

Advanced Calculus II - Fall 2016

Midterm Exam I, September 23, 2016

In all multiple choice problems you don't have to show your work. In all non-multiple choice problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

ParallelFace05c 10.0 points

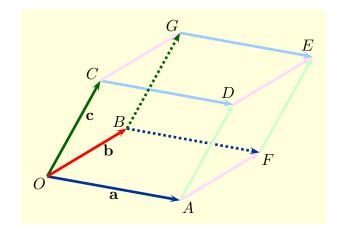
The vectors

$$\mathbf{a} = \langle 4, -3, -4 \rangle, \quad \mathbf{b} = \langle 2, -1, -3 \rangle,$$

and

$$\mathbf{c} = \langle 1, 2, 1 \rangle,$$

shown below form adjacent edges of the parallelepiped



Describe the face CDEG in vector form.

1.

$$\langle 2 + 4s + t, -1 - 3s + 2t, -3 - 4s + t \rangle$$
,
for, $0 \le s, t \le 1$.

2.

$$\langle 1 + 4s + 2t, 2 - 3s - t, 1 - 4s - 3t \rangle$$
,
for $0 \le s, t \le 1$.

3.

$$\langle 4 + 2s + t, -3 - s + 2t, -4 - 3s + t \rangle$$
,
for, $-1 \le s, t \le 1$.

4.
$$\langle s + 2t, 2s - t, s - 3t \rangle$$
, for $0 < s, t < 1$.

5. $\langle 2s+t, -s+2t, -3s+t \rangle$ for $-1 \le s, t \le 1$.

6.
$$\langle 1+4s+2t, \, 2-3s-t, \, 1-4s-3t \rangle,$$
 for $-1 \leq s, \, t \leq 1.$

CalC13a30aNC 10.0 points 002

Find an equation for the set of all points in 3-space equidistant from the points

$$A(-2, 2, -1), B(2, 4, 2).$$

1.
$$4x - 6y + 8z + 15 = 0$$

2.
$$4x + 8y - 6z - 15 = 0$$

3.
$$8x + 4y + 6z - 15 = 0$$

4.
$$6x + 4y + 8z + 15 = 0$$

5.
$$6x - 8y - 4z - 15 = 0$$

6.
$$8x + 4y + 6z + 15 = 0$$

If **a** is a vector parallel to the xy-plane and **b** is a vector parallel to **k**, determine $\|\mathbf{a} \times \mathbf{b}\|$ when $\|\mathbf{a}\| = 3$ and $\|\mathbf{b}\| = 2$.

1.
$$\|\mathbf{a} \times \mathbf{b}\| = -3$$

2.
$$\|\mathbf{a} \times \mathbf{b}\| = 6$$

3.
$$\|\mathbf{a} \times \mathbf{b}\| = 3\sqrt{2}$$

4.
$$\|\mathbf{a} \times \mathbf{b}\| = 0$$

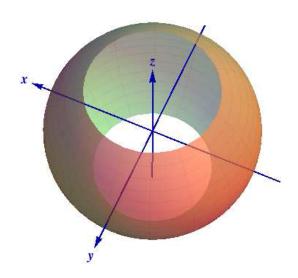
5.
$$\|\mathbf{a} \times \mathbf{b}\| = 3$$

6.
$$\|\mathbf{a} \times \mathbf{b}\| = -3\sqrt{2}$$

7.
$$\|\mathbf{a} \times \mathbf{b}\| = -6$$

SphericalCoords04click 004 10.0 points

The surface S shown in



consists of the portion of the sphere

$$x^2 + y^2 + z^2 = 16$$

where

$$x^2 + y^2 \ge 4.$$

Use spherical polar coordinates (ρ, θ, ϕ) to describe S.

