This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

ParallelFace05c
001 10.0 points

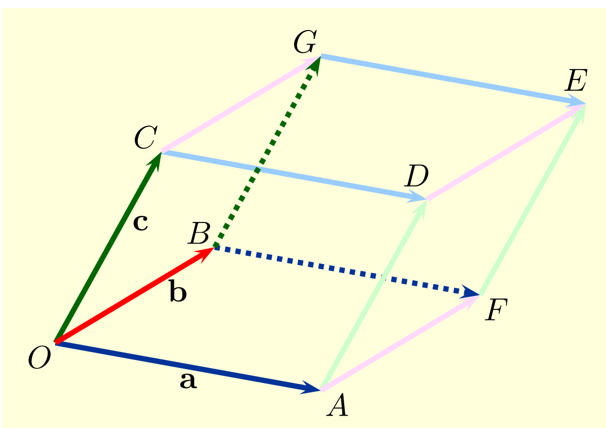
The vectors

$$\mathbf{a} = \langle 3, -1, -3\rangle, \quad \mathbf{b} = \langle 2, -1, -1\rangle,$$

and

$$\mathbf{c} = \langle 1, -1, 4\rangle,$$

shown below form adjacent edges of the parallelepiped



Describe the face $CDEG$ in vector form.

1.

$$\langle 2+3s+t, -1-s-t, -1-3s+4t\rangle,$$

for, $0 \le s, t \le 1$.

2.

$$\langle 3+2s+t, -1-s-t, -3-s+4t\rangle,$$

for, $-1 \le s, t \le 1$.

3.

$$\langle s+2t, -s-t, 4s-t\rangle,$$

for $0 \le s, t \le 1$.

4.

$$\langle 1+3s+2t, -1-s-t, 4-3s-t\rangle,$$

for $-1 \le s, t \le 1$.

5.

$$\langle 1+3s+2t, -1-s-t, 4-3s-t\rangle,$$

for $0 \le s, t \le 1$.

correct

6.

$$\langle 2s+t, -s-t, -s+4t\rangle,$$

for $-1 \le s, t \le 1$.

Explanation:

The face $CDEG$ of the parallelepiped lies in the unique plane in which the vertices C, D, and G lie. Now in vector form this plane is

$$\mathbf{x} = \mathbf{c} + s\mathbf{a} + t\mathbf{b}, \quad -\infty \le s, t \le \infty.$$

But the points in the parallelogram $CDEG$ lying in this plane correspond to $0 \le s \le 1$ and $0 \le t \le 1$, *i.e.*, to

$$\mathbf{c} + s\mathbf{a} + t\mathbf{b}, \quad 0 \le s, t \le 1.$$

Consequently, the face $CDEG$ is given in vector form by

$$\boxed{\langle 1+3s+2t, -1-s-t, 4-3s-t\rangle,}.$$

for $0 \le s, t \le 1$.

CalC13a30aNC
002 10.0 points

Find an equation for the set of all points in 3-space equidistant from the points

$$A(1, -3, 2), \qquad B(4, 1, 3).$$

1. $4x + 3y - z - 6 = 0$

2. $3x + 4y + z + 6 = 0$

3. $3x + 4y + z - 6 = 0$ **correct**

4. $x + 4y + 3z + 6 = 0$

5. $4x - y + 3z + 6 = 0$

6. $x - 3y - 4z - 6 = 0$

Explanation:

We have to find the set of points $P(x, y, z)$ such that

$$\|\overline{AP}\| = \|\overline{BP}\|.$$

Now by the distance formula in 3-space,

$$\|\overline{AP}\|^2 = (x-1)^2 + (y+3)^2 + (z-2)^2,$$

while

$$\|\overline{BP}\|^2 = (x-4)^2 + (y-1)^2 + (z-3)^2.$$

After expansion therefore,

$$\|\overline{AP}\|^2 = x^2 - 2x + y^2 + 6y + z^2 - 4z + 14,$$

while

$$\|\overline{BP}\|^2 = x^2 - 8x + y^2 - 2y + z^2 - 6z + 26.$$

Thus $\|\overline{AP}\| = \|\overline{BP}\|$ when

$$x^2 - 2x + y^2 + 6y + z^2 - 4z + 14 = x^2 - 8x + y^2 - 2y + z^2 - 6z + 26.$$

Consequently, the set of all points equidistant from A and B satisfies the equation

$$\boxed{3x + 4y + z - 6 = 0}.$$

Notice that this is a plane perpendicular to the line segment joining A and B (since it must contain the perpendicular bisector of the line segment $\overline{AB}$).

