This print-out should have 12 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

MatrixProp01a
001 10.0 points

Compute $AA^T - A^TA$ for the matrix

$$A = \begin{bmatrix} -3 & -2 \\ -3 & -2 \end{bmatrix}.$$

1. $AA^T - A^TA = \begin{bmatrix} -5 & -1 \\ -1 & 5 \end{bmatrix}$

2. $AA^T - A^TA = \begin{bmatrix} 5 & -1 \\ -1 & -5 \end{bmatrix}$

3. $AA^T - A^TA = \begin{bmatrix} 5 & 1 \\ 1 & -5 \end{bmatrix}$

4. $AA^T - A^TA = \begin{bmatrix} 5 & -1 \\ 1 & -5 \end{bmatrix}$

5. $AA^T - A^TA = \begin{bmatrix} -5 & 1 \\ 1 & 5 \end{bmatrix}$

6. $AA^T - A^TA = \begin{bmatrix} -5 & -1 \\ 1 & 5 \end{bmatrix}$

InverseMatrix01b
002 10.0 points

Solve for X when $A(X + B) = C$,

$$A = \begin{bmatrix} 5 & 2 \\ -3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}.$$

LUDecomp06h
003 10.0 points

Find L in an LU decomposition of

$$A = \begin{bmatrix} 3 & -2 & 5 & 2 \\ -6 & 4 & -6 & 1 \\ 12 & -8 & 4 & -17 \end{bmatrix}.$$

1. $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & -4 & 1 \end{bmatrix}$

2. $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -4 & 1 \end{bmatrix}$

3. $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 4 & 1 \end{bmatrix}$

4. $L = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 2 & 0 \\ 4 & -4 & 2 \end{bmatrix}$

5. $L = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -1 & 0 \\ 4 & -4 & -1 \end{bmatrix}$

6. $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -4 & 4 & 1 \end{bmatrix}$

Subspace02a
004 10.0 points

Which of the following describes

$$H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$$

when

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}.$$

1. $H = \mathbb{R}^3$

2. H is a plane not through origin

3. H is a plane through origin

4. H is a line

Invertible02
005 10.0 points

A is an $n \times n$ matrix. Which of the following statements are equivalent to A being invertible?

(i) $\text{Col}\,A = \{\mathbf{0}\}$.

(ii) *The columns of A do not form a basis of $\mathbb{R}^n$.*

(iii) $\dim(\text{Col}\,A) = n$.

1. iii

2. None of these

3. i and ii

4. i

5. ii

6. All of these

Rank02e
006 10.0 points

Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & 2 & 5 \end{bmatrix}.$$

1. $\text{rank}(A) = 1$

2. $\text{rank}(A) = 4$

3. $\text{rank}(A) = 5$

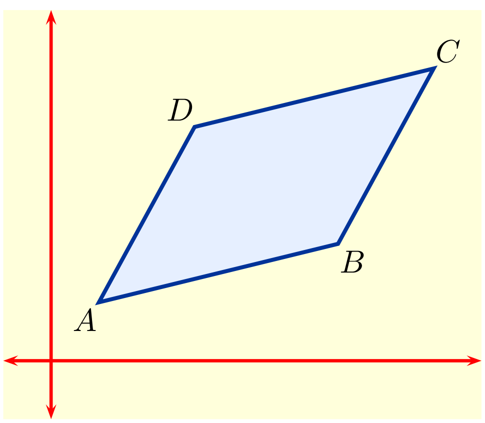
4. $\text{rank}(A) = 2$

5. $\text{rank}(A) = 3$

5. $\text{area} = 14$

DetArea03a
007 10.0 points

Compute the area of the parallelogram $ABCD$ shown in



having vertices at

$$A = (1, 1), \qquad B = (6, 2),$$

and

$$C = (8, 5), \qquad D = (3, 4).$$

1. $\text{area} = 13$

2. $\text{area} = 10$

3. $\text{area} = 11$

4. $\text{area} = 12$

BasisNull01b
008 10.0 points

Find a basis for the Null space of the matrix

$$A = \begin{bmatrix} 2 & -2 & 10 & 2 \\ -1 & -1 & -1 & -3 \\ 2 & -3 & 12 & -1 \end{bmatrix}.$$

Basis03a
009 10.0 points

In the vector space V of all real-valued functions, find a basis for the subspace

$$H = \mathrm{Span}\{\sin t,\ \sin 2t,\ \sin t\cos t\}.$$

1. $\{\sin 2t,\ \sin t\cos t\}$

2. $\{\sin t,\ \sin 2t,\ \sin t\cos t\}$

3. $\{\sin t,\ \sin 2t\}$

4. $\{\cos t,\ \sin 2t,\ \sin t\cos t\}$

5. $\{\cos t,\ \sin 2t\}$

PolyCoordVec01b
010 10.0 points

Find the coordinate vector $[\mathbf{p}]_{\mathcal{B}}$ in $\mathbb{R}^3$ for the polynomial

$$\mathbf{p}(t) = 2 + 3t - 6t^2$$

with respect to the basis

$$\mathcal{B} = \left\{1 - t^2,\ t - t^2,\ 1 - t + t^2\right\}$$

for $\mathbb{P}_2$.

1. $[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$

2. $[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$

3. $[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

4. $[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$

5. $[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$

6. $[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

ChangeBasis01c
011 (part 1 of 2) 10.0 points

Determine the change of coordinates matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ to $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ of a vector space V when

$$\mathbf{b}_1 = -\mathbf{c}_1 + 4\mathbf{c}_2, \qquad \mathbf{b}_2 = 5\mathbf{c}_1 - 3c_2 .$$

4. $[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix} 11 \\ 10 \end{bmatrix}$

5. $[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix} -10 \\ 11 \end{bmatrix}$

6. $[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix} 10 \\ -11 \end{bmatrix}$

Explanation:
When

$$\mathbf{x} = 5\mathbf{b}_1 + 3\mathbf{b}_2,$$

then

$$\mathbf{x} = 5(-\mathbf{c}_1 + 4\mathbf{c}_2) + 3(5\mathbf{c}_1 - 3c_2).$$

Consequently,

$$\boxed{[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}}.$$