

This print-out should have 17 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

---

**FinM4a24**  
**001 10.0 points**

Pandit is an aging dog who has to be kept on a strict diet containing, among other things, 2.5 grams of protein and 1.8 grams of fat. Two dog foods are available to Pandit's owner.

Food  $A$  has 8% protein and 6% fat, while

Food  $B$  has 3% protein and 2% fat.

How many grams of food  $A$  should Pandit's owner use in his diet?

1. # grams food  $A$  = 20 **correct**

2. # grams food  $A$  = 21

3. # grams food  $A$  = 19

4. # grams food  $A$  = 22

5. # grams food  $A$  = 23

**Explanation:**

Let  $x$  and  $y$  be the amounts (in grams) of foods  $A$  and  $B$  respectively used in Pandit's diet. Then

$$0.08x + 0.03y = 2.5,$$

$$0.06x + 0.02y = 1.8.$$

Solving for  $x, y$  we see that

$$x = 20, \quad y = 30,$$

so

# grams food  $A$  = 20.

---

**FitParabola01b**  
**002 10.0 points**

When the graph of the function

$$y = ax^2 + bx + c$$

is the unique parabola passing through the points

$$(1, 7), \quad (-1, 1), \quad (-3, 3),$$

determine  $b$ .

1.  $b = 2$

2.  $b = 6$

3.  $b = 4$

4.  $b = 5$

5.  $b = 3$  **correct**

**Explanation:**

Since the parabola passes through the points

$$(1, 7), \quad (-1, 1), \quad (-3, 3),$$

the coefficients  $a, b$  and  $c$  must satisfy the equations

$$a + b + c = 7$$

$$a - b + c = 1$$

$$9a - 3b + c = 3$$

To solve these equations for  $c$  we reduce the augmented matrix

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 1 & -1 & 1 & 1 \\ 9 & -3 & 1 & 3 \end{array} \right]$$

to echelon form by successive row operations:

$$\xrightarrow{R_2 - R_1} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 0 & -6 \\ 9 & -3 & 1 & 3 \end{array} \right]$$

$$\xrightarrow{R_3 - 9R_1} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 0 & -6 \\ 0 & -12 & -8 & -60 \end{array} \right]$$

$$\xrightarrow{R_3 - 6R_2} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 0 & -6 \\ 0 & 0 & -8 & -24 \end{array} \right].$$

Thus

$$\boxed{b = 3}.$$

---

**LinearSystemT/F01a**  
**003 10.0 points**

Elementary row operations on an augmented matrix never change the the solution set of the associated linear system.

True or False?

1. TRUE **correct**

2. FALSE

**Explanation:**

Elementary row operations on an augmented matrix

(i) replace one row by the sum of itself and the multiple of another row,

(ii) interchange rows,

(iii) multiply all entries in a row by a non-zero constant.

For the linear system associated with the original augmented matrix these respective operations

(i) add a constant times one equation to another,

(ii) interchange two equations,

(iii) multiply an equation through by a non-zero constant.

But none of these will affect the solutions of the equations.

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$

---

**LinSysUniqueTF02**  
**004 10.0 points**

If a system of linear equations has no free variables, then it has a unique solution.

True or False?

1. TRUE

2. FALSE **correct**

**Explanation:**

A linear system must have no solutions, unique solutions, or infinitely many solutions. Having no free variables indicates that the system does not have infinitely many solutions, but it does not determine which of the other two cases it might be. Consider the following counterexample:

$$\begin{aligned}x_1 + x_2 &= 1 \\x_2 &= 5 \\x_1 + x_2 &= 2\end{aligned}$$

Subtracting the first row from the third row obtains the equation  $0 = 1$ , so this is clearly not consistent and hence has no solution. You can also show the system has no free variables. Hence it has no free variables and no solution.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

---

**RowReduceMan02a**  
**005 10.0 points**

The augmented matrix of a linear system of equations has been reduced by row operations to

$$\begin{bmatrix} 1 & 2 & -3 & 7 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

(a) *Continue row operations to write the matrix in reduced row echelon form.*

(b) *Then determine the solution set of the original system.*

$$(a) \quad \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

1.

$$(b) \quad x_1 = -1, \quad x_2 = 1, \quad x_3 = -2$$

**correct**

**Explanation:**

(a) Using row operations we see that

$$\begin{aligned} & \begin{bmatrix} 1 & 2 & -3 & 7 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix} \\ & \sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix} \\ & \sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}. \end{aligned}$$

The augmented matrix is now in reduced row echelon form.

(b) By row reduction the original system is equivalent to the system

$$\begin{aligned} x_1 &= -1 \\ x_2 &= 1 \\ x_3 &= -2, \end{aligned}$$

so the solution set of the original system is

$$\boxed{x_1 = -1, \ x_2 = 1, \ x_3 = -2}.$$

---

**AxisIntersect01a**  
**006 10.0 points**

When  $P$  is the plane in  $\mathbb{R}^3$  given in vector form by

$$\mathbf{x} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix} + t \begin{bmatrix} 3 \\ -4 \\ -4 \end{bmatrix},$$

determine where  $P$  intersects the  $z$ -axis.

1.  $z = -7$  **correct**

2.  $z = -8$

3.  $z = -9$

4.  $z = -5$

5.  $z = -6$

**Explanation:**

Since the  $z$ -axis consists of points in  $\mathbb{R}^3$  with  $x = y = 0$ ,  $P$  intersects the  $z$ -axis when

$$\begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix} + t \begin{bmatrix} 3 \\ -4 \\ -4 \end{bmatrix}$$

for some choice of  $s$  and  $t$ , *i.e.*, when

$$s \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix} + t \begin{bmatrix} 3 \\ -4 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ z - 1 \end{bmatrix}$$

is consistent as a vector equation in  $s, t$ . This will be true if and only if the associated augmented matrix

$$A = \begin{bmatrix} -2 & 3 & 1 \\ 2 & -4 & 2 \\ 4 & -4 & z - 1 \end{bmatrix}$$

is row equivalent to an echelon matrix whose entries in the last row are all zero.

Now by row reduction in the first column of  $A$ , we obtain

$$A \sim \begin{bmatrix} -2 & 3 & 1 \\ 0 & -1 & 3 \\ 0 & 2 & z + 1 \end{bmatrix},$$

and after row reduction in the second column

$$A \sim \begin{bmatrix} -2 & 3 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & z + 7 \end{bmatrix}.$$

Consequently,  $P$  intersects the  $z$ -axis at

$$\boxed{z = -7}.$$

---

**M340LSpanM02**  
**007 10.0 points**

Given

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$

determine all values of  $\lambda$  for which

$$\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ \lambda \end{bmatrix}$$

is a vector in  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ ?

1.  $\lambda = 3$
2.  $\lambda = 1, 3$
3.  $\lambda = -1, 1$
4.  $\lambda = -1$  **correct**
5.  $\lambda = 1$
6.  $\lambda = -1, 3$

**Explanation:**

The vector  $\mathbf{w}$  is in  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  if there exist weights  $x_1, x_2, x_3$  such that

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}.$$

Such weights exist when the rightmost column in the augmented matrix

$$[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{w}] = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 4 & 0 & 1 \\ 0 & 2 & -1 & \lambda \end{bmatrix}$$

is not a pivot column. But

$$\begin{aligned} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 4 & 0 & 1 \\ 0 & 2 & -1 & \lambda \end{bmatrix} &\sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -1 & -1 \\ 0 & 2 & -1 & \lambda \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & \lambda + 1 \end{bmatrix} \end{aligned}$$

Thus the rightmost column is not a pivot column when  $\lambda + 1 = 0$ . Consequently,  $\mathbf{w}$  lies in  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  when

$$\lambda = -1.$$

---

**VectorEquTF01e**  
008 10.0 points

If  $\mathbf{u}, \mathbf{v}$  are vectors in  $\mathbb{R}^3$ , when can  $\text{Span}\{\mathbf{u}, \mathbf{v}\}$  be visualized as a plane through the origin in  $\mathbb{R}^3$ .

True or False?

1. ALWAYS
2. SOMETIMES **correct**
3. NEVER

**Explanation:**

If  $\mathbf{u}, \mathbf{v}$  are linearly independent, then  $\text{Span}\{\mathbf{u}, \mathbf{v}\}$  can be visualized as a plane through the origin in  $\mathbb{R}^3$ . But if  $\mathbf{v}$  is a non-zero scalar multiple of  $\mathbf{u}$ , or when  $\mathbf{v} = \mathbf{0}$ , then  $\text{Span}\{\mathbf{u}, \mathbf{v}\}$  consists of all scalar multiples of  $\mathbf{u}$ , and hence can be identified with a *line* through the origin.

Consequently,  $\text{Span}\{\mathbf{u}, \mathbf{v}\}$  can

SOMETIMES

be visualized as a plane through the origin in  $\mathbb{R}^3$ , but not always.

---

**Consistent01d**  
009 10.0 points

Describe geometrically the conditions on a vector  $\mathbf{b}$  in  $\mathbb{R}^2$  under which the equation

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

has a solution in  $\mathbb{R}^2$ .

1.  $\mathbf{b}$  lies on line  $y + 3x = 0$
2.  $\mathbf{b}$  lies on line  $y - 3x = 0$  **correct**
3. arbitrary  $\mathbf{b}$  in  $\mathbb{R}^2$
4. any  $\mathbf{b}$  not on line  $y + 3x = 0$
5. any  $\mathbf{b}$  not on line  $y - 3x = 0$

**Explanation:**

A matrix equation  $A\mathbf{x} = \mathbf{b}$  has a solution when the last column of the associated augmented matrix  $[A \ \mathbf{b}]$  is not a pivot column.

Now for the given equation,

$$[A \ \mathbf{b}] = \begin{bmatrix} 2 & 3 & b_1 \\ 6 & 9 & b_2 \end{bmatrix} \\ \sim \begin{bmatrix} 2 & 3 & b_1 \\ 0 & 0 & b_2 - 3b_1 \end{bmatrix}.$$

Thus the equation has a solution when

$$b_2 - 3b_1 = 0,$$

*i.e.*,  $(b_1, b_2)$  satisfies the linear equation

$$y - 3x = 0.$$

Consequently,  $A\mathbf{x} = \mathbf{b}$  has a solution if and only if  $\mathbf{b}$  lies on the line

$$y - 3x = 0.$$

---

**MatEquTF02b**  
**010 10.0 points**

If the matrix equation  $A\mathbf{x} = \mathbf{b}$  is consistent, then  $\mathbf{b}$  is in the set spanned by the columns of  $A$ .

True or False?

1. TRUE correct

2. FALSE

**Explanation:**

Since

$$A\mathbf{x} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \\ = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \dots + x_n\mathbf{a}_n,$$

the equation  $A\mathbf{x} = \mathbf{b}$  is consistent, *i.e.*, has a solution, if and only if  $\mathbf{b}$  is a linear combination of the columns of  $A$ .

On the other hand,

$$\text{Span}\{\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_n\}$$

consists of all linear combination of the columns of  $A$ . Hence,  $A\mathbf{x} = \mathbf{b}$  is consistent if and only if  $\mathbf{b}$  is in the span of the columns of  $A$ .

Consequently, the statement is

TRUE

---

**SolSetsLinSysTF03**  
**011 10.0 points**

If the equation  $A\mathbf{x} = \mathbf{b}$  has more than one solution, then so does the homogeneous equation  $A\mathbf{x} = \mathbf{0}$ .

True or False?

1. FALSE

2. TRUE correct

**Explanation:**

The homogeneous equation  $A\mathbf{x} = \mathbf{0}$  always has the trivial solution  $\mathbf{x} = \mathbf{0}$ . Now suppose  $\mathbf{p}_1, \mathbf{p}_2$  are solutions of  $A\mathbf{x} = \mathbf{b}$  with  $\mathbf{p}_1 \neq \mathbf{p}_2$ , *i.e.*,  $\mathbf{w} = \mathbf{p}_1 - \mathbf{p}_2 \neq \mathbf{0}$  and

$$A\mathbf{p}_1 = \mathbf{b}, \quad A\mathbf{p}_2 = \mathbf{b}.$$

But then

$$A\mathbf{w} = A\mathbf{p}_1 - A\mathbf{p}_2 = \mathbf{b} - \mathbf{b} = \mathbf{0}.$$

So  $\mathbf{x} = \mathbf{0}$  and  $\mathbf{x} = \mathbf{w}$  are two different solutions of  $A\mathbf{x} = \mathbf{0}$ .

Consequently, the statement is

TRUE

---

**ThreePoints01a**  
**012 10.0 points**

Determine the linear equation of the unique plane in  $\mathbb{R}^3$  containing the points

$$P(1, 2, 1), \quad Q(-1, -1, 0),$$

and

$$R(-1, -4, -2).$$

1.  $3x - 4y + 6z = 1$  **correct**

2.  $3x + 4y - 6z + 1 = 0$

3.  $3x - 4y - 6z + 1 = 0$

4.  $3x + 4y + 6z = 1$

5.  $3x + 4y - 6z = 1$

6.  $3x - 4y + 6z + 1 = 0$

**Explanation:**

A linear equation in  $x, y, z$  has the form

$$ax + by + cz + d = 0.$$

Its graph will contain the points  $P, Q$ , and  $R$  when  $a, b, c$ , and  $d$  satisfy the homogeneous system

$$a + 2b + z + d = 0,$$

$$-a - b + 0z + d = 0,$$

$$-x - 4y - 2z + d = 0,$$

of 3 linear equations in 4 variables. Now the associated augmented matrix is

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ -1 & -4 & -2 & 1 & 0 \end{bmatrix},$$

and

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 1 & 6 & 0 \end{bmatrix}.$$

So  $d$  is a free variable, say  $d = s$ , while

$$a = -3s, \quad b = 4s, \quad c = -6s.$$

Although there are infinitely many solutions, the parameter  $s$  is common to each of  $a, b, c$ , and  $d$ . Thus there is a unique plane passing through  $P, Q$ , and  $R$ , namely the graph of

$$\boxed{3x - 4y + 6z = 1},$$

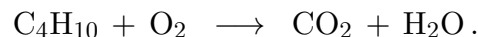
obtained by cancelling the common factor  $s$ .

---

**BalChemEq01a**

**013 10.0 points**

When butane  $\text{C}_4\text{H}_{10}$  burns in the presence of oxygen  $\text{O}_2$  it produces carbon dioxide  $\text{CO}_2$  and water  $\text{H}_2\text{O}$ , represented chemically by



If 60 molecules of water were produced in one particular reaction, how many molecules of butane were burned in that reaction?

1. # molecules = 15

2. # molecules = 14

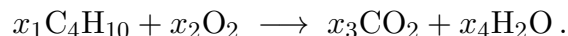
3. # molecules = 11

4. # molecules = 13

5. # molecules = 12 **correct**

**Explanation:**

We need to solve first for the relative numbers  $x_1, \dots, x_4$  of molecules in the balanced chemical equation



Now the fundamental rule governing this reaction is that the left and right hand sides contain the same number of the respective carbon, hydrogen and oxygen atoms. Thus

$$4x_1 + 0x_2 = x_3 + 0x_4,$$

$$10x_1 + 0x_2 = 0x_3 + 2x_4,$$

$$0x_1 + 2x_2 = 2x_3 + x_4,$$

which as a homogeneous system can be written in augmented matrix form

$$[A \ 0] = \begin{bmatrix} 4 & 0 & -1 & 0 & 0 \\ 10 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}.$$

But

$$\text{rref}([A \ 0]) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{5} & 0 \\ 0 & 1 & 0 & -\frac{13}{10} & 0 \\ 0 & 0 & 1 & -\frac{4}{5} & 0 \end{bmatrix}.$$

So  $x_4$  is a free variable, say  $x_4 = s$ , and

$$x_1 = \frac{1}{5}s, \quad x_2 = \frac{13}{10}s, \quad x_3 = \frac{4}{5}s,$$

give the respective proportions of the other molecules in the reaction with respect to  $x_4$ .

Consequently, if 60 molecules of water were produced, then

12 molecules

of butane were burned.

---

**LinIndependMan01a**  
**014 10.0 points**

Find all values  $h$  for which the vectors

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$$

are linearly independent.

1.  $h \neq -4$  **correct**

**Explanation:**

The vectors

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix},$$

are linearly independent if and only if the only solutions of

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{0}$$

are  $x_1 = x_2 = x_3 = 0$ .

To determine the solutions of this vector equation we use row operations to reduce the corresponding augmented matrix to echelon

form:

$$\begin{aligned} & \begin{bmatrix} 2 & 4 & -2 & 0 \\ -2 & -6 & 2 & 0 \\ 4 & 7 & h & 0 \end{bmatrix} \\ & \sim \begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -1 & h+4 & 0 \end{bmatrix} \\ & \sim \begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & h+4 & 0 \end{bmatrix} \\ & \sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h+4 & 0 \end{bmatrix}, \end{aligned}$$

showing that the solutions of the original vector equation are

$$x_3(h+4) = 0, \quad x_2 = 0, \quad x_1 = x_3.$$

But then if  $h+4 \neq 0$ , the only solutions are

$$x_3 = 0, \quad x_2 = 0, \quad x_1 = 0,$$

while if  $h+4 = 0$ ,  $x_3$  is a free variable. Consequently, the vectors are linearly independent if and only if

$$h \neq -4.$$

---

**LinIndepTF01c**  
**015 10.0 points**

The columns of any  $4 \times 5$  matrix are linearly dependent.

True or False?

1. FALSE

2. TRUE **correct**

**Explanation:**

If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set

$$\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\}$$

in  $\mathbb{R}^n$  is linearly dependent if  $p > n$ .

Now when

$$A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_5]$$

is a  $4 \times 5$  matrix, there are 5 columns and each is a vector in  $\mathbb{R}^4$ . Because  $5 > 4$ , the columns must therefore be linearly dependent.

Consequently, the statement is

TRUE

### LinTransform01e

**016 10.0 points**

A transformation  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is linear if and only if

$$T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1T(\mathbf{v}_1) + c_2T(\mathbf{v}_2)$$

for all vectors  $\mathbf{v}_1, \mathbf{v}_2$  in  $\mathbb{R}^n$  and all scalars  $c_1, c_2$ .

True or False?

1. FALSE

2. TRUE correct

#### Explanation:

A transformation  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  by definition is linear when:

(i)  $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ ,

(ii)  $T(c\mathbf{u}) = cT(\mathbf{u})$ ,

for all  $\mathbf{u}, \mathbf{v}$  in  $\mathbb{R}^n$  and scalars  $c$ .

The property

$$T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1T(\mathbf{v}_1) + c_2T(\mathbf{v}_2)$$

combines (i) and (ii) into a single condition.

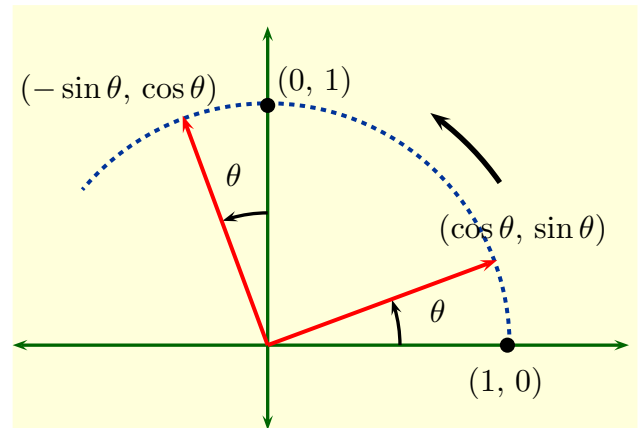
Consequently, the statement is

TRUE

### MatrixTrans01a

**017 10.0 points**

Determine the Standard Matrix for the transformation rotating the plane counter-clockwise about the origin through  $60^\circ$ .



1.  $A = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$

2.  $A = \frac{1}{2} \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$

3.  $A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

4.  $A = \frac{1}{2} \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$

5.  $A = \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$

6.  $A = \frac{1}{2} \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$  correct

#### Explanation:

When  $T$  is rotation counter-clockwise through  $\theta$  about the origin in the plane, then by right-angle trig in the graphic above, its associated Standard Matrix is

$$[T(1, 0) \ T(0, 1)] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$

When  $\theta = 60^\circ$ , therefore, this is the matrix

$$\frac{1}{2} \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}.$$