This print-out should have 17 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

FinM4a24
001   10.0 points

Pandit is an aging dog who has to be kept on a strict diet containing, among other things, 1.8 grams of protein and 5.5 grams of fat. Two dog foods are available to Pandit's owner.

Food A has 2% protein and 5% fat, while

Food B has 2% protein and 7% fat.

How many grams of food A should Pandit's owner use in his diet?

1. # grams food $A = 43$

2. # grams food $A = 39$

3. # grams food $A = 41$

4. # grams food $A = 42$

5. # grams food $A = 40$ **correct**

Explanation:
Let x and y be the amounts (in grams) of foods A and B respectively used in Pandit's diet. Then

$$0.02x + 0.02y = 1.8,$$
$$0.05x + 0.07y = 5.5.$$

Solving for x, y we see that

$$x = 40, \quad y = 50,$$

so

$$\boxed{\text{\# grams food } A = 40.}$$

FitParabola01b
002   10.0 points

When the graph of the function

$$y = ax^2 + bx + c$$

is the unique parabola passing through the points

$$(1,\, 9), \quad (-1,\, 3), \quad (-3,\, 13),$$

determine b.

1. $b = 3$ **correct**

2. $b = 5$

3. $b = 4$

4. $b = 6$

5. $b = 2$

Explanation:
Since the parabola passes through the points

$$(1,\, 9), \quad (-1,\, 3), \quad (-3,\, 13),$$

the coefficients a, b and c must satisfy the equations

$$a + b + c = 9$$
$$a - b + c = 3$$
$$9a - 3b + c = 13$$

To solve these equations for c we reduce the augmented matrix

$$\begin{bmatrix} 1 & 1 & 1 & | & 9 \\ 1 & -1 & 1 & | & 3 \\ 9 & -3 & 1 & | & 13 \end{bmatrix}$$

to echelon form by successive row operations:

$$\xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 & | & 9 \\ 0 & -2 & 0 & | & -6 \\ 9 & -3 & 1 & | & 13 \end{bmatrix}$$

$$\xrightarrow{R_3 - 9R_1} \begin{bmatrix} 1 & 1 & 1 & | & 9 \\ 0 & -2 & 0 & | & -6 \\ 0 & -12 & -8 & | & -68 \end{bmatrix}$$

$$\xrightarrow{R_3 - 6R_2} \begin{bmatrix} 1 & 1 & 1 & | & 9 \\ 0 & -2 & 0 & | & -6 \\ 0 & 0 & -8 & | & -32 \end{bmatrix}.$$

Thus

$$\boxed{b \;=\; 3}\;.$$

LinearSystemT/F01a
003   10.0 points

Elementary row operations on an augmented matrix never change the the solution set of the associated linear system.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
Elementary row operations on an augmented matrix

(i) replace one row by the sum of itself and the multiple of another row,

(ii) interchange rows,

(iii) multiply all entries in a row by a non-zero constant.

For the linear system associated with the original augmented matrix these respective operations

(i) add a constant times one equation to another,

(ii) interchange two equations,

(iii) multiply an equation through by by a non-zero constant.

But none of these will affect the solutions of the equations.

Consequently, the statement is

$$\boxed{\text{TRUE}}\;.$$

LinSysUniqueTF02
004   10.0 points

If a system of linear equations has no free variables, then it has a unique solution.

True or False?

1. FALSE **correct**

2. TRUE

Explanation:
A linear system must have no solutions, unique solutions, or infinitely many solutions. Having no free variables indicates that the system does not have infinitely many solutions, but it does not determine which of the other two cases it might be.   Consider the following counterexample:

$$\begin{aligned}
x_1 \;+\; x_2 \;&=\; 1 \\
x_2 \;&=\; 5 \\
x_1 \;+\; x_2 \;&=\; 2
\end{aligned}$$

Subtracting the first row from the third row obtains the equation $0 = 1$, so this is clearly not consistent and hence has no solution. You can also show the system has no free variables. Hence it has no free variables and no solution.

