This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

QuadApprox02a
001 10.0 points

Find the quadratic approximation to

$$f(x,\, y) \;=\; \cos(x+y) - 2\sin(x-y)$$

at $P(0,\, 0)$.

1. $Q(x,\, y) \;=\; 1 + 2x - 2y + \dfrac{1}{2}x^2 - xy + y^2$

2. $Q(x,\, y) \;=\; 2 - 2x + 2y + \dfrac{1}{2}x^2 - xy + y^2$

3. $Q(x,\, y) \;=\; 1 - 2x + 2y - \dfrac{1}{2}x^2 - xy - \dfrac{1}{2}y^2$ **correct**

4. $Q(x,\, y) \;=\; 2 - 2x + 2y + \dfrac{1}{2}x^2 + xy - \dfrac{1}{2}y^2$

5. $Q(x,\, y) \;=\; 2 - 2x + 2y - \dfrac{1}{2}x^2 + xy - \dfrac{1}{2}y^2$

6. $Q(x,\, y) \;=\; 1 + 2x - 2y - \dfrac{1}{2}x^2 + xy + y^2$

Explanation:

The Quadratic Approximation to $f(x,\, y)$ at $P(0,\, 0)$ is given by

$$Q(x,\, y) \;=\; f(0,\, 0) + f_x(0,\, 0)x + f_y(0,\, 0)y$$
$$+\, \frac{1}{2}f_{xx}(0,\, 0)x^2 + f_{xy}(0,\, 0)xy + \frac{1}{2}f_{yy}(0,\, 0)y^2.$$

But when

$$f(x,\, y) \;=\; \cos(x+y) - 2\sin(x-y)$$

we see that

$$f_x \;=\; -\sin(x+y) - 2\cos(x-y)\,,$$
$$f_y \;=\; -\sin(x+y) + 2\cos(x-y)\,,$$

so that $f(0,\, 0) = 1$ and

$$f_x(0,\, 0) = -2, \quad f_y(0,\, 0) = 2\,,$$

while

$$f_{xx} \;=\; -\cos(x+y) + 2\sin(x-y)\,,$$
$$f_{xy} \;=\; \cos(x+y) - 2\sin(x-y)\,,$$
$$f_{yy} \;=\; \cos(x+y) + 2\sin(x-y)\,,$$

so that $f_{xx}(0,\, 0) = 1$ and

$$f_{xy}(0,\, 0) = -1, \quad f_{yy}(0,\, 0) = -1\,,$$

Consequently, the Quadratic Approximation to f at $P(0,\, 0)$ is

$$\boxed{Q(x,\, y) \;=\; 1 - 2x + 2y - \frac{1}{2}x^2 - xy - \frac{1}{2}y^2}\,.$$

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keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

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CalC15g19b
002 10.0 points

Locate and classify the critical point of

$$f(x, y) \;=\; \ln(xy) + 4y^2 - 2y - 2xy + 5,$$

for $x, y > 0$.

1. local maximum at $\left(\dfrac{1}{4},\, 2\right)$

2. saddle-point at $\left(2,\, \dfrac{1}{4}\right)$ **correct**

3. local minimum at $\left(2,\, \dfrac{1}{4}\right)$

4. local minimum at $\left(\dfrac{1}{4},\, 2\right)$

5. local maximum at $\left(2,\, \dfrac{1}{4}\right)$

6. saddle-point at $\left(\dfrac{1}{4},\, 2\right)$

Explanation:

The critical point of f is the common solution of the equations

$$\frac{\partial f}{\partial x} = \frac{1}{x} - 2y = 0\,,$$

$$\frac{\partial f}{\partial y} = \frac{1}{y} + 8y - 2 - 2x = 0\,.$$

By the first equation, $2x = 1/y$. Using this in the second equation, we see that

$$8y - 2 = 0 \quad i.e.,\ y = \frac{1}{4}\,.$$

So f has a critical point at

$$\left(2,\ \frac{1}{4}\right).$$

Now after differentiation,

$$f_{xx} = -\frac{1}{x^2}, \quad f_{xy} = -2, \quad f_{yy} = 8 - \frac{1}{y^2}\,.$$

Thus at the critical point $\left(2,\ \frac{1}{4}\right)$,

$$A = f_{xx}\Big|_{\left(2,\frac{1}{4}\right)} = -\frac{1}{4} < 0, \qquad B = -2\,,$$

$$C = f_{yy}\Big|_{\left(2,\frac{1}{4}\right)} = -8 < 0\,,$$

in which case

$$AC - B^2 = -2 < 0\,,$$

Consequently, by the second derivative test f has a

$$\boxed{\text{saddle-point at } \left(2,\ \frac{1}{4}\right)}\,.$$

CalC14d16s
003 10.0 points

Determine the position vector, $\mathbf{r}(t)$, of a particle having acceleration

$$\mathbf{a}(t) = -8\,\mathbf{k}$$

when its initial velocity and position are given by

$$\mathbf{v}(0) = \mathbf{i} + \mathbf{j} - 4\,\mathbf{k}\,, \quad \mathbf{r}(0) = 5\,\mathbf{i} + 2\,\mathbf{j}$$

respectively.

1. $\mathbf{r}(t) = (t-5)\,\mathbf{i} + (t+2)\,\mathbf{j} - (4t^2 - 4t)\,\mathbf{k}$

2. $\mathbf{r}(t) = (t+5)\,\mathbf{i} + (t+2)\,\mathbf{j} - (4t^2 + 4t)\,\mathbf{k}$ **correct**

3. $\mathbf{r}(t) = (t-5)\,\mathbf{i} + (t+2)\,\mathbf{j} - (4t^2 + 4t)\,\mathbf{k}$

4. $\mathbf{r}(t) = (t+5)\,\mathbf{i} - (t-2)\,\mathbf{j} - (4t^2 + 4t)\,\mathbf{k}$

5. $\mathbf{r}(t) = (t+5)\,\mathbf{i} + (t+2)\,\mathbf{j} - (4t^2 - 4t)\,\mathbf{k}$

6. $\mathbf{r}(t) = (t+5)\,\mathbf{i} - (t-2)\,\mathbf{j} - (4t^2 - 4t)\,\mathbf{k}$

Explanation:
Since

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -8\,\mathbf{k}\,,$$

we see that

$$\mathbf{v}(t) = -8t\,\mathbf{k} + C$$

where C is a constant vector such that

$$\mathbf{v}(0) = C = \mathbf{i} + \mathbf{j} - 4\,\mathbf{k}\,.$$

Thus

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \mathbf{i} + \mathbf{j} - (8t+4)\,\mathbf{k}\,.$$

But then

$$\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} - (4t^2 + 4t)\,\mathbf{k} + D$$

where D is a constant vector such that

$$\mathbf{r}(0) = D = 5\,\mathbf{i} + 2\,\mathbf{j}\,.$$

Consequently,

$$\boxed{\mathbf{r}(t) = (t+5)\,\mathbf{i} + (t+2)\,\mathbf{j} - (4t^2 + 4t)\,\mathbf{k}}\,.$$

CalC14c04a
004 10.0 points

The curve C is parametrized by

$$\mathbf{c}(t) = (4-2t)\mathbf{i} + \ln(2t)\,\mathbf{j} + (5+t^2)\mathbf{k}\,.$$

Find the arc length of C between $\mathbf{c}(1)$ and $\mathbf{c}(3)$.

