

This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixVecProd03
001 10.0 points

Determine \mathbf{uv}^T when

$$\mathbf{u} = \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

1. $\mathbf{uv}^T = -2a - 3b + 4c$

2. $\mathbf{uv}^T = \begin{bmatrix} -2a & -2b & -2c \\ -3a & -3b & -3c \\ 4a & 4b & 4c \end{bmatrix}$ **correct**

3. $\mathbf{uv}^T = 4a - 3b - 2c$

4. $\mathbf{uv}^T = \begin{bmatrix} -2a & -3a & 4a \\ -2b & -3b & 4b \\ -2c & -3c & 4c \end{bmatrix}$

Explanation:

Since

$$\mathbf{v}^T = [a \ b \ c],$$

we see that

$$\begin{aligned} \mathbf{uv}^T &= \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix} [a \ b \ c] \\ &= \begin{bmatrix} -2a & -2b & -2c \\ -3a & -3b & -3c \\ 4a & 4b & 4c \end{bmatrix}. \end{aligned}$$

Consequently,

$$\mathbf{uv}^T = \begin{bmatrix} -2a & -2b & -2c \\ -3a & -3b & -3c \\ 4a & 4b & 4c \end{bmatrix}.$$

M340LInverseTF04
002 10.0 points

Suppose $AB = AC$ for some A, B, C matrices. Suppose that A is invertible. Then, $B = C$.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:

If $AB = AC$ for some A, B, C matrices and A is invertible. Then, $A^{-1}AB = A^{-1}AC$, since $A^{-1}A = I$, it follows that $B = C$.

Consequently, the statement is

TRUE

MatrixAlg02aT/F
003 10.0 points

All $n \times n$ invertible matrices A, B have the property

$$(AB)^{-1} = A^{-1}B^{-1}.$$

True or False?

1. FALSE **correct**

2. TRUE

Explanation:

Set

$$A = \begin{bmatrix} a & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 \\ 1 & b \end{bmatrix}.$$

Then

$$AB = \begin{bmatrix} 1 & b-a \\ 0 & 1 \end{bmatrix}$$

and

$$A^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & a \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} b & 1 \\ -1 & 0 \end{bmatrix}.$$

In this case,

$$(AB)^{-1} = \begin{bmatrix} 1 & a-b \\ 0 & 1 \end{bmatrix},$$

while

$$A^{-1}B^{-1} = \begin{bmatrix} 1 & 0 \\ b-a & 1 \end{bmatrix},$$

so $(AB)^{-1} \neq A^{-1}B^{-1}$ when $a \neq b$.

Consequently, the statement is

FALSE

.

InvertibleTF02a
004 10.0 points

If A and D are $n \times n$ matrices such that $AD = I$, then $DA = I$

True or False?

1. TRUE **correct**

2. FALSE

Explanation:

Because A and D are square matrices and $AD = I$, then A and D are both invertible, with $D = A^{-1}$ and $A = D^{-1}$. So using this substitution, the first equation can be rewritten as $AA^{-1} = I$, and the second as $DD^{-1} = I$. Both of these statements are true by the definition of inverse matrices.

Consequently, the statement is

TRUE

.

LUDecomp3x4a
005 10.0 points

Determine the Lower Triangular Matrix L in an LU -decomposition of the matrix

$$A = \begin{bmatrix} 4 & -3 & 4 & -3 \\ -8 & 11 & -10 & 9 \\ 16 & -17 & 18 & -17 \end{bmatrix}.$$

1. $L = \begin{bmatrix} 4 & -3 & -3 \\ 0 & 5 & 3 \\ 0 & 0 & -2 \end{bmatrix}$

2. $L = \begin{bmatrix} 1 & -3 & 4 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$

3. $L = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

4. $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & 1 & 1 \end{bmatrix}$

5. $L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}$

6. $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix}$ **correct**

Explanation:

We first determine the elementary matrices reducing A to an echelon form U by row reductions *downwards*.

