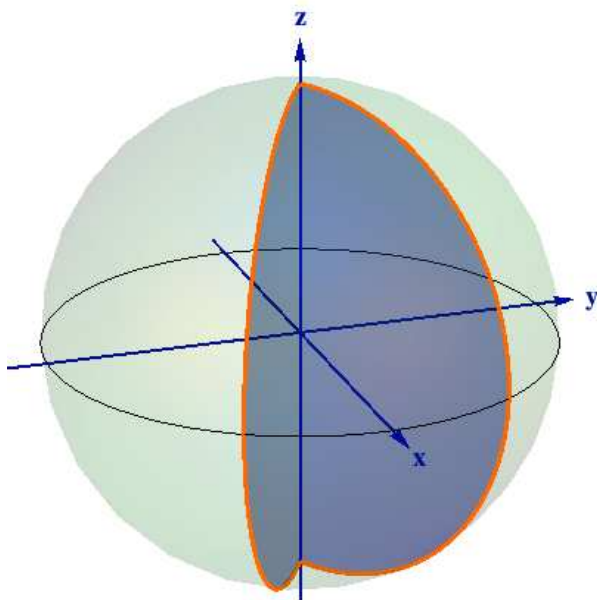
III is well-defined because both terms in the dot product are cross products, hence vectors.

keywords: vectors, dot product, cross product, T/F, length,

---

**SphericalCoords05a**  
**003 10.0 points**

The spine of the ‘math taco’  $T$  shown in



lies on the  $z$ -axis, while the faces lie in the planes  $y = \pm(\tan \alpha)x$  for fixed  $\alpha$ .

Use spherical polar coordinates to describe  $T$  as a set of points  $P(\rho, \theta, \phi)$  when the taco has radius 3.

1.  $T = \{P(\rho, \theta, \phi)\}$  with

$$0 \leq \rho \leq 6, \quad \theta = \pm\alpha, \quad 0 \leq \phi \leq \pi.$$

2.  $T = \{P(\rho, \theta, \phi)\}$  with

$$0 \leq \rho \leq 3, \quad \theta = \pm\alpha, \quad 0 \leq \phi \leq \pi.$$

correct

3.  $T = \{P(\rho, \theta, \phi)\}$  with

$$0 \leq \rho \leq 6, \quad 0 \leq \theta \leq \frac{\pi}{2}, \quad \phi = \pm\alpha.$$

4.  $T = \{(\rho, \theta, \phi)\}$  with

$$0 \leq \rho \leq 6, \quad \theta = \pm\alpha, \quad 0 \leq \phi \leq \frac{\pi}{2}.$$

5.  $T = \{P(\rho, \theta, \phi)\}$  with

$$0 \leq \rho \leq 3, \quad 0 \leq \theta \leq \pi, \quad \phi = \pm\alpha.$$

6.  $T = \{P(\rho, \theta, \phi)\}$  with

$$0 \leq \rho \leq 3, \quad 0 \leq \theta \leq \frac{\pi}{2}, \quad \phi = \pm\alpha.$$

**Explanation:**

In spherical polar coordinates  $(\rho, \theta, \phi)$ ,

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta,$$

and

$$z = \rho \cos \phi,$$

with  $0 \leq \theta \leq 2\pi$  and  $0 \leq \psi \leq \pi$ . We need to find further restrictions on  $\rho$ ,  $\theta$ , and  $\phi$  so that

the taco shown has radius 3 and its faces lie in the planes

$$y = \pm(\tan \alpha)x.$$

But

$$\frac{y}{x} = \tan \theta = \pm \tan \alpha,$$

so  $\theta = \pm\alpha$ . On the other hand,  $z$  ranges from  $-3$  to  $3$ . Thus  $\rho$  ranges from  $0$  to  $3$ , while  $0 \leq \phi \leq \pi$ .

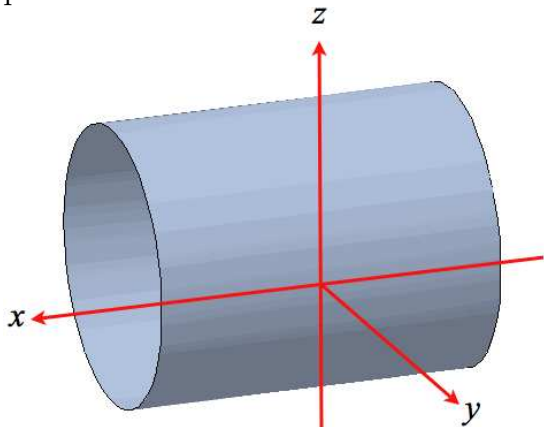
Consequently, the taco consists of all points  $P(\rho, \theta, \phi)$  with

$$0 \leq \rho \leq 3, \quad \theta = \pm\alpha, \quad 0 \leq \phi \leq \pi.$$

---

**CalC13f03d**  
**004 10.0 points**

Which one of the following equations has graph



when the circular cylinder has radius 2?

1.  $x^2 + y^2 - 4y = 0$
2.  $y^2 + z^2 - 4z = 0$
3.  $y^2 + z^2 + 4y = 0$  **correct**
4.  $y^2 + z^2 - 2z = 0$
5.  $x^2 + y^2 - 2y = 0$
6.  $y^2 + z^2 + 2y = 0$

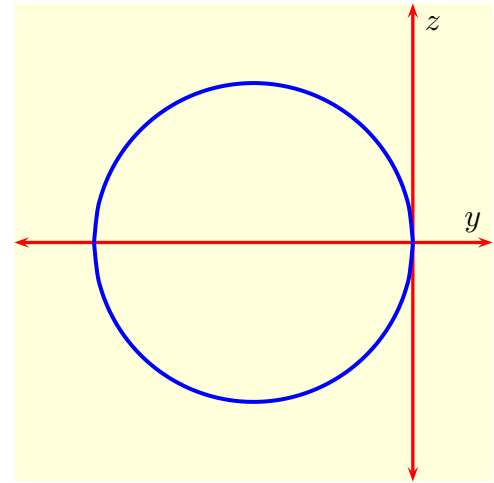
**Explanation:**

The graph is a circular cylinder whose axis of symmetry is parallel to the  $x$ -axis, so it

will be the graph of an equation containing no  $x$ -term. This already eliminates the equations

$$x^2 + y^2 - 2y = 0, \quad x^2 + y^2 - 4y = 0.$$

On the other hand, the intersection of the graph with the  $yz$ -plane, *i.e.* the  $x = 0$  plane, is a circle centered on the  $y$ -axis and passing through the origin as shown in



But this circle has radius 2 because the cylinder has radius 2, and so its equation is

$$(y + 2)^2 + z^2 = 4$$

as a circle in the  $yz$ -plane.

Consequently, the graph is that of the equation

$$y^2 + z^2 + 4y = 0.$$

---

keywords: quadric surface, graph of equation, cylinder, 3D graph, circular cylinder, trace

---

**CalC15b19s**  
**005 10.0 points**

Find  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + 4yz^2 + 2xz^2}{x^2 + y^2 + z^4}$ , if it exists.

1. 4

2. 6

3. 0

4. The limit does not exist. **correct**

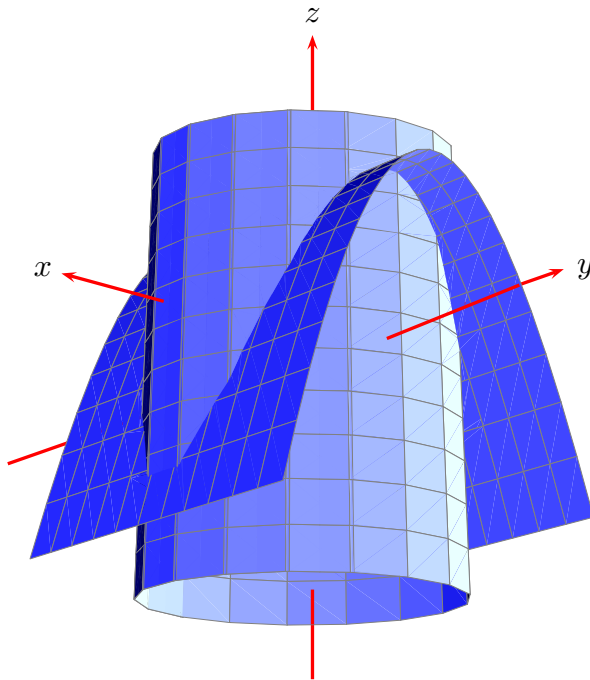
5. 2

**Explanation:**


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**Intersection01a**  
**006    10.0 points**

The curve of intersection of the surfaces shown in



is the graph of which of the following vector functions?

1.  $\mathbf{r}(t) = \langle \sin t, \cos t, \cos 2t - 1 \rangle$
2.  $\mathbf{r}(t) = \langle \sin t, \cos t, 1 - \cos 2t \rangle$
3.  $\mathbf{r}(t) = \langle \cos t, \sin t, 1 - \cos 2t \rangle$
4.  $\mathbf{r}(t) = \langle \sin t, \cos t, \cos 2t \rangle$  **correct**
5.  $\mathbf{r}(t) = \langle \cos t, \sin t, \cos 2t \rangle$
6.  $\mathbf{r}(t) = \langle \cos t, \sin t, \cos 2t - 1 \rangle$

**Explanation:**

If we write

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle,$$

then each of the given vector functions has the property that

$$x(t)^2 + y(t)^2 = 1.$$

This is consistent with the fact that the curve of intersection lies on a circular cylinder

$$x^2 + y^2 = 1$$

with the  $z$ -axis the line of symmetry as is shown in the figure. Thus we need to look more carefully at the vector functions to determine which lie on the parabolic cylinder shown in the figure.

Recall first that

$$\cos 2\theta = 1 - 2\sin^2 \theta = 2\cos^2 \theta - 1.$$

Now the cross-sections of the parabolic cylinder perpendicular to the  $y$ -axis are parabolas *opening downwards* with *vertex above* the  $y$ -axis, so the parabolic cylinder is the graph of

$$z = a - bx^2, \quad a, b > 0.$$

As a result, the components of  $\mathbf{r}(t)$  have to satisfy an equation

$$z(t) = a - bx(t)^2, \quad a, b > 0.$$

Consequently, the curve of intersection of the two surfaces is the graph of the vector function

$$\mathbf{r}(t) = \langle \sin t, \cos t, \cos 2t \rangle.$$

---

keywords: surface, space curve, parametric equation, 3D graph, circular cylinder, paraboloid,

---

**CalC15e21s**  
**007    10.0 points**

Use the Chain Rule to find the partial derivative  $\frac{\partial w}{\partial s}$  for

$$w = x^2 + y^2 + z^2, \quad x = st,$$

$$y = s \cos t, \quad z = s \sin t$$

when  $s = 4, t = 0$ .

$$1. \frac{\partial w}{\partial s} = 8 \text{ correct}$$

$$2. \frac{\partial w}{\partial s} = 11$$

$$3. \frac{\partial w}{\partial s} = 12$$

$$4. \frac{\partial w}{\partial s} = 7$$

$$5. \frac{\partial w}{\partial s} = 10$$

**Explanation:**

By the Chain Rule for Partial Differentiation

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial s}.$$

Here, we have

$$\frac{\partial w}{\partial x} = 2x, \quad \frac{\partial x}{\partial s} = t$$

while

$$\frac{\partial w}{\partial y} = 2y, \quad \frac{\partial y}{\partial s} = \cos t$$

and

$$\frac{\partial w}{\partial z} = 2z, \quad \frac{\partial z}{\partial s} = \sin t.$$

Thus

$$\frac{\partial w}{\partial s} = 2xt + 2y \cos t + 2z \sin t.$$

Note that when  $s = 4$  and  $t = 0$ , it follows that  $x = 0, y = 4, z = 0$ . Consequently, for these values,

$$\boxed{\frac{\partial w}{\partial s} = 8}.$$

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keywords:

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CalC15f19s  
008 10.0 points

Find the directional derivative,  $f_{\mathbf{v}}$ , of

$$f(x, y) = 5\left(\frac{y}{x}\right)^{1/2}$$

at  $P = (2, 2)$  in the direction of the vector  $\overrightarrow{PQ}$  when  $Q = (6, 5)$ .

$$1. f_{\mathbf{v}} = -\frac{1}{4} \text{ correct}$$

$$2. f_{\mathbf{v}} = -\frac{1}{5}$$

$$3. f_{\mathbf{v}} = -\frac{3}{10}$$

$$4. f_{\mathbf{v}} = -\frac{7}{20}$$

$$5. f_{\mathbf{v}} = -\frac{2}{5}$$

**Explanation:**

The directional derivative of  $f(x, y)$  at  $P$  in the direction of  $\mathbf{v} = \overrightarrow{PQ}$  is given by the dot product

$$f_{\mathbf{v}}|_P = \nabla f|_P \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right).$$

But when

$$f(x, y) = 5\left(\frac{y}{x}\right)^{1/2},$$

we see that

$$\frac{\partial f}{\partial x} = -\frac{5}{2}\left(\frac{y}{x^3}\right)^{1/2}, \quad \frac{\partial f}{\partial y} = \frac{5}{2}\left(\frac{1}{xy}\right)^{1/2},$$

so that at  $P(2, 2)$ ,

$$(\nabla f)(2, 2) = -\frac{5}{4}\mathbf{i} + \frac{5}{4}\mathbf{j} = \left\langle -\frac{5}{4}, \frac{5}{4} \right\rangle.$$

On the other hand,  $\mathbf{v} = \langle 4, 3 \rangle$  which as a vector of unit length becomes

$$\frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle.$$

Consequently,

$$\nabla f|_P = \left\langle -\frac{5}{4}, \frac{5}{4} \right\rangle \cdot \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle = -\frac{1}{4}.$$

---

**CalC15f39s**

**009 10.0 points**

Find the equation of the tangent plane to the surface

$$4x^2 + 5y^2 + 5z^2 = 66$$

at the point  $(2, -1, 3)$ .

1.  $4x - 5y + 5z = 66$

2.  $8x + 5y + 15z = 66$

3.  $8x + 5y + 15z = 56$

4.  $8x - 5y + 15z = 66$  **correct**

5.  $8x - 5y + 15z = 56$

**Explanation:**

Let

$$F(x) = 4x^2 + 5y^2 + 5z^2.$$

The equation to the tangent plane to the surface at the point  $P(2, -1, 3)$  is given by

$$F_x|_P(x-2) + F_y|_P(y+1) + F_z|_P(z-3) = 0.$$

Since

$$F_x = 8x, \quad F_x|_P = 16,$$

$$F_y = 10y, \quad F_y|_P = -10,$$

and

$$F_z = 10z, \quad F_z|_P = 30$$

it follows that the equation of the tangent plane is

$$8x - 5y + 15z = 66.$$

keywords:

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**QuadApprox04a**  
**010 10.0 points**

Find the quadratic approximation to

$$f(x, y) = e^{-2x-y^2}$$

at  $P(0, 0)$ .

1.  $Q(x, y) = 1 - 2x + 2x^2 - y^2$  **correct**

2.  $Q(x, y) = 1 - x + 2x^2 - y^2$

3.  $Q(x, y) = 1 - y - xy + 2y^2$

4.  $Q(x, y) = 1 - 2x + 2x^2 + y^2$

5.  $Q(x, y) = 1 + x + 2x^2 + y^2$

6.  $Q(x, y) = 1 + 2x + 2xy - y^2$

**Explanation:**

The Quadratic Approximation to  $f(x, y)$  at  $P(0, 0)$  is given by

$$Q(x, y) = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y + \frac{1}{2}f_{xx}(0, 0)x^2 + f_{xy}(0, 0)xy + \frac{1}{2}f_{yy}(0, 0)y^2.$$

But when

$$f(x, y) = e^{-2x-y^2}$$

we see that

$$f_x = -2e^{-2x-y^2}, \quad f_y = -2ye^{-2x-y^2},$$

so that  $f(0, 0) = 1$  and

$$f_x(0, 0) = -2, \quad f_y(0, 0) = 0,$$

while

$$f_{xx} = 4e^{-2x-y^2}, \quad f_{xy} = 4ye^{-2x-y^2},$$

and

$$f_{yy} = -2e^{-2x-y^2} + 4y^2e^{-2x-y^2},$$

so that

$$f_{xx}(0, 0) = 4, \quad f_{xy}(0, 0) = 0,$$

and  $f_{yy}(0, 0) = -2$ .

Consequently, the Quadratic Approximation to  $f$  at  $P(0, 0)$  is

$$Q(x, y) = 1 - 2x + 2x^2 - y^2.$$

keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

---

**CalC15g06a**  
**011 10.0 points**

Locate and classify all the local extrema of

$$f(x, y) = 2x^3 + 2y^3 + 6xy - 9.$$

1. local min at  $(-1, -1)$ , saddle at  $(0, 0)$
2. saddle at  $(-1, -1)$ , local max at  $(0, 0)$
3. local max at  $(1, 1)$ , saddle at  $(0, 0)$
4. local min at  $(0, 0)$ , local max at  $(-1, -1)$
5. local max at  $(-1, -1)$ , saddle at  $(0, 0)$   
**correct**

**Explanation:**

Local extrema occur at the critical points of  $f$ . Now after differentiation of  $f$  we obtain

$$f_x = 6(x^2 + y), \quad f_y = 6(y^2 + x).$$

The critical points of  $f$  are thus the common solutions of the equations

$$x^2 + y = 0, \quad y^2 + x = 0.$$

This yields only the two extremum points  $(-1, -1)$  and  $(0, 0)$ . But after differentiating again we see that

$$f_{xx} = 12x, \quad f_{xy} = 6, \quad f_{yy} = 12y;$$

consequently,

$$f_{xx}f_{yy} - (f_{xy})^2 = (12)^2xy - 36.$$

Hence by the second derivative test there are

local max at  $(-1, -1)$ , saddle at  $(0, 0)$ .

---

**CalC15g28a**  
**012 10.0 points**

Find the absolute maximum value of the function

$$f(x, y) = 7 + xy - 2x - 3y$$

over the closed triangular region  $\mathcal{D}$  having vertices

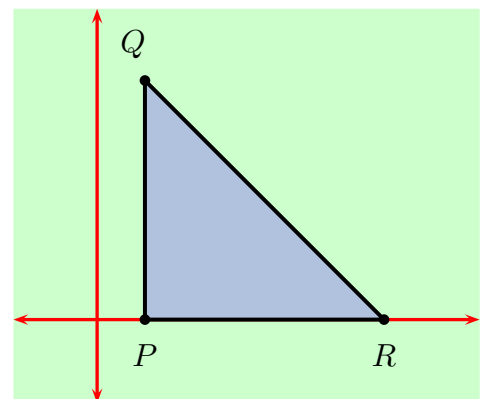
$$P(1, 0), \quad Q(1, 4), \quad R(5, 0).$$

1. abs max value = 4
2. abs max value = 8
3. abs max value = 6
4. abs max value = 7
5. abs max value = 9
6. abs max value = 5 **correct**

**Explanation:**

Since  $f$  is continuous everywhere its absolute maximum on  $\mathcal{D}$  will exist and be attained at a critical point of  $f$  inside  $\mathcal{D}$  or at a point on the boundary of  $\mathcal{D}$ .

Now  $\mathcal{D}$  is similar to the shaded region in



and its boundary consists of the line segments

$$L_1 = \overline{PQ}, \quad L_2 = \overline{QR}, \quad L_3 = \overline{PR}.$$

The critical points of  $f$  are the common solutions of the equations

$$f_x = y - 2 = 0, \quad f_y = x - 3 = 0.$$

Thus  $(3, 2)$  is the only critical point; it is inside  $\mathcal{D}$  and at this critical point

$$f(3, 2) = 1.$$

On the other hand,

- (i)  $L_1 = \{(1, y) : 0 \leq y \leq 4\},$
- (ii)  $L_2 = \{(x, 5 - x) : 1 \leq x \leq 5\},$
- (iii)  $L_3 = \{(x, 0) : 1 \leq x \leq 5\}.$

But on  $L_1$ ,

$$f(x, y) = 7 + y - 2 - 3y = 5 - 2y,$$

while on  $L_2$

$$\begin{aligned} f(x, y) &= 7 + x(5 - x) - 2x - 3(5 - x) \\ &= 7 - 2x - (3 - x)(5 - x), \end{aligned}$$

and on  $L_3$

$$f(x, y) = 7 - 2x.$$

Thus

- (i) on  $L_1$  abs max value of  $f$  is 5,
- (ii) on  $L_2$  abs max value of  $f$  is 1,
- (iii) on  $L_3$  abs max value of  $f$  is 5.

Consequently, taking the largest of

$$1, \quad 5, \quad 1, \quad 5,$$

we see that on  $\mathcal{D}$

$\text{abs max value of } f = 5$

---

keywords: partial differentiation, critical point, absolute extremum,

---

### CalC15h06b

**013    10.0 points**

Use Lagrange Multipliers to determine the maximum value of

$$f(x, y) = 6xy$$

subject to the constraint

$$g(x, y) = \frac{x^2}{4} + \frac{y^2}{9} - 1 = 0.$$

**1.** maximum = 18 **correct**

**2.** maximum = 16

**3.** maximum = 17

**4.** maximum = 19

**5.** maximum = 15

#### Explanation:

By the Method of Lagrange multipliers, the extreme values of  $f$  occur at the common solutions of

$$(\nabla f)(x, y) = \lambda(\nabla g)(x, y), \quad g(x, y) = 0.$$

Now

$$(\nabla f)(x, y) = \langle 6y, 6x \rangle,$$

while

$$(\nabla g)(x, y) = \left\langle \frac{1}{2}x, \frac{2}{9}y \right\rangle.$$

But then by the condition on  $\nabla f$  and  $\nabla g$ ,

$$6y = \frac{1}{2}\lambda x, \quad 6x = \frac{2}{9}\lambda y,$$

which after simplification gives

$$\lambda = \frac{12y}{x} = \frac{27x}{y}, \quad \text{i.e., } y = \pm \frac{3}{2}x.$$

Thus by the constraint equation,

$$g(x, \pm \frac{3}{2}x) = \frac{x^2}{4} + \frac{x^2}{4} - 1 = 0,$$



i.e.,  $x = \pm\sqrt{2}$ . Consequently, the extreme points are

$$\left(\sqrt{2}, \frac{3\sqrt{2}}{2}\right), \quad \left(\sqrt{2}, -\frac{3\sqrt{2}}{2}\right),$$

and

$$\left(-\sqrt{2}, -\frac{3\sqrt{2}}{2}\right), \quad \left(-\sqrt{2}, \frac{3\sqrt{2}}{2}\right).$$

Since

$$f\left(\sqrt{2}, \frac{3\sqrt{2}}{2}\right) = f\left(-\sqrt{2}, -\frac{3\sqrt{2}}{2}\right) = 18,$$

while

$$f\left(\sqrt{2}, -\frac{3\sqrt{2}}{2}\right) = f\left(-\sqrt{2}, \frac{3\sqrt{2}}{2}\right) = -18,$$

we thus see that  $f$  has

$$\text{max value} = 18$$

subject to the constraint  $g(x, y) = 0$ .

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keywords:

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**CalC14d05s**  
**014 10.0 points**

Find the velocity of a particle with the given position function

$$\mathbf{r}(t) = 6e^{9t}\mathbf{i} + 2e^{-10t}\mathbf{j}.$$

1.  $\mathbf{v}(t) = 6e^{9t}\mathbf{i} - 2e^{-10t}\mathbf{j}$
2.  $\mathbf{v}(t) = 54e^{9t}\mathbf{i} - 20e^{-10t}\mathbf{j}$  **correct**
3.  $\mathbf{v}(t) = 15e^{9t}\mathbf{i} - 12e^{-10t}\mathbf{j}$
4.  $\mathbf{v}(t) = 54e^t\mathbf{i} - 20e^{-t}\mathbf{j}$
5.  $\mathbf{v}(t) = 6e^{9t}\mathbf{i} + 20e^{-10t}\mathbf{j}$

**Explanation:**

The velocity of a particle with position function

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$$

is given by

$$\mathbf{r}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j}.$$

Thus, since

$$\mathbf{r}'(t) = 54e^{9t}\mathbf{i} - 20e^{-10t}\mathbf{j},$$

we have that

$$\mathbf{v}(t) = 54e^{9t}\mathbf{i} - 20e^{-10t}\mathbf{j}.$$

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keywords:

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**CalC14c01s**  
**015 10.0 points**

When  $C$  is parametrized by

$$\mathbf{c}(t) = (\sin 3t)\mathbf{i} + 4t\mathbf{j} + (\cos 3t)\mathbf{k},$$

find its arc length between  $\mathbf{c}(0)$  and  $\mathbf{c}(2)$ .

1. arc length = 6
2. arc length = 8
3. arc length = 12
4. arc length = 4
5. arc length = 10 **correct**

**Explanation:**

The length of the curve between  $\mathbf{c}(t_0)$  and  $\mathbf{c}(t_1)$  is given by the integral

$$L = \int_{t_0}^{t_1} \|\mathbf{c}'(t)\| dt.$$

Now when

$$\mathbf{c}(t) = (\sin 3t)\mathbf{i} + 4t\mathbf{j} + (\cos 3t)\mathbf{k},$$

we see that

$$\mathbf{c}'(t) = (3 \cos 3t)\mathbf{i} + 4\mathbf{j} - (3 \sin 3t)\mathbf{k}.$$

But then by the Pythagorean identity,

$$\|\mathbf{c}'(t)\| = (9 + 16)^{1/2} = 5.$$

Thus

$$L = \int_0^2 5 \, dt = \left[ 5t \right]_0^2.$$

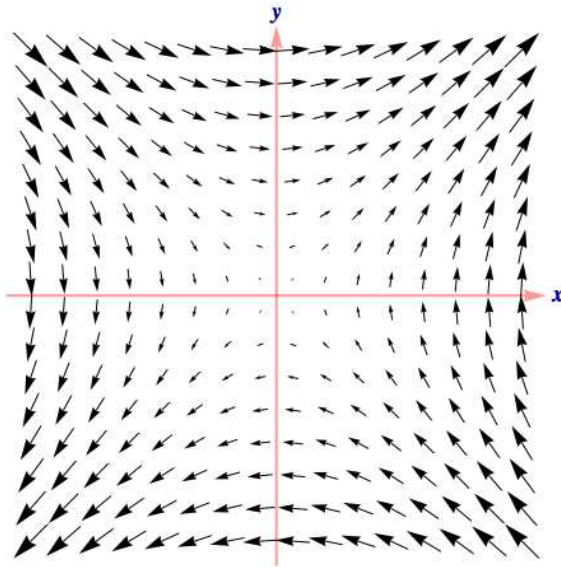
Consequently,

$$\boxed{\text{arc length} = L = 10}.$$

---

**VectorField01e**  
**016 10.0 points**

Which vector field  $\mathbf{F}$  has graph



1.  $\mathbf{F}(x, y) = x\mathbf{i} - y\mathbf{j}$
2.  $\mathbf{F}(x, y) = y\mathbf{i} - x\mathbf{j}$
3.  $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$  **correct**
4.  $\mathbf{F}(x, y) = -x\mathbf{i} + y\mathbf{j}$
5.  $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$
6.  $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$

**Explanation:**

We determine  $\mathbf{F}(x, y)$  by looking at a few points on the graph. Now on the  $x$ -axis,

$$\mathbf{F}(x, 0) = x\mathbf{j},$$

while on the  $y$ -axis,

$$\mathbf{F}(0, y) = y\mathbf{i}.$$

The only one of the given vector fields satisfying these conditions is

$$\boxed{\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}}.$$

---

**CalC16c16s**  
**017 10.0 points**

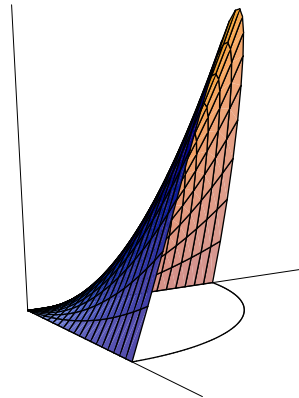
The graph of

$$f(x, y) = 4xy$$

over the bounded region  $A$  in the first quadrant enclosed by

$$y = \sqrt{4 - x^2}$$

and the  $x$ ,  $y$ -axes is the surface



Find the volume of the solid under this graph over the region  $A$ .

1. Volume =  $\frac{16}{3}$  cu. units
2. Volume = 4 cu. units
3. Volume = 8 cu. units **correct**
4. Volume = 2 cu. units
5. Volume = 16 cu. units

**Explanation:**

The volume of the solid under the graph of  $f$  is given by the double integral

$$V = \int \int_A f(x, y) dx dy,$$

which in turn can be written as the repeated integral

$$\int_0^2 \left( \int_0^{\sqrt{4-x^2}} 4xy dy \right) dx.$$

Now the inner integral is equal to

$$\left[ 2xy^2 \right]_0^{\sqrt{4-x^2}} = 2x(4-x^2).$$

Thus

$$V = 2 \int_0^2 x(4-x^2) dx = \left[ -\frac{1}{2}(4-x^2)^2 \right]_0^2.$$

Consequently,

Volume = 8 cu. units

---

**CalC16g07a**

**018 10.0 points**

Evaluate the triple integral

$$I = \int \int \int_E x dx dy dz$$

when  $E$  is the set of points  $(x, y, z)$  in 3-space such that

$$0 \leq x \leq \sqrt{4-y^2}, \quad 0 \leq z \leq y \leq 1.$$

1.  $I = \frac{7}{8}$  **correct**

2.  $I = \frac{9}{8}$

3.  $I = \frac{11}{8}$

4.  $I = \frac{13}{8}$

5.  $I = \frac{15}{8}$

**Explanation:**

As a repeated integral

$$I = \int_0^1 \left( \int_0^y \left( \int_0^{\sqrt{4-y^2}} x dx \right) dz \right) dy.$$

Now

$$\int_0^{\sqrt{4-y^2}} x dx = \left[ \frac{1}{2}x^2 \right]_0^{\sqrt{4-y^2}} = \frac{1}{2}(4-y^2),$$

while

$$\int_0^y \frac{1}{2}(4-y^2) dz = \frac{1}{2}(4-y^2)y.$$

Thus

$$I = \frac{1}{2} \int_0^1 (4y - y^3) dy = \frac{1}{2} \left[ 2y^2 - \frac{1}{4}y^4 \right]_0^1$$

Consequently,

$I = \frac{7}{8}$

---

**CalC16i04a**

**019 10.0 points**

Find the Jacobian of the transformation

$$T : (u, v) \longrightarrow (x, y)$$

when

$$x = 4u \sin v, \quad y = 2u \cos v.$$

1.  $\frac{\partial(x, y)}{\partial(u, v)} = -8u \sin v$

2.  $\frac{\partial(x, y)}{\partial(u, v)} = -8u$  **correct**

3.  $\frac{\partial(x, y)}{\partial(u, v)} = 6u \sin v \cos v$

4.  $\frac{\partial(x, y)}{\partial(u, v)} = 8u$

5.  $\frac{\partial(x, y)}{\partial(u, v)} = -6u$

$$6. \frac{\partial(x, y)}{\partial(u, v)} = 6u \cos v$$

**Explanation:**

For general functions the Jacobian of the transformation

$$T : (u, v) \longrightarrow (x, y)$$

is defined by

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}.$$

Now when

$$x = 4u \sin v, \quad y = 2v u \cos v,$$

then by the Product Rule,

$$x_u = 4 \sin v, \quad x_v = 4u \cos v,$$

while

$$y_u = 2 \cos v, \quad y_v = -2u \sin v.$$

In this case,

$$\frac{\partial(x, y)}{\partial(u, v)} = 8u \begin{vmatrix} \sin v & \cos v \\ \cos v & -\sin v \end{vmatrix}.$$

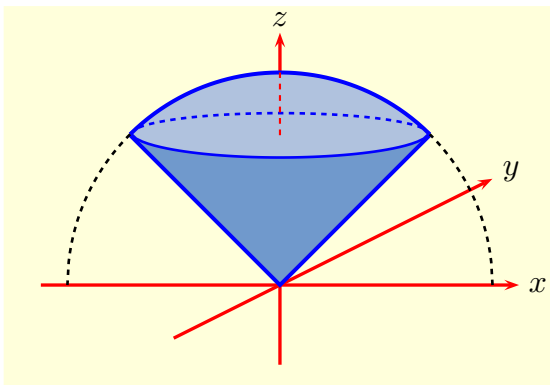
Consequently,

$$\boxed{\frac{\partial(x, y)}{\partial(u, v)} = -8v}.$$

---

**CalC16d25s**  
**020 10.0 points**

Use polar coordinates to find the volume of the solid shown in



above the cone

$$z = \sqrt{x^2 + y^2}$$

and below the sphere

$$x^2 + y^2 + z^2 = 4.$$

$$1. V = \frac{8\pi}{3}\sqrt{2}$$

$$2. V = \frac{16\pi}{3}(2 - \sqrt{2})$$

$$3. V = \frac{8\pi}{3}(2 - \sqrt{2}) \text{ correct}$$

$$4. V = \frac{4\pi}{3}\sqrt{2}$$

$$5. V = \frac{16\pi}{3}\sqrt{2}$$

$$6. V = \frac{4\pi}{3}(2 - \sqrt{2})$$

**Explanation:**

The cone and the sphere intersect when  $z = 2/\sqrt{2}$ ; in particular, the top of the cone is the disk formed by the a circle of radius  $2/\sqrt{2}$  and its interior, while the height of the cone is  $2/\sqrt{2}$ . The cone has volume

$$V_{\text{cone}} = \frac{1}{3}\pi \frac{8}{2\sqrt{2}}.$$

On the other hand, the volume of the cap of the sphere is given by

$$V_{\text{cap}} = \int \int_R \left( \sqrt{4 - x^2 - y^2} - \frac{2}{\sqrt{2}} \right) dx dy$$

where  $R$  is the region

$$\left\{ (x, y) : x^2 + y^2 \leq \left( \frac{2}{\sqrt{2}} \right)^2 \right\}.$$

Thus after changing to polar coordinates we see that the volume,  $V_{\text{cap}}$ , of the cap of the sphere is given by

$$\begin{aligned} & \int_0^{2/\sqrt{2}} \int_0^{2\pi} r \left( \sqrt{4 - r^2} - \frac{2}{\sqrt{2}} \right) d\theta dr \\ &= 2\pi \int_0^{2/\sqrt{2}} r \left( \sqrt{4 - r^2} - \frac{2}{\sqrt{2}} \right) dr. \end{aligned}$$

Thus

$$V_{cap} = \pi \left[ -\frac{2}{3}(4-r^2)^{3/2} - \frac{2r^2}{\sqrt{2}} \right]_0^{2/\sqrt{2}}.$$

Consequently, the volume of the solid is

$$\boxed{V_{cone} + V_{cap} = \frac{8\pi}{3}(2 - \sqrt{2})}.$$

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keywords:

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**CalC16i11a**  
**021 10.0 points**

Use the transformation  $T : (u, v) \rightarrow (x, y)$  with

$$x = \frac{1}{3}(u+v), \quad y = \frac{1}{3}(v-2u),$$

to evaluate the integral

$$I = \iint_D (3x - y) \, dx \, dy$$

when  $D$  is the region bounded by the lines

$$y = x, \quad y = x - 2$$

and

$$y + 2x = 0, \quad y + 2x = 3.$$

1.  $I = 6$
2.  $I = \frac{16}{3}$  **correct**
3.  $I = 5$
4.  $I = \frac{14}{3}$
5.  $I = \frac{17}{3}$

**Explanation:**

The Jacobian of the transformation

$$T : (u, v) \rightarrow \left( \frac{1}{3}(u+v), \frac{1}{3}(v-2u) \right)$$

is given by

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{1}{9} \begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = \frac{1}{3}.$$

On the other hand,

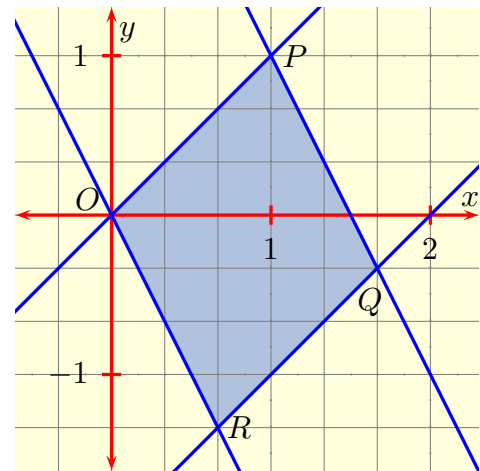
$$\begin{aligned} 3x - y &= (u+v) - \frac{1}{3}(v-2u) \\ &= \frac{2}{3}v + \frac{5}{3}u. \end{aligned}$$

Thus

$$I = \frac{1}{3} \iint_{\mathcal{D}} \left( \frac{2}{3}v + \frac{5}{3}u \right) \, du \, dv$$

where  $\mathcal{D}$  is the rectangle in the  $uv$ -plane that  $T$  maps onto  $D$ .

Now  $D$  is the shaded parallelogram shown in



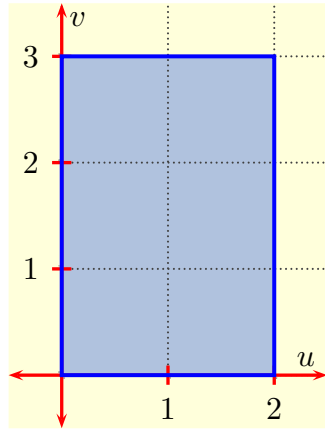
On the other hand, by solving for  $u, v$  in terms of  $x, y$  we see that

$$u = x - y, \quad v = 2x + y.$$

Thus  $T$  maps the lines

$$u = 0, \quad u = 2, \quad v = 0, \quad v = 3,$$

in the  $uv$ -plane to the lines enclosing  $D$ , so  $\mathcal{D}$  is the shaded rectangle in the  $uv$ -plane shown in



and

$$I = \frac{1}{3} \int_0^2 \left( \int_0^3 \left( \frac{2}{3}v + \frac{5}{3}u \right) dv \right) du.$$

Consequently,

$$\boxed{I = \frac{16}{3}}.$$

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keywords:

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**SphTripleInt01a**  
**022 10.0 points**

Use spherical coordinates to evaluate the integral

$$I = \iiint_B x^2 + y^2 + z^2 dV$$

when  $B$  is the ball

$$x^2 + y^2 + z^2 \leq 4.$$

1.  $I = \frac{32\pi}{3}$

2.  $I = \frac{128\pi}{5}$  **correct**

3.  $I = 16\pi$

4.  $I = 128\pi$

5.  $I = 32\pi$

**Explanation:**

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**ScalarLineInt03a**  
**023 10.0 points**

Evaluate the integral

$$I = \int_C x e^{yz} ds$$

when  $C$  is the line segment from  $(0, 0, 0)$  to  $(1, 2, 2)$ .

1.  $I = \frac{3}{4}e^2$

2.  $I = \frac{3}{4}(e^4 - 1)$

3.  $I = \frac{3}{4}(e - 1)$

4.  $I = \frac{3}{8}e^4$

5.  $I = \frac{3}{8}(e^4 - 1)$  **correct**

6.  $I = \frac{3}{8}e$

**Explanation:**

The line segment from  $(0, 0, 0)$  to  $(1, 2, 2)$  can be parametrized by

$$\mathbf{c}(t) = (0, 0, 0) + t(1, 2, 2) = (t, 2t, 2t)$$

with  $0 \leq t \leq 1$ . In this case

$$\|\mathbf{c}'(t)\| = \|\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}\| = 3,$$

while

$$x e^{yz} = t e^{4t^2}.$$

on  $C$ . Thus

$$I = \int_0^1 t e^{4t^2} \|\mathbf{c}'(t)\| dt = 3 \int_0^1 t e^{4t^2} dt.$$

To evaluate this last integral we use the substitution  $u = t^2$ . For then

$$du = 2t dt, \quad \frac{1}{2} du = t dt,$$

and so

$$I = \frac{3}{2} \int_0^1 e^{4u} du = \left[ \frac{3}{8} e^{4u} \right]_0^1.$$

Consequently,

$$I = \frac{3}{8}(e^4 - 1).$$

---

**LineIntegral01a**  
**024 10.0 points**

Evaluate the integral

$$I = \int_C (4xe^y dx - 3e^x dy)$$

when  $C$  is the parabola parametrized by

$$\mathbf{c}(t) = (t, t^2), \quad 0 \leq t \leq 1.$$

1.  $I = 2e - 8$  correct

2.  $I = 4e + 8$

3.  $I = 4e + 4$

4.  $I = 2e - 4$

5.  $I = 4e - 8$

6.  $I = 2e + 8$

**Explanation:**

When  $C$  is parametrized by

$$\mathbf{c}(t) = (x(t), y(t)) = (t, t^2),$$

then

$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 2t.$$

Thus on  $C$ ,

$$4xe^y dx = 4te^{t^2} dt, \quad 3e^x dy = 6te^t dt,$$

and so

$$I = \int_0^1 (4te^{t^2} - 6te^t) dt = I_1 + I_2.$$

To evaluate  $I_1$  we use the substitution  $u = t^2$ .  
For then,

$$I_1 = 2 \int_0^1 e^u du = 2e - 2.$$

On the other hand, to evaluate  $I_2$  we integrate by parts:

$$\begin{aligned} I_2 &= -6 \int_0^1 te^t dt \\ &= -6 \left[ te^t \right]_0^1 + 6 \int_0^1 e^t dt = -6 \left[ te^t - e^t \right]_0^1. \end{aligned}$$

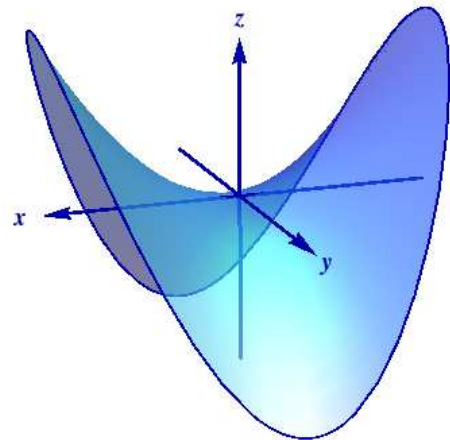
Consequently,

$$I = 2e - 2 - 6 = 2e - 8.$$

---

**SurfaceArea01a**  
**025 10.0 points**

The surface  $S$  shown in



is the portion of the graph of

$$z = f(x, y) = x^2 - y^2$$

lying inside the cylinder

$$x^2 + y^2 = 2$$

Determine the surface area of  $S$ .

1. Surface Area =  $\frac{13}{3}\pi$  sq. units **correct**

2. Surface Area =  $\frac{11}{3}\pi$  sq. units

3. Surface Area =  $\frac{10}{3}\pi$  sq. units

4. Surface Area =  $3\pi$  sq. units

5. Surface Area =  $4\pi$  sq. units

**Explanation:**

As  $S$  is enclosed by the cylinder

$$x^2 + y^2 = 2,$$

it is the graph of the function

$$f(x, y) = x^2 - y^2,$$

over the disk

$$D = \{(x, y) : x^2 + y^2 \leq 2\}$$

in the  $xy$ -plane. Its surface area element is

$$dS = (f_x^2 + f_y^2 + 1)^{1/2} dx dy$$

where

$$f_x = 2x, \quad f_y = -2y.$$

Thus

$$dS = (4x^2 + 4y^2 + 1)^{1/2} dx dy,$$

and so its surface area is given by the integral

$$I = \int \int_D (4x^2 + 4y^2 + 1)^{1/2} dx dy.$$

Because of rotational symmetry, the integral is most easily evaluated using polar coordinates. For then

$$D = \{(r, \theta) : 0 \leq r \leq 2, \ 0 \leq \theta \leq 2\pi\}$$

so that

$$\begin{aligned} I &= \int_0^2 \int_0^{2\pi} (4r^2 + 1)^{1/2} r \, d\theta \, dr \\ &= 2\pi \int_0^{\sqrt{2}} (4r^2 + 1)^{1/2} r \, dr. \end{aligned}$$

To evaluate this last integral we use the substitution  $u^2 = 1 + 4r^2$ . For then

$$I = \frac{\pi}{2} \int_1^3 u^2 \, du = \frac{\pi}{6} \left[ u^3 \right]_1^3 = \frac{13}{3} \pi.$$

---

**SurfaceInt04a**  
**026    10.0 points**

Evaluate the integral

$$I = \frac{1}{8} \int_S dS$$

when  $S$  is the surface given parametrically by

$$\Phi(u, v) = (2uv, u + v, u - v)$$

for  $u^2 + v^2 \leq 4$ .

1.  $I = 2\pi$

2.  $I = \frac{11}{6}\pi$

3.  $I = \frac{13}{6}\pi$  **correct**

4.  $I = \frac{5}{2}\pi$

5.  $I = \frac{7}{3}\pi$

**Explanation:**

When  $S$  is parametrized by

$$\Phi(u, v) = (2uv, u + v, u - v)$$

for  $u^2 + v^2 \leq 4$ , then

$$I = \frac{1}{8} \int \int_D \|\mathbf{T}_u \times \mathbf{T}_v\| \, du \, dv,$$

where

$$D = \{(u, v) : u^2 + v^2 \leq 4\}.$$



Now

$$\mathbf{T}_u = \frac{\partial \Phi}{\partial u} = (2v, 1, 1),$$

while

$$\mathbf{T}_v = \frac{\partial \Phi}{\partial v} = (2u, 1, -1).$$

In this case,

$$\begin{aligned} \mathbf{T}_u \times \mathbf{T}_v &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2v & 1 & 1 \\ 2u & 1 & -1 \end{vmatrix} \\ &= -2\mathbf{i} + 2(u+v)\mathbf{j} + 2(v-u)\mathbf{k}. \end{aligned}$$

Thus

$$\begin{aligned} \|\mathbf{T}_u \times \mathbf{T}_v\| &= 2(1 + (u+v)^2 + (v-u)^2)^{1/2} \\ &= 2(1 + 2(u^2 + v^2))^{1/2}. \end{aligned}$$

So, finally, we arrive at

$$I = \frac{1}{4} \int \int_D (1 + 2(u^2 + v^2))^{1/2} du dv.$$

Because of the rotational symmetry, we'll use polar coordinates with

$$u = r \cos \theta, \quad v = r \sin \theta,$$

to evaluate  $I$ . For then

$$\begin{aligned} I &= \frac{1}{4} \int_0^2 \int_0^{2\pi} (1 + 2r^2)^{1/2} r d\theta dr \\ &= \frac{1}{4} \pi \int_0^4 (1 + 2t)^{1/2} dt \\ &= \frac{1}{12} \pi \left[ (1 + 2t)^{3/2} \right]_0^4, \end{aligned}$$

using the substitution  $t = r^2$ . Consequently,

$$\boxed{I = \frac{13}{6} \pi}.$$

for the vector field

$$\mathbf{F} = x\mathbf{i} - 2y\mathbf{j} + z\mathbf{k}$$

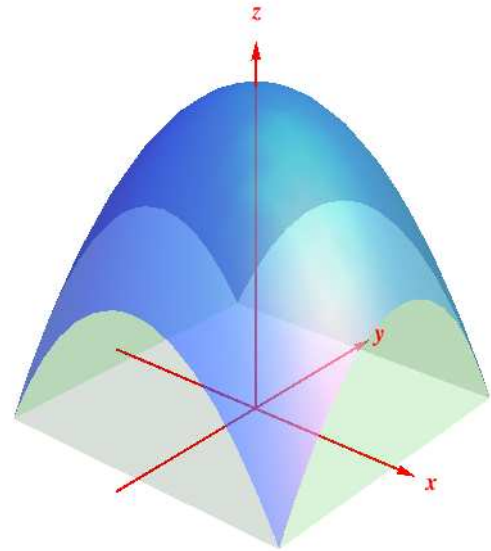
when  $S$  is the part of the paraboloid

$$z = 2 - x^2 - y^2,$$

oriented upwards, lying above the square

$$-1 \leq x \leq 1, \quad -1 \leq y \leq 1,$$

as shown in



1.  $I = 1$

2.  $I = \frac{4}{3}$

3.  $I = \frac{2}{3}$

4.  $I = 2$

5.  $I = \frac{8}{3}$  correct

**Explanation:**

If  $S$  is the graph of  $z = f(x, y)$ , then

$$d\mathbf{S} = (-f_x \mathbf{i} - f_y \mathbf{j} + \mathbf{k}) dx dy.$$

---

StewartC5 17 07 19  
027 10.0 points

Evaluate the integral

$$I = \int \int_S \mathbf{F} \cdot d\mathbf{S}$$

So when  $z = 2 - x^2 - y^2$ , and

$$\mathbf{F} = x\mathbf{i} - 2y\mathbf{j} + z\mathbf{k}$$

we see that

$$d\mathbf{S} = 2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k},$$

while

$$\begin{aligned}\mathbf{F} \cdot d\mathbf{S} &= (2x^2 - 4y^2 + (2 - x^2 - y^2))dxdy \\ &= (x^2 - 5y^2 + 2)dxdy.\end{aligned}$$

Consequently,

$$I = \int_{-1}^1 \int_{-1}^1 (x^2 - 5y^2 + 2) dxdy = \frac{8}{3}.$$

keywords:

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**GreensThm01a**  
**028 10.0 points**

Use Green's Theorem to evaluate the integral

$$I = \int_C (2xy^2 dx + x^3 dy)$$

when  $C$  is the rectangle in the  $xy$ -plane having vertices at

$$(0, 0), \quad (2, 0), \quad (2, 1), \quad (0, 1).$$

1.  $I = 4$  **correct**
2.  $I = 7$
3.  $I = 5$
4.  $I = 6$
5.  $I = 8$

**Explanation:**

Since  $C$  is a piecewise smooth curve, Green's Theorem applies and says that

$$\int_C (P dx + Q dy) = \int \int_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy$$

for all smooth functions  $P, Q$ , where  $D$  is the rectangular region in the  $xy$ -plane enclosed by  $C$ . When

$$P(x, y) = 2xy^2, \quad Q(x, y) = x^3,$$

therefore,

$$\begin{aligned}I &= \int_0^2 \left( \int_0^1 (3x^2 - 4xy) dy \right) dx \\ &= \int_0^2 \left[ 3x^2 y - 2xy^2 \right]_0^1 dx \\ &= \int_0^2 (3x^2 - 2x) dx.\end{aligned}$$

Consequently,

$$I = \left[ x^3 - x^2 \right]_0^2 = 4.$$

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**StokesThm02a**  
**029 10.0 points**

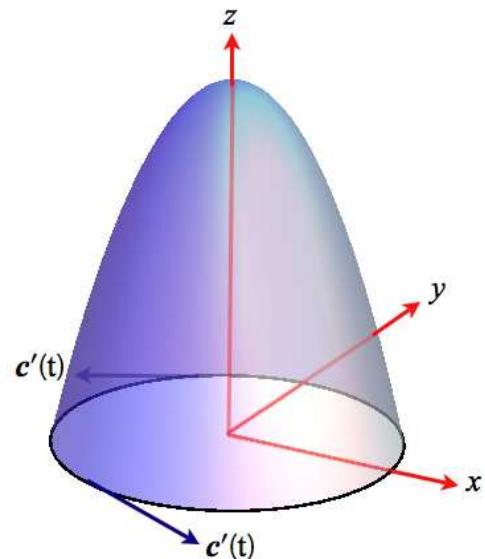
Use Stokes' theorem to evaluate the integral

$$I = \int \int_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

when  $\mathbf{F}$  is the vector field

$$\mathbf{F} = 3zx\mathbf{i} - xy\mathbf{j} + 2yz\mathbf{k}$$

and  $S$  is the surface shown in



whose boundary is the circle

$$\mathbf{c}(t) = \cos t \mathbf{i} + \sin t \mathbf{j}$$

in the  $xy$ -plane.

1.  $I = -4$
2.  $I = -1$
3.  $I = -2$
4.  $I = -3$
5.  $I = 0$  **correct**

**Explanation:**

Since the boundary of  $S$  is the circle  $C$  parametrized by

$$\mathbf{c}(t) = \cos t \mathbf{i} + \sin t \mathbf{j}, \quad 0 \leq t \leq 2\pi,$$

and oriented counterclockwise as seen from above, Stokes Theorem can be used to reduce the integral over the surface  $S$  to a line integral

$$I = \int_C \mathbf{F} \cdot d\mathbf{s} = \int_0^{2\pi} \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t) dt$$

over  $C$ .

Now

$$\mathbf{F}(\mathbf{c}(t)) = -\cos t \sin t \mathbf{j},$$

while

$$\mathbf{c}'(t) = -\sin t \mathbf{i} + \cos t \mathbf{j},$$

so

$$\mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t) = -\cos^2 t \sin t.$$

Thus

$$I = - \int_0^{2\pi} \cos^2 t \sin t dt.$$

Consequently,

$$I = \left[ \frac{1}{3} \cos^3 t \right]_0^{2\pi} = 0.$$

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**DivThm02a**  
**030 10.0 points**

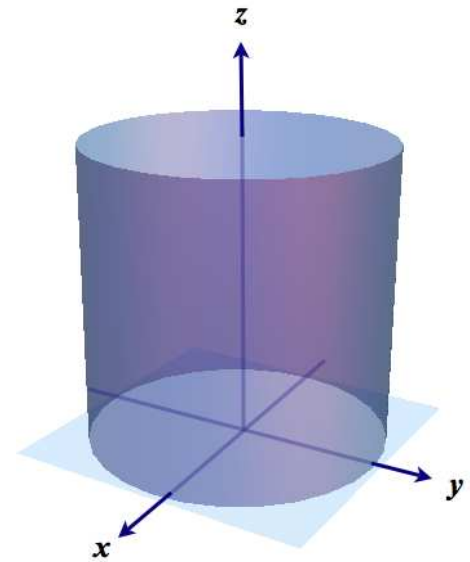
Evaluate the integral

$$I = \int \int_{\partial W} \mathbf{F} \cdot d\mathbf{S}$$

when

$$\mathbf{F}(x, y, z) = y \mathbf{i} - 2yz \mathbf{j} + 3z^2 \mathbf{k}$$

and  $\partial W$  is the boundary of the solid  $W$  shown in



enclosed by the cylinder

$$x^2 + y^2 = 1,$$

the  $xy$ -plane, and the plane  $z = 3$ .

1.  $I = 18\pi$  **correct**
2.  $I = 19\pi$
3.  $I = 18$
4.  $I = 20\pi$
5.  $I = 20$
6.  $I = 19$

**Explanation:**

As shown, the boundary  $\partial W$  of  $W$  is piecewise-smooth, so the Divergence theorem can be applied:

$$I = \int \int_{\partial W} \mathbf{F} \cdot d\mathbf{S} = \int \int \int_W \operatorname{div}(\mathbf{F}) dV .$$

Now

$$\begin{aligned} \operatorname{div}(\mathbf{F}) &= \nabla \cdot \mathbf{F} \\ &= \frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(2yz) + \frac{\partial}{\partial z}(3z^2) \\ &= -2z + 6z = 4z . \end{aligned}$$

On the other hand,  $W$  consists of all points  $(r, \theta, z)$  in cylindrical polar coordinates such that

$$0 \leq \theta \leq 2\pi, \quad 0 \leq r \leq 1, \quad 0 \leq z \leq 3 .$$

So as a repeated integral in cylindrical polar coordinates,

$$I = \int_0^3 4z \left( \int_0^1 \left( \int_0^{2\pi} d\theta \right) r dr \right) dz .$$

But

$$\int_0^1 \left( \int_0^{2\pi} d\theta \right) r dr = \left[ \pi r^2 \right]_0^1 = \pi .$$

Consequently,

$$\boxed{I = 4\pi \int_0^3 z dz = 18\pi} .$$