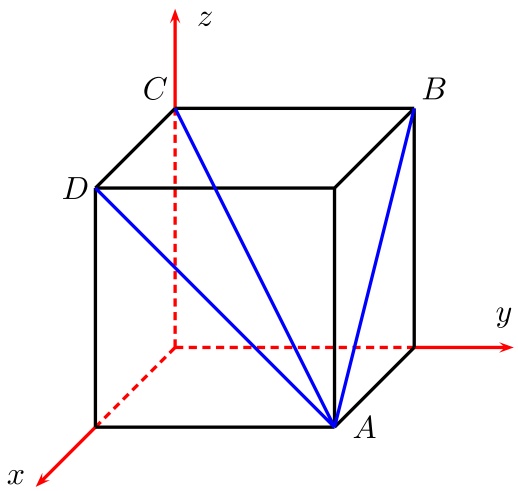
This print-out should have 30 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

CalC13c52a
001 10.0 points

The box shown in



is the unit cube having one corner at the origin and the coordinate planes for three of its adjacent faces.

Determine the projection of $\overrightarrow{AD}$ on $\overrightarrow{AB}$.

1. projection $= \dfrac{1}{2}(\mathbf{j}-\mathbf{k})$

2. projection $= -\dfrac{2}{3}(\mathbf{i}+\mathbf{j}-\mathbf{k})$

3. projection $= \dfrac{1}{2}(\mathbf{i}-\mathbf{k})$

4. projection $= -\dfrac{1}{2}(\mathbf{j}-\mathbf{k})$

5. projection $= \dfrac{2}{3}(\mathbf{i}+\mathbf{j}-\mathbf{k})$

6. projection $= -\dfrac{1}{2}(\mathbf{i}-\mathbf{k})$ **correct**

Explanation:

The projection of a vector $\mathbf{b}$ onto a vector $\mathbf{a}$ is given in terms of the dot product by

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|^2}\right)\mathbf{a}.$$

On the other hand, since the unit cube has side-length 1,

$$A = (1, 1, 0), \quad B = (0, 1, 1),$$

while $D = (1, 0, 1)$. In this case $\overrightarrow{AB}$ is a directed line segment determining the vector

$$\mathbf{a} = \langle -1, 0, 1 \rangle = -\mathbf{i}+\mathbf{k},$$

while $\overrightarrow{AD}$ determines the vector

$$\mathbf{b} = \langle 0, -1, 1 \rangle = -\mathbf{j}+\mathbf{k}.$$

For these choices of $\mathbf{a}$ and $\mathbf{b}$,

$$\mathbf{a}\cdot\mathbf{b} = 1, \qquad \|\mathbf{a}\|^2 = 2.$$

Consequently, the projection of $\overrightarrow{AD}$ onto $\overrightarrow{AB}$ is given by

$$\boxed{\text{proj}_{\mathbf{a}}\mathbf{b} = -\frac{1}{2}(\mathbf{i}-\mathbf{k})}.$$

keywords: projection, dot product, unit cube, component,

CalC13d04a
002 10.0 points

Which of the following expressions are well-defined for all vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$?

I $\mathbf{a}\times(\mathbf{b}\cdot\mathbf{c})$,

II $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d})$,

III $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$.

1. all of them

2. II only

3. I and II only

4. II and III only **correct**

5. III only

6. I and III only

7. none of them

8. I only

Explanation:

The cross product is defined only for two vectors, and its value is a vector; on the other hand, the dot product is defined only for two vectors, and its value is a scalar.

For the three given expressions, therefore, we see that

I *is not well-defined because the second term in the cross product is a dot product, hence not a vector.*

II *is well-defined because both terms in the dot product are cross products, hence vectors.*

III *is well-defined because it is the cross product of two vectors.*

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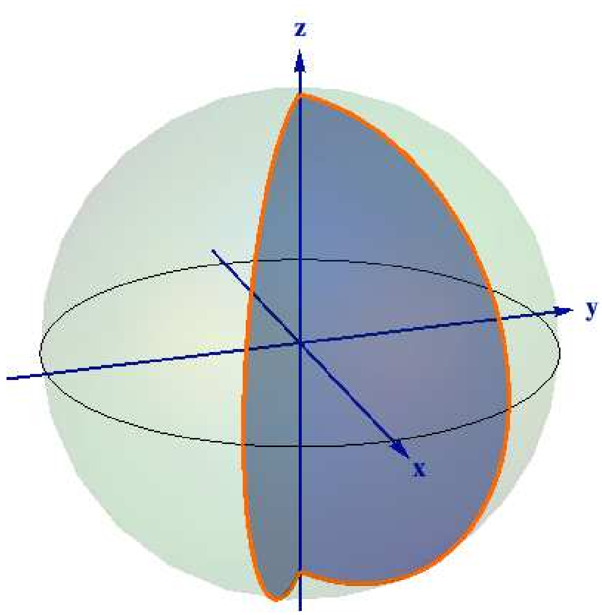
keywords: vectors, dot product, cross product, T/F, length,

SphericalCoords05a
003 10.0 points

The spine of the 'math taco' T shown in



lies on the z-axis, while the faces lie in the planes $y = \pm(\tan\alpha)x$ for fixed α .

Use spherical polar coordinates to describe T as a set of points $P(\rho, \theta, \phi)$ when the taco has radius 3.

1. $T = \{(\rho, \theta, \phi)\}$ with

$$0 \le \rho \le 6, \quad \theta = \pm\alpha, \quad 0 \le \phi \le \frac{\pi}{2}.$$

2. $T = \{P(\rho, \theta, \phi)\}$ with

$$0 \le \rho \le 3, \quad 0 \le \theta \le \frac{\pi}{2}, \quad \phi = \pm\alpha.$$

3. $T = \{P(\rho, \theta, \phi)\}$ with

$$0 \le \rho \le 6, \quad 0 \le \theta \le \frac{\pi}{2}, \quad \phi = \pm\alpha.$$

4. $T = \{P(\rho, \theta, \phi)\}$ with

$$0 \le \rho \le 6, \quad \theta = \pm\alpha, \quad 0 \le \phi \le \pi.$$

5. $T = \{P(\rho, \theta, \phi)\}$ with

$$0 \le \rho \le 3, \quad \theta = \pm\alpha, \quad 0 \le \phi \le \pi.$$

correct

6. $T = \{P(\rho, \theta, \phi)\}$ with

$$0 \le \rho \le 3, \quad 0 \le \theta \le \pi, \quad \phi = \pm\alpha.$$

Explanation:

In spherical polar coordinates (ρ, θ, ϕ),

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta,$$

and

$$z = \rho \cos\phi,$$

with $0 \le \theta \le 2\pi$ and $0 \le \psi \le \pi$. We need to find further restrictions on ρ, θ, and ϕ so that the taco shown has radius 3 and its faces lie in the planes

$$y = \pm(\tan\alpha)x\,.$$

But

$$\frac{y}{x} = \tan\theta = \pm\tan\alpha\,,$$

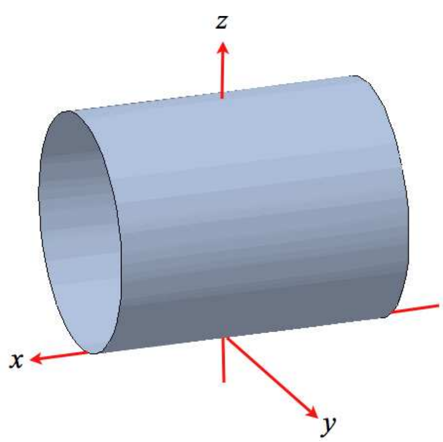
so $\theta = \pm\alpha$. On the other hand, z ranges from -3 to 3. Thus ρ ranges from 0 to 3, while $0 \le \phi \le \pi$.

Consequently, the taco consists of all points $P(\rho,\,\theta,\,\phi)$ with

$$\boxed{0 \le \rho \le 3, \quad \theta = \pm\alpha, \quad 0 \le \phi \le \pi}\,.$$

CalC13f03d
004 10.0 points

Which one of the following equations has graph



when the circular cylinder has radius 1?

1. $x^2 + y^2 - 2y = 0$

2. $y^2 + z^2 + 4y = 0$

3. $y^2 + z^2 + 2y = 0$

4. $y^2 + z^2 - 2z = 0$ **correct**

5. $y^2 + z^2 - 4z = 0$

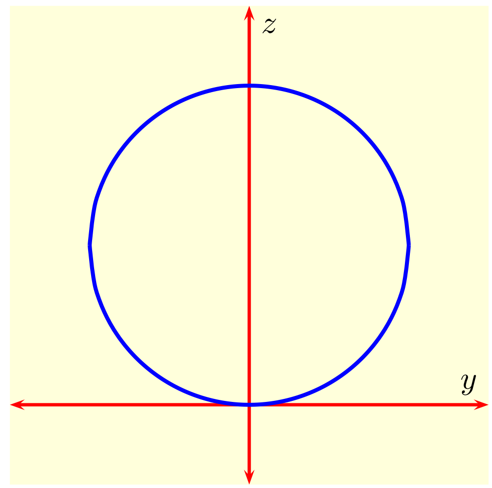
6. $x^2 + y^2 - 4y = 0$

Explanation:

The graph is a circular cylinder whose axis of symmetry is parallel to the x-axis, so it will be the graph of an equation containing no x-term. This already eliminates the equations

$$x^2 + y^2 - 2y = 0, \quad x^2 + y^2 - 4y = 0\,.$$

On the other hand, the intersection of the graph with the yz-plane, *i.e.* the $x = 0$ plane, is a circle centered on the z-axis and passing through the origin as shown in



But this circle has radius 1 because the cylinder has radius 1, and so its equation is

$$y^2 + (z-1)^2 = 1$$

as a circle in the yz-plane.

Consequently, the graph is that of the equation

$$\boxed{y^2 + z^2 - 2z = 0}\,.$$

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keywords: quadric surface, graph of equation, cylinder, 3D graph, circular cylinder, trace

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CalC15b19s
005 10.0 points

Find $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy + 7yz^2 + 4xz^2}{x^2 + y^2 + z^4}$, if it exists.

1. The limit does not exist. **correct**

2. 11

3. 7

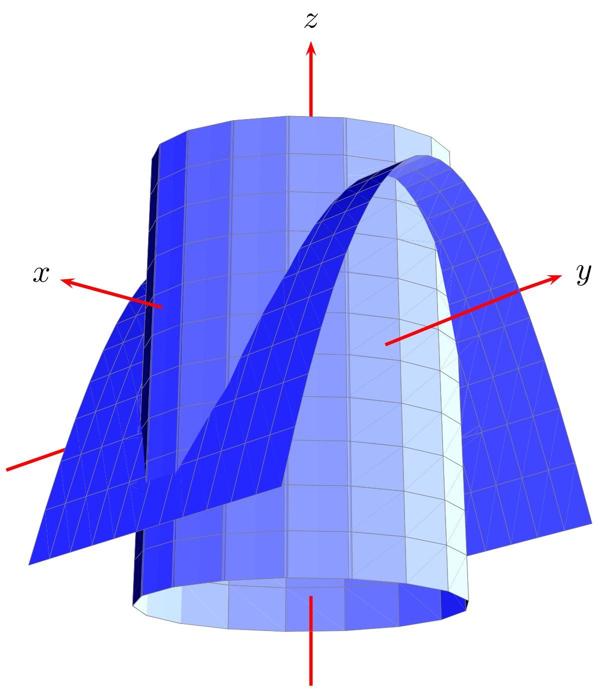
4. 0

5. 4

Explanation:

Intersection01a
006 10.0 points

The curve of intersection of the surfaces shown in



is the graph of which of the following vector functions?

1. $\mathbf{r}(t) = \langle \cos t, \sin t, 1 - \cos 2t \rangle$

2. $\mathbf{r}(t) = \langle \sin t, \cos t, \cos 2t - 1 \rangle$

3. $\mathbf{r}(t) = \langle \sin t, \cos t, \cos 2t \rangle$ **correct**

4. $\mathbf{r}(t) = \langle \sin t, \cos t, 1 - \cos 2t \rangle$

5. $\mathbf{r}(t) = \langle \cos t, \sin t, \cos 2t \rangle$

6. $\mathbf{r}(t) = \langle \cos t, \sin t, \cos 2t - 1 \rangle$

Explanation:

If we write

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle,$$

then each of the given vector functions has the property that

$$x(t)^2 + y(t)^2 = 1.$$

This is consistent with the fact that the curve of intersection lies on a circular cylinder

$$x^2 + y^2 = 1$$

with the z-axis the line of symmetry as is shown in the figure. Thus we need to look more carefully at the vector functions to determine which lie on the parabolic cylinder shown in the figure.

Recall first that

$$\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1.$$

Now the cross-sections of the parabolic cylinder perpendicular to the y-axis are parabolas *opening downwards* with *vertex above* the y-axis, so the parabolic cylinder is the graph of

$$z = a - bx^2, \qquad a, b > 0.$$

As a result, the components of $\mathbf{r}(t)$ have to satisfy an equation

$$z(t) = a - bx(t)^2, \qquad a, b > 0.$$

Consequently, the curve of intersection of the two surfaces is the graph of the vector function

$$\boxed{\mathbf{r}(t) = \langle \sin t, \cos t, \cos 2t \rangle}.$$

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keywords: surface, space curve, parametric equation, 3D graph, circular cylinder, paraboloid,

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CalC15e21s
007 10.0 points

Use the Chain Rule to find the partial derivative $\dfrac{\partial w}{\partial s}$ for

$$w = x^2 + y^2 + z^2,\ x = st,$$

$$y = s\cos t,\ z = s\sin t$$

when $s = 12$, $t = 0$.

1. $\dfrac{\partial w}{\partial s} = 21$

2. $\dfrac{\partial w}{\partial s} = 25$

3. $\dfrac{\partial w}{\partial s} = 26$

4. $\dfrac{\partial w}{\partial s} = 24$ **correct**

5. $\dfrac{\partial w}{\partial s} = 20$

Explanation:

By the Chain Rule for Partial Differentiation

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}.$$

Here, we have

$$\frac{\partial w}{\partial x} = 2x, \quad \frac{\partial x}{\partial s} = t$$

while

$$\frac{\partial w}{\partial y} = 2y, \quad \frac{\partial y}{\partial s} = \cos t$$

and

$$\frac{\partial w}{\partial z} = 2z, \quad \frac{\partial z}{\partial s} = \sin t.$$

Thus

$$\frac{\partial w}{\partial s} = 2xt + 2y\cos t + 2z\sin t.$$

Note that when $s = 12$ and $t = 0$, it follows that $x = 0$, $y = 12$, $z = 0$. Consequently, for these values,

$$\boxed{\frac{\partial w}{\partial s} = 24}.$$

keywords:

CalC15f19s

008 10.0 points

Find the directional derivative, $f_{\mathbf{v}}$, of

$$f(x, y) = 5\left(\frac{y}{x}\right)^{1/2}$$

at $P = (2, 2)$ in the direction of the vector $\overrightarrow{PQ}$ when $Q = (6, 5)$.

1. $f_{\mathbf{v}} = -\dfrac{3}{10}$

2. $f_{\mathbf{v}} = -\dfrac{1}{4}$ **correct**

3. $f_{\mathbf{v}} = -\dfrac{1}{5}$

4. $f_{\mathbf{v}} = -\dfrac{3}{20}$

5. $f_{\mathbf{v}} = -\dfrac{1}{10}$

Explanation:

The directional derivative of $f(x, y)$ at P in the direction of $\mathbf{v} = \overrightarrow{PQ}$ is given by the dot product

$$f_{\mathbf{v}}|_P = \nabla f|_P \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right).$$

But when

$$f(x, y) = 5\left(\frac{y}{x}\right)^{1/2},$$

we see that

$$\frac{\partial f}{\partial x} = -\frac{5}{2}\left(\frac{y}{x^3}\right)^{1/2}, \quad \frac{\partial f}{\partial y} = \frac{5}{2}\left(\frac{1}{xy}\right)^{1/2},$$

so that at $P(2, 2)$,

$$(\nabla f)(2, 2) = -\frac{5}{4}\mathbf{i} + \frac{5}{4}\mathbf{j} = \left\langle -\frac{5}{4}, \frac{5}{4}\right\rangle.$$

On the other hand, $\mathbf{v} = \langle 4, 3\rangle$ which as a vector of unit length becomes

$$\frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \frac{4}{5}, \frac{3}{5}\right\rangle.$$

Consequently,

$$\boxed{\nabla f|_P = \left\langle -\frac{5}{4}, \frac{5}{4}\right\rangle . \left\langle \frac{4}{5}, \frac{3}{5}\right\rangle = -\frac{1}{4}}.$$

CalC15f39s
009 10.0 points

Find the equation of the tangent plane to the surface

$$4x^2 + 3y^2 + 3z^2 = 28$$

at the point $(1, -2, 2)$.

1. $2x + 3y + 3z = 4$

2. $2x - 3y + 3z = 14$ **correct**

3. $2x + 3y + 3z = 14$

4. $2x - 3y + 3z = 4$

5. $4x - 3y + 3z = 28$

Explanation:
Let

$$F(x) = 4x^2 + 3y^2 + 3z^2 .$$

The equation to the tangent plane to the surface at the point $P(1, -2, 2)$ is given by

$$F_x\Big|_P(x-1) + F_y\Big|_P(y+2) + F_z\Big|_P(z-2) = 0 .$$

Since

$$F_x = 8x , \qquad F_x\Big|_P = 8,$$

$$F_y = 6y , \qquad F_y\Big|_P = -12,$$

and

$$F_z = 6z , \qquad F_z\Big|_P = 12$$

it follows that the equation of the tangent plane is

$$\boxed{2x - 3y + 3z = 14}.$$

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keywords:

QuadApprox04a
010 10.0 points

Find the quadratic approximation to

$$f(x, y) = e^{x+2y^2}$$

at $P(0, 0)$.

1. $Q(x, y) = 1 - x + \frac{1}{2}xy + 2y^2$

2. $Q(x, y) = 1 - 2x + \frac{1}{2}x^2 - 2y^2$

3. $Q(x, y) = 1 + x + \frac{1}{2}x^2 + 2y^2$ **correct**

4. $Q(x, y) = 1 + 2x + \frac{1}{2}x^2 + 2y^2$

5. $Q(x, y) = 1 + x + \frac{1}{2}x^2 - 2y^2$

6. $Q(x, y) = 1 + 2y + 2xy + \frac{1}{2}y^2$

Explanation:
The Quadratic Approximation to $f(x, y)$ at $P(0, 0)$ is given by

$$Q(x, y) = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y$$

$$+ \frac{1}{2}f_{xx}(0, 0)x^2 + f_{xy}(0, 0)xy + \frac{1}{2}f_{yy}(0, 0)y^2.$$

But when

$$f(x, y) = e^{x+2y^2}$$

we see that

$$f_x = e^{x+2y^2} , \quad f_y = 4ye^{x+2y^2} ,$$

so that $f(0, 0) = 1$ and

$$f_x(0, 0) = 1, \quad f_y(0, 0) = 0 ,$$

while

$$f_{xx} = e^{x+2y^2}, \quad f_{xy} = 4ye^{x+2y^2} ,$$

and

$$f_{yy} = 4e^{x+2y^2} + 16y^2e^{x+2y^2} ,$$

so that

$$f_{xx}(0,\,0) = 1, \quad f_{xy}(0,\,0) = 0\,,$$

and $f_{yy}(0,\,0) = 4$.

Consequently, the Quadratic Approximation to f at $P(0,\,0)$ is

$$\boxed{Q(x,\,y) = 1 + x + \frac{1}{2}x^2 + 2y^2}\,.$$

keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

CalC15g06a
011 10.0 points

Locate and classify all the local extrema of

$$f(x,y) = 2x^3 + 2y^3 + 6xy - 4.$$

1. local max at $(-1,-1)$, saddle at $(0,0)$ **correct**

2. local min at $(0,0)$, local max at $(-1,-1)$

3. saddle at $(-1,-1)$, local max at $(0,0)$

4. local min at $(-1,-1)$, saddle at $(0,0)$

5. local max at $(1,1)$, saddle at $(0,0)$

Explanation:

Local extrema occur at the critical points of f. Now after differentiation of f we obtain

$$f_x = 6(x^2 + y), \quad f_y = 6(y^2 + x).$$

The critical points of f are thus the common solutions of the equations

$$x^2 + y \;=\; 0, \quad y^2 + x \;=\; 0.$$

This yields only the two extremum points $(-1,-1)$ and $(0,0)$. But after differentiating again we see that

$$f_{xx} = 12x, \quad f_{xy} = 6, \quad f_{yy} = 12y;$$

consequently,

$$f_{xx}f_{yy} - (f_{xy})^2 = (12)^2 xy - 36.$$

Hence by the second derivative test there are

$$\boxed{\text{local max at } (-1,-1), \ \ \text{saddle at } (0,0).}$$

CalC15g28a
012 10.0 points

Find the absolute maximim value of the function

$$f(x,y) \;=\; 2 + xy - 3x - 3y$$

over the closed triangular region $\mathcal{D}$ having vertices

$$P(1,\,0)\,, \quad Q(1,\,4)\,, \quad R(5,\,0)\,.$$

1. abs max value $=$ 3

2. abs max value $=$ -2

3. abs max value $=$ 2

4. abs max value $=$ 1

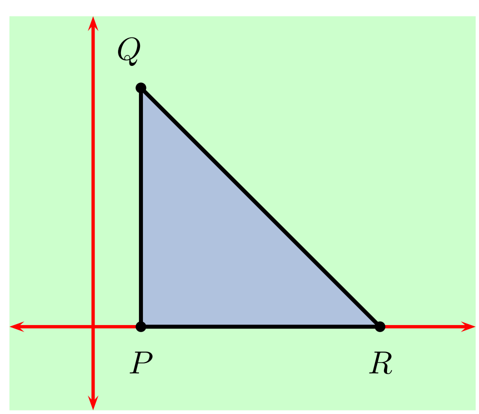
5. abs max value $=$ 0

6. abs max value $=$ -1 **correct**

Explanation:

Since f is continuous everywhere its absolute maximum on $\mathcal{D}$ will exist and be attained at a critical point of f inside $\mathcal{D}$ or at a point on the boundary of $\mathcal{D}$.

Now $\mathcal{D}$ is similar to the shaded region in



and its boundary consists of the line segments

$$L_1 = \overline{PQ}\,, \quad L_2 = \overline{QR}\,, \quad L_3 = \overline{PR}\,.$$

The critical points of f are the common solutions of the equations

$$f_x = y - 3 = 0\,, \quad f_y = x - 3 = 0\,.$$

Thus (3, 3) is the only critical point; it is inside $\mathcal{D}$ and at this critical point

$$f(3, 3) = -7\,.$$

On the other hand,

(i) $L_1 = \{(1, y) : 0 \le y \le 4\}$,

(ii) $L_2 = \{(x, 5 - x) : 1 \le x \le 5\}$,

(iii) $L_3 = \{(x, 0) : 1 \le x \le 5\}$.

But on L_1,

$$f(x, y) = 2 + y - 3 - 3y = -1 - 2y\,,$$

while on L_2

$$\begin{aligned} f(x, y) &= 2 + x(5 - x) - 3x - 3(5 - x) \\ &= 2 - 3x - (3 - x)(5 - x)\,, \end{aligned}$$

and on L_3

$$f(x, y) = 2 - 3x\,.$$

Thus

(i) on L_1 abs max value of f is -1 ,

(ii) on L_2 abs max value of f is -6.75 ,

(iii)on L_3 abs max value of f is -1 .

Consequently, taking the largest of

$$-7\,, \quad -1\,, \quad -6.75\,, \quad -1\,,$$

we see that on $\mathcal{D}$

$$\boxed{\text{abs max value of } f = -1}\,.$$

keywords: partial differentiation, critical point, absolute extremum,

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CalC15h06b
013 10.0 points

Use Lagrange Multipliers to determine the maximum value of

$$f(x, y) = 8xy$$

subject to the constraint

$$g(x, y) = \frac{x^2}{1} + \frac{y^2}{4} - 1 = 0\,.$$

1. maximum = 8 **correct**
2. maximum = 7
3. maximum = 9
4. maximum = 5
5. maximum = 6

Explanation:

By the Method of Lagrange multipliers, the extreme values of f occur at the common solutions of

$$(\nabla f)(x, y) = \lambda(\nabla g)(x, y), \quad g(x, y) = 0\,.$$

Now

$$(\nabla f)(x, y) = \langle 8y, 8x \rangle\,,$$

while

$$(\nabla g)(x, y) = \left\langle 2x, \frac{1}{2}y \right\rangle\,.$$

But then by the condition on ∇f and ∇g,

$$8y = 2\lambda x\,, \quad 8x = \frac{1}{2}\lambda y\,,$$

which after simplification gives

$$\lambda = \frac{4y}{x} = \frac{16x}{y}\,, \quad i.e.,\ y = \pm 2x\,.$$

Thus by the constraint equation,

$$g(x, \pm 2x) = \frac{x^2}{1} + \frac{x^2}{1} - 1 = 0\,,$$

i.e., $x = \pm\dfrac{\sqrt{2}}{2}$. Consequently, the extreme points are

$$\Big(\frac{\sqrt{2}}{2},\, \sqrt{2}\Big), \quad \Big(\frac{\sqrt{2}}{2},\, -\sqrt{2}\Big),$$

and

$$\Big(-\frac{\sqrt{2}}{2},\, -\sqrt{2}\Big), \quad \Big(-\frac{\sqrt{2}}{2},\, -\sqrt{2}\Big).$$

Since

$$f\Big(\frac{\sqrt{2}}{2},\, \sqrt{2}\Big) = f\Big(-\frac{\sqrt{2}}{2},\, -\sqrt{2}\Big) = 8\,,$$

while

$$f\Big(\frac{\sqrt{2}}{2},\, -\sqrt{2}\Big) = f\Big(-\frac{\sqrt{2}}{2},\, \sqrt{2}\Big) = -8\,,$$

we thus see that f has

$$\boxed{\text{max value} \;=\; 8}$$

subject to the constraint $g(x, y) = 0$.

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keywords:

CalC14d05s
014 10.0 points

Find the velocity of a particle with the given position function

$$\mathbf{r}(t) = 2e^{7t}\mathbf{i} + 5e^{-2t}\mathbf{j}\,.$$

1. $\mathbf{v}(t) = 14e^{7t}\mathbf{i} - 10e^{-2t}\mathbf{j}$ **correct**

2. $\mathbf{v}(t) = 9e^{7t}\mathbf{i} - 7e^{-2t}\mathbf{j}$

3. $\mathbf{v}(t) = 14e^{7t}\mathbf{i} + 5e^{-2t}\mathbf{j}$

4. $\mathbf{v}(t) = 2e^{7t}\mathbf{i} - 5e^{-2t}\mathbf{j}$

5. $\mathbf{v}(t) = 14e^{t}\mathbf{i} - 10e^{-t}\mathbf{j}$

Explanation:

The velocity of a particle with position function

$$\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$$

is given by

$$\mathbf{r}'(t) = x'(t)\,\mathbf{i} + y'(t)\,\mathbf{j}\,.$$

Thus, since

$$\mathbf{r}'(t) = 14e^{7t}\mathbf{i} - 10e^{-2t}\mathbf{j},$$

we have that

$$\boxed{\mathbf{v}(t) = 14e^{7t}\mathbf{i} - 10e^{-2t}\mathbf{j}}\,.$$

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keywords:

CalC14c01s
015 10.0 points

When C is parametrized by

$$\mathbf{c}(t) \;=\; (\sin 4t)\,\mathbf{i} + 3t\,\mathbf{j} + (\cos 4t)\,\mathbf{k}\,,$$

find its arc length between $\mathbf{c}(0)$ and $\mathbf{c}(4)$.

1. arc length $=$ 20 **correct**

2. arc length $=$ 24

3. arc length $=$ 8

4. arc length $=$ 12

5. arc length $=$ 16

Explanation:

The length of the curve between $\mathbf{c}(t_0)$ and $\mathbf{c}(t_1)$ is given by the integral

$$L \;=\; \int_{t_0}^{t_1} \|\mathbf{c}'(t)\|\, dt\,.$$

Now when

$$\mathbf{c}(t) \;=\; (\sin 4t)\,\mathbf{i} + 3t\,\mathbf{j} + (\cos 4t)\,\mathbf{k}\,,$$

we see that

$$\mathbf{c}'(t) \;=\; (4\cos 4t)\,\mathbf{i} + 3\,\mathbf{j} - (4\sin 4t)\,bk\,.$$

But then by the Pythagorean identity,

$$\|\mathbf{c}'(t)\| = (16+9)^{1/2} = 5\,.$$

Thus

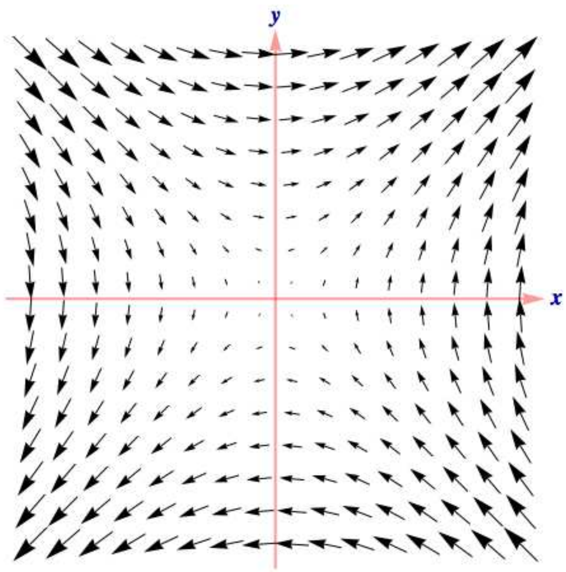
$$L = \int_0^4 5\,dt = \Big[5\,t\Big]_0^4.$$

Consequently,

$$\boxed{\text{arc length} = L = 20}\,.$$

VectorField01e
016 10.0 points

Which vector field **F** has graph



1. $\mathbf{F}(x,\,y) = x\,\mathbf{i} + y\,\mathbf{j}$

2. $\mathbf{F}(x,\,y) = -y\,\mathbf{i} + x\,\mathbf{j}$

3. $\mathbf{F}(x,\,y = y\,\mathbf{i} + x\,\mathbf{j}$ **correct**

4. $\mathbf{F}(x,\,y = -x\,\mathbf{i} + y\,\mathbf{j}$

5. $\mathbf{F}(x,\,y = x\,\mathbf{i} - y\,\mathbf{j}$

6. $\mathbf{F}(x,\,y = y\,\mathbf{i} - x\,\mathbf{j}$

Explanation:

We determine $\mathbf{F}(x,\,y)$ by looking at a few points on the graph. Now on the x-axis,

$$\mathbf{F}(x,\,0) = x\,\mathbf{j}\,,$$

while on the y-axis,

$$\mathbf{F}(0,\,y) = y\,\mathbf{i}\,.$$

The only one of the given vector fields satisfying these conditions is

$$\boxed{\mathbf{F}(x,\,y) = y\,\mathbf{i} + x\,\mathbf{j}}\,.$$

CalC16c16s
017 10.0 points

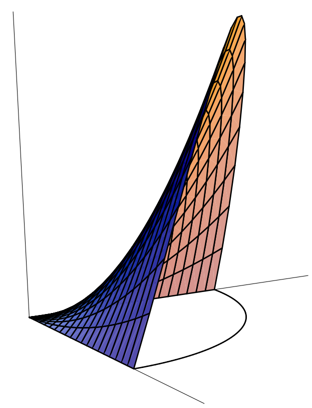
The graph of

$$f(x,y) = 3xy$$

over the bounded region A in the first quadrant enclosed by

$$y = \sqrt{16-x^2}$$

and the x, y-axes is the surface



Find the volume of the solid under this graph over the region A.

1. Volume $= 64$ cu. units

2. Volume $= 192$ cu. units

3. Volume $= 48$ cu. units

4. Volume $= 24$ cu. units

5. Volume $= 96$ cu. units **correct**

Explanation:

The volume of the solid under the graph of f is given by the double integral

$$V = \int\int_A f(x, y)\, dxdy,$$

which in turn can be written as the repeated integral

$$\int_0^4 \left(\int_0^{\sqrt{16-x^2}} 3xy\, dy\right) dx.$$

Now the inner integral is equal to

$$\left[\frac{3}{2}xy^2\right]_0^{\sqrt{16-x^2}} = \frac{3}{2}x(16-x^2).$$

Thus

$$V = \frac{3}{2}\int_0^4 x(16-x^2)\, dx = \left[-\frac{3}{8}(16-x^2)^2\right]_0^4.$$

Consequently,

$$\boxed{\text{Volume } = 96 \text{ cu. units}}\,.$$

CalC16g07a
018 10.0 points

Evaluate the triple integral

$$I = \int\int\int_E 3\,x\, dxdydz$$

when E is the set of points (x, y, z) in 3-space such that

$$0 \le x \le \sqrt{4-y^2}, \quad 0 \le z \le y \le 1.$$

1. $I = \dfrac{27}{8}$

2. $I = \dfrac{25}{8}$

3. $I = \dfrac{21}{8}$ **correct**

4. $I = \dfrac{19}{8}$

5. $I = \dfrac{23}{8}$

Explanation:

As a repeated integral

$$I = \int_0^1 \Big(\int_0^y \Big(\int_0^{\sqrt{4-y^2}} 3\,x\, dx\Big)\, dz\Big)\, dy\,.$$

Now

$$\int_0^{\sqrt{4-y^2}} 3\,x\, dx = \left[\frac{3}{2}x^2\right]_0^{\sqrt{4-y^2}} = \frac{3}{2}(4-y^2)\,,$$

while

$$\int_0^y \frac{3}{2}(4-y^2)\, dz = \frac{3}{2}(4-y^2)\, y\,.$$

Thus

$$I = \frac{3}{2}\int_0^1 (4\,y - y^3)\, dy = \frac{3}{2}\left[2\,y^2 - \frac{1}{4}\,y^4\right]_0^1$$

Consequently,

$$\boxed{I = \frac{21}{8}}\,.$$

CalC16i04a
019 10.0 points

Find the Jacobian of the transformation

$$T : (u,\, v) \longrightarrow (x,\, y)$$

when

$$x = 5u \sin v\,, \quad y = 4u \cos v\,.$$

1. $\dfrac{\partial(x,\, y)}{\partial(u,\, v)} = 9u \sin v \cos v$

2. $\dfrac{\partial(x,\, y)}{\partial(u,\, v)} = 9u \cos v$

3. $\dfrac{\partial(x,\, y)}{\partial(u,\, v)} = -9u$

4. $\dfrac{\partial(x,\, y)}{\partial(u,\, v)} = -20u \sin v$

5. $\dfrac{\partial(x,\, y)}{\partial(u,\, v)} = -20u$ **correct**

6. $\dfrac{\partial(x,\, y)}{\partial(u,\, v)} \;=\; 20u$

Explanation:

For general functions the Jacobian of the transformation

$$T\,:\,(u,\,v)\;\longrightarrow\;(x,\,y)$$

is defined by

$$\frac{\partial(x,\,y)}{\partial(u,\,v)} \;=\; \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}\,.$$

Now when

$$x \;=\; 5u\sin v\,, \quad y \;=\; 4v\;u\cos v\,,$$

then by the Product Rule,

$$x_u \;=\; 5\sin v\,, \quad x_v \;=\; 5u\cos v\,,$$

while

$$y_u \;=\; 4\cos v\,, \quad y_v \;=\; -4u\sin v\,.$$

In this case,

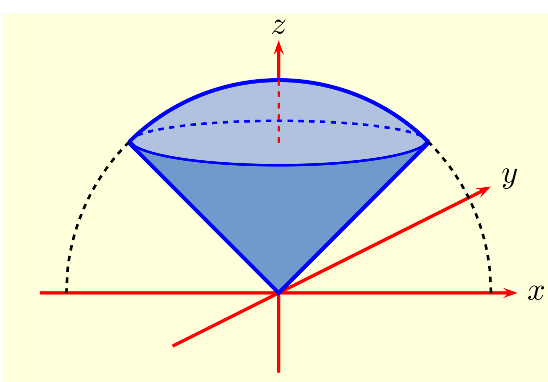
$$\frac{\partial(x,\,y)}{\partial(u,\,v)} \;=\; 20u\begin{vmatrix} \sin v & \cos v \\ \cos v & -\sin v \end{vmatrix}\,.$$

Consequently,

$$\boxed{\frac{\partial(x,\,y)}{\partial(u,\,v)} \;=\; -20v}\,.$$

CalC16d25s
020 10.0 points

Use polar coordinates to find the volume of the solid shown in



above the cone

$$z \;=\; \sqrt{x^2+y^2}$$

and below the sphere

$$x^2+y^2+z^2 \;=\; 49\,.$$

1. $V \;=\; \dfrac{2401\pi}{3}\left(2-\sqrt{2}\right)$

2. $V \;=\; \dfrac{49\pi}{3}\sqrt{2}$

3. $V \;=\; \dfrac{49\pi}{3}\left(2-\sqrt{2}\right)$

4. $V \;=\; \dfrac{343\pi}{3}\left(2-\sqrt{2}\right)$ **correct**

5. $V \;=\; \dfrac{343\pi}{3}\sqrt{2}$

6. $V \;=\; \dfrac{2401\pi}{3}\sqrt{2}$

Explanation:

The cone and the sphere intersect when $z = 7/\sqrt{2}$; in particular, the top of the cone is the disk formed by the a circle of radius $7/\sqrt{2}$ and its interior, while the height of the cone is $7/\sqrt{2}$. The cone has volume

$$V_{cone} \;=\; \frac{1}{3}\pi\frac{343}{2\sqrt{2}}\,.$$

On the other hand, the volume of the cap of the sphere is given by

$$V_{cap} \;=\; \int\!\!\int_R \left(\sqrt{49-x^2-y^2}-\frac{7}{\sqrt{2}}\right)dxdy$$

where R is the region

$$\left\{(x,\,y)\,:\,x^2+y^2\;\le\;\left(\frac{7}{\sqrt{2}}\right)^2\right\}.$$

Thus after changing to polar coordinates we see that the volume, V_{cap}, of the cap of the sphere is given by

$$\int_0^{7/\sqrt{2}}\int_0^{2\pi} r\left(\sqrt{49-r^2}-\frac{7}{\sqrt{2}}\right)d\theta dr$$

$$=\;2\pi\int_0^{7/\sqrt{2}} r\left(\sqrt{49-r^2}-\frac{7}{\sqrt{2}}\right)dr\,.$$

Thus

$$V_{cap} = \pi\Big[-\frac{2}{3}(49-r^2)^{3/2} - \frac{7r^2}{\sqrt{2}}\Big]_0^{7/\sqrt{2}}.$$

Consequently, the volume of the solid is

$$\boxed{V_{cone} + V_{cap} = \frac{343\pi}{3}(2-\sqrt{2})}.$$

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keywords:

CalC16i11a
021 10.0 points

Use the transformation $T : (u, v) \to (x, y)$ with

$$x = \frac{1}{3}(u+v), \qquad y = \frac{1}{3}(v-2u),$$

to evaluate the integral

$$I = \int\int_D (3x+2y)\, dxdy$$

when D is the region bounded by the lines

$$y = x, \qquad y = x-2$$

and

$$y + 2x = 0, \qquad y + 2x = 3.$$

1. $I = \dfrac{14}{3}$

2. $I = 4$

3. $I = \dfrac{10}{3}$

4. $I = \dfrac{13}{3}$ **correct**

5. $I = \dfrac{11}{3}$

Explanation:

The Jacobian of the transformation

$$T : (u, v) \to \Big(\frac{1}{3}(u+v), \frac{1}{3}(v-2u)\Big)$$

is given by

$$\begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{1}{9}\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = \frac{1}{3}.$$

On the other hand,

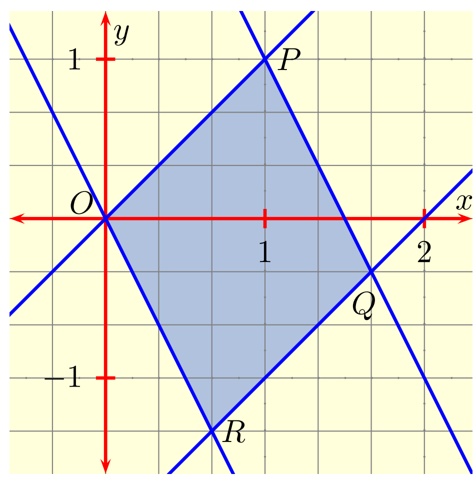
$$\begin{aligned} 3x + 2y &= (u+v) + \frac{2}{3}(v-2u) \\ &= \frac{5}{3}v - \frac{1}{3}u. \end{aligned}$$

Thus

$$I = \frac{1}{3}\int\int_{\mathcal{D}} \Big(\frac{5}{3}v - \frac{1}{3}u\Big)\, dudv$$

where $\mathcal{D}$ is the rectangle in the uv-plane that T maps onto D.

Now D is the shaded parallelogram shown in



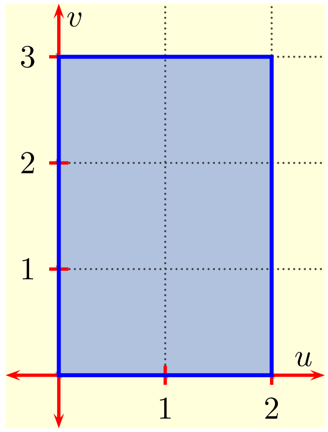
On the other hand, by solving for u, v in terms of x, y we see that

$$u = x - y, \qquad v = 2x + y.$$

Thus T maps the lines

$$u = 0, \quad u = 2, \quad v = 0, \quad v = 3,$$

in the uv-plane to the lines enclosing D, so $\mathcal{D}$ is the shaded rectangle in the uv-plane shown in



and

$$I = \frac{1}{3}\int_0^2 \left(\int_0^3 \left(\frac{5}{3}v - \frac{1}{3}u\right) dv\right) du\,.$$

Consequently,

$$\boxed{I = \frac{13}{3}}\,.$$

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keywords:

SphTripleInt01a
022 10.0 points

Use spherical coordinates to evaluate the integral

$$I = \int\int\int_B x^2 + y^2 + z^2\, dV$$

when B is the ball

$$x^2 + y^2 + z^2 \le 9\,.$$

1. $I = \dfrac{972\pi}{5}$ **correct**

2. $I = 12\pi$

3. $I = 972\pi$

4. $I = \dfrac{8\pi}{3}$

5. $I = 8\pi$

Explanation:

ScalarLineInt03a
023 10.0 points

Evaluate the integral

$$I = \int_C xe^{yz}\, ds$$

when C is the line segment from $(0, 0, 0)$ to $(2, 1, 2)$.

1. $I = \dfrac{3}{2}(e^2 - 1)$ **correct**

2. $I = 3(e^2 - 1)$

3. $I = 3(e - 1)$

4. $I = \dfrac{3}{2}e^2$

5. $I = 3e^2$

6. $I = \dfrac{3}{2}e$

Explanation:
The line segment from $(0, 0, 0)$ to $(2, 1, 2)$ can be parametrized by

$$\mathbf{c}(t) = (0, 0, 0) + t(2, 1, 2) = (2t, t, 2t)$$

with $0 \le t \le 1$. In this case

$$\|\mathbf{c}'(t)\| = \|2\,\mathbf{i} + \mathbf{j} + 2\,\mathbf{k}\| = 3\,,$$

while

$$xe^{yz} = 2te^{2t^2}\,.$$

on C. Thus

$$I = \int_0^1 2te^{2t^2}\|\mathbf{c}'(t)\|\, dt = 6\int_0^1 te^{2t^2}\, dt\,.$$

To evaluate this last integral we use the substitution $u = t^2$. For then

$$du = 2t\, dt\,, \qquad \frac{1}{2}du = t\, dt\,,$$

and so

$$I = 3\int_0^1 e^{2u}\,du = \Big[\frac{3}{2}e^{2u}\Big]_0^1.$$

Consequently,

$$\boxed{I = \frac{3}{2}(e^2-1)}.$$

LineIntegral01a
024 10.0 points

Evaluate the integral

$$I = \int_C (2xe^y\,dx - 3e^x\,dy)$$

when C is the parabola parametrized by

$$\mathbf{c}(t) = (t,\, t^2), \quad 0 \le t \le 1.$$

1. $I = 2e + \frac{7}{2}$

2. $I = e - \frac{7}{2}$

3. $I = 2e + 7$

4. $I = e + 7$

5. $I = e - 7$ **correct**

6. $I = 2e - 7$

Explanation:

When C is parametrized by

$$\mathbf{c}(t) = (x(t), y(t)) = (t,\, t^2),$$

then

$$\frac{dx}{dt} = 1, \qquad \frac{dy}{dt} = 2t.$$

Thus on C,

$$2xe^y\,dx = 2te^{t^2}\,dt, \quad 3e^x\,dy = 6te^t\,dt,$$

and so

$$I = \int_0^1 (2te^{t^2} - 6te^t)\,dt = I_1 + I_2.$$

To evaluate I_1 we use the substitution $u = t^2$. For then,

$$I_1 = \int_0^1 e^u\,du = e - 1.$$

On the other hand, to evaluate I_2 we integrate by parts:

$$I_2 = -6\int_0^1 te^t\,dt$$

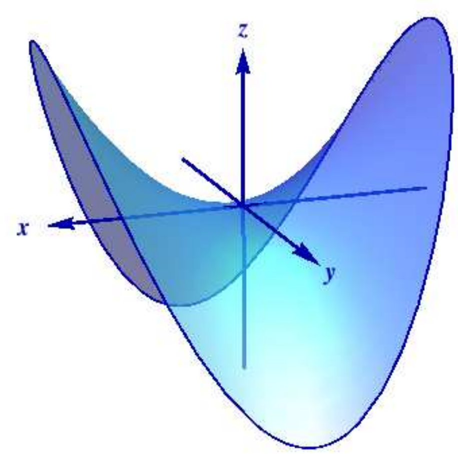
$$= -6\Big[te^t\Big]_0^1 + 6\int_0^1 e^t\,dt = -6\Big[te^t - e^t\Big]_0^1.$$

Consequently,

$$\boxed{I = e - 1 - 6 = e - 7}.$$

SurfaceArea01a
025 10.0 points

The surface S shown in



is the portion of the graph of

$$z = f(x, y) = x^2 - y^2$$

lying inside the cylinder

$$x^2 + y^2 = 2$$

Determine the surface area of S.

1. Surface Area $= \dfrac{13}{3}\pi$ sq. units **correct**

2. Surface Area $= \dfrac{16}{3}\pi$ sq. units

3. Surface Area $= 5\pi$ sq. units

4. Surface Area $= \dfrac{14}{3}\pi$ sq. units

5. Surface Area $= \dfrac{17}{3}\pi$ sq. units

Explanation:

As S is enclosed by the cylinder

$$x^2 + y^2 = 2\,,$$

it is the graph of the function

$$f(x, y) = x^2 - y^2\,,$$

over the disk

$$D = \{(x, y) : x^2 + y^2 \le 2\}$$

in the xy-plane. Its surface area element is

$$dS = (f_x^2 + f_y^2 + 1)^{1/2}\, dxdy$$

where

$$f_x = 2x\,, \qquad f_y = -2y\,.$$

Thus

$$dS = (4x^2 + 4y^2 + 1)^{1/2}\, dxdy\,,$$

and so its surface area is given by the integral

$$I = \int\int_D (4x^2 + 4y^2 + 1)^{1/2}\, dxdy\,.$$

Because of rotational symmetry, the integral is most easily evaluated using polar coordinates. For then

$$D = \{(r, \theta) : 0 \le r \le 2\,, \ \ 0 \le \theta \le 2\pi\}$$

so that

$$I = \int_0^2 \int_0^{2\pi} (4r^2 + 1)^{1/2} r\, d\theta dr$$

$$= 2\pi \int_0^{\sqrt{2}} (4r^2 + 1)^{1/2} r\, dr\,.$$

To evaluate this last integral we use the substitution $u^2 = 1 + 4r^2$. For then

$$\boxed{I = \frac{\pi}{2}\int_1^3 u^2\, du = \frac{\pi}{6}\Big[u^3\Big]_1^3 = \frac{13}{3}\pi}\,.$$

SurfaceInt04a
026 10.0 points

Evaluate the integral

$$I = \frac{1}{4}\int_S dS$$

when S is the surface given parametrically by

$$\mathbf{\Phi}(u, v) = (2uv,\, u + v,\, u - v)$$

for $u^2 + v^2 \le 4$.

1. $I = \dfrac{10}{3}\pi$

2. $I = \dfrac{11}{3}\pi$

3. $I = 3\pi$

4. $I = 4\pi$

5. $I = \dfrac{13}{3}\pi$ **correct**

Explanation:

When S is parametrized by

$$\mathbf{\Phi}(u, v) = (2uv,\, u + v,\, u - v)$$

for $u^2 + v^2 \le 4$, then

$$I = \frac{1}{4}\int\int_D \|\mathbf{T}_u \times \mathbf{T}_v\|\, dudv\,,$$

where

$$D = \{(u, v) :\, u^2 + v^2 \le 4\}\,.$$

Now

$$\mathbf{T}_u = \frac{\partial \boldsymbol{\Phi}}{\partial u} = (2v, 1, 1),$$

while

$$\mathbf{T}_v = \frac{\partial \boldsymbol{\Phi}}{\partial v} = (2u, 1, -1).$$

In this case,

$$\mathbf{T}_u \times \mathbf{T}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2v & 1 & 1 \\ 2u & 1 & -1 \end{vmatrix}$$

$$= -2\,\mathbf{i} + 2(u+v)\,\mathbf{j} + 2(v-u)\,\mathbf{k}.$$

Thus

$$\|\mathbf{T}_u \times \mathbf{T}_v\| = 2(1 + (u+v)^2 + (v-u)^2)^{1/2}$$

$$= 2(1 + 2(u^2+v^2))^{1/2}.$$

So, finally, we arrive at

$$I = \frac{1}{2}\int\int_D (1 + 2(u^2+v^2))^{1/2}\, du dv.$$

Because of the rotational symmetry, we'll use polar coordinates with

$$u = r\cos\theta, \qquad v = r\sin\theta,$$

to evaluate I. For then

$$I = \frac{1}{2}\int_0^2 \int_0^{2\pi} (1+2r^2)^{1/2}\, r\, d\theta dr$$

$$= \frac{1}{2}\pi \int_0^4 (1+2t)^{1/2}\, dt$$

$$= \frac{1}{6}\pi\Big[(1+2t)^{3/2}\Big]_0^4,$$

using the substitution $t = r^2$. Consequently,

$$\boxed{I = \frac{13}{3}\pi}.$$

StewartC5 17 07 19
027 10.0 points

Evaluate the integral

$$I = \int\int_S \mathbf{F}\cdot d\mathbf{S}$$

for the vector field

$$\mathbf{F} = 3x\,\mathbf{i} + 2y\,\mathbf{j} + 2z\mathbf{k}$$

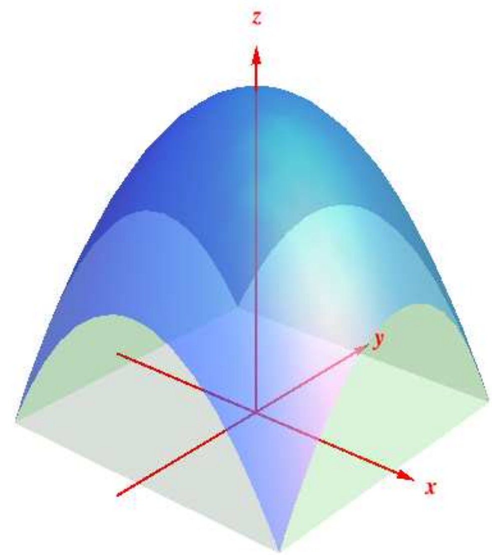
when S is the part of the paraboloid

$$z = 2 - x^2 - y^2,$$

oriented upwards, lying above the square

$$-1 \le x \le 1, \quad -1 \le y \le 1,$$

as shown in



1. $I = 24$ **correct**

2. $I = 9$

3. $I = 6$

4. $I = 18$

5. $I = 12$

Explanation:

If S is the graph of $z = f(x, y)$, then

$$d\mathbf{S} = (-f_x\,\mathbf{i} - f_y\,\mathbf{j} + \mathbf{k})dxdy.$$

So when $z = 2 - x^2 - y^2$, and

$$\mathbf{F} = 3x\,\mathbf{i} + 2y\,\mathbf{j} + 2z\mathbf{k}$$

we see that

$$dS = 2x\,\mathbf{i} + 2y\,\mathbf{j} + \mathbf{k}\,,$$

while

$$\begin{aligned}\mathbf{F}\cdot d\mathbf{S} &= (6x^2 + 4y^2 + 2(2 - x^2 - y^2))dxdy \\ &= (4x^2 + 2y^2 + 4)dxdy\,.\end{aligned}$$

Consequently,

$$\boxed{I = \int_{-1}^{1}\int_{-1}^{1}(4x^2 + 2y^2 + 4)\,dxdy = 24}\,.$$

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keywords:

GreensThm01a
028 10.0 points

Use Green's Theorem to evaluate the integral

$$I = \int_C (3xy^2dx + x^3dy)$$

when C is is the rectangle in the xy-plane having vertices at

$$(0,\,0), \quad (1,\,0), \quad (1,\,2), \quad (0,\,2)\,.$$

1. $I = -4$ **correct**

2. $I = -2$

3. $I = -6$

4. $I = -3$

5. $I = -5$

Explanation:

Since C is a piecewise smooth curve, Green's Theorem applies and says that

$$\int_C (P\,dx + Q\,dy) = \int\int_D \Big(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\Big)\,dxdy$$

for all smooth functions P, Q, where D is the rectangular region in the xy-plane enclosed by C. When

$$P(x,\,y) = 3xy^2\,, \qquad Q(x,\,y) = x^3\,,$$

therefore,

$$\begin{aligned} I &= \int_0^1\Big(\int_0^2 (3x^2 - 6xy)\,dy\Big)dx \\ &= \int_0^1 \Big[3x^2y - 3xy^2\Big]_0^2\,dx \\ &= \int_0^1 (6x^2 - 12x)\,dx\,.\end{aligned}$$

Consequently,

$$\boxed{I = \Big[2x^3 - 6x^2\Big]_0^1 = -4}\,.$$

StokesThm02a
029 10.0 points

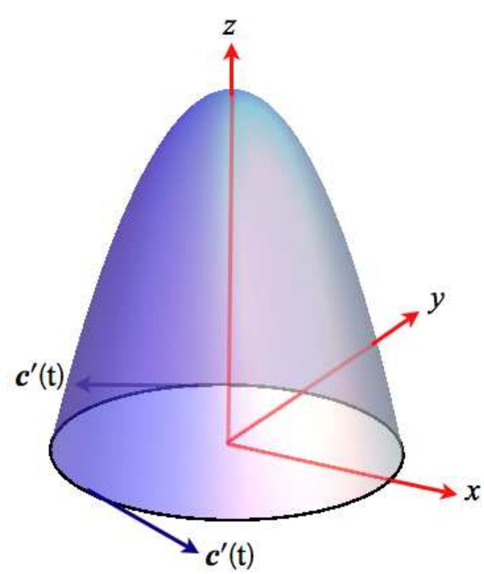
Use Stokes' theorem to evaluate the integral

$$I = \int\int_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}$$

when $\mathbf{F}$ is the vector field

$$\mathbf{F} = 3zx\,\mathbf{i} - xy\,\mathbf{j} - 2yz\,\mathbf{k}$$

and S is the surface shown in



whose boundary is the circle

$$\mathbf{c}(t) \;=\; \cos t\,\mathbf{i} + \sin t\,\mathbf{j}$$

in the xy-plane.

1. $I \;=\; -2$

2. $I \;=\; 0$ **correct**

3. $I \;=\; 2$

4. $I \;=\; 1$

5. $I \;=\; -1$

Explanation:

Since the boundary of S is the circle C parametrized by

$$\mathbf{c}(t) \;=\; \cos t\,\mathbf{i} + \sin t\,\mathbf{j}, \quad 0 \le t \le 2\pi\,,$$

and oriented counterclockwise as seen from above, Stokes Theorem can be used to reduce the integral over the surface S to a line integral

$$I \;=\; \int_C \mathbf{F}\cdot d\mathbf{s} \;=\; \int_0^{2\pi} \mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t)\,dt$$

over C.

Now

$$\mathbf{F}(\mathbf{c}(t)) \;=\; -\cos t\sin t\,\mathbf{j}\,,$$

while

$$\mathbf{c}'(t) \;=\; -\sin t\,\mathbf{i} + \cos t\,\mathbf{j}\,,$$

so

$$\mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t) \;=\; -\cos^2 t\sin t\,.$$

Thus

$$I = -\int_0^{2\pi} \cos^2 t\sin t\,dt\,.$$

Consequently,

$$\boxed{I \;=\; \Big[\frac{1}{3}\cos^3 t\Big]_0^{2\pi} \;=\; 0}\,.$$

DivThm02a
030 10.0 points

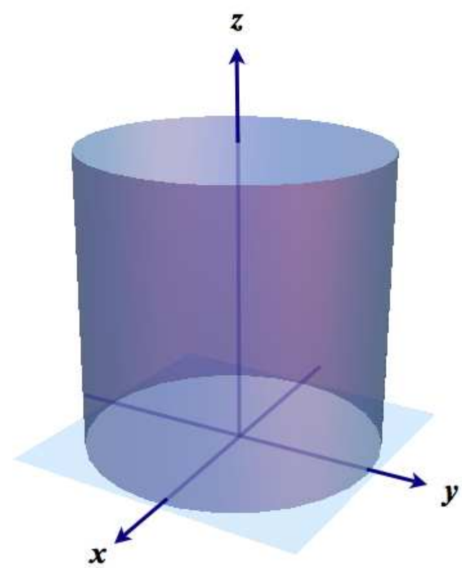
Evaluate the integral

$$I \;=\; \int\!\!\int_{\partial W} \mathbf{F}\cdot d\mathbf{S}$$

when

$$\mathbf{F}(x,\,y,\,z) \;=\; y\,\mathbf{i} - yz\,\mathbf{j} + 2z^2\,\mathbf{k}$$

and ∂W is the boundary of the solid W shown in



enclosed by the cylinder

$$x^2 + y^2 \;=\; 4\,,$$

the xy-plane, and the plane $z = 3$.

1. $I \;=\; 56$

2. $I \;=\; 56\pi$

3. $I \;=\; 54$

4. $I \;=\; 54\pi$ **correct**

5. $I \;=\; 55$

6. $I \;=\; 55\pi$

Explanation:

As shown, the boundary ∂W of W is piecewise-smooth, so the Divergence theorem can be applied:

$$I = \int\int_{\partial W} \mathbf{F} \cdot d\mathbf{S} = \int\int\int_{W} \operatorname{div}(\mathbf{F})\, dV\,.$$

Now

$$\begin{aligned}\operatorname{div}(\mathbf{F}) &= \nabla \cdot \mathbf{F} \\ &= \frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(2z^2) \\ &= -z + 4z = 3z\,.\end{aligned}$$

On the other hand, W consists of all points (r, θ, z) in cylindrical polar coordinates such that

$$0 \le \theta \le 2\pi, \quad 0 \le r \le 2, \quad 0 \le z \le 3\,.$$

So as a repeated integral in cylindrical polar coordinates,

$$I = \int_0^3 3z \left(\int_0^2 \left(\int_0^{2\pi} d\theta\right) r\, dr\right) dz\,.$$

But

$$\int_0^2 \left(\int_0^{2\pi} d\theta\right) r\, dr = \Big[\pi r^2\Big]_0^2 = 4\pi\,.$$

Consequently,

$$\boxed{I = 12\pi \int_0^3 z\, dz = 54\pi}\,.$$