keywords: plane, locus points, equidistant two points

CalC13d12s
003 10.0 points

If $\mathbf{a}$ is a vector parallel to the xy-plane and $\mathbf{b}$ is a vector parallel to $\mathbf{k}$, determine $\|\mathbf{a} \times \mathbf{b}\|$ when $\|\mathbf{a}\| = 1$ and $\|\mathbf{b}\| = 4$.

1. $\|\mathbf{a} \times \mathbf{b}\| = 2$

2. $\|\mathbf{a} \times \mathbf{b}\| = 0$

3. $\|\mathbf{a} \times \mathbf{b}\| = -2$

4. $\|\mathbf{a} \times \mathbf{b}\| = -4$

5. $\|\mathbf{a} \times \mathbf{b}\| = -2\sqrt{2}$

6. $\|\mathbf{a} \times \mathbf{b}\| = 2\sqrt{2}$

7. $\|\mathbf{a} \times \mathbf{b}\| = 4$ **correct**

Explanation:

For vectors $\mathbf{a}$ and $\mathbf{b}$,

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta$$

when the angle between them is θ, $0 \le \theta < \pi$.

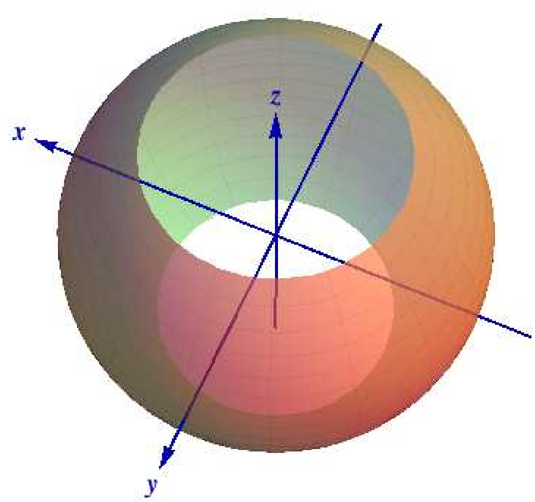
But $\theta = \pi/2$ in the case when $\mathbf{a}$ is parallel to the xy-plane and $\mathbf{b}$ is parallel to $\mathbf{k}$ because $\mathbf{k}$ is then perpendicular to the xy-plane. Consequently, for the given vectors,

$$\boxed{\|\mathbf{a} \times \mathbf{b}\| = 4}.$$

keywords: cross product, length, angle,

SphericalCoords04click
004 10.0 points

The surface S shown in



consists of the portion of the sphere

$$x^2 + y^2 + z^2 \;=\; 16$$

where

$$x^2 + y^2 \;\geq\; 4\,.$$

Use spherical polar coordinates $(\rho,\, \theta,\, \phi)$ to describe S.

1. $S =$ all points $P(\rho,\, \theta,\, \phi)\}$ with

$$\rho = 4,\;\; 0 \leq \theta \leq 2\pi,\;\; \frac{\pi}{6} \leq \phi \leq \pi\,.$$

2. $S =$ all points $P(\rho,\, \theta,\, \phi)\}$ with

$$\rho = 4,\;\; 0 \leq \theta \leq 2\pi,\;\; \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}\,.$$

3. $S =$ all points $P(\rho,\, \theta,\, \phi)$ with

$$\rho = 4,\;\; 0 \leq \theta \leq 2\pi,\;\; \frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}\,.$$

correct

4. $S =$ all points $P(\rho,\, \theta,\, \phi)\}$ with

$$\rho = 2,\;\; 0 \leq \theta \leq 2\pi,\;\; \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}\,.$$

5. $S =$ all points $P(\rho,\, \theta,\, \phi)\}$ with

$$\rho = 2,\;\; 0 \leq \theta \leq 2\pi,\;\; \frac{\pi}{3} \leq \phi \leq \pi\,.$$

6. $S =$ all points $P(\rho,\, \theta,\, \phi)\}$ with

$$\rho = 2,\;\; 0 \leq \theta \leq 2\pi,\;\; \frac{\pi}{6} \leq \frac{5\pi}{6}\,.$$

Explanation:

In spherical polar coordinates $(\rho,\, \theta,\, \phi)$,

$$x \;=\; \rho \sin\phi \cos\theta, \quad y \;=\; \rho \sin\phi \sin\theta\,,$$

and

$$z \;=\; \rho \cos\phi\,,$$

with $0 \leq \theta \leq 2\pi$ and $0 \leq \psi \leq \pi$. We need to find further restrictions on ρ, θ, and ϕ so that

$$x^2 + y^2 + z^2 \;=\; 16, \quad x^2 + y^2 \;\geq\; 4\,.$$

Now

$$\rho^2 \;=\; x^2 + y^2 + z^2 \;=\; 16\,,$$

i.e., $\rho = 4$. But then,

$$z^2 \;=\; 16\cos^2\phi \;=\; 16 - x^2 - y^2 \;\leq\; 12\,.$$

Consequently, S consists of all points P with $\rho = 4$ and

$$\boxed{0 \leq \theta \leq 2\pi,\;\; \frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}}\,.$$

FinM4e05
005 10.0 points

Solve for X when $AX + B = C$,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix},\;\; B = \begin{bmatrix} 4 & 2 \\ -1 & 4 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}.$$

1. $X \;=\; \begin{bmatrix} 3 & -4 \\ -2 & 6 \end{bmatrix}$

2. $X \;=\; \begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}$ **correct**

3. $X \;=\; \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

4. $X \;=\; \begin{bmatrix} 4 & -4 \\ -1 & 3 \end{bmatrix}$

5. $X \;=\; \begin{bmatrix} 4 & -4 \\ 2 & 6 \end{bmatrix}$

Explanation:

By the algbra of matrices,

$$X \;=\; A^{-1}(C - B).$$

But the inverse of any 2×2 matrix

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

is given by

$$D^{-1} = \begin{bmatrix} \dfrac{d_{22}}{\Delta} & -\dfrac{d_{12}}{\Delta} \\ -\dfrac{d_{21}}{\Delta} & \dfrac{d_{11}}{\Delta} \end{bmatrix}$$

with $\Delta = d_{11}d_{22} - d_{12}d_{21}$, so

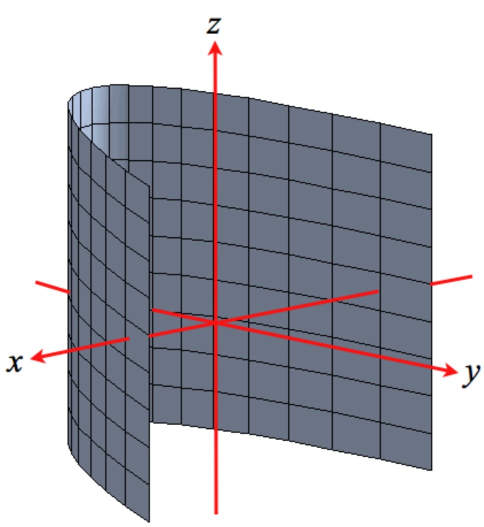
$$X = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \left(\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 4 \end{bmatrix} \right) = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix}.$$

Thus

$$\boxed{X = \begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}.}$$

CalC13f04c
006 10.0 points

Which one of the following equations has graph



1. $x + y^2 - 4 = 0$

2. $x - z^2 + 4 = 0$

3. $z - y^2 + 4 = 0$

4. $y - x^2 + 4 = 0$ **correct**

5. $y + z^2 - 4 = 0$

6. $z + x^2 - 4 = 0$

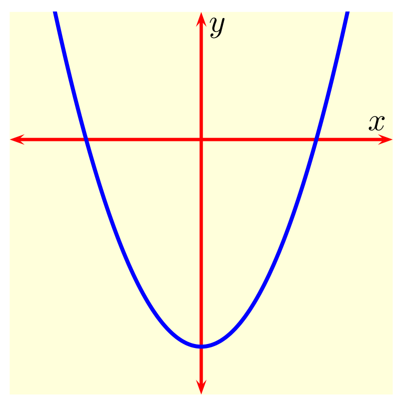
Explanation:

The graph is a parabolic cylinder that has constant value on any line parallel to the z-axis, so it will be the graph of an equation containing no z-term. This already eliminates the equations

$$z - y^2 + 4 = 0, \quad z + x^2 - 4 = 0,$$

$$x - z^2 + 4 = 0, \quad y + z^2 - 4 = 0.$$

On the other hand, the intersection of the graph with the xy-plane, *i.e.* the $z = 0$ plane, is a parabola opening upwards on the y-axis as shown in



Consequently, the graph is that of the equation

$$\boxed{y - x^2 + 4 = 0}.$$

keywords: quadric surface, graph of equation, cylinder, 3D graph, parabolic cylinder, trace

CalC15b16s
007 10.0 points

Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy^4}{x^2 + y^8}$, if it exists.

1. 6

2. 0

3. 3

4. 1.5

5. The limit does not exist. **correct**

Explanation:

CalC15d11s
008 10.0 points

Find the linearization, $L(x, y)$, of

$$f(x, y) \;=\; x\sqrt{y}$$

at the point $(2, 9)$.

1. $L(x, y) \;=\; 6 - \dfrac{1}{3}x - \dfrac{3}{2}y$

2. $L(x, y) \;=\; 6 + \dfrac{3}{2}x - \dfrac{1}{3}y$

3. $L(x, y) \;=\; -3 + \dfrac{1}{3}x + 3y$

4. $L(x, y) \;=\; -3 + 3x + \dfrac{1}{3}y$ **correct**

5. $L(x, y) \;=\; 3 + 3x - \dfrac{1}{3}y$

6. $L(x, y) \;=\; 3 - \dfrac{1}{3}x + 3y$

Explanation:
The linearization of $f = f(x, y)$ at a point (a, b) is given by

$$L(x, y) = f(a, b) + (x-a)\frac{\partial f}{\partial x}\Big|_{(a,b)} + (y-b)\frac{\partial f}{\partial y}\Big|_{(a,b)}.$$

But when $f(x, y) = x\sqrt{y}$,

$$\frac{\partial f}{\partial x} \;=\; \sqrt{y}\,, \qquad \frac{\partial f}{\partial y} \;=\; \frac{x}{2\sqrt{y}}\,;$$

thus when $(a, b) = (2, 9)$,

$$\frac{\partial f}{\partial x}\Big|_{(a,b)} = 3\,, \qquad \frac{\partial f}{\partial y}\Big|_{(a,b)} \;=\; \frac{1}{3}\,,$$

while $f(a, b) = 6$. Consequently,

$$\boxed{L(x, y) \;=\; -3 + 3x + \frac{1}{3}y}\,.$$

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keywords:

Tangent01a
009 10.0 points

If $\mathbf{r}(x)$ is the vector function whose graph is trace of the surface

$$z \;=\; f(x, y) \;=\; 2x^2 - y^2 + 2x - 3y$$

on the plane $y = 2x$, determine the tangent vector to $\mathbf{r}(x)$ at $x = 1$.

1. tangent vector $=\; \langle\, 1, 0, -8 \,\rangle$

2. tangent vector $=\; \langle\, 2, 1, -4 \,\rangle$

3. tangent vector $=\; \langle\, 2, 2, -8 \,\rangle$

4. tangent vector $=\; \langle\, 1, 2, -4 \,\rangle$

5. tangent vector $=\; \langle\, 2, 0, -4 \,\rangle$

6. tangent vector $=\; \langle\, 1, 2, -8 \,\rangle$ **correct**

Explanation:
The graph of

$$z \;=\; f(x, y) \;=\; 2x^2 - y^2 + 2x - 3y$$

is the set of all points

$$(x,\, y,\, f(x, y))$$

as x, y vary in 3-space. So the intersection of the surface with the plane $y = 2x$ is the set of all points

