This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixVecProd03
001   10.0 points

Determine $\mathbf{uv}^T$ when

$$\mathbf{u} = \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

1. $\mathbf{uv}^T = \begin{bmatrix} -2a & -3a & -4a \\ -2b & -3b & -4b \\ -2c & -3c & -4c \end{bmatrix}$

2. $\mathbf{uv}^T = -2a - 3b - 4c$

3. $\mathbf{uv}^T = \begin{bmatrix} -2a & -2b & -2c \\ -3a & -3b & -3c \\ -4a & -4b & -4c \end{bmatrix}$ **correct**

4. $\mathbf{uv}^T = -4a - 3b - 2c$

Explanation:
Since
$$\mathbf{v}^T = \begin{bmatrix} a & b & c \end{bmatrix},$$
we see that

$$\mathbf{uv}^T = \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix}$$

$$= \begin{bmatrix} -2a & -2b & -2c \\ -3a & -3b & -3c \\ -4a & -4b & -4c \end{bmatrix}.$$

Consequently,

$$\boxed{\mathbf{uv}^T = \begin{bmatrix} -2a & -2b & -2c \\ -3a & -3b & -3c \\ -4a & -4b & -4c \end{bmatrix}.}$$

M340LInverseTF04
002   10.0 points

There are some matrices A, B, C with $AB = AC$, A invertible and $B \neq C$.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:
For all matrices such that $AB = AC$ and A invertible, it is always true that $A^{-1}AB = A^{-1}AC$. Since $A^{-1}A = I$ it follows that $B = C$.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

MatrixAlg02aT/F
003   10.0 points

There exist invertible matrices A, B such that
$$(AB)^{-1} \neq A^{-1}B^{-1}.$$

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
Set
$$A = \begin{bmatrix} a & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 \\ 1 & b \end{bmatrix}.$$

Then
$$AB = \begin{bmatrix} 1 & b - a \\ 0 & 1 \end{bmatrix}$$

and
$$A^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & a \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} b & 1 \\ -1 & 0 \end{bmatrix}.$$

In this case,
$$(AB)^{-1} = \begin{bmatrix} 1 & a - b \\ 0 & 1 \end{bmatrix},$$

while
$$A^{-1}B^{-1} = \begin{bmatrix} 1 & 0 \\ b - a & 1 \end{bmatrix},$$

so $(AB)^{-1} \neq A^{-1}B^{-1}$ when $a \neq b$.

Consequently, the statement is

$$\boxed{\text{TRUE}}\ .$$

InvertibleTF02a
004　10.0 points

If A and D are $n \times n$ matrices such that $AD = I$, then $DA = I$

True or False?

1. TRUE correct

2. FALSE

Explanation:
Because A and D are square matrices and $AD = I$, then A and D are both invertible, with $D = A^{-1}$ and $A = D^{-1}$. So using this substitution, the first equation can be rewritten as $AA^{-1} = I$, and the second as $DD^{-1} = I$. Both of these statements are true by the definition of inverse matrices.

Consequently, the statement is

$$\boxed{\text{TRUE}}\ .$$

LUDecomp3x4a
005　10.0 points

Determine the Lower Triangular Matrix L in an LU-decomposition of the matrix

$$A = \begin{bmatrix} -4 & -2 & 5 & -3 \\ 20 & 13 & -24 & 16 \\ 16 & 11 & -19 & 8 \end{bmatrix}.$$

1. $L = \begin{bmatrix} 1 & -5 & -4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

2. $L = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix}$

3. $L = \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -24 \\ 0 & 0 & 1 \end{bmatrix}$

4. $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$

5. $L = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -4 & 1 & 1 \end{bmatrix}$ **correct**

6. $L = \begin{bmatrix} -4 & -2 & -3 \\ 0 & 3 & 1 \\ 0 & 0 & -5 \end{bmatrix}$

Explanation:
We first determine the elementary matrices reducing A to an echelon form U by row reductions *downwards*.

Set

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that

$$E_1 A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -2 & 5 & -3 \\ 20 & 13 & -24 & 16 \\ 16 & 11 & -19 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 & 5 & -3 \\ 0 & 3 & 1 & 1 \\ 16 & 11 & -19 & 8 \end{bmatrix} = A_1,$$

say. Next set

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

so that

$$E_2 A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -2 & 5 & -3 \\ 0 & 3 & 1 & 1 \\ 16 & 11 & -19 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 & 5 & -3 \\ 0 & 3 & 1 & 1 \\ 0 & 3 & 1 & -4 \end{bmatrix} = A_2,$$

say. Finally, set

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

so that the product

$$E_3 A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -4 & -2 & 5 & -3 \\ 0 & 3 & 1 & 1 \\ 0 & 3 & 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 & 5 & -3 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & -5 \end{bmatrix} = U,$$

and

$$E_3 E_2 E_1 A = E_3 E_2 A_1 = E_3 A_2 = U$$

is an echelon form of A. But every elementary matrix is invertible. Thus $A = LU$, setting

$$L = E_1^{-1} E_2^{-1} E_3^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -4 & 1 & 1 \end{bmatrix}.$$

Consequently,

$$\boxed{L = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -4 & 1 & 1 \end{bmatrix}.}$$

Subspace05a
006   10.0 points

Let H be the set of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

where a and b are real. Determine if H is a subspace of $\mathbb{R}^3$, and then check the correct answer below.

1. H is not a subspace of $\mathbb{R}^3$ because it is not closed under vector addition. **correct**

2. H is not a subspace of $\mathbb{R}^3$ because it does not contain $\mathbf{0}$.

3. H is a subspace of $\mathbb{R}^3$ because it can be written as $Span\{\mathbf{v}_1, \mathbf{v}_2\}$ with $\mathbf{v}_1, \mathbf{v}_2$ in $\mathbb{R}^3$.

4. H is a subspace of $\mathbb{R}^3$ because it can be written as $Nul(A)$ for some matrix A.

Explanation:
To check if the set H of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

is a subspace of $\mathbb{R}^3$ we check the properties defining a subspace:

1. *the zero vector $\mathbf{0}$ is in H:* set $a = b = 0$. Then

$$\begin{bmatrix} 0 - 0 \\ 0 + 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

so H contains $\mathbf{0}$.

2. *for each $\mathbf{u}$, $\mathbf{v}$ in H the sum $\mathbf{u} + \mathbf{v}$ is in H:* set

$$\mathbf{v}_1 = \begin{bmatrix} a_1 - 2b_1 \\ a_1 b_1 + 3a_1 \\ b_1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} a_2 - 2b_2 \\ a_2 b_2 + 3a_2 \\ b_2 \end{bmatrix},$$

in H. Then

$$\mathbf{v}_1 + \mathbf{v}_2 = \begin{bmatrix} a_1 - 2b_1 \\ a_1 b_1 + 3a_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} a_2 - 2b_2 \\ a_2 b_2 + 3a_2 \\ b_2 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1 + a_2) - 2(b_1 + b_2) \\ a_1 b_1 + a_2 b_2 + 3(a_1 + a_2) \\ (b_1 + b_2) \end{bmatrix}.$$

But in general,

$$a_1 b_1 + a_2 b_2 \neq (a_1 + a_2)(b_1 + b_2),$$

in which case $\mathbf{u} + \mathbf{v}$ is not in H.

Consequently, H is not a subspace of $\mathbb{R}^3$ because it is

$$\boxed{\textit{not closed under vector addition}}\,.$$

DimRankTF02a
007   10.0 points

If $\mathcal{B}$ is a basis for a subspace H, then each vector in H can be written in only one way as a linear combination of the vectors in $\mathcal{B}$.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
If H is a p-dimensional subspace of $\mathbb{R}^n$, then $\mathcal{B}$ must be a set of p elements $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$. To represent any vector $\mathbf{b}$ in H is to find the coefficients c_j in the linear combination

$$\mathbf{b} \;=\; c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_p\mathbf{v}_p$$

of vectors in $\mathcal{B}$ whose sum is $\mathbf{b}$. This is equivalent to solving a system of equations with p equations and p unknowns. Because the vectors are linearly independent by the definition of a basis, this means there can only be one solution.

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

DetElemOps02TF
008   10.0 points

When the matrix

$$B \;=\; \begin{bmatrix} a & b \\ c + ka & d + kb \end{bmatrix}$$

is obtained from

$$A \;=\; \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

by adding k times row 1 to row 2, then

$$\det[B] \;=\; \det[A]\,.$$

True or False?

1. TRUE **correct**

2. FALSE

Explanation:
As 2×2 matrices,

$$\det[B] \;=\; \begin{vmatrix} a & b \\ c + ka & d + kb \end{vmatrix}$$

$$=\; a(d + kb) - b(c + ka) \;=\; ad - bc\,,$$

while

$$\det[A] \;=\; \begin{vmatrix} a & b \\ c & d \end{vmatrix} \;=\; ad - bc.$$

Thus

$$\det[B] \;=\; \det[A]\,.$$

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

DetInverseT/F01b
009   10.0 points

The matrix

$$A \;=\; \begin{bmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & 5 & 3 \end{bmatrix}$$

is invertible.

True or False?

1. FALSE **correct**

2. TRUE

Explanation:
The matrix A will be invertible if and only if $\det(A) \neq 0$. Now

$$\det(A) \;=\; \begin{vmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & 5 & 3 \end{vmatrix}$$

$$=\; 5\begin{vmatrix} -3 & -2 \\ 5 & 3 \end{vmatrix} - \begin{vmatrix} 1 & -3 \\ 0 & 5 \end{vmatrix}$$

$$=\; 5 \times 1 - 1 \times 5 \;=\; 0\,.$$

Consequently, the statement is

$$\boxed{\text{FALSE}}\ .$$

VectorSpace01aT/F
010   10.0 points

The subset

$$V \ = \ \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : ab \geq 0 \right\}$$

of $\mathbb{R}^2$ is closed under scalar multiplication.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
For each scalar k,

$$k \begin{bmatrix} a \\ b \end{bmatrix} \ = \ \begin{bmatrix} ka \\ kb \end{bmatrix}.$$

Now $(ka)(kb) = k^2(ab)$, so

$$ab \ \geq \ 0 \ \implies \ (ka)(kb) \ = \ k^2 ab \ \geq \ 0$$

because $k^2 \geq 0$ for all k.

Consequently, the statement is

$$\boxed{\text{TRUE}}\ .$$

BasisNull02b
011   10.0 points

Find a basis for the Null space of the matrix

$$A \ = \ \begin{bmatrix} 2 & -6 & -4 & 4 \\ 1 & -3 & -5 & 8 \\ -2 & 6 & 5 & -6 \end{bmatrix}.$$

1. $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

2. $\left\{ \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

3. $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$ **correct**

4. $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

5. $\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

6. $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

Explanation:
We first row reduce $[A \ \ \mathbf{0}]$:

$$\text{rref}([A \ \ \mathbf{0}]) \ = \ \begin{bmatrix} 1 & -3 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

to identify the free variables for $\mathbf{x}$ in the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

Thus x_1 and x_3 are basic variables, while x_2 and x_4 are free variables. So set $x_2 = s$ and $x_4 = t$. Then

$$x_1 \ = \ 3s + 2t, \quad x_3 \ = \ 2t\,,$$

and

$$\text{Nul}(A) \ = \ \text{Span} \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}.$$

Consequently,

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

is a basis for $\text{Nul}(A)$.

LinIndSetsTF02e
012   10.0 points

If B is an echelon form of a matrix A, then the pivot columns of B form a basis for Col A.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:
The pivot columns of A are the same as the pivot columns for B, and the pivot columns of A form a basis for $\mathrm{Col}(A)$. But row operations on a matrix can change the entries in a column, so the columns of B need not be in the column space of A. Thus the pivot columns of B need not form a basis for $\mathrm{Col}(A)$.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

CoordVec01b
013   10.0 points

Find the coordinate vector $[\mathbf{x}]_\mathcal{B}$ in $\mathbb{R}^2$ for the vector

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \end{bmatrix} \right\}$$

for $\mathbb{R}^2$.

1. $[\mathbf{x}]_\mathcal{B} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

2. $[\mathbf{x}]_\mathcal{B} = \begin{bmatrix} -6 \\ -2 \end{bmatrix}$

3. $[\mathbf{x}]_\mathcal{B} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

4. $[\mathbf{x}]_\mathcal{B} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$ **correct**

5. $[\mathbf{x}]_\mathcal{B} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

6. $[\mathbf{x}]_\mathcal{B} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$

Explanation:
The coordinate vector $[\mathbf{x}]_\mathcal{B}$ of a vector $\mathbf{x}$ in $\mathbb{R}^2$ with respect to a basis

$$\mathcal{B} = \{\mathbf{b}_1, \ \mathbf{b}_2\}$$

for $\mathbb{R}^2$ satisfies the matrix equation

$$A[\mathbf{x}]_\mathcal{B} = \mathbf{x}, \qquad A = [\,\mathbf{b}_1 \quad \mathbf{b}_2\,].$$

Consequently, when

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \end{bmatrix} \right\},$$

and

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix},$$

the associated augmented matrix is

$$[A \ \mathbf{x}] = \begin{bmatrix} 1 & 5 & 4 \\ -2 & -6 & 0 \end{bmatrix}.$$

But then

$$\mathrm{rref}[A \ \mathbf{x}] = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 2 \end{bmatrix}.$$

Thus

$$\boxed{[\mathbf{x}]_\mathcal{B} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}}.$$

DimSubspace01b
014   10.0 points

Determine the dimension of the subspace

$$\mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix} \right\}$$

of $\mathbb{R}^3$.

1. $\dim = 1$

2. $\dim = 5$

3. dim = 4

4. dim = 2 **correct**

5. dim = 3

Explanation:
Since the subspace is the column space of the matrix

$$A = \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix}.$$

its dimension is the number of pivot columns in A. Now

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus A has 2 pivot columns, so

$$\boxed{\text{dimension} = 2}.$$

RankTF06c
015   10.0 points

The dimensions of the row space and column space of an $m \times n$ matrix A are the same, even if $m \neq n$.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
Recall that the rank A is the number of pivot columns in A. Equivalently, rank A is the number of pivot positions in an echelon form B of A. Furthermore, since B has a nonzero row for each pivot, and since these rows form a basis for the row space of A, rank A is also the dimension of the row space.

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$