Consequently, the statement is

$$\boxed{\text{FALSE}}\;.$$

RowReduceMan02a
005   10.0 points

The augmented matrix of a linear system of equations has been reduced by row operations to

$$\left[\begin{array}{ccc|c}
1 & 2 & -3 & 7 \\
0 & 1 & -1 & 3 \\
0 & 0 & 1 & -2
\end{array}\right]\;.$$

(a) *Continue row operations to write the matrix in reduced row echelon form.*

(b) *Then determine the solution set of the original system.*

1. (a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$

(b) $x_1 = -1,\; x_2 = 1,\; x_3 = -2$

correct

Explanation:

(a) Using row operations we see that

$$\begin{bmatrix} 1 & 2 & -3 & 7 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

The augmented matrix is now in reduced row echelon form.

(b) By row reduction the original system is equivalent to the system

$$\begin{aligned} x_1 & & & = & -1 \\ & x_2 & & = & 1 \\ & & x_3 & = & -2, \end{aligned}$$

so the solution set of the original system is

$$\boxed{x_1 = -1, \ \ x_2 = 1, \ \ x_3 = -2}.$$

AxisIntersect01a
006   10.0 points

When P is the plane in $\mathbb{R}^3$ given in vector form by

$$\mathbf{x} = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} + s \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} + t \begin{bmatrix} -2 \\ -1 \\ -5 \end{bmatrix},$$

determine where P intersects the z-axis.

1. $z = -15$

2. $z = -14$

3. $z = -11$ **correct**

4. $z = -12$

5. $z = -13$

Explanation:

Since the z-axis consists of points in $\mathbb{R}^3$ with $x = y = 0$, P intersects the z-axis when

$$\begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} + s \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} + t \begin{bmatrix} -2 \\ -1 \\ -5 \end{bmatrix}$$

for some choice of s and t, *i.e.*, when

$$s \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} + t \begin{bmatrix} -2 \\ -1 \\ -5 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ z+2 \end{bmatrix}$$

is consistent as a vector equation in s, t. This will be true if and only if the associated augmented matrix

$$A = \begin{bmatrix} -1 & -2 & -2 \\ -1 & -1 & 3 \\ -2 & -5 & z+2 \end{bmatrix}$$

is row equivalent to an echelon matrix whose entries in the last row are all zero.

Now by row reduction in the first column of A, we obtain

$$A \sim \begin{bmatrix} -1 & -2 & -2 \\ 0 & 1 & 5 \\ 0 & -1 & z+6 \end{bmatrix},$$

and after row reduction in the second column

$$A \sim \begin{bmatrix} -1 & -2 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & z+11 \end{bmatrix}.$$

Consequently, P intersects the z-axis at

$$\boxed{z = -11}.$$

M340LSpanM02
007   10.0 points

Given

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \ \ \mathbf{v_2} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \ \ \mathbf{v_3} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$

determine all values of λ for which

$$\mathbf{w} \;=\; \begin{bmatrix} 3 \\ 2 \\ \lambda \end{bmatrix}$$

is a vector in $\mathrm{Span}\{\mathbf{v_1},\, \mathbf{v_2},\, \mathbf{v_3}\}$?

1. $\lambda \;=\; 5$

2. $\lambda \;=\; -1$ **correct**

3. $\lambda \;=\; 1,\, 5$

4. $\lambda \;=\; -1,\, 5$

5. $\lambda \;=\; 1$

6. $\lambda \;=\; -1,\, 1$

Explanation:
The vector $\mathbf{w}$ is in $\mathrm{Span}\{\mathbf{v_1},\, \mathbf{v_2},\, \mathbf{v_3}\}$ if there exist weights x_1, x_2, x_3 such that

$$x_1\mathbf{v_1} + x_2\mathbf{v_2} + x_3\mathbf{v_3} \;=\; \mathbf{w}\,.$$

Such weights exist when the rightmost column in the augmented matrix

$$\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{w}\end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & 4 & 0 & 2 \\ 0 & 2 & -1 & \lambda \end{bmatrix}$$

is not a pivot column. But

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & 4 & 0 & 2 \\ 0 & 2 & -1 & \lambda \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -1 & -1 \\ 0 & 2 & -1 & \lambda \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & \lambda+1 \end{bmatrix}$$

Thus the rightmost column is not a pivot column when $\lambda + 1 = 0$. Consequently, $\mathbf{w}$ lies in $\mathrm{Span}\{\mathbf{v_1},\, \mathbf{v_2},\, \mathbf{v_3}\}$ when

$$\boxed{\;\lambda \;=\; -1\;}.$$

VectorEquTF01e
008   10.0 points

If $\mathbf{u}$, $\mathbf{v}$ are vectors in $\mathbb{R}^3$, when can $\mathrm{Span}\{\mathbf{u},\, \mathbf{v}\}$ be visualized as a plane through the origin in $\mathbb{R}^3$.

True or False?

1. ALWAYS

2. SOMETIMES **correct**

3. NEVER

Explanation:
If $\mathbf{u}$, $\mathbf{v}$ are linearly independent, then $\mathrm{Span}\{\mathbf{u},\, \mathbf{v}\}$ can be visualized as a plane through the origin in $\mathbb{R}^3$. But if $\mathbf{v}$ is a non-zero scalar multiple of $\mathbf{u}$, or when $\mathbf{v} = \mathbf{0}$, then $\mathrm{Span}\{\mathbf{u},\, \mathbf{v}\}$ consists of all scalar multiples of $\mathbf{u}$, and hence can be identified with a *line* through the origin.

Consequently, $\mathrm{Span}\{\mathbf{u},\, \mathbf{v}\}$ can

$$\boxed{\;\text{SOMETIMES}\;}$$

be visualized as a plane through the origin in $\mathbb{R}^3$, but not always.

Consistent01d
009   10.0 points

Describe geometrically the conditions on a vector $\mathbf{b}$ in $\mathbb{R}^2$ under which the equation

$$\begin{bmatrix} 3 & 2 \\ -12 & -8 \end{bmatrix} \mathbf{x} \;=\; \mathbf{b}$$

has a solution in $\mathbb{R}^2$.

1. any $\mathbf{b}$ not on line $y + 4x \;=\; 0$

2. arbitrary $\mathbf{b}$ in $\mathbb{R}^2$

3. any $\mathbf{b}$ not on line $y - 4x \;=\; 0$

4. $\mathbf{b}$ lies on line $y - 4x \;=\; 0$

5. $\mathbf{b}$ lies on line $y + 4x \;=\; 0$ **correct**

Explanation:

A matrix equation $A\mathbf{x} = \mathbf{b}$ has a solution when the last column of the associated augmented matrix $[A \ \mathbf{b}]$ is not a pivot column.

Now for the given equation,

$$[A \ \mathbf{b}] = \begin{bmatrix} 3 & 2 & b_1 \\ -12 & -8 & b_2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 2 & b_1 \\ 0 & 0 & b_2 + 4b_1 \end{bmatrix}.$$

Thus the equation has a solution when

$$b_2 + 4b_1 \ = \ 0 \,,$$

i.e., $(b_1, \ b_2)$ satisfies the linear equation

$$y + 4x \ = \ 0 \,.$$

Consequently, $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ lies on the line

$$\boxed{y + 4x \ = \ 0} \,.$$

MatEquTF02b
010   10.0 points

If the matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b}$ is in the set spanned by the columns of A.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
Since

$$A\mathbf{x} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \ldots \ \mathbf{a}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \ x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots + x_n\mathbf{a}_n \,,$$

the equation $A\mathbf{x} = \mathbf{b}$ is consistent, *i.e.*, has a solution, if and only if $\mathbf{b}$ is a linear combination of the columns of A.

On the other hand,

$$\text{Span}\{\mathbf{a}_1 \ \mathbf{a}_2 \ \ldots \ \mathbf{a}_n\}$$

consists of all linear combination of the columns of A. Hence, $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ is in the span of the columns of A.

Consequently, the statement is

$$\boxed{\text{TRUE}} \,.$$

SolSetsLinSysTF03
011   10.0 points

If the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

True or False?

1. TRUE **correct**

2. FALSE

Explanation:
The homogeneous equation $A\mathbf{x} = \mathbf{0}$ always has the trivial solution $\mathbf{x} = \mathbf{0}$. Now suppose $\mathbf{p}_1, \mathbf{p}_2$ are solutions of $A\mathbf{x} = \mathbf{b}$ with $\mathbf{p}_1 \neq \mathbf{p}_2$, *i.e.*, $\mathbf{w} = \mathbf{p}_1 - \mathbf{p}_2 \neq \mathbf{0}$ and

$$A\mathbf{p}_1 \ = \ \mathbf{b} \,, \qquad A\mathbf{p}_2 \ = \ \mathbf{b} \,.$$

But then

$$A\mathbf{w} \ = \ A\mathbf{p}_1 - A\mathbf{p}_2 \ = \ \mathbf{b} - \mathbf{b} \ = \ \mathbf{0} \,.$$

So $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{w}$ are two different solutions of $A\mathbf{x} = \mathbf{0}$.

Consequently, the statement is

$$\boxed{\text{TRUE}} \,.$$

ThreePoints01a
012   10.0 points

Determine the linear equation of the unique plane in $\mathbb{R}^3$ containing the points

$$P(1, \ -1, \ 1), \quad Q(-3, \ 4, \ 0),$$

and

$$R(3, \ -2, \ 7) \,.$$

1. $29x - 22y + 6z + 1 = 0$

2. $29x - 22y - 6z = 1$

3. $29x - 22y + 6z = 1$

4. $29x + 22y - 6z + 1 = 0$

5. $29x + 22y - 6z = 1$ **correct**

6. $29x + 22y + 6z + 1 = 0$

Explanation:
A linear equation in x, y, z has the form

$$ax + by + cz + d = 0.$$

Its graph will contain the points P, Q, and R when a, b, c, and d satisfy the homogeneous system

$$a - b + z + d = 0,$$

$$-3a + 4b + 0z + d = 0,$$

$$3x - 2y + 7z + d = 0,$$

of 3 linear equations in 4 variables. Now the associated augmented matrix is

$$A = \begin{bmatrix} 1 & -1 & 1 & 1 & 0 \\ -3 & 4 & 0 & 1 & 0 \\ 3 & -2 & 7 & 1 & 0 \end{bmatrix},$$

and

$$\mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 29 & 0 \\ 0 & 1 & 0 & 22 & 0 \\ 0 & 0 & 1 & -6 & 0 \end{bmatrix}.$$

So d is a free variable, say $d = s$, while

$$a = -29s, \quad b = -22s, \quad c = 6s.$$

Although there are infinitely many solutions, the parameter s is common to each of a, b, c, and d. Thus there is a unique plane passing through P, Q, and R, namely the graph of

$$\boxed{29x + 22y - 6z = 1},$$

obtained by cancelling the common factor s.

BalChemEqt01a
013   10.0 points

When butane C_4H_{10} burns in the presence of oxygen O_2 it produces carbon dioxide CO_2 and water H_2O, represented chemically by

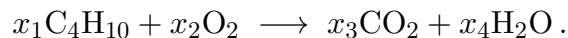$$C_4H_{10} + O_2 \longrightarrow CO_2 + H_2O.$$
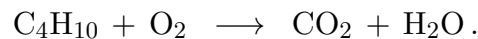
If 55 molecules of water were produced in one particular reaction, how many molecules of butane were burned in that reaction?

1. # molecules $= 13$

2. # molecules $= 12$

3. # molecules $= 14$

4. # molecules $= 11$ **correct**

5. # molecules $= 15$

Explanation:
We need to solve first for the relative numbers $x_1, \ldots, x_4$ of molecules in the balanced chemical equation

$$x_1 C_4H_{10} + x_2 O_2 \longrightarrow x_3 CO_2 + x_4 H_2O.$$

Now the fundamental rule governing this reaction is that the left and right hand sides contain the same number of the respective carbon, hydrogen and oxygen atoms. Thus

$$4x_1 + 0x_2 = x_3 + 0x_4,$$

$$10x_1 + 0x_2 = 0x_3 + 2x_4,$$

$$0x_1 + 2x_2 = 2x_3 + x_4,$$

which as a homogeneous system can be written in augmented matrix form

$$[A \ \mathbf{0}] = \begin{bmatrix} 4 & 0 & -1 & 0 & 0 \\ 10 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}.$$

But

$$\mathrm{rref}([A \ \mathbf{0}]) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{5} & 0 \\ 0 & 1 & 0 & -\frac{13}{10} & 0 \\ 0 & 0 & 1 & -\frac{4}{5} & 0 \end{bmatrix}.$$

So x_4 is a free variable, say $x_4 = s$, and

$$x_1 = \frac{1}{5}s, \quad x_2 = \frac{13}{10}s, \quad x_3 = \frac{4}{5}s,$$

give the respective proportions of the other molecules in the reaction with respect to x_4.

Consequently, if 55 molecules of water were produced, then

$$\boxed{11 \text{ molecules}}$$

of butane were burned.

LinIndependMan01a
014   10.0 points

Find all values h for which the vectors

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$$

are linearly independent.

1. $h \neq -4$ **correct**

Explanation:
The vectors

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \ \mathbf{a}_3 = \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix},$$

are linearly independent if and only if the only solutions of

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{0}$$

are $x_1 = x_2 = x_3 = 0$.

To determine the solutions of this vector equation we use row operations to reduce the corresponding augmented matrix to echelon form:

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ -2 & -6 & 2 & 0 \\ 4 & 7 & h & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -1 & h+4 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & h+4 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h+4 & 0 \end{bmatrix},$$

showing that the solutions of the original vector equation are

$$x_3(h+4) = 0, \quad x_2 = 0, \quad x_1 = x_3.$$

But then if $h + 4 \neq 0$, the only solutions are

$$x_3 = 0, \quad x_2 = 0, \quad x_1 = 0,$$

while if $h + 4 = 0$, x_3 is a free variable. Consequently, the vectors are linearly independent if and only if

$$\boxed{h \neq -4}.$$

LinIndepTF01c
015   10.0 points

The columns of any 4×5 matrix are linearly dependent.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set

$$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$$

in $\mathbb{R}^n$ is linearly dependent if $p > n$.

Now when

$$A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \ldots \ \mathbf{a}_5]$$

is a 4×5 matrix, there are 5 columns and each is a vector in $\mathbb{R}^4$. Because $5 > 4$, the columns must therefore be linearly dependent.

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$

LinTransform01e
016   10.0 points

A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear if and only if

$$T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1 T(\mathbf{v}_1) + c_2 T(\mathbf{v}_2)$$

for all vectors $\mathbf{v}_1$, $\mathbf{v}_2$ in $\mathbb{R}^n$ and all scalars c_1, c_2.

True or False?

1. TRUE **correct**

2. FALSE

Explanation:
A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ by definition is linear when:

(i) $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$,

(ii) $T(c\mathbf{u}) = cT(\mathbf{u})$,

for all $\mathbf{u}$, $\mathbf{v}$ in $\mathbb{R}^n$ and scalars c.

The property

$$T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1 T(\mathbf{v}_1) + c_2 T(\mathbf{v}_2)$$
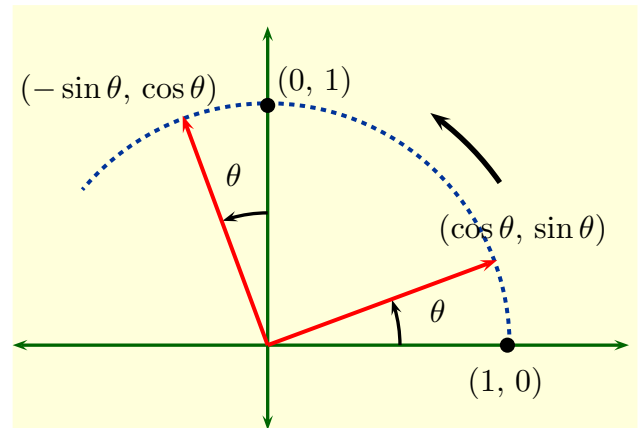
combines (i) and (ii) into a single condition.

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$

MatrixTrans01a
017   10.0 points

Determine the Standard Matrix for the transformation rotating the plane counterclockwise about the origin through $60°$.

1. $A = \dfrac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

2. $A = \dfrac{1}{2} \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$ **correct**

3. $A = \dfrac{1}{2} \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$

4. $A = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$

6. $A = \dfrac{1}{2} \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$

Explanation:
When T is rotation counter-clockwise through θ about the origin in the plane, then by right-angle trig in the graphic above, its associated Standard Matrix is

$$[T(1,\, 0) \quad T(0,\, 1)] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

When $\theta = 60°$, therefore, this is the matrix

$$\boxed{\dfrac{1}{2} \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}}.$$