$$(x,\, 2x,\, f(x, 2x))\,, \quad -\infty < x < \infty\,.$$

But

$$f(x, 2x) \;=\; -2x^2 - 4x\,.$$

Thus the surface and the plane $y = 2x$ intersect in the graph of

$$\mathbf{r}(x) \;=\; \langle\, x,\, 2x,\, -2x^2 - 4x \,\rangle\,.$$

Now the tangent vector to the graph of $\mathbf{r}(x)$ is the derivative

$$\mathbf{r}'(x) \;=\; \langle\, 1,\, 2,\, -4x-4 \,\rangle\,.$$

Consequently, at $x = 1$ the graph of $\mathbf{r}(x)$ has

$$\boxed{\text{tangent vector} \;=\; \langle\, 1,\, 2,\, -8 \,\rangle}\,.$$

keywords:

CalC15e07s

010 10.0 points

Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ when

$$z \;=\; x^2 - xy + y^2\,,$$

and

$$x \;=\; 3s + t\,, \qquad y \;=\; st\,.$$

1. $\dfrac{\partial z}{\partial s} \;=\; 6x - y - xt + 2yt$

2. $\dfrac{\partial z}{\partial s} \;=\; 2x - y - xt + 2yt$

3. $\dfrac{\partial z}{\partial s} \;=\; 6x - 3y - xt + 2yt$ **correct**

4. $\dfrac{\partial z}{\partial s} \;=\; 2x - y - xs + 2ys$

5. $\dfrac{\partial z}{\partial s} \;=\; 6x - 3y - xs + 2ys$

6. $\dfrac{\partial z}{\partial s} \;=\; 2x - 3y - xs + 2ys$

Explanation:

By the Chain Rule for Partial Differentiation,

$$\frac{\partial z}{\partial s} \;=\; \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}\,.$$

Now

$$\frac{\partial z}{\partial x} \;=\; 2x - y, \qquad \frac{\partial x}{\partial s} \;=\; 3$$

while

$$\frac{\partial z}{\partial y} \;=\; -x + 2y, \qquad \frac{\partial y}{\partial s} \;=\; t\,.$$

Thus

$$\frac{\partial z}{\partial s} \;=\; 3(2x - y) + t(-x + 2y)\,.$$

Consequently,

$$\boxed{\frac{\partial z}{\partial s} \;=\; 6x - 3y - xt + 2yt}\,.$$

CalC15f11s

011 10.0 points

Find the directional derivative, $f_{\mathbf{v}}$, of the function

$$f(x,\, y) \;=\; 4 + x\sqrt{y}$$

at the point $P(1,\, 1)$ in the direction of the vector

$$\mathbf{v} \;=\; \langle\, 3,\, 4 \,\rangle\,.$$

1. $f_{\mathbf{v}} \;=\; \dfrac{6}{5}$

2. $f_{\mathbf{v}} \;=\; \dfrac{2}{5}$

3. $f_{\mathbf{v}} \;=\; \dfrac{4}{5}$

4. $f_{\mathbf{v}} \;=\; \dfrac{3}{5}$

5. $f_{\mathbf{v}} \;=\; 1$ **correct**

Explanation:

Now for an arbitrary vector $\mathbf{v}$,

$$f_{\mathbf{v}} \;=\; \nabla f \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right)\,,$$

where we have normalized so that the direction vector has unit length. But when

$$f(x,\, y) \;=\; 4 + x\sqrt{y}\,,$$

then

$$\nabla f \;=\; (\sqrt{y})\,\mathbf{i} + \frac{1}{2}\left(\frac{x}{\sqrt{y}}\right)\mathbf{j}\,.$$

At $P(1,\, 1)$, therefore,

$$\nabla f\Big|_P \;=\; \mathbf{i} + \frac{1}{2}\,\mathbf{j}\,.$$

Consequently, when $\mathbf{v} = \langle 3,\, 4\rangle$,

$$\boxed{f_{\mathbf{v}}(1,\, 1) \;=\; \left\langle\, ,\, \frac{1}{2} \right\rangle \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) \;=\; 1}\,.$$

keywords: