1

This print-out should have 12 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixVecProd04 001 10.0 points

Determine $\mathbf{v}\mathbf{u}^T$ when

$$\mathbf{u} = \begin{bmatrix} -3\\2\\-5 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} a\\b\\c \end{bmatrix}.$$

1.
$$\mathbf{v}\mathbf{u}^T = -3a + 2b - 5c$$

$$\mathbf{2.} \ \mathbf{v}\mathbf{u}^{T} = \begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$$

3.
$$\mathbf{v}\mathbf{u}^T = \begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$$

4.
$$\mathbf{v}\mathbf{u}^{T} = -5a + 2b - 3c$$

$\begin{array}{cc} Inverse Matrix 01 a \\ 002 & 10.0 \ points \end{array}$

Solve for X when AX + B = C,

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 2 \\ -1 & 5 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}.$$

Find L in an LU decomposition of

$$A = \begin{bmatrix} -4 & 0 & 4 & -4 \\ 4 & 0 & -7 & 9 \\ -12 & 0 & 0 & 3 \end{bmatrix}.$$

1.
$$L = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$
2. $L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & -4 & 1 \end{bmatrix}$

2.
$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & -4 & 1 \end{bmatrix}$$

$$\mathbf{3.} \ L = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & 4 & 2 \end{bmatrix}$$

4.
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}$$

5.
$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

6.
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 4 & 1 \end{bmatrix}$$

Let H be the set of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

where a and b are real. Determine if H is a subspace of \mathbb{R}^3 , and then check the correct answer below.

- 1. H is not a subspace of \mathbb{R}^3 because it is not closed under vector addition.
- **2.** *H* is a subspace of \mathbb{R}^3 because it can be written as Nul(A) for some matrix *A*.
- **3.** *H* is a subspace of \mathbb{R}^3 because it can be written as $Span\{\mathbf{v}_1, \mathbf{v}_2\}$ with $\mathbf{v}_1, \mathbf{v}_2$ in \mathbb{R}^3 .
- **4.** *H* is not a subspace of \mathbb{R}^3 because it does not contain **0**.

$\begin{array}{cc} Invertible 01/02 \\ 005 & 10.0 \ points \end{array}$

A is an $n \times n$ matrix. Which of the following statements are equivalent to A being invertible?

- (i) There is no $n \times n$ matrix D such that AD = I.
- (ii) $\operatorname{Col} A = \{ \mathbf{0} \}.$
- (iii) A is row equivalent to the $n \times n$ identity matrix.
- **1.** ii
- **2.** i
- 3. i and ii
- **4.** iii
- **5.** All of these
- **6.** None of these

where O is the origin in 3-space.

- 1. volume = 6
- 2. volume = 5
- 3. volume = 8
- 4. volume = 7
- 5. volume = 9

Rank02c 006 10.0 points

Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & -2 & -7 \\ 1 & 1 & 3 \end{bmatrix}.$$

- 1. rank(A) = 1
- **2.** rank(A) = 3
- 3. rank(A) = 2
- 4. $\operatorname{rank}(A) = 5$
- 5. $\operatorname{rank}(A) = 4$

$\begin{array}{cc} Basis Nul 01a \\ 008 & 10.0 \ points \end{array}$

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 3 & -3 & -6 & 3 \\ 2 & -2 & -7 & -1 \\ -1 & 1 & 1 & -4 \end{bmatrix}.$$

DetVolume01a 007 10.0 points

Compute the volume of the parallelepiped with adjacent edges \overline{OP} , \overline{OQ} , and \overline{OR} determined by vertices

$$P(2, -1, -1), Q(1, 4, 3), R(1, 4, 4),$$

- 1. $\{\cos t, \sin 2t\}$
- 2. $\{\cos t, \sin 2t, \sin t \cos t\}$
- 3. $\{\sin 2t, \sin t \cos t\}$
- **4.** $\{ \sin t, \sin 2t \}$
- 5. $\{\sin t, \sin 2t, \sin t \cos t\}$

$\begin{array}{cc} PolyCoordVec 01a \\ 010 & 10.0 \ points \end{array}$

Find the coordinate vector $[\mathbf{p}]_{\mathcal{B}}$ in \mathbb{R}^3 for the polynomial

$$\mathbf{p}(t) = 1 + 4t + 7t^2$$

with respect to the basis

$$\mathcal{B} = \left\{1 + t^2, \ t + t^2, \ 1 + 2t + t^2\right\}$$

for \mathbb{P}_2 .

$$\mathbf{1.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -6\\1\\2 \end{bmatrix}$$

$\begin{array}{c} {\rm Basis 03a} \\ 009 & 10.0 \ {\rm points} \end{array}$

In the vector space V of all real-valued functions, find a basis for the subspace

 $H \ = \ {\rm Span} \{ \, \sin t, \, \sin 2t, \, \sin t \cos t \, \} \, .$

$$\mathbf{2.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -2\\-6\\1 \end{bmatrix}$$

3.
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix}$$

4.
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$$

$$\mathbf{5.} \ [\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix}$$

6.
$$[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -1\\2\\6 \end{bmatrix}$$

ChangeBasis01b 011 10.0 points

Determine the change of coordinates matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$ from basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ to $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ of a vector space V when

$$\mathbf{b}_1 = 6\mathbf{c}_1 - 2\mathbf{c}_2, \quad \mathbf{b}_2 = 9\mathbf{c}_1 - 4c_2.$$