Set

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that

$$\begin{aligned} E_1 A &= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & -3 & 4 & -3 \\ -8 & 11 & -10 & 9 \\ 16 & -17 & 18 & -17 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -3 & 4 & -3 \\ 0 & 5 & -2 & 3 \\ 16 & -17 & 18 & -17 \end{bmatrix} = A_1, \end{aligned}$$

say. Next set

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

so that

$$\begin{aligned} E_2 A_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & -3 & 4 & -3 \\ 0 & 5 & -2 & 3 \\ 16 & -17 & 18 & -17 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -3 & 4 & -3 \\ 0 & 5 & -2 & 3 \\ 0 & -5 & 2 & -5 \end{bmatrix} = A_2, \end{aligned}$$

say. Finally, set

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

so that the product

$$\begin{aligned} E_3 A_2 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -3 & 4 & -3 \\ 0 & 5 & -2 & 3 \\ 0 & -5 & 2 & -5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -3 & 4 & -3 \\ 0 & 5 & -2 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix} = U, \end{aligned}$$

and

$$E_3 E_2 E_1 A = E_3 E_2 A_1 = E_3 A_2 = U$$

is an echelon form of A . But every elementary matrix is invertible. Thus $A = LU$, setting

$$\begin{aligned} L &= E_1^{-1} E_2^{-1} E_3^{-1} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix}. \end{aligned}$$

Consequently,

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix}.$$

Subspace05a
006 10.0 points

Let H be the set of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

where a and b are real. Determine if H is a subspace of \mathbb{R}^3 , and then check the correct answer below.

1. H is not a subspace of \mathbb{R}^3 because it does not contain $\mathbf{0}$.

2. H is a subspace of \mathbb{R}^3 because it can be written as $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ with $\mathbf{v}_1, \mathbf{v}_2$ in \mathbb{R}^3 .

3. H is not a subspace of \mathbb{R}^3 because it is not closed under vector addition. **correct**

4. H is a subspace of \mathbb{R}^3 because it can be written as $\text{Nul}(A)$ for some matrix A .

Explanation:

To check if the set H of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

is a subspace of \mathbb{R}^3 we check the properties defining a subspace:

1. the zero vector $\mathbf{0}$ is in H : set $a = b = 0$. Then

$$\begin{bmatrix} 0 - 0 \\ 0 + 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

so H contains $\mathbf{0}$.

2. for each \mathbf{u}, \mathbf{v} in H the sum $\mathbf{u} + \mathbf{v}$ is in H : set

$$\mathbf{v}_1 = \begin{bmatrix} a_1 - 2b_1 \\ a_1 b_1 + 3a_1 \\ b_1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} a_2 - 2b_2 \\ a_2 b_2 + 3a_2 \\ b_2 \end{bmatrix},$$

in H . Then

$$\begin{aligned} \mathbf{v}_1 + \mathbf{v}_2 &= \begin{bmatrix} a_1 - 2b_1 \\ a_1 b_1 + 3a_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} a_2 - 2b_2 \\ a_2 b_2 + 3a_2 \\ b_2 \end{bmatrix} \\ &= \begin{bmatrix} (a_1 + a_2) - 2(b_1 + b_2) \\ a_1 b_1 + a_2 b_2 + 3(a_1 + a_2) \\ (b_1 + b_2) \end{bmatrix}. \end{aligned}$$

But in general,

$$a_1b_1 + a_2b_2 \neq (a_1 + a_2)(b_1 + b_2),$$

in which case $\mathbf{u} + \mathbf{v}$ is not in H .

Consequently, H is not a subspace of \mathbb{R}^3 because it is

not closed under vector addition

DimRankTF02a
007 10.0 points

If \mathcal{B} is a basis for a subspace H , then each vector in H can be written in only one way as a linear combination of the vectors in \mathcal{B} .

True or False?

1. TRUE correct

2. FALSE

Explanation:

If H is a p -dimensional subspace of \mathbb{R}^n , then \mathcal{B} must be a set of p elements $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\}$. To represent any vector \mathbf{b} in H is to find the coefficients c_j in the linear combination

$$\mathbf{b} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_p\mathbf{v}_p$$

of vectors in \mathcal{B} whose sum is \mathbf{b} . This is equivalent to solving a system of equations with p equations and p unknowns. Because the vectors are linearly independent by the definition of a basis, this means there can only be one solution.

Consequently, the statement is

TRUE

DetElemOps02TF
008 10.0 points

When the matrix

$$B = \begin{bmatrix} a & b \\ c + ka & d + kb \end{bmatrix}$$

is obtained from

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

by adding k times row 1 to row 2, then

$$\det[B] = \det[A].$$

True or False?

1. FALSE

2. TRUE correct

Explanation:

As 2×2 matrices,

$$\begin{aligned} \det[B] &= \begin{vmatrix} a & b \\ c + ka & d + kb \end{vmatrix} \\ &= a(d + kb) - b(c + ka) = ad - bc, \end{aligned}$$

while

$$\det[A] = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Thus

$$\det[B] = \det[A].$$

Consequently, the statement is

TRUE

DetInverseT/F01b
009 10.0 points

The matrix

$$A = \begin{bmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & 5 & 3 \end{bmatrix}$$

is invertible.

True or False?

1. FALSE correct

2. TRUE

Explanation:

The matrix A will be invertible if and only if $\det(A) \neq 0$. Now

$$\begin{aligned}\det(A) &= \begin{vmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & 5 & 3 \end{vmatrix} \\ &= 5 \begin{vmatrix} -3 & -2 \\ 5 & 3 \end{vmatrix} - \begin{vmatrix} 1 & -3 \\ 0 & 5 \end{vmatrix} \\ &= 5 \times 1 - 1 \times 5 = 0.\end{aligned}$$

Consequently, the statement is

FALSE

.

VectorSpace01aT/F
010 10.0 points

The subset

$$V = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : ab \geq 0 \right\}$$

of \mathbb{R}^2 is closed under scalar multiplication.

True or False?

1. TRUE correct

2. FALSE

Explanation:

For each scalar k ,

$$k \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ka \\ kb \end{bmatrix}.$$

Now $(ka)(kb) = k^2(ab)$, so

$$ab \geq 0 \implies (ka)(kb) = k^2 ab \geq 0$$

because $k^2 \geq 0$ for all k .

Consequently, the statement is

TRUE

.

BasisNull02b
011 10.0 points

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 3 & 9 & -3 & 0 \\ -3 & -9 & 6 & 6 \\ -3 & -9 & 5 & 4 \end{bmatrix}.$$

1. $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$
2. $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$ **correct**
3. $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$
4. $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$
5. $\left\{ \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$
6. $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

Explanation:

We first row reduce $[A \ 0]$:

$$\text{rref}([A \ 0]) = \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

to identify the free variables for \mathbf{x} in the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

Thus x_1 and x_3 are basic variables, while x_2 and x_4 are free variables. So set $x_2 = s$ and $x_4 = t$. Then

$$x_1 = -3s - 2t, \quad x_3 = -2t,$$

and

$$\text{Nul}(A) = \text{Span} \left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}.$$

Consequently,

$$\mathcal{B} = \left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$$

is a basis for $\text{Nul}(A)$.

LinIndSetsTF02e
012 10.0 points

If B is an echelon form of a matrix A , then the pivot columns of B form a basis for $\text{Col } A$.

True or False?

1. TRUE

2. FALSE correct

Explanation:

The pivot columns of A are the same as the pivot columns for B , and the pivot columns of A form a basis for $\text{Col}(A)$. But row operations on a matrix can change the entries in a column, so the columns of B need not be in the column space of A . Thus the pivot columns of B need not form a basis for $\text{Col}(A)$.

Consequently, the statement is

FALSE

CoordVec01b
013 10.0 points

Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ in \mathbb{R}^2 for the vector

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \end{bmatrix} \right\}$$

for \mathbb{R}^2 .

1. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

2. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

3. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ -2 \end{bmatrix}$

4. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$

5. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

6. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$ correct

Explanation:

The coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of a vector \mathbf{x} in \mathbb{R}^2 with respect to a basis

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$$

for \mathbb{R}^2 satisfies the matrix equation

$$A[\mathbf{x}]_{\mathcal{B}} = \mathbf{x}, \quad A = [\mathbf{b}_1 \quad \mathbf{b}_2].$$

Consequently, when

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \end{bmatrix} \right\},$$

and

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix},$$

the associated augmented matrix is

$$[A \quad \mathbf{x}] = \begin{bmatrix} 1 & 5 & 4 \\ -2 & -6 & 0 \end{bmatrix}.$$

But then

$$\text{rref}[A \quad \mathbf{x}] = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 2 \end{bmatrix}.$$

Thus

$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

DimSubspace01b
014 10.0 points

Determine the dimension of the subspace

$$\text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix} \right\}$$

of \mathbb{R}^3 .

1. $\dim = 4$
2. $\dim = 5$
3. $\dim = 1$
4. $\dim = 2$ **correct**
5. $\dim = 3$

Explanation:

Since the subspace is the column space of the matrix

$$A = \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix}.$$

its dimension is the number of pivot columns in A . Now

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus A has 2 pivot columns, so

$\text{dimension} = 2$

RankTF06c

015 10.0 points

The dimensions of the row space and column space of an $m \times n$ matrix A are the same, even if $m \neq n$.

True or False?

1. FALSE
2. TRUE **correct**

Explanation:

Recall that the rank A is the number of pivot columns in A . Equivalently, rank A is the number of pivot positions in an echelon form B of A . Furthermore, since B has a nonzero row for each pivot, and since these rows form a basis for the row space of A , rank A is also the dimension of the row space.

Consequently, the statement is

TRUE