

This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

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**ParallelFace05c**  
**001 10.0 points**

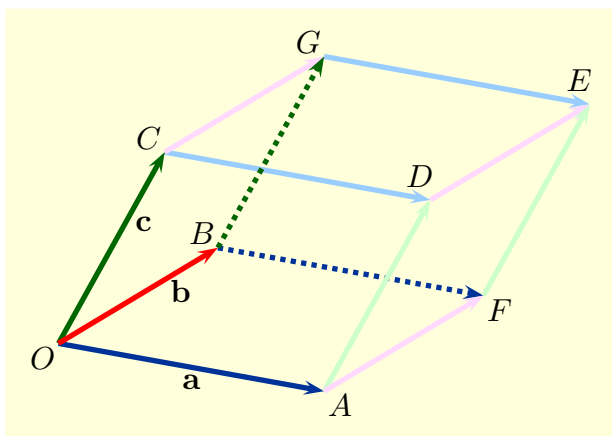
The vectors

$$\mathbf{a} = \langle 4, -4, -4 \rangle, \quad \mathbf{b} = \langle 2, 2, -1 \rangle,$$

and

$$\mathbf{c} = \langle 1, 4, 4 \rangle,$$

shown below form adjacent edges of the parallelepiped



Describe the face  $CDEG$  in vector form.

1.

$$\langle 4 + 2s + t, -4 + 2s + 4t, -4 - s + 4t \rangle,$$

for,  $-1 \leq s, t \leq 1$ .

2.

$$\langle s + 2t, 4s + 2t, 4s - t \rangle,$$

for  $0 \leq s, t \leq 1$ .

3.

$$\langle 2s + t, 2s + 4t, -s + 4t \rangle,$$

for  $-1 \leq s, t \leq 1$ .

4.

$$\langle 2 + 4s + t, 2 - 4s + 4t, -1 - 4s + 4t \rangle,$$

for,  $0 \leq s, t \leq 1$ .

5.

$$\langle 1 + 4s + 2t, 4 - 4s + 2t, 4 - 4s - t \rangle,$$

for  $0 \leq s, t \leq 1$ .

**correct**

6.

$$\langle 1 + 4s + 2t, 4 - 4s + 2t, 4 - 4s - t \rangle,$$

for  $-1 \leq s, t \leq 1$ .

**Explanation:**

The face  $CDEG$  of the parallelepiped lies in the unique plane in which the vertices  $C$ ,  $D$ , and  $G$  lie. Now in vector form this plane is

$$\mathbf{x} = \mathbf{c} + s\mathbf{a} + t\mathbf{b}, \quad -\infty \leq s, t \leq \infty.$$

But the points in the parallelogram  $CDEG$  lying in this plane correspond to  $0 \leq s \leq 1$  and  $0 \leq t \leq 1$ , *i.e.*, to

$$\mathbf{c} + s\mathbf{a} + t\mathbf{b}, \quad 0 \leq s, t \leq 1.$$

Consequently, the face  $CDEG$  is given in vector form by

$\langle 1 + 4s + 2t, 4 - 4s + 2t, 4 - 4s - t \rangle,$

for  $0 \leq s, t \leq 1$ .

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**CalC13a30aNC**  
**002 10.0 points**

Find an equation for the set of all points in 3-space equidistant from the points

$$A(-3, -1, 3), \quad B(1, 2, 4).$$

1.  $3x - y + 4z + 1 = 0$

2.  $x + 3y + 4z + 1 = 0$
3.  $4x + 3y + z + 1 = 0$
4.  $3x + 4y - z - 1 = 0$
5.  $4x + 3y + z - 1 = 0$  **correct**
6.  $x - 4y - 3z - 1 = 0$

**Explanation:**

We have to find the set of points  $P(x, y, z)$  such that

$$\|\overline{AP}\| = \|\overline{BP}\|.$$

Now by the distance formula in 3-space,

$$\|\overline{AP}\|^2 = (x+3)^2 + (y+1)^2 + (z-3)^2,$$

while

$$\|\overline{BP}\|^2 = (x-1)^2 + (y-2)^2 + (z-4)^2.$$

After expansion therefore,

$$\|\overline{AP}\|^2 = x^2 + 6x + y^2 + 2y + z^2 - 6z + 19,$$

while

$$\|\overline{BP}\|^2 = x^2 - 2x + y^2 - 4y + z^2 - 8z + 21.$$

Thus  $\|\overline{AP}\| = \|\overline{BP}\|$  when

$$\begin{aligned} x^2 + 6x + y^2 + 2y + z^2 - 6z + 19 \\ = x^2 - 2x + y^2 - 4y + z^2 - 8z + 21. \end{aligned}$$

Consequently, the set of all points equidistant from  $A$  and  $B$  satisfies the equation

$$4x + 3y + z - 1 = 0.$$

Notice that this is a plane perpendicular to the line segment joining  $A$  and  $B$  (since it must contain the perpendicular bisector of the line segment  $\overline{AB}$ ).

keywords: plane, locus points, equidistant two points

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**CalC13d12s**  
**003 10.0 points**

If  $\mathbf{a}$  is a vector parallel to the  $xy$ -plane and  $\mathbf{b}$  is a vector parallel to  $\mathbf{k}$ , determine  $\|\mathbf{a} \times \mathbf{b}\|$  when  $\|\mathbf{a}\| = 2$  and  $\|\mathbf{b}\| = 3$ .

1.  $\|\mathbf{a} \times \mathbf{b}\| = -6$
2.  $\|\mathbf{a} \times \mathbf{b}\| = -3\sqrt{2}$
3.  $\|\mathbf{a} \times \mathbf{b}\| = 3$
4.  $\|\mathbf{a} \times \mathbf{b}\| = 0$
5.  $\|\mathbf{a} \times \mathbf{b}\| = 6$  **correct**
6.  $\|\mathbf{a} \times \mathbf{b}\| = -3$
7.  $\|\mathbf{a} \times \mathbf{b}\| = 3\sqrt{2}$

**Explanation:**

For vectors  $\mathbf{a}$  and  $\mathbf{b}$ ,

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \sin \theta$$

when the angle between them is  $\theta$ ,  $0 \leq \theta < \pi$ .

But  $\theta = \pi/2$  in the case when  $\mathbf{a}$  is parallel to the  $xy$ -plane and  $\mathbf{b}$  is parallel to  $\mathbf{k}$  because  $\mathbf{k}$  is then perpendicular to the  $xy$ -plane. Consequently, for the given vectors,

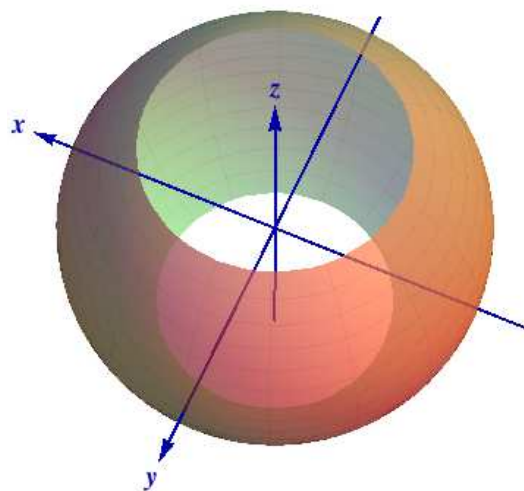
$$\|\mathbf{a} \times \mathbf{b}\| = 6.$$

keywords: cross product, length, angle,

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**SphericalCoords04click**  
**004 10.0 points**

The surface  $S$  shown in



consists of the portion of the sphere

$$x^2 + y^2 + z^2 = 16$$

where

$$x^2 + y^2 \geq 4.$$

Use spherical polar coordinates  $(\rho, \theta, \phi)$  to describe  $S$ .

1.  $S$  = all points  $P(\rho, \theta, \phi)$  with

$$\rho = 2, \quad 0 \leq \theta \leq 2\pi, \quad \frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}.$$

2.  $S$  = all points  $P(\rho, \theta, \phi)$  with

$$\rho = 4, \quad 0 \leq \theta \leq 2\pi, \quad \frac{\pi}{6} \leq \phi \leq \pi.$$

3.  $S$  = all points  $P(\rho, \theta, \phi)$  with

$$\rho = 4, \quad 0 \leq \theta \leq 2\pi, \quad \frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}.$$

correct

4.  $S$  = all points  $P(\rho, \theta, \phi)$  with

$$\rho = 4, \quad 0 \leq \theta \leq 2\pi, \quad \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}.$$

5.  $S$  = all points  $P(\rho, \theta, \phi)$  with

$$\rho = 2, \quad 0 \leq \theta \leq 2\pi, \quad \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}.$$

6.  $S$  = all points  $P(\rho, \theta, \phi)$  with

$$\rho = 2, \quad 0 \leq \theta \leq 2\pi, \quad \frac{\pi}{3} \leq \phi \leq \pi.$$

**Explanation:**

In spherical polar coordinates  $(\rho, \theta, \phi)$ ,

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta,$$

and

$$z = \rho \cos \phi,$$

with  $0 \leq \theta \leq 2\pi$  and  $0 \leq \phi \leq \pi$ . We need to find further restrictions on  $\rho$ ,  $\theta$ , and  $\phi$  so that

$$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 \geq 4.$$

Now

$$\rho^2 = x^2 + y^2 + z^2 = 16,$$

i.e.,  $\rho = 4$ . But then,

$$z^2 = 16 \cos^2 \phi = 16 - x^2 - y^2 \leq 12.$$

Consequently,  $S$  consists of all points  $P$  with  $\rho = 4$  and

$$0 \leq \theta \leq 2\pi, \quad \frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}.$$

**FinM4e05**  
**005 10.0 points**

Solve for  $X$  when  $AX + B = C$ ,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 2 \\ -1 & 2 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}.$$

1.  $X = \begin{bmatrix} 13 & 4 \\ -8 & -2 \end{bmatrix}$  **correct**

2.  $X = \begin{bmatrix} 12 & 4 \\ -7 & -2 \end{bmatrix}$

3.  $X = \begin{bmatrix} 13 & 4 \\ -7 & -1 \end{bmatrix}$

4.  $X = \begin{bmatrix} 13 & 4 \\ -4 & 6 \end{bmatrix}$

5.  $X = \begin{bmatrix} 12 & 4 \\ -8 & 6 \end{bmatrix}$

**Explanation:**

By the algebra of matrices,

$$X = A^{-1}(C - B).$$

But the inverse of any  $2 \times 2$  matrix

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

is given by

$$D^{-1} = \begin{bmatrix} \frac{d_{22}}{\Delta} & -\frac{d_{12}}{\Delta} \\ -\frac{d_{21}}{\Delta} & \frac{d_{11}}{\Delta} \end{bmatrix}$$

with  $\Delta = d_{11}d_{22} - d_{12}d_{21}$ , so

$$\begin{aligned} X &= \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \left( \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ -1 & 2 \end{bmatrix} \right) \\ &= \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 2 & 2 \end{bmatrix}. \end{aligned}$$

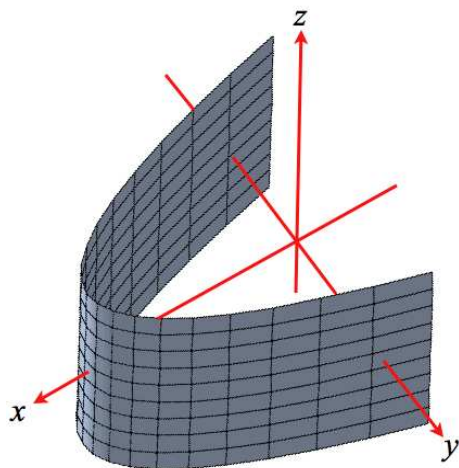
Thus

$$X = \begin{bmatrix} 13 & 4 \\ -8 & -2 \end{bmatrix}.$$

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**CalC13f04c**  
**006 10.0 points**

Which one of the following equations has graph



1.  $x - z^2 + 4 = 0$
2.  $y - x^2 + 4 = 0$
3.  $z - y^2 + 4 = 0$

4.  $y + z^2 - 4 = 0$

5.  $z + x^2 - 4 = 0$

6.  $x + y^2 - 4 = 0$  **correct**

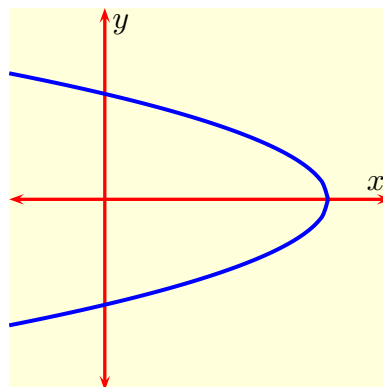
**Explanation:**

The graph is a parabolic cylinder that has constant value on any line parallel to the  $z$ -axis, so it will be the graph of an equation containing no  $z$ -term. This already eliminates the equations

$$y + z^2 - 4 = 0, \quad z - y^2 + 4 = 0,$$

$$z + x^2 - 4 = 0, \quad x - z^2 + 4 = 0.$$

On the other hand, the intersection of the graph with the  $xy$ -plane, *i.e.* the  $z = 0$  plane, is a parabola opening to the left on the  $x$ -axis as shown in



Consequently, the graph is that of the equation

$$x + y^2 - 4 = 0.$$

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keywords: quadric surface, graph of equation, cylinder, 3D graph, parabolic cylinder, trace

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**CalC15b16s**  
**007 10.0 points**

Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{7xy^4}{x^2 + y^8}$ , if it exists.

1. 7

2. The limit does not exist. **correct**

3. 0

4. 14

5. 3.5

**Explanation:**


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**CalC15d11s**  
**008    10.0 points**

Find the linearization,  $L(x, y)$ , of

$$f(x, y) = y\sqrt{x}$$

at the point  $(9, -2)$ .

1.  $L(x, y) = -6 + \frac{3}{2}x + \frac{1}{3}y$

2.  $L(x, y) = 3 - \frac{1}{3}x + 3y$  **correct**

3.  $L(x, y) = -6 + \frac{1}{3}x - \frac{3}{2}y$

4.  $L(x, y) = -3 + 3x + \frac{1}{3}y$

5.  $L(x, y) = 3 + 3x - \frac{1}{3}y$

6.  $L(x, y) = -3 + \frac{1}{3}x + 3y$

**Explanation:**The linearization of  $f = f(x, y)$  at a point  $(a, b)$  is given by

$$L(x, y) = f(a, b) + (x-a)\frac{\partial f}{\partial x}\Big|_{(a,b)} + (y-b)\frac{\partial f}{\partial y}\Big|_{(a,b)}.$$

But when  $f(x, y) = y\sqrt{x}$ ,

$$\frac{\partial f}{\partial x} = \frac{y}{2\sqrt{x}}, \quad \frac{\partial f}{\partial y} = \sqrt{x};$$

thus when  $(a, b) = (9, -2)$ ,

$$\frac{\partial f}{\partial x}\Big|_{(a,b)} = -\frac{1}{3}, \quad \frac{\partial f}{\partial y}\Big|_{(a,b)} = 3,$$

while  $f(a, b) = -6$ . Consequently,

$$L(x, y) = 3 - \frac{1}{3}x + 3y$$

keywords:

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**Tangent01a**  
**009    10.0 points**

If  $\mathbf{r}(x)$  is the vector function whose graph is trace of the surface

$$z = f(x, y) = 3x^2 - y^2 - x + 2y$$

on the plane  $y = 2x$ , determine the tangent vector to  $\mathbf{r}(x)$  at  $x = 1$ .

1. tangent vector =  $\langle 1, 0, 1 \rangle$

2. tangent vector =  $\langle 1, 2, 1 \rangle$  **correct**

3. tangent vector =  $\langle 2, 0, 3 \rangle$

4. tangent vector =  $\langle 1, 2, 3 \rangle$

5. tangent vector =  $\langle 2, 2, 1 \rangle$

6. tangent vector =  $\langle 2, 1, 3 \rangle$

**Explanation:**

The graph of

$$z = f(x, y) = 3x^2 - y^2 - x + 2y$$

is the set of all points

$$(x, y, f(x, y))$$

as  $x, y$  vary in 3-space. So the intersection of the surface with the plane  $y = 2x$  is the set of all points

$$(x, 2x, f(x, 2x)), \quad -\infty < x < \infty.$$

But

$$f(x, 2x) = -x^2 + 3x.$$

Thus the surface and the plane  $y = 2x$  intersect in the graph of

$$\mathbf{r}(x) = \langle x, 2x, -x^2 + 3x \rangle.$$

Now the tangent vector to the graph of  $\mathbf{r}(x)$  is the derivative

$$\mathbf{r}'(x) = \langle 1, 2, -2x + 3 \rangle.$$

Consequently, at  $x = 1$  the graph of  $\mathbf{r}(x)$  has

$$\text{tangent vector} = \langle 1, 2, 1 \rangle.$$

keywords:

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**CalC15e07s**  
**010 10.0 points**

Use the Chain Rule to find  $\frac{\partial z}{\partial s}$  when

$$z = x^2 - 4xy + y^2,$$

and

$$x = 2s - t, \quad y = st.$$

1.  $\frac{\partial z}{\partial s} = -2x + 4y - 4xs + 2ys$
2.  $\frac{\partial z}{\partial s} = -2x - 8y - 4xs + 2ys$
3.  $\frac{\partial z}{\partial s} = -2x + 4y - 4xt + 2yt$
4.  $\frac{\partial z}{\partial s} = 4x - 8y - 4xs + 2ys$
5.  $\frac{\partial z}{\partial s} = 4x + 4y - 4xt + 2yt$
6.  $\frac{\partial z}{\partial s} = 4x - 8y - 4xt + 2yt$  **correct**

**Explanation:**

By the Chain Rule for Partial Differentiation,

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s}.$$

Now

$$\frac{\partial z}{\partial x} = 2x - 4y, \quad \frac{\partial x}{\partial s} = 2$$

while

$$\frac{\partial z}{\partial y} = -4x + 2y, \quad \frac{\partial y}{\partial s} = t.$$

Thus

$$\frac{\partial z}{\partial s} = 2(2x - 4y) + t(-4x + 2y).$$

Consequently,

$$\frac{\partial z}{\partial s} = 4x - 8y - 4xt + 2yt.$$

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**CalC15f11s**  
**011 10.0 points**

Find the directional derivative,  $f_{\mathbf{v}}$ , of the function

$$f(x, y) = 6 + 2x\sqrt{y}$$

at the point  $P(3, 4)$  in the direction of the vector

$$\mathbf{v} = \langle 3, -4 \rangle.$$

1.  $f_{\mathbf{v}} = \frac{4}{5}$
2.  $f_{\mathbf{v}} = \frac{11}{10}$
3.  $f_{\mathbf{v}} = 1$
4.  $f_{\mathbf{v}} = \frac{9}{10}$
5.  $f_{\mathbf{v}} = \frac{6}{5}$  **correct**

**Explanation:**

Now for an arbitrary vector  $\mathbf{v}$ ,

$$f_{\mathbf{v}} = \nabla f \cdot \left( \frac{\mathbf{v}}{|\mathbf{v}|} \right),$$

where we have normalized so that the direction vector has unit length. But when

$$f(x, y) = 6 + 2x\sqrt{y},$$

then

$$\nabla f = (2\sqrt{y})\mathbf{i} + \left( \frac{x}{\sqrt{y}} \right)\mathbf{j}.$$

At  $P(3, 4)$ , therefore,

$$\nabla f|_P = 4\mathbf{i} + \frac{3}{2}\mathbf{j}.$$

Consequently, when  $\mathbf{v} = \langle 3, -4 \rangle$ ,

$$f_{\mathbf{v}}(3, 4) = \left\langle 4, \frac{3}{2} \right\rangle \cdot \left( \frac{\mathbf{v}}{|\mathbf{v}|} \right) = \frac{6}{5}.$$

keywords: