This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixOpsTF01b
001 10.0 points

When

$$A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \ldots \ \mathbf{a}_n]$$

and B are matrices such that the product AB is defined, then

$$AB = [B\mathbf{a}_1 \ B\mathbf{a}_2 \ \ldots \ B\mathbf{a}_n].$$

True or False?

1. FALSE **correct**

2. TRUE

Explanation:

The matrix

$$[B\mathbf{a}_1 \ B\mathbf{a}_2 \ \ldots \ B\mathbf{a}_n]$$

is the product BA, *not* AB, and $AB \neq BA$ in general.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

InverseProp01a
002 10.0 points

Which of the following shows that if A is an invertible $n \times n$ matrix and B is any $n \times n$ matrix such that $BA = I$, then $B = A^{-1}$.

1. *Left-multiply each side of the equation $I = BA$ by A^{-1}. Then*

$$A^{-1} = A^{-1}BA = BI = B,$$

so $B = A^{-1}$.

2. *Subtract A^{-1} from both sides of the equation $I = BA$. Then*

$$I - A^{-1} = BA - A^{-1},$$

so $A^{-1} = BI = B$.

3. *Add A^{-1} to both sides of the equation $I = BA$. Then*

$$I + A^{-1} = BA + A^{-1},$$

so $A^{-1} = BI = B$.

4. *Right-multiply each side of the equation $I = BA$ by A^{-1}. Then*

$$A^{-1} = BAA^{-1} = BI = B,$$

so $B = A^{-1}$. **correct**

Explanation:

By definition, an $n \times n$ matrix A is invertible if there exists an $n \times n$ matrix C such that $AC = I = CA$ where I is the $n \times n$ identity matrix; we usually write A^{-1} for this matrix C.

So if B is any $n \times n$ matrix such that $BA = I$, then after multiplying the equation $I = BA$ on the right by A^{-1} we see that

$$A^{-1} = (BA)A^{-1} = B(AA^{-1}) = BI = B$$

because of the associativity of matrix multiplication and the fact that I is the identity matrix.

InverseTF02a
003 10.0 points

If A is an $n \times n$ invertible matrix, then the same sequence of elementary row operations that row reduces A to the identity I_n also reduces A^{-1} to I_n.

True or False?

1. FALSE **correct**

2. TRUE

Explanation:

If an $n\times n$ matrix A is invertible, then there exist elementary matrices $E_1, E_2, \ldots, E_p$ such that

$$E_pE_{p-1}\ldots E_2E_1A \;=\; I_n\,.$$

Now every elementary matrix is invertible, so

$$\begin{aligned} A \;&=\; E_1^{-1}E_2{-}1\ldots E_{p-1}^{-1}E_pI_n \\ &=\; E_1^{-1}E_2^{-1}\ldots E_{p-1}^{-1}E_p^{-1}\,. \end{aligned}$$

But then

$$A^{-1} \;=\; E_pE_{p-1}\ldots E_2E_1\,,$$

so

$$E_1^{-1}E_2{-}1\ldots E_{p-1}^{-1}E_p^{-1}A^{-1} \;=\; I_n\,.$$

Thus A^{-1} can be row reduced to I_n by the same elementary row operations as used to row reduce A to I_n, but we need to use their inverses and to apply them in the reverse order.

Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

Invertible01
004 10.0 points

A is an $n \times n$ matrix. Which of the following statements are equivalent to A being invertible?

(i) *There is no $n \times n$ matrix C such that $CA = I$.*

(ii) *The equation $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.*

(iii) *The linear transformation $\mathbf{x} \to A\mathbf{x}$ is not one-to-one.*

1. i and ii

2. i and iii

3. ii

4. i

5. None of these. **correct**

6. All of these.

Explanation:

(i) A is said to be invertible if there is an $n\times n$ matrix C such that $CA = I$ and $AC = I$ where I is the $n \times n$ identity matrix.

(ii) Because A is invertible, there is some matrix C such that $CA = I_n$. Now we will suppose there is some $\mathbf{x}$ that satisfies the equation $A\mathbf{x} = \mathbf{0}$. Then by right-multiplying both sides of $CA = I_n$ by $\mathbf{x}$, we obtain the equation $CA\mathbf{x} = I_n\mathbf{x}$. Because $A\mathbf{x} = \mathbf{0}$, $CA\mathbf{x} = C\mathbf{0} = \mathbf{0}$. Also $I_n\mathbf{x} = \mathbf{x}$ by the definition of the identity matrix. Hence, $\mathbf{x} = \mathbf{0}$. This implies $A\mathbf{x} = \mathbf{0}$ has *only* the trivial solution.

(iii) Since the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, the linear transformation $\mathbf{x} \to A\mathbf{x}$ is one-to-one.

LUDecomp2x3b
005 10.0 points

Determine the Lower Triangular matrix L in an LU-Decomposition of

$$A \;=\; \begin{bmatrix} 3 & 2 & 0 \\ -9 & -6 & 1 \end{bmatrix}.$$

1. $L \;=\; \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ **correct**

2. $L \;=\; \begin{bmatrix} 3 & 0 \\ 9 & 1 \end{bmatrix}$

3. $L \;=\; \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

4. $L \;=\; \begin{bmatrix} 3 & 0 \\ 3 & 1 \end{bmatrix}$

5. $L \;=\; \begin{bmatrix} 3 & 0 \\ -3 & 1 \end{bmatrix}$

6. $L \;=\; \begin{bmatrix} 3 & 0 \\ -9 & 1 \end{bmatrix}$

Explanation:

We first determine the elementary matrix reducing A to echelon form U by row reductions *downwards.*

$$\begin{aligned} A \;&\sim\; E_1 A \\ &= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ -9 & -6 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = U\,. \end{aligned}$$

But an elementary matrix is always invertible. Thus

$$L \;=\; E_1^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}.$$

Consequently,

$$\boxed{\begin{aligned} L &= \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \\ U &= \begin{bmatrix} 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}}\,.$$

NullSpace01a

006 10.0 points

Find a matrix A so that $\mathrm{Nul}(A)$ is the set of all vectors

$$H \;=\; \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \quad \begin{aligned} a - b \;&=\; 2c\,, \\ 3a \;&=\; c - 4d\,, \end{aligned} \right\}$$

in $\mathbb{R}^4$.

1. $A \;=\; \begin{bmatrix} 1 & 1 & 2 & 0 \\ 3 & 0 & -1 & 4 \end{bmatrix}$

2. $A \;=\; \begin{bmatrix} 1 & 1 & 2 & 0 \\ 3 & 0 & 1 & -4 \end{bmatrix}$

3. $A \;=\; \begin{bmatrix} 1 & -1 & 2 & 0 \\ 3 & 0 & 1 & -4 \end{bmatrix}$

4. $A \;=\; \begin{bmatrix} 1 & -1 & -2 & 0 \\ 3 & 0 & -1 & 4 \end{bmatrix}$ **correct**

5. $A \;=\; \begin{bmatrix} 1 & 1 & -2 & 0 \\ 3 & 0 & 1 & -4 \end{bmatrix}$

6. $A \;=\; \begin{bmatrix} 1 & -1 & -2 & 0 \\ 3 & 0 & -1 & -4 \end{bmatrix}$

Explanation:

Rewrite the conditions

$$a - b \;=\; 2c, \quad 3a \;=\; c - 4d$$

as

$$\begin{aligned} a - b - 2c \;&=\; 0, \\ 3a - c + 4d \;&=\; 0\,, \end{aligned}$$

and set

$$A \;=\; \begin{bmatrix} 1 & -1 & -2 & 0 \\ 3 & 0 & -1 & 4 \end{bmatrix}.$$

Then

$$\begin{aligned} A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \;&=\; \begin{bmatrix} 1 & -1 & -2 & 0 \\ 3 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \\ &= \begin{bmatrix} a - b - 2c \\ 3a - c - 4d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

if and only if

$$\begin{aligned} a - b - 2c \;&=\; 0, \\ 3a - c + 4d \;&=\; 0\,. \end{aligned}$$

Consequently,

$$\boxed{\mathrm{Nul}(A) \;=\; H}\,.$$

Rank02b

007 10.0 points

Determine the rank of the matrix

$$A \;=\; \begin{bmatrix} 1 & -1 & 2 \\ -3 & 4 & -8 \\ 3 & -6 & 12 \end{bmatrix}.$$

1. $\mathrm{rank}(A) \;=\; 4$

2. $\mathrm{rank}(A) \;=\; 2$ **correct**

3. $\mathrm{rank}(A) \;=\; 3$

4. $\mathrm{rank}(A) \;=\; 1$

5. $\text{rank}(A) = 5$

Explanation:

Since

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix},$$

the first two rows of rref(A) contain leading 1's, so

$$\boxed{\text{Rank}(A) = 2}.$$

DetElemOps01TF
008 10.0 points

When the matrix

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is obtained from

$$A = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

by interchanging rows, then

$$\det[B] = \det[A].$$

True or False?

1. FALSE **correct**

2. TRUE

Explanation:

As 2×2 matrices,

$$\det[B] = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc,$$

while

$$\det[A] = \begin{vmatrix} c & d \\ a & b \end{vmatrix} = bc - ad.$$

Thus

$$\det[B] = -\det[A].$$

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

DetInverseT/F01a
009 10.0 points

The matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & -4 \\ 1 & 0 & 2 \end{bmatrix}$$

is invertible.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

The matrix A will be invertible if and only if $\det(A) \neq 0$. Now

$$\det(A) = \begin{vmatrix} 2 & 1 & 3 \\ 0 & 4 & -4 \\ 1 & 0 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 4 & -4 \\ 0 & 2 \end{vmatrix} - \begin{vmatrix} 0 & -4 \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 2 \times 8 - 4 - 3 \times 4 = 0.$$

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

SubspaceTF01
010 10.0 points

Let H be the set of points inside and on the unit circle in the xy-plane. That is, let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}$.

H is a subspace of $\mathbb{R}^2$. True or false?

1. FALSE **correct**

2. TRUE

Explanation:

If $\mathbf{u} = \begin{bmatrix} .5 \\ .5 \end{bmatrix}$ and $c = 4$, then $\mathbf{u}$ is in H but $c\mathbf{u}$ is not in H. Since H is not closed under scalar multiplication, H is not a subspace of $\mathbb{R}^2$. Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

BasisNull02a
011 10.0 points

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 3 & -3 & 6 & -15 \\ -1 & -2 & 7 & -1 \\ 3 & -2 & 3 & -13 \end{bmatrix}.$$

1. $\left\{ \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

2. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

3. $\left\{ \begin{bmatrix} -1 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

4. $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ **correct**

5. $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix} \right\}$

6. $\left\{ \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

Explanation:

We first row reduce $[A \;\; \mathbf{0}]$:

$$\text{rref}([A \;\; \mathbf{0}]) = \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

to identify the free variables for $\mathbf{x}$ in the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

This shows that x_1, x_2 are basic variables, while x_3, x_4 are free variables. So set $x_3 = s$, $x_4 = t$. Then

$$x_1 = s + 3t, \quad x_2 = 3s - 2t,$$

and

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} s+3t \\ 3s-2t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} 1 \\ 3 \\ s \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix}.$$

Thus

$$\text{Nul}(A) = \text{Span} \left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Consequently,

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is a basis for $\text{Nul}(A)$.

LinIndSetsTF01b
012 10.0 points

When $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p$ are vectors in $\mathbb{R}^n$ and

$$H = \text{Span}\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\},$$

then $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}$ is a basis for H.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

For the set $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}$ to be a basis for H, BOTH of the conditions

(i) $H = \text{Span}\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}$,

(ii) *the set is linearly independent*,

have to be satisfied. Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

CoordVec01a
013 10.0 points

Find the vector $\mathbf{x}$ in $\mathbb{R}^2$ having coordinate vector

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}$$

for $\mathbb{R}^2$.

1. $\mathbf{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ **correct**

2. $\mathbf{x} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

3. $\mathbf{x} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

4. no such $\mathbf{x}$ exists

5. $\mathbf{x} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$

Explanation:

The coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of a vector $\mathbf{x}$ in $\mathbb{R}^2$ with respect to a basis

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$$

for $\mathbb{R}^2$ satisfies the matrix equation

$$A[\mathbf{x}]_{\mathcal{B}} = \mathbf{x}, \qquad A = [\,\mathbf{b}_1 \quad \mathbf{b}_2\,]\,.$$

Consequently, when

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}, \quad [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix},$$

$$\boxed{\mathbf{x} = \begin{bmatrix} 4 & 6 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}}\,.$$

DimSubspace01a
014 10.0 points

Determine the dimension of the subspace

$$\text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix} \right\}$$

of $\mathbb{R}^3$.

1. dim $= 3$

2. dim $= 4$

3. dim $= 2$ **correct**

4. dim $= 1$

5. dim $= 5$

Explanation:

Since the subspace is the column space of the matrix

$$A = \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix}.$$

its dimension is the number of pivot columns in A. Now

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus A has 2 pivot columns, so

$$\boxed{\text{dimension} = 2}\,.$$

RankTF06b
015 10.0 points

If B is an echelon form of an an $m \times n$ matrix A, and if B has three nonzero rows, then the first three row of A form a basis for Row(A).

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

Although row operations cannot change the linear dependence relations among *columns* of a matrix, they can change the linear dependence among *rows* of a matrix. For example, when

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$B = \text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

then $\text{Row}(A) = \mathbb{R}^3$. But the first 3 rows of A do not form a basis for $\mathbb{R}^3$. Notice that B is obtained from A by the row operation of interchanging rows 3 and 4 of A.

Consequently, the answer is

$$\boxed{\text{FALSE}}.$$