This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixOpsTF01b
001 10.0 points

When

$$A = [\mathbf{a}_1 \;\; \mathbf{a}_2 \;\; \dots \;\; \mathbf{a}_n]$$

and B are matrices such that the product AB is defined, then

$$AB = [B\mathbf{a}_1 \;\; B\mathbf{a}_2 \;\; \dots \;\; B\mathbf{a}_n] .$$

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

The matrix

$$[B\mathbf{a}_1 \;\; B\mathbf{a}_2 \;\; \dots \;\; B\mathbf{a}_n]$$

is the product BA, *not* AB, and $AB \neq BA$ in general.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

InverseProp01a
002 10.0 points

Which of the following shows that if A is an invertible $n \times n$ matrix and B is any $n \times n$ matrix such that $BA = I$, then $B = A^{-1}$.

1. *Right-multiply each side of the equation $I = BA$ by A^{-1}. Then*

$$A^{-1} = BAA^{-1} = BI = B,$$

so $B = A^{-1}$. **correct**

2. *Left-multiply each side of the equation $I = BA$ by A^{-1}. Then*

$$A^{-1} = A^{-1}BA = BI = B,$$

so $B = A^{-1}$.

3. *Add A^{-1} to both sides of the equation $I = BA$. Then*

$$I + A^{-1} = BA + A^{-1},$$

so $A^{-1} = BI = B$.

4. *Subtract A^{-1} from both sides of the equation $I = BA$. Then*

$$I - A^{-1} = BA - A^{-1},$$

so $A^{-1} = BI = B$.

Explanation:

By definition, an $n \times n$ matrix A is invertible if there exists an $n \times n$ matrix C such that $AC = I = CA$ where I is the $n \times n$ identity matrix; we usually write A^{-1} for this matrix C.

So if B is any $n \times n$ matrix such that $BA = I$, then after multiplying the equation $I = BA$ on the right by A^{-1} we see that

$$A^{-1} = (BA)A^{-1} = B(AA^{-1}) = BI = B$$

because of the associativity of matrix multiplication and the fact that I is the identity matrix.

InverseTF02a
003 10.0 points

If A is an $n \times n$ invertible matrix, then the same sequence of elementary row operations that row reduces A to the identity I_n also reduces A^{-1} to I_n.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

If an $n \times n$ matrix A is invertible, then there exist elementary matrices $E_1, E_2, \ldots, E_p$ such that

$$E_p E_{p-1} \ldots E_2 E_1 A \; = \; I_n \,.$$

Now every elementary matrix is invertible, so

$$\begin{aligned} A \; &= \; E_1^{-1} E_2{-1} \ldots E_{p-1}^{-1} E_p I_n \\ &= \; E_1^{-1} E_2^{-1} \ldots E_{p-1}^{-1} E_p^{-1} \,. \end{aligned}$$

But then

$$A^{-1} \; = \; E_p E_{p-1} \ldots E_2 E_1 \,,$$

so

$$E_1^{-1} E_2{-1} \ldots E_{p-1}^{-1} E_p^{-1} A^{-1} \; = \; I_n \,.$$

Thus A^{-1} can be row reduced to I_n by the same elementary row operations as used to row reduce A to I_n, but we need to use their inverses and to apply them in the reverse order.

Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

Invertible01
004 10.0 points

A is an $n \times n$ matrix. Which of the following statements are equivalent to A being invertible?

(i) *A^T is an invertible matrix.*
(ii) *The columns of A form a linearly dependent set.*
(iii) *A is not row equivalent to the $n \times n$ identity matrix.*

1. i and iii

2. None of these.

3. iii

4. i **correct**

5. All of these.

6. i and ii

Explanation:

(i) Because A is an invertible matrix, then so is A^T, and the inverse of A^T is the transpose of A^{-1}. Mathematically, we can see that $I = I^T = (AA^{-1})^T = (A^{-1})^T A^T$. Hence because A is invertible, A^T must be invertible to satisfy this equation.

(ii) Since the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, the columns of A must form a linearly independent set.

(iii) Since A is invertible, A has n pivot positions. With n pivot positions, the pivots must lie on the main diagonal, in which case the reduced echelon form of A is I_n.

LUDecomp2x3b
005 10.0 points

Determine the Lower Triangular matrix L in an LU-Decomposition of

$$A \; = \; \begin{bmatrix} 4 & 2 & -4 \\ -20 & -10 & 22 \end{bmatrix}.$$

1. $L \; = \; \begin{bmatrix} 4 & 0 \\ -20 & 2 \end{bmatrix}$

2. $L \; = \; \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$ **correct**

3. $L \; = \; \begin{bmatrix} 4 & 0 \\ 5 & 2 \end{bmatrix}$

4. $L \; = \; \begin{bmatrix} 4 & 0 \\ -5 & 2 \end{bmatrix}$

5. $L \; = \; \begin{bmatrix} 4 & 0 \\ 20 & 2 \end{bmatrix}$

6. $L \; = \; \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

Explanation:

We first determine the elementary matrix reducing A to echelon form U by row reductions *downwards*.

$$\begin{aligned} A &\sim E_1 A \\ &= \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & -4 \\ -20 & -10 & 22 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 & -4 \\ 0 & 0 & 2 \end{bmatrix} = U\,. \end{aligned}$$

But an elementary matrix is always invertible. Thus

$$L = E_1^{-1} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}.$$

Consequently,

$$\boxed{\begin{aligned} L &= \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \\ U &= \begin{bmatrix} 4 & 2 & -4 \\ 0 & 0 & 2 \end{bmatrix} \end{aligned}}\,.$$

NullSpace01a

006 10.0 points

Find a matrix A so that $\mathrm{Nul}(A)$ is the set of all vectors

$$H = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \quad \begin{aligned} a + 3b &= 4c\,, \\ 2a &= c + d\,, \end{aligned} \right\}$$

in $\mathbb{R}^4$.

1. $A = \begin{bmatrix} 1 & 3 & -4 & 0 \\ 2 & 0 & -1 & 1 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & -3 & 4 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 3 & 4 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 3 & -4 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix}$ **correct**

5. $A = \begin{bmatrix} 1 & -3 & 4 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & -3 & -4 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}$

Explanation:

Rewrite the conditions

$$a + 3b = 4c, \quad 2a = c + d$$

as

$$\begin{aligned} a + 3b - 4c &= 0, \\ 2a - c - d &= 0\,, \end{aligned}$$

and set

$$A = \begin{bmatrix} 1 & 3 & -4 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix}.$$

Then

$$\begin{aligned} A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} &= \begin{bmatrix} 1 & 3 & -4 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \\ &= \begin{bmatrix} a + 3b - 4c \\ 2a - c + d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

if and only if

$$\begin{aligned} a + 3b - 4c &= 0, \\ 2a - c - d &= 0\,. \end{aligned}$$

Consequently,

$$\boxed{\mathrm{Nul}(A) = H}\,.$$

Rank02b

007 10.0 points

Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ 3 & -7 & -7 \end{bmatrix}.$$

1. $\mathrm{rank}(A) = 1$

2. $\mathrm{rank}(A) = 2$ **correct**

3. $\mathrm{rank}(A) = 4$

4. $\mathrm{rank}(A) = 5$

5. $\text{rank}(A) = 3$

Explanation:

Since

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

the first two rows of rref(A) contain leading 1's, so

$$\boxed{\text{Rank}(A) = 2}.$$

DetElemOps01TF
008 10.0 points

When the matrix

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is obtained from

$$A = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

by interchanging rows, then

$$\det[B] = \det[A].$$

True or False?

1. FALSE **correct**

2. TRUE

Explanation:

As 2×2 matrices,

$$\det[B] = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc,$$

while

$$\det[A] = \begin{vmatrix} c & d \\ a & b \end{vmatrix} = bc - ad.$$

Thus

$$\det[B] = -\det[A].$$

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

DetInverseT/F01a
009 10.0 points

The matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & -2 \\ 1 & 0 & 2 \end{bmatrix}$$

is invertible.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:

The matrix A will be invertible if and only if $\det(A) \neq 0$. Now

$$\det(A) = \begin{vmatrix} 2 & 1 & 3 \\ 0 & 4 & -2 \\ 1 & 0 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 4 & -2 \\ 0 & 2 \end{vmatrix} - \begin{vmatrix} 0 & -2 \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 2 \times 4 - 2 - 3 \times 4 = 2.$$

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$

SubspaceTF01
010 10.0 points

Let H be the set of points inside and on the unit circle in the xy-plane. That is, let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}$.

H is a subspace of $\mathbb{R}^2$. True or false?

1. FALSE **correct**

2. TRUE

Explanation:

If $\mathbf{u} = \begin{bmatrix} .5 \\ .5 \end{bmatrix}$ and $c = 4$, then $\mathbf{u}$ is in H but $c\mathbf{u}$ is not in H. Since H is not closed under scalar multiplication, H is not a subspace of $\mathbb{R}^2$. Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

BasisNull02a
011 10.0 points

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 3 & -6 & 12 & 0 \\ 1 & -3 & 7 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}.$$

1. $\left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

2. $\left\{ \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

3. $\left\{ \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

4. $\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} \right\}$

5. $\left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

6. $\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ **correct**

Explanation:

We first row reduce $[A\ \ \mathbf{0}]$:

$$\text{rref}([A\ \ \mathbf{0}]) = \begin{bmatrix} 1 & 0 & -2 & -2 & 0 \\ 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

to identify the free variables for $\mathbf{x}$ in the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

This shows that x_1, x_2 are basic variables, while x_3, x_4 are free variables. So set $x_3 = s$, $x_4 = t$. Then

$$x_1 = 2s + 2t, \quad x_2 = 3s + t\,,$$

and

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2s + 2t \\ 3s + t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} 2 \\ 3 \\ s \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Thus

$$\text{Nul}(A) = \text{Span} \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Consequently,

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is a basis for $\text{Nul}(A)$.

LinIndSetsTF01b
012 10.0 points

When $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p$ are vectors in $\mathbb{R}^n$ and

$$H = \text{Span}\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}\,,$$

then $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}$ is a basis for H.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

For the set $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}$ to be a basis for H, BOTH of the conditions

(i) $H = \text{Span}\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}\,,$

(ii) *the set is linearly independent*,

have to be satisfied. Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

CoordVec01a
013 10.0 points

Find the vector $\mathbf{x}$ in $\mathbb{R}^2$ having coordinate vector

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}$$

for $\mathbb{R}^2$.

1. $\mathbf{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ **correct**

2. $\mathbf{x} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

3. $\mathbf{x} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$

4. no such $\mathbf{x}$ exists

5. $\mathbf{x} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

Explanation:

The coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of a vector $\mathbf{x}$ in $\mathbb{R}^2$ with respect to a basis

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$$

for $\mathbb{R}^2$ satisfies the matrix equation

$$A[\mathbf{x}]_{\mathcal{B}} = \mathbf{x}, \qquad A = [\,\mathbf{b}_1 \quad \mathbf{b}_2\,]\,.$$

Consequently, when

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}, \quad [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix},$$

$$\boxed{\mathbf{x} = \begin{bmatrix} 4 & 6 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}}\,.$$

DimSubspace01a
014 10.0 points

Determine the dimension of the subspace

$$\text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix} \right\}$$

of $\mathbb{R}^3$.

1. dim $= 3$

2. dim $= 2$ **correct**

3. dim $= 1$

4. dim $= 4$

5. dim $= 5$

Explanation:

Since the subspace is the column space of the matrix

$$A = \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix}.$$

its dimension is the number of pivot columns in A. Now

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus A has 2 pivot columns, so

$$\boxed{\text{dimension} = 2}\,.$$

RankTF06b
015 10.0 points

If B is an echelon form of an an $m \times n$ matrix A, and if B has three nonzero rows, then the first three row of A form a basis for Row(A).

True or False?

1. FALSE **correct**

2. TRUE

Explanation:

Although row operations cannot change the linear dependence relations among *columns* of a matrix, they can change the linear dependence among *rows* of a matrix. For example, when

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$B = \text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

then $\text{Row}(A) = \mathbb{R}^3$. But the first 3 rows of A do not form a basis for $\mathbb{R}^3$. Notice that B is obtained from A by the row operation of interchanging rows 3 and 4 of A.

Consequently, the answer is

$$\boxed{\text{FALSE}}.$$