[Review Koefisien Aerodinamika]

Persamaan Gaya

$$-W\sin\theta + X = m(\dot{u} + qw - rv)$$
$$W\cos\theta\sin\varphi + Y = m(\dot{v} + ru - pw)$$
$$W\cos\theta\cos\varphi + Z = m(\dot{w} + pv - qu)$$

Persamaan Momen

$$\mathcal{L} = I_x\dot{p} + (I_z - I_y)qr - J_{xz}(\dot{r} + pq)$$
$$\mathcal{M} = I_y\dot{q} + (I_x - I_z)rp + J_{xz}(p^2 - r^2)$$
$$\mathcal{N} = I_z\dot{r} + (I_y - I_x)pq - J_{xz}(\dot{p} - rq)$$

Gaya/Momen pada pesawat, dapat diwakilkan sebagai fungsi dengan **parameter aerodinamika** (di sumbu angin) sebagai variable pembentuknya.

$$X = f\left\{V_\infty, \rho, u, \alpha, \beta, p, q, r, \dot{\alpha}, \dot{\beta}, i_H, \delta_f, \ldots\right\}$$

Dan dapat di representasikan sebagai **