1. arc length $= 8 - 2\ln 3$

2. arc length $= 3 + \ln 6$

3. arc length $= 9 + 2\ln 3$

4. arc length $= 8 + \ln 3$ **correct**

5. arc length $= 8 - \ln 3$

6. arc length $= 6 - \ln 3$

Explanation:

The arc length of C between $\mathbf{c}(t_0)$ and $\mathbf{c}(t_1)$ is given by the integral

$$L = \int_{t_0}^{t_1} \|\mathbf{c}'(t)\|\,dt\,.$$

Now when

$$\mathbf{c}(t) = (4-2t)\mathbf{i} + \ln(2t)\,\mathbf{j} + (5+t^2)\mathbf{k}$$

we see that

$$\mathbf{c}'(t) = -2\,\mathbf{i} + \frac{1}{t}\,\mathbf{j} + 2t\,\mathbf{k}\,.$$

But then

$$\|\mathbf{c}'(t)\| = \left(4 + \frac{1}{t^2} + 4t^2\right)^{1/2} = \frac{2t^2+1}{t}\,.$$

Thus

$$L = \int_1^3 \left(2t + \frac{1}{t}\right) dt = \Big[t^2 + \ln t\Big]_1^3.$$

Consequently,

$$\boxed{\text{arc length} = L = 8 + \ln 3}\,.$$

GradVectorField01a
005 10.0 points

If $f(x, y)$ is a potential function for the gradient vector field

$$\mathbf{F}(x, y) = (3x+y)\,\mathbf{i} + (x+2y)\,\mathbf{j}\,,$$

evaluate

$$f(1, 2) - f(0, 1)\,.$$

1. $f(1, 2) - f(0, 1) = \dfrac{13}{2}$ **correct**

2. $f(1, 2) - f(0, 1) = 5$

3. $f(1, 2) - f(0, 1) = \dfrac{11}{2}$

4. $f(1, 2) - f(0, 1) = 6$

5. $f(1, 2) - f(0, 1) = \dfrac{9}{2}$

Explanation:

If $f(x, y)$ is a potential function for the gradient vector field

$$\mathbf{F}(x, y) = (3x+y)\,\mathbf{i} + (x+2y)\,\mathbf{j}\,,$$

then

$$\frac{\partial f}{\partial x} = 3x + y\,, \quad \frac{\partial f}{\partial y} = x + 2y\,.$$

Now by the first equation,

$$f(x, y) = \frac{3}{2}x^2 + xy + D(y)$$

for an arbitrary function $D(y)$, which by the second equation satisfies

$$x + D'(y) = x + 2y\,, \quad i.e., \quad D(y) = y^2 + K\,,$$

for an arbitrary constant K. Thus

$$f(x, y) = \frac{3}{2}x^2 + xy + y^2 + K\,.$$

But then

$$f(0, 1) = 1 + K\,,$$

while

$$f(1, 2) = \frac{3}{2} + 2 + 4 + K = \frac{15}{2} + K\,.$$

Consequently,

$$\boxed{f(1,\,2) - f(0,\,1) \;=\; \frac{13}{2}}\,.$$

Curl01a
006 10.0 points

Find the curl of the vector field

$$\mathbf{F}(x,\,y,\,z) \;=\; 2zx\,\mathbf{i} + 3xy\,\mathbf{j} + yz\,\mathbf{k}\,.$$

1. $\operatorname{curl}\mathbf{F} \;=\; 2z\,\mathbf{i} - 3x\,\mathbf{j} + y\,\mathbf{k}$

2. $\operatorname{curl}\mathbf{F} \;=\; 3x\,\mathbf{i} + y\,\mathbf{j} + 2z\,\mathbf{k}$

3. $\operatorname{curl}\mathbf{F} \;=\; z\,\mathbf{i} + 2x\,\mathbf{j} + 3y\,\mathbf{k}$ **correct**

4. $\operatorname{curl}\mathbf{F} \;=\; 2x\,\mathbf{i} + 3y\,\mathbf{j} + z\,\mathbf{k}$

5. $\operatorname{curl}\mathbf{F} \;=\; z\,\mathbf{i} - 2x\,\mathbf{j} + 3y\,\mathbf{k}$

6. $\operatorname{curl}\mathbf{F} \;=\; 3x\,\mathbf{i} - y\,\mathbf{j} - 2z\,\mathbf{k}$

Explanation:
The curl of $\mathbf{F}$ is given symbolically by

$$\operatorname{curl}\mathbf{F} \;=\; \nabla\times\mathbf{F} \;=\; \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2zx & 3xy & yz \end{vmatrix}$$

$$= X\,\mathbf{i} - Y\,\mathbf{j} + Z\,\mathbf{k}$$

where

$$X \;=\; \begin{vmatrix} \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3xy & yz \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}(yz) - \frac{\partial}{\partial z}(3xy)\right) \;=\; z\,,$$

$$Y \;=\; \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial z} \\ 2zx & yz \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial z}(2zx)\right) \;=\; -2x\,,$$

and

$$Z \;=\; \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \\ 2zx & 3xy \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial x}(3xy) - \frac{\partial}{\partial y}(2zx)\right) \;=\; 3y\,.$$

Consequently,

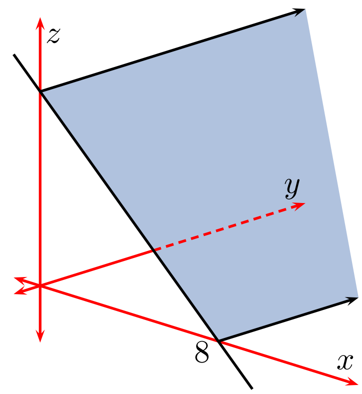
$$\boxed{\operatorname{curl}\mathbf{F} \;=\; z\,\mathbf{i} + 2x\,\mathbf{j} + 3y\,\mathbf{k}}\,.$$

CalC16b01a
007 10.0 points

The graph of the function

$$z \;=\; f(x,\,y) \;=\; 8 - x$$

is the plane shown in



Determine the value of the double integral

$$I \;=\; \int\!\!\int_A f(x,\,y)\,dxdy$$

over the region

$$A \;=\; \Big\{(x, y) : 0 \le x \le 3,\;\; 0 \le y \le 2\Big\}$$

in the xy-plane by first identifying it as the volume of a solid below the graph of f.

1. $I \;=\; 43$ cu. units

2. $I \;=\; 42$ cu. units

3. $I \;=\; 41$ cu. units

4. $I = 39$ cu. units **correct**

5. $I = 40$ cu. units

Explanation:

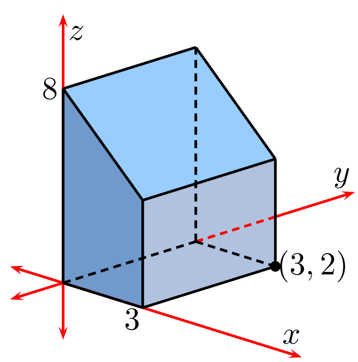
The double integral

$$I = \int\int_A f(x, y)\, dx dy$$

is the volume of the solid below the graph of f having the rectangle

$$A = \Big\{(x, y) : 0 \le x \le 3,\ \ 0 \le y \le 2\Big\}$$

for its base. Thus the solid is the wedge



and so its volume is the area of trapezoidal face multiplied by the thickness of the wedge. Consequently,

$$\boxed{I = 39 \text{ cu. units}}.$$

keywords:

CalC16c05s
008 10.0 points

Evaluate the iterated integral

$$I = \int_0^{\pi/2} \int_0^{\cos(\theta)} 4\, e^{\sin(\theta)}\, dr\, d\theta\,.$$

1. $I = 4e$

2. $I = 4(e-1)$ **correct**

3. $I = e - 4$

4. $I = \dfrac{1}{e} - 4$

5. $I = 0$

6. $I = 4\left(\dfrac{1}{e} - 1\right)$

Explanation:

After simple integration

$$\int_0^{\cos(\theta)} 4\, e^{\sin(\theta)}\, dr = \Big[4\, r\, e^{\sin(\theta)}\Big]_0^{\cos(\theta)}$$

$$= 4\cos(\theta)\, e^{\sin(\theta)}\,.$$

In this case,

$$I = \int_0^{\pi/2} 4\cos(\theta)\, e^{\sin(\theta)}\, d\theta = \Big[4\, e^{\sin(\theta)}\Big]_0^{\pi/2}\,.$$

Consequently,

$$\boxed{I = 4(e-1)}.$$

CalC16g01a
009 10.0 points

Evaluate the triple integral

$$I = \int_0^1 \int_0^x \int_0^{x+y} (3x - 2y)\, dz dy dx\,.$$

1. $I = \dfrac{17}{24}$ **correct**

2. $I = \dfrac{19}{24}$

3. $I = \dfrac{25}{24}$

4. $I = \dfrac{7}{8}$

5. $I = \dfrac{23}{24}$

Explanation:

As a repeated integral,

$$I = \int_0^1 \Big(\int_0^x \Big(\int_0^{x+y} (3x-2y)\,dz\Big)dy\Big)dx\,.$$

Now

$$\int_0^{x+y} (3x-2y)\,dz = \Big[(3x-2y)z\Big]_0^{x+y}$$

$$= (3x-2y)(x+y) = 3x^2+xy-2y^2\,,$$

while

$$\int_0^x (3x^2+xy-2y^2)\,dy$$

$$= \Big[3x^2y+\frac{1}{2}xy^2-\frac{2}{3}y^3\Big]_0^x = \frac{17}{6}x^3\,.$$

Consequently,

$$\boxed{I = \int_0^1 \frac{17}{6}x^3\,dx = \frac{17}{24}}\,.$$

keywords: integral, triple integral, repeated integral, linear function,polynomial integrand, binomial integrand, evaluation of triple integral

Div01a
010 10.0 points

Find the divergence of the vector field

$$\mathbf{F}(x,\,y,\,z) = x^2yz\,\mathbf{i} - 2xy^2z\,\mathbf{j} - 3xyz^2\,\mathbf{k}\,.$$

1. $\operatorname{div}\mathbf{F} = -6xyz$

2. $\operatorname{div}\mathbf{F} = -5xyz$

3. $\operatorname{div}\mathbf{F} = -8xyz$ **correct**

4. $\operatorname{div}\mathbf{F} = -7xyz$

5. $\operatorname{div}\mathbf{F} = -4xyz$

Explanation:

The div of **F** is given symbolically by

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F}$$

$$= \frac{\partial}{\partial x}(x^2yz) - 2\frac{\partial}{\partial y}(xy^2z) - 3\frac{\partial}{\partial z}(xyz^2)\,.$$

Thus

$$\boxed{\operatorname{div}\mathbf{F} = (2-4-6)xyz = -8xyz}\,.$$

CalC15h04exam
011 10.0 points

Determine the minimum value of

$$f(x,\,y) = 3x+4y+2$$

subject to the constraint

$$g(x,\,y) = x^2+y^2-1 = 0\,.$$

1. min value $= -2$

2. min value $= -1$

3. min value $= -4$

4. min value $= -5$

5. min value $= -3$ **correct**

Explanation:

The extreme values of f subject to the constraint $g = 0$ occur at solutions of

$$(\nabla f)(x,\,y) = \lambda(\nabla g)(x,\,y)\,, \quad g(x,\,y) = 0\,.$$

Now

$$(\nabla f)(x,\,y) = \langle\,3,\,4\,\rangle\,,$$

while

$$(\nabla g)(x,\,y) = \langle\,2x,\,2y\,\rangle\,.$$

Thus

$$3 = 2\lambda x\,, \qquad 4 = 2\lambda y\,,$$

and so

$$\lambda = \frac{3}{2x} = \frac{2}{y}\,, \quad i.e.,\ y = \frac{4}{3}x\,.$$

But

$$g\left(x,\, \frac{4}{3}x\right) \;=\; x^2 + \frac{16}{9}x^2 - 1 \;=\; 0\,,$$

i.e., $x = \pm 3/5$. Consequently, the extreme points are

$$\left(\frac{3}{5},\, \frac{4}{5}\right), \quad \left(-\frac{3}{5},\, -\frac{4}{5}\right).$$

Since

$$f\left(\frac{3}{5},\, \frac{4}{5}\right) = 7\,, \quad f\left(-\frac{3}{5},\, -\frac{4}{5}\right) = -3\,,$$

we thus see that

$$\boxed{\text{min value} \;=\; -3}\,.$$

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keywords: