This print-out should have 12 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

#### MatrixVecProd04 001 10.0 points

Determine  $\mathbf{v}\mathbf{u}^T$  when

$$\mathbf{u} = \begin{bmatrix} -3\\2\\-5 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} a\\b\\c \end{bmatrix}.$$

1. 
$$\mathbf{v}\mathbf{u}^T = \begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$$

**2.** 
$$\mathbf{v}\mathbf{u}^T = -5a + 2b - 3c$$

**3.** 
$$\mathbf{v}\mathbf{u}^T = \begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$$

4. 
$$\mathbf{v}\mathbf{u}^T = -3a + 2b - 5c$$

### InverseMatrix01a 002 10.0 points

Solve for X when AX + B = C,

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}.$$

Find L in an LU decomposition of

$$A = \begin{bmatrix} 4 & -2 & 2 & -2 \\ 16 & -8 & 11 & -7 \\ -12 & 6 & -3 & 4 \end{bmatrix}.$$

$$\mathbf{1.} \ L = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ -3 & 1 & 2 \end{bmatrix}$$

**2.** 
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$$

**3.** 
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

4. 
$$L = \begin{bmatrix} -1 & 0 & 0 \\ 4 & -1 & 0 \\ -3 & 1 & -1 \end{bmatrix}$$
5. 
$$L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$\mathbf{5.} \ L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

**6.** 
$$L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -3 & -1 & 1 \end{bmatrix}$$

Let H be the set of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

where a and b are real. Determine if H is a subspace of  $\mathbb{R}^3$ , and then check the correct answer below.

- 1. *H* is a subspace of  $\mathbb{R}^3$  because it can be written as  $Span\{\mathbf{v}_1, \mathbf{v}_2\}$  with  $\mathbf{v}_1, \mathbf{v}_2$  in  $\mathbb{R}^3$ .
- **2.** H is not a subspace of  $\mathbb{R}^3$  because it is not closed under vector addition.
- **3.** *H* is not a subspace of  $\mathbb{R}^3$  because it does not contain **0**.
- **4.** *H* is a subspace of  $\mathbb{R}^3$  because it can be written as Nul(A) for some matrix A.

#### $\begin{array}{cc} Invertible 01/02 \\ 005 & 10.0 \ points \end{array}$

A is an  $n \times n$  matrix. Which of the following statements are equivalent to A being invertible?

- (i) The equation  $A\mathbf{x} = \mathbf{0}$  has infinitely many solutions.
- (ii) A is row equivalent to the  $n \times n$  identity matrix.
- (iii) The columns of A do not form a basis of  $\mathbb{R}^n$ .
- **1.** ii
- **2.** i
- **3.** i and iii
- **4.** iii
- **5.** All of these
- 6. ii and iii

Compute the volume of the parallelepiped with adjacent edges  $\overline{OP}$ ,  $\overline{OQ}$ , and  $\overline{OR}$  determined by vertices

$$P(4, -2, -4), \quad Q(2, -1, -3), \quad R(2, 2, -3),$$

where O is the origin in 3-space.

- 1. volume = 13
- $\mathbf{2.}$  volume = 14
- 3. volume = 12
- 4. volume = 11
- 5. volume = 10

### $\begin{array}{c} Rank02c \\ 006 \quad 10.0 \ points \end{array}$

Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & -1 & -1 \\ -2 & 2 & -1 \\ -1 & 1 & -1 \end{bmatrix}.$$

- 1.  $\operatorname{rank}(A) = 5$
- 2. rank(A) = 3
- 3. rank(A) = 2
- **4.** rank(A) = 1
- 5.  $\operatorname{rank}(A) = 4$

#### $\begin{array}{cc} Basis Nul 01a \\ 008 & 10.0 \ points \end{array}$

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 3 & -3 & 6 & -3 \\ -1 & 1 & 0 & 7 \\ -3 & 3 & -9 & -3 \end{bmatrix}.$$

is a basis for Nul(A).

#### $\begin{array}{c} {\rm Basis 03a} \\ 009 & 10.0 \ {\rm points} \end{array}$

In the vector space V of all real-valued functions, find a basis for the subspace

 $H \ = \ {\rm Span} \{ \, \sin t, \, \sin 2t, \, \sin t \cos t \, \} \, .$ 

- 1.  $\{\sin t, \sin 2t, \sin t \cos t\}$
- **2.**  $\{\cos t, \sin 2t\}$
- 3.  $\{\cos t, \sin 2t, \sin t \cos t\}$
- 4.  $\{\sin 2t, \sin t \cos t\}$
- **5.**  $\{ \sin t, \sin 2t \}$

## $\begin{array}{cc} PolyCoordVec 01a \\ 010 & 10.0 \ points \end{array}$

Find the coordinate vector  $[\mathbf{p}]_{\mathcal{B}}$  in  $\mathbb{R}^3$  for the polynomial

$$\mathbf{p}(t) = 1 + 4t + 7t^2$$

with respect to the basis

$$\mathcal{B} = \left\{ 1 + t^2, \ t + t^2, \ 1 + 2t + t^2 \right\}$$

for  $\mathbb{P}_2$ .

$$\mathbf{1.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ -6 \\ 1 \end{bmatrix}$$

$$\mathbf{2.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$$

$$\mathbf{3.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$$

4. 
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix}$$

5. 
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -6\\1\\2 \end{bmatrix}$$

$$\mathbf{6.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix}$$

# ChangeBasis01b 011 10.0 points

Determine the change of coordinates matrix  $P_{\mathcal{C}\leftarrow\mathcal{B}}$  from basis  $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$  to  $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$  of a vector space V when

$$\mathbf{b}_1 = 6\mathbf{c}_1 - 2\mathbf{c}_2, \quad \mathbf{b}_2 = 9\mathbf{c}_1 - 4c_2.$$