This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

QuadApprox02a
001 10.0 points

Find the quadratic approximation to

$$f(x, y) = \cos(2x + y) - \sin(x - y)$$

at $P(0, 0)$.

1. $Q(x, y) = 2 - x + y + 2x^2 - 2xy + y^2$

2. $Q(x, y) = 1 + x - y + 2x^2 - 2xy + y^2$

3. $Q(x, y) = 2 - x + y - 2x^2 + 2xy - \frac{1}{2}y^2$

4. $Q(x, y) = 2 - x + y + 2x^2 + 2xy - \frac{1}{2}y^2$

5. $Q(x, y) = 1 - x + y - 2x^2 - 2xy - \frac{1}{2}y^2$ **correct**

6. $Q(x, y) = 1 + x - y - 2x^2 + 2xy + y^2$

Explanation:

The Quadratic Approximation to $f(x, y)$ at $P(0, 0)$ is given by

$$Q(x, y) = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y + \frac{1}{2}f_{xx}(0, 0)x^2 + f_{xy}(0, 0)xy + \frac{1}{2}f_{yy}(0, 0)y^2.$$

But when

$$f(x, y) = \cos(2x + y) - \sin(x - y)$$

we see that

$$f_x = -2\sin(2x + y) - \cos(x - y)\,,$$
$$f_y = -\sin(2x + y) + \cos(x - y)\,,$$

so that $f(0, 0) = 1$ and

$$f_x(0, 0) = -1, \quad f_y(0, 0) = 1\,,$$

while

$$f_{xx} = -4\cos(2x + y) + \sin(x - y)\,,$$
$$f_{xy} = 2\cos(2x + y) - \sin(x - y)\,,$$
$$f_{yy} = \cos(2x + y) + \sin(x - y)\,,$$

so that $f_{xx}(0, 0) = 4$ and

$$f_{xy}(0, 0) = -2, \quad f_{yy}(0, 0) = -1\,,$$

Consequently, the Quadratic Approximation to f at $P(0, 0)$ is

$$\boxed{Q(x, y) = 1 - x + y - 2x^2 - 2xy - \frac{1}{2}y^2}\,.$$

keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

CalC15g19b
002 10.0 points

Locate and classify the critical point of

$$f(x, y) = \ln(xy) + 2y^2 - 2y - 2xy + 4,$$

for $x, y > 0$.

1. local minimum at $\left(\frac{1}{2}, 1\right)$

2. saddle-point at $\left(1, \frac{1}{2}\right)$ **correct**

3. local maximum at $\left(1, \frac{1}{2}\right)$

4. local maximum at $\left(\frac{1}{2}, 1\right)$

5. saddle-point at $\left(\frac{1}{2}, 1\right)$

6. local minimum at $\left(1, \frac{1}{2}\right)$

Explanation:

The critical point of f is the common solution of the equations

$$\frac{\partial f}{\partial x} = \frac{1}{x} - 2y = 0\,,$$
$$\frac{\partial f}{\partial y} = \frac{1}{y} + 4y - 2 - 2x = 0\,.$$

By the first equation, $2x = 1/y$. Using this in the second equation, we see that

$$4y - 2 = 0 \quad i.e., \ y = \frac{1}{2}.$$

So f has a critical point at

$$\left(1, \frac{1}{2}\right).$$

Now after differentiation,

$$f_{xx} = -\frac{1}{x^2}, \quad f_{xy} = -2, \quad f_{yy} = 4 - \frac{1}{y^2}.$$

Thus at the critical point $\left(1, \frac{1}{2}\right)$,

$$A = f_{xx}\Big|_{\left(1, \frac{1}{2}\right)} = -1 < 0, \qquad B = -2,$$

$$C = f_{yy}\Big|_{\left(1, \frac{1}{2}\right)} = 0 < 0,$$

in which case

$$AC - B^2 = -4 < 0,$$

Consequently, by the second derivative test f has a

$$\boxed{\text{saddle-point at } \left(1, \frac{1}{2}\right)}.$$

CalC14d16s
003 10.0 points

Determine the position vector, $\mathbf{r}(t)$, of a particle having acceleration

$$\mathbf{a}(t) = -4\,\mathbf{k}$$

when its initial velocity and position are given by

$$\mathbf{v}(0) = \mathbf{i} + \mathbf{j} - 2\,\mathbf{k}, \quad \mathbf{r}(0) = 2\,\mathbf{i} + 5\,\mathbf{j}$$

respectively.

1. $\mathbf{r}(t) = (t-2)\,\mathbf{i} + (t+5)\,\mathbf{j} - (2t^2 - 2t)\,\mathbf{k}$

2. $\mathbf{r}(t) = (t+2)\,\mathbf{i} + (t+5)\,\mathbf{j} - (2t^2 + 2t)\,\mathbf{k}$ **correct**

3. $\mathbf{r}(t) = (t+2)\,\mathbf{i} + (t+5)\,\mathbf{j} - (2t^2 - 2t)\,\mathbf{k}$

4. $\mathbf{r}(t) = (t+2)\,\mathbf{i} - (t-5)\,\mathbf{j} - (2t^2 + 2t)\,\mathbf{k}$

5. $\mathbf{r}(t) = (t-2)\,\mathbf{i} + (t+5)\,\mathbf{j} - (2t^2 + 2t)\,\mathbf{k}$

6. $\mathbf{r}(t) = (t+2)\,\mathbf{i} - (t-5)\,\mathbf{j} - (2t^2 - 2t)\,\mathbf{k}$

Explanation:

Since

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -4\,\mathbf{k},$$

we see that

$$\mathbf{v}(t) = -4t\,\mathbf{k} + C$$

where C is a constant vector such that

$$\mathbf{v}(0) = C = \mathbf{i} + \mathbf{j} - 2\,\mathbf{k}.$$

Thus

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \mathbf{i} + \mathbf{j} - (4t+2)\,\mathbf{k}.$$

But then

$$\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} - (2t^2 + 2t)\,\mathbf{k} + D$$

where D is a constant vector such that

$$\mathbf{r}(0) = D = 2\,\mathbf{i} + 5\,\mathbf{j}.$$

Consequently,

$$\boxed{\mathbf{r}(t) = (t+2)\,\mathbf{i} + (t+5)\,\mathbf{j} - (2t^2 + 2t)\,\mathbf{k}}.$$

CalC14c04a
004 10.0 points

The curve C is parametrized by

$$\mathbf{c}(t) = (4 + 2t)\mathbf{i} + \ln(2t)\,\mathbf{j} + (3 + t^2)\mathbf{k}.$$

Find the arc length of C between $\mathbf{c}(1)$ and $\mathbf{c}(3)$.

1. arc length $= 8 - 2\ln 3$

2. arc length $= 3 + \ln 6$

3. arc length $= 6 - \ln 3$

4. arc length $= 8 + \ln 3$ **correct**

5. arc length $= 9 + 2\ln 3$

6. arc length $= 8 - \ln 3$

Explanation:

The arc length of C between $\mathbf{c}(t_0)$ and $\mathbf{c}(t_1)$ is given by the integral

$$L = \int_{t_0}^{t_1} \|\mathbf{c}'(t)\|\, dt\,.$$

Now when

$$\mathbf{c}(t) = (4+2t)\mathbf{i} + \ln(2t)\,\mathbf{j} + (3+t^2)\mathbf{k}$$

we see that

$$\mathbf{c}'(t) = 2\,\mathbf{i} + \frac{1}{t}\,\mathbf{j} + 2t\,\mathbf{k}\,.$$

But then

$$\|\mathbf{c}'(t)\| = \left(4 + \frac{1}{t^2} + 4t^2\right)^{1/2} = \frac{2t^2+1}{t}\,.$$

Thus

$$L = \int_1^3 \left(2t + \frac{1}{t}\right) dt = \Big[t^2 + \ln t\Big]_1^3\,.$$

Consequently,

$$\boxed{\text{arc length} = L = 8 + \ln 3}\,.$$

GradVectorField01a
005 10.0 points

If $f(x, y)$ is a potential function for the gradient vector field

$$\mathbf{F}(x, y) = (3x - y)\,\mathbf{i} - (x + 2y)\,\mathbf{j}\,,$$

evaluate

$$f(1, 2) - f(0, 1)\,.$$

1. $f(1, 2) - f(0, 1) = -\dfrac{5}{2}$

2. $f(1, 2) - f(0, 1) = -\dfrac{7}{2}$ **correct**

3. $f(1, 2) - f(0, 1) = -4$

4. $f(1, 2) - f(0, 1) = -3$

5. $f(1, 2) - f(0, 1) = -\dfrac{9}{2}$

Explanation:

If $f(x, y)$ is a potential function for the gradient vector field

$$\mathbf{F}(x, y) = (3x - y)\,\mathbf{i} - (x + 2y)\,\mathbf{j}\,,$$

then

$$\frac{\partial f}{\partial x} = 3x - y\,, \quad \frac{\partial f}{\partial y} = -x - 2y\,.$$

Now by the first equation,

$$f(x, y) = \frac{3}{2}x^2 - xy + D(y)$$

for an arbitrary function $D(y)$, which by the second equation satisfies

$$-x + D'(y) = -x - 2y\,, \quad i.e.,\ D(y) = -y^2 + K\,,$$

for an arbitrary constant K. Thus

$$f(x, y) = \frac{3}{2}x^2 - xy - y^2 + K\,.$$

But then

$$f(0, 1) = -1 + K\,,$$

while

$$f(1, 2) = \frac{3}{2} - 2 - 4 + K = -\frac{9}{2} + K\,.$$

Consequently,

$$\boxed{f(1, 2) - f(0, 1) = -\frac{7}{2}}\,.$$

Curl01a
006 10.0 points

Find the curl of the vector field

$$\mathbf{F}(x, y, z) = 3zx\,\mathbf{i} + xy\,\mathbf{j} - 2yz\,\mathbf{k}\,.$$

1. $\operatorname{curl}\mathbf{F} \;=\; -2z\,\mathbf{i}+3x\,\mathbf{j}+y\,\mathbf{k}$ **correct**

2. $\operatorname{curl}\mathbf{F} \;=\; -2z\,\mathbf{i}-3x\,\mathbf{j}+y\,\mathbf{k}$

3. $\operatorname{curl}\mathbf{F} \;=\; 3x\,\mathbf{i}+y\,\mathbf{j}-2z\,\mathbf{k}$

4. $\operatorname{curl}\mathbf{F} \;=\; x\,\mathbf{i}+2y\,\mathbf{j}-3z\,\mathbf{k}$

5. $\operatorname{curl}\mathbf{F} \;=\; x\,\mathbf{i}-2y\,\mathbf{j}+3z\,\mathbf{k}$

6. $\operatorname{curl}\mathbf{F} \;=\; 3z\,\mathbf{i}-x\,\mathbf{j}-2y\,\mathbf{k}$

Explanation:
The curl of $\mathbf{F}$ is given symbolically by

$$\operatorname{curl}\mathbf{F} \;=\; \nabla\times\mathbf{F} \;=\; \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3zx & xy & -2yz \end{vmatrix}$$

$$= X\,\mathbf{i}-Y\,\mathbf{j}+Z\,\mathbf{k}$$

where

$$X \;=\; \begin{vmatrix} \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xy & -2yz \end{vmatrix}$$

$$= \Big(\frac{\partial}{\partial y}(-2yz)-\frac{\partial}{\partial z}(xy)\Big) \;=\; -2z\,,$$

$$Y \;=\; \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial z} \\ 3zx & -2yz \end{vmatrix}$$

$$= \Big(\frac{\partial}{\partial x}(-2yz)-\frac{\partial}{\partial z}(3zx)\Big) \;=\; -3x\,,$$

and

$$Z \;=\; \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \\ 3zx & xy \end{vmatrix}$$

$$= \Big(\frac{\partial}{\partial x}(xy)-\frac{\partial}{\partial y}(3zx)\Big) \;=\; y\,.$$

Consequently,

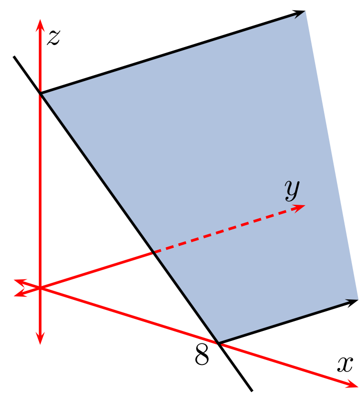
$$\boxed{\operatorname{curl}\mathbf{F} \;=\; -2z\,\mathbf{i}+3x\,\mathbf{j}+y\,\mathbf{k}}\,.$$

CalC16b01a
007 10.0 points

The graph of the function

$$z \;=\; f(x,\,y) \;=\; 8-x$$

is the plane shown in



Determine the value of the double integral

$$I \;=\; \int\!\!\int_A f(x,\,y)\,dxdy$$

over the region

$$A \;=\; \Big\{(x,y) : 0\le x\le 3,\;\; 0\le y\le 4\Big\}$$

in the xy-plane by first identifying it as the volume of a solid below the graph of f.

1. $I \;=\; 77$ cu. units

2. $I \;=\; 78$ cu. units **correct**

3. $I \;=\; 76$ cu. units

4. $I \;=\; 75$ cu. units

5. $I \;=\; 74$ cu. units

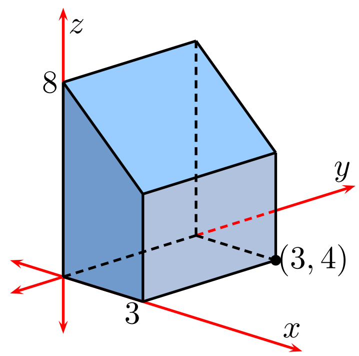
Explanation:
The double integral

$$I \;=\; \int\!\!\int_A f(x,\,y)\,dxdy$$

is the volume of the solid below the graph of f having the rectangle

$$A \;=\; \Big\{(x,y) : 0\le x\le 3,\;\; 0\le y\le 4\Big\}$$

for its base. Thus the solid is the wedge



and so its volume is the area of trapezoidal face multiplied by the thickness of the wedge. Consequently,

$$\boxed{I = 78 \text{ cu. units}}.$$

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keywords:

CalC16c05s
008 10.0 points

Evaluate the iterated integral

$$I = \int_0^{3\pi/2} \int_0^{\cos(\theta)} 2\, e^{\sin(\theta)}\, dr\, d\theta\,.$$

1. $I = 2e$

2. $I = 2\left(\dfrac{1}{e} - 1\right)$ **correct**

3. $I = 2(e-1)$

4. $I = 0$

5. $I = e - 2$

6. $I = \dfrac{1}{e} - 2$

Explanation:

After simple integration

$$\int_0^{\cos(\theta)} 2\, e^{\sin(\theta)}\, dr = \Big[2\, r\, e^{\sin(\theta)}\Big]_0^{\cos(\theta)}$$

$$= 2\cos(\theta)\, e^{\sin(\theta)}\,.$$

In this case,

$$I = \int_0^{3\pi/2} 2\cos(\theta)\, e^{\sin(\theta)}\, d\theta = \Big[2\, e^{\sin(\theta)}\Big]_0^{3\pi/2}.$$

Consequently,

$$\boxed{I = 2\left(\frac{1}{e} - 1\right)}.$$

CalC16g01a
009 10.0 points

Evaluate the triple integral

$$I = \int_0^1 \int_0^x \int_0^{x+y} (2x - y)\, dz dy dx\,.$$

1. $I = \dfrac{3}{8}$

2. $I = \dfrac{5}{8}$

3. $I = \dfrac{17}{24}$

4. $I = \dfrac{13}{24}$ **correct**

5. $I = \dfrac{11}{24}$

Explanation:

As a repeated integral,

$$I = \int_0^1 \Big(\int_0^x \Big(\int_0^{x+y} (2x-y)\, dz\Big) dy\Big) dx\,.$$

Now

$$\int_0^{x+y} (2x-y)\, dz = \Big[(2x-y)z\Big]_0^{x+y}$$

$$= (2x-y)(x+y) = 2x^2 + xy - y^2\,,$$

while

$$\int_0^x (2x^2 + xy - y^2)\, dy$$

$$= \Big[2x^2y + \frac{1}{2}xy^2 - \frac{1}{3}y^3\Big]_0^x = \frac{13}{6}x^3\,.$$

Consequently,

$$\boxed{I = \int_0^1 \frac{13}{6}x^3\,dx = \frac{13}{24}}.$$

keywords: integral, triple integral, repeated integral, linear function,polynomial integrand, binomial integrand, evaluation of triple integral

Div01a
010 10.0 points

Find the divergence of the vector field

$$\mathbf{F}(x,\,y,\,z) \;=\; 2x^2yz\,\mathbf{i} - xy^2z\,\mathbf{j} + 3xyz^2\,\mathbf{k}\,.$$

1. $\operatorname{div}\mathbf{F} \;=\; 8xyz$ **correct**

2. $\operatorname{div}\mathbf{F} \;=\; 11xyz$

3. $\operatorname{div}\mathbf{F} \;=\; 12xyz$

4. $\operatorname{div}\mathbf{F} \;=\; 10xyz$

5. $\operatorname{div}\mathbf{F} \;=\; 9xyz$

Explanation:
The div of $\mathbf{F}$ is given symbolically by

$$\operatorname{div}\mathbf{F} \;=\; \nabla\cdot\mathbf{F}$$

$$= 2\frac{\partial}{\partial x}(x^2yz) - \frac{\partial}{\partial y}(xy^2z) + 3\frac{\partial}{\partial z}(xyz^2)\,.$$

Thus

$$\boxed{\operatorname{div}\mathbf{F} \;=\; (4-2+6)xyz \;=\; 8xyz}.$$

CalC15h04exam
011 10.0 points

Determine the maximum value of

$$f(x,\,y) \;=\; 4x - 3y + 2$$

subject to the constraint

$$g(x,\,y) \;=\; x^2 + y^2 - 1 \;=\; 0\,.$$

1. max value $=$ 6

2. max value $=$ 7 **correct**

3. max value $=$ 5

4. max value $=$ 9

5. max value $=$ 8

Explanation:
The extreme values of f subject to the constraint $g = 0$ occur at solutions of

$$(\nabla f)(x,\,y) \;=\; \lambda(\nabla g)(x,\,y)\,, \quad g(x,\,y) \;=\; 0\,.$$

Now

$$(\nabla f)(x,\,y) \;=\; \langle\, 4,\,-3\,\rangle\,,$$

while

$$(\nabla g)(x,\,y) \;=\; \langle\, 2x,\,2y\,\rangle\,.$$

Thus

$$4 \;=\; 2\lambda x\,, \qquad -3 \;=\; 2\lambda y\,,$$

and so

$$\lambda \;=\; \frac{2}{x} \;=\; -\frac{3}{2y}\,, \quad i.e.,\; y \;=\; -\frac{3}{4}x\,.$$

But

$$g\left(x,\,-\frac{3}{4}x\right) \;=\; x^2 + \frac{9}{16}x^2 - 1 \;=\; 0\,,$$

$i.e.,\; x \;=\; \pm 4/5$. Consequently, the extreme points are

$$\left(\frac{4}{5},\,-\frac{3}{5}\right), \quad \left(-\frac{4}{5},\,\frac{3}{5}\right).$$

Since

$$f\left(\frac{4}{5},\,-\frac{3}{5}\right) = 7\,, \quad f\left(-\frac{4}{5},\,\frac{3}{5}\right) = -3\,,$$

we thus see that

$$\boxed{\text{max value} \;=\; 7}.$$

keywords: