This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixVecProd03
001 10.0 points

Determine $\mathbf{u}\mathbf{v}^T$ when

$$\mathbf{u} = \begin{bmatrix} -2 \\ -4 \\ 3 \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

1. $\mathbf{u}\mathbf{v}^T = \begin{bmatrix} -2a & -2b & -2c \\ -4a & -4b & -4c \\ 3a & 3b & 3c \end{bmatrix}$ **correct**

2. $\mathbf{u}\mathbf{v}^T = \begin{bmatrix} -2a & -4a & 3a \\ -2b & -4b & 3b \\ -2c & -4c & 3c \end{bmatrix}$

3. $\mathbf{u}\mathbf{v}^T = 3a - 4b - 2c$

4. $\mathbf{u}\mathbf{v}^T = -2a - 4b + 3c$

Explanation:
Since

$$\mathbf{v}^T = [\,a \;\; b \;\; c\,],$$

we see that

$$\mathbf{u}\mathbf{v}^T = \begin{bmatrix} -2 \\ -4 \\ 3 \end{bmatrix} [\,a \;\; b \;\; c\,]$$

$$= \begin{bmatrix} -2a & -2b & -2c \\ -4a & -4b & -4c \\ 3a & 3b & 3c \end{bmatrix}.$$

Consequently,

$$\boxed{\mathbf{u}\mathbf{v}^T = \begin{bmatrix} -2a & -2b & -2c \\ -4a & -4b & -4c \\ 3a & 3b & 3c \end{bmatrix}}.$$

M340LInverseTF04
002 10.0 points

There are some matrices A, B, C with $AB = AC$, A invertible and $B \neq C$.

True or False?

1. FALSE **correct**

2. TRUE

Explanation:
For all matrices such that $AB = AC$ and A invertible, it is always true that $A^{-1}AB = A^{-1}AC$. Since $A^{-1}A = I$ it follows that $B = C$.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

MatrixAlg02aT/F
003 10.0 points

There exist invertible matrices A, B such that

$$(AB)^{-1} \neq A^{-1}B^{-1}.$$

True or False?

1. TRUE **correct**

2. FALSE

Explanation:
Set

$$A = \begin{bmatrix} a & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 \\ 1 & b \end{bmatrix}.$$

Then

$$AB = \begin{bmatrix} 1 & b-a \\ 0 & 1 \end{bmatrix}$$

and

$$A^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & a \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} b & 1 \\ -1 & 0 \end{bmatrix}.$$

In this case,

$$(AB)^{-1} = \begin{bmatrix} 1 & a-b \\ 0 & 1 \end{bmatrix},$$

while

$$A^{-1}B^{-1} = \begin{bmatrix} 1 & 0 \\ b-a & 1 \end{bmatrix},$$

so $(AB)^{-1} \neq A^{-1}B^{-1}$ when $a \neq b$.

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

InvertibleTF02a
004 10.0 points

If A and D are $n \times n$ matrices such that $AD = I$, then $DA = I$

True or False?

1. TRUE **correct**

2. FALSE

Explanation:

Because A and D are square matrices and $AD = I$, then A and D are both invertible, with $D = A^{-1}$ and $A = D^{-1}$. So using this substitution, the first equation can be rewritten as $AA^{-1} = I$, and the second as $DD^{-1} = I$. Both of these statements are true by the definition of inverse matrices.

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

LUDecomp3x4a
005 10.0 points

Determine the Lower Triangular Matrix L in an LU-decomposition of the matrix

$$A = \begin{bmatrix} 4 & 1 & -2 & 4 \\ 16 & 7 & -7 & 21 \\ -4 & 11 & 6 & 21 \end{bmatrix}.$$

1. $L = \begin{bmatrix} 4 & 1 & 4 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix}$

2. $L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}$ **correct**

3. $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$

4. $L = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$

5. $L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 1 & -4 & 1 \end{bmatrix}$

6. $L = \begin{bmatrix} 1 & 4 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

Explanation:

We first determine the elementary matrices reducing A to an echelon form U by row reductions *downwards.*

Set

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that

$$E_1A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 & 4 \\ 16 & 7 & -7 & 21 \\ -4 & 11 & 6 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 3 & 1 & 5 \\ -4 & 11 & 6 & 21 \end{bmatrix} = A_1\,,$$

say. Next set

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

so that

$$E_2A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 3 & 1 & 5 \\ -4 & 11 & 6 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 3 & 1 & 5 \\ 0 & 12 & 4 & 25 \end{bmatrix} = A_2\,,$$

say. Finally, set

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$$

so that the product

$$E_3A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 3 & 1 & 5 \\ 0 & 12 & 4 & 25 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 3 & 1 & 5 \\ 0 & 0 & 0 & 5 \end{bmatrix} = U\,,$$

and

$$E_3E_2E_1A = E_3E_2A_1 = E_3A_2 = U$$

is an echelon form of A. But every elementary matrix is invertible. Thus $A = LU$, setting

$$L = E_1^{-1}E_2^{-1}E_3^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}.$$

Consequently,

$$\boxed{L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}}.$$

Subspace05a
006 10.0 points

Let H be the set of all vectors

$$\begin{bmatrix} a-2b \\ ab+3a \\ b \end{bmatrix}$$

where a and b are real. Determine if H is a subspace of $\mathbb{R}^3$, and then check the correct answer below.

1. *H is a subspace of $\mathbb{R}^3$ because it can be written as Span$\{\mathbf{v}_1,\, \mathbf{v}_2\}$ with $\mathbf{v}_1,\, \mathbf{v}_2$ in $\mathbb{R}^3$.*

2. *H is a subspace of $\mathbb{R}^3$ because it can be written as Nul(A) for some matrix A.*

3. *H is not a subspace of $\mathbb{R}^3$ because it is not closed under vector addition.* **correct**

4. *H is not a subspace of $\mathbb{R}^3$ because it does not contain* $\mathbf{0}$.

Explanation:

To check if the set H of all vectors

$$\begin{bmatrix} a-2b \\ ab+3a \\ b \end{bmatrix}$$

is a subspace of $\mathbb{R}^3$ we check the properties defining a subspace:

1. *the zero vector* $\mathbf{0}$ *is in H*: set $a = b = 0$. Then

$$\begin{bmatrix} 0-0 \\ 0+0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

so H contains $\mathbf{0}$.

2. *for each* $\mathbf{u}$, $\mathbf{v}$ *in H the sum* $\mathbf{u}+\mathbf{v}$ *is in H*: set

$$\mathbf{v}_1 = \begin{bmatrix} a_1-2b_1 \\ a_1b_1+3a_1 \\ b_1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} a_2-2b_2 \\ a_2b_2+3a_2 \\ b_2 \end{bmatrix},$$

in H. Then

$$\mathbf{v}_1+\mathbf{v}_2 = \begin{bmatrix} a_1-2b_1 \\ a_1b_1+3a_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} a_2-2b_2 \\ a_2b_2+3a_2 \\ b_2 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1+a_2)-2(b_1+b_2) \\ a_1b_1+a_2b_2+3(a_1+a_2) \\ (b_1+b_2) \end{bmatrix}.$$

But in general,

$$a_1b_1+a_2b_2 \neq (a_1+a_2)(b_1+b_2)\,,$$

in which case $\mathbf{u}+\mathbf{v}$ is not in H.

Consequently, H is not a subspace of $\mathbb{R}^3$ because it is

$$\boxed{\textit{not closed under vector addition}}\,.$$

DimRankTF02a
007 10.0 points

If $\mathcal{B}$ is a basis for a subspace H, then each vector in H can be written in only one way as a linear combination of the vectors in $\mathcal{B}$.

True or False?

1. TRUE **correct**

2. FALSE

Explanation:
If H is a p-dimensional subspace of $\mathbb{R}^n$, then $\mathcal{B}$ must be a set of p elements $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$. To represent any vector $\mathbf{b}$ in H is to find the coefficients c_j in the linear combination

$$\mathbf{b} \;=\; c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_p\mathbf{v}_p$$

of vectors in $\mathcal{B}$ whose sum is $\mathbf{b}$. This is equivalent to solving a system of equations with p equations and p unknowns. Because the vectors are linearly independent by the definition of a basis, this means there can only be one solution.

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

DetElemOps02TF
008 10.0 points

When the matrix

$$B \;=\; \begin{bmatrix} a & b \\ c+ka & d+kb \end{bmatrix}$$

is obtained from

$$A \;=\; \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

by adding k times row 1 to row 2, then

$$\det[B] \;=\; \det[A]\,.$$

True or False?

1. TRUE **correct**

2. FALSE

Explanation:
As 2×2 matrices,

$$\det[B] \;=\; \begin{vmatrix} a & b \\ c+ka & d+kb \end{vmatrix}$$

$$=\; a(d+kb) - b(c+ka) \;=\; ad - bc\,,$$

while

$$\det[A] \;=\; \begin{vmatrix} a & b \\ c & d \end{vmatrix} \;=\; ad - bc.$$

Thus

$$\det[B] \;=\; \det[A]\,.$$

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

DetInverseT/F01b
009 10.0 points

The matrix

$$A \;=\; \begin{bmatrix} 2 & 3 & 0 \\ 1 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

is invertible.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:
The matrix A will be invertible if and only if $\det(A) \neq 0$. Now

$$\det(A) \;=\; \begin{vmatrix} 2 & 3 & 0 \\ 1 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}$$

$$=\; 2\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix}$$

$$= 2 \times (-5) - 3 \times (-3) \;=\; -1\,.$$

Consequently, the statement is

$$\boxed{\text{TRUE}}\,.$$

VectorSpace01aT/F
010 10.0 points

The subset

$$V = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : ab \geq 0 \right\}$$

of $\mathbb{R}^2$ is closed under vector addition.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:
Set

$$\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$$

Then $\mathbf{u}$, $\mathbf{v}$ are vectors in V. On the other hand,

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

But $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is not in V.

Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

BasisNull02b
011 10.0 points

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 2 & -2 & 4 & -6 \\ 3 & -3 & 9 & -18 \\ 2 & -2 & 3 & -3 \end{bmatrix}.$$

1. $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

2. $\left\{ \begin{bmatrix} -3 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$

3. $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$

4. $\left\{ \begin{bmatrix} 3 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$

5. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$

6. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$ **correct**

Explanation:
We first row reduce $[A\ \mathbf{0}]$:

$$\text{rref}([A\ \mathbf{0}]) = \begin{bmatrix} 1 & -1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

to identify the free variables for $\mathbf{x}$ in the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

Thus x_1 and x_3 are basic variables, while x_2 and x_4 are free variables. So set $x_2 = s$ and $x_4 = t$. Then

$$x_1 = s - 3t, \quad x_3 = 3t,$$

and

$$\text{Nul}(A) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}.$$

Consequently,

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$$

is a basis for Nul(A).

LinIndSetsTF02e
012 10.0 points

If B is an echelon form of a matrix A, then the pivot columns of B form a basis for Col A.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

The pivot columns of A are the same as the pivot columns for B, and the pivot columns of A form a basis for Col(A). But row operations on a matrix can change the entries in a column, so the columns of B need not be in the column space of A. Thus the pivot columns of B need not form a basis for Col(A).

Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

CoordVec01b
013 10.0 points

Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ in $\mathbb{R}^2$ for the vector

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \end{bmatrix} \right\}$$

for $\mathbb{R}^2$.

1. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ -2 \end{bmatrix}$

2. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$

3. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$ **correct**

4. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

5. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

6. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

Explanation:

The coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of a vector $\mathbf{x}$ in $\mathbb{R}^2$ with respect to a basis

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$$

for $\mathbb{R}^2$ satisfies the matrix equation

$$A[\mathbf{x}]_{\mathcal{B}} = \mathbf{x}, \qquad A = [\,\mathbf{b}_1 \quad \mathbf{b}_2\,]\,.$$

Consequently, when

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \end{bmatrix} \right\},$$

and

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix},$$

the associated augmented matrix is

$$[\,A \;\; \mathbf{x}\,] = \begin{bmatrix} 1 & 5 & 4 \\ -2 & -6 & 0 \end{bmatrix}.$$

But then

$$\text{rref}[A \;\; \mathbf{x}] = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 2 \end{bmatrix}.$$

Thus

$$\boxed{[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}}\,.$$

DimSubspace01b
014 10.0 points

Determine the dimension of the subspace

$$\text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix} \right\}$$

of $\mathbb{R}^3$.

1. dim $= 2$ **correct**

2. dim $= 3$

3. dim $=$ 4

4. dim $=$ 5

5. dim $=$ 1

Explanation:

Since the subspace is the column space of the matrix

$$A = \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix}.$$

its dimension is the number of pivot columns in A. Now

$$\mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus A has 2 pivot columns, so

$$\boxed{\text{dimension} = 2}.$$

RankTF06c
015 10.0 points

The dimensions of the row space and column space of an $m \times n$ matrix A are the same, even if $m \neq n$.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:

Recall that the rank A is the number of pivot columns in A. Equivalently, rank A is the number of pivot positions in an echelon form B of A. Furthermore, since B has a nonzero row for each pivot, and since these rows form a basis for the row space of A, rank A is also the dimension of the row space.

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$