- 1. $S = \text{all points } P(\rho, \theta, \phi)$ with $\rho = 4, \ \ 0 \le \theta \le 2\pi, \ \ \frac{\pi}{3} \le \phi \le \frac{2\pi}{3} \, .$
- 2. $S = \text{all points } P(\rho, \, \theta, \, \phi) \}$ with $\rho = 2, \;\; 0 \le \theta \le 2\pi, \;\; \frac{\pi}{3} \le \phi \le \pi \; .$
- 3. $S = \text{all points } P(\rho, \, \theta, \, \phi) \}$ with $\rho = 2, \ \ 0 \le \theta \le 2\pi, \ \ \frac{\pi}{3} \le \phi \le \frac{2\pi}{3} \, .$
- **4.** $S = \text{all points } P(\rho, \, \theta, \, \phi) \text{ with}$ $\rho = 4, \ \ 0 \le \theta \le 2\pi, \ \ \frac{\pi}{6} \le \phi \le \frac{5\pi}{6}.$
- 5. S= all points $P(\rho,\,\theta,\,\phi)\}$ with $\rho=4,\ \ 0\leq\theta\leq2\pi,\ \ \frac{\pi}{6}\leq\phi\leq\pi\,.$
- **6.** $S = \text{all points } P(\rho, \theta, \phi) \}$ with $\rho = 2, \quad 0 \le \theta \le 2\pi, \quad \frac{\pi}{6} \le \frac{5\pi}{6}.$

$\begin{array}{cc} Fin M4e 05 \\ 005 & 10.0 \ points \end{array}$

Solve for X when AX + B = C,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 2 \\ -1 & 2 \end{bmatrix},$$

and

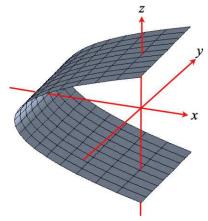
$$C = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}.$$

5.
$$x + y^2 - 4 = 0$$

6.
$$z + x^2 - 4 = 0$$

$\begin{array}{cc} CalC13f04c \\ 006 & 10.0 \ points \end{array}$

Which one of the following equations has graph



1.
$$x - z^2 + 4 = 0$$

2.
$$z - y^2 + 4 = 0$$

$$3. \ y - x^2 + 4 = 0$$

4.
$$y + z^2 - 4 = 0$$

$\begin{array}{cc} CalC15b16s \\ 007 & 10.0 \ points \end{array}$

Find
$$\lim_{(x,y)\to(0,0)} \frac{7xy^4}{x^2+y^8}$$
, if it exists.

$\begin{array}{cc} CalC15d11s \\ 008 & 10.0 \ points \end{array}$

Find the linearization, L(x, y), of

$$f(x, y) = y\sqrt{x}$$

at the point (4, -2).

1.
$$L(x, y) = -4 + x + \frac{1}{2}y$$

2.
$$L(x, y) = 2 - \frac{1}{2}x + 2y$$

3.
$$L(x, y) = -2 + \frac{1}{2}x + 2y$$

4.
$$L(x, y) = -2 + 2x + \frac{1}{2}y$$

5.
$$L(x, y) = -4 + \frac{1}{2}x - y$$

6.
$$L(x, y) = 2 + 2x - \frac{1}{2}y$$

Tangent01a 009 10.0 points

If $\mathbf{r}(x)$ is the vector function whose graph is trace of the surface

$$z = f(x, y) = 3x^2 - y^2 - 3x + 3y$$

on the plane y = 2x, determine the tangent vector to $\mathbf{r}(x)$ at x = 1.

$\begin{array}{cc} CalC15e07s \\ 010 & 10.0 \text{ points} \end{array}$

Use the Chain Rule to find $\frac{\partial z}{\partial s}$ when

$$z = x^2 + 3xy + y^2,$$

and

$$x = 4s - t, \qquad y = st.$$

1.
$$\frac{\partial z}{\partial s} = -2x + 12y + 3xs + 2ys$$

2.
$$\frac{\partial z}{\partial s} = -2x - 3y + 3xs + 2ys$$

3.
$$\frac{\partial z}{\partial s} = 8x + 12y + 3xs + 2ys$$

4.
$$\frac{\partial z}{\partial s} = 8x + 12y + 3xt + 2yt$$

5.
$$\frac{\partial z}{\partial s} = 8x - 3y + 3xt + 2yt$$

6.
$$\frac{\partial z}{\partial s} = -2x - 3y + 3xt + 2yt$$

Find the directional derivative, $f_{\mathbf{v}}$, of the function

$$f(x, y) = 4 + x\sqrt{y}$$

at the point P(3, 9) in the direction of the vector

$$\mathbf{v} = \langle 3, 4 \rangle$$
.

1.
$$f_{\mathbf{v}} = \frac{32}{15}$$

2.
$$f_{\mathbf{v}} = \frac{31}{15}$$

3.
$$f_{\mathbf{v}} = \frac{11}{5}$$

4.
$$f_{\mathbf{v}} = \frac{34}{15}$$

5.
$$f_{\mathbf{v}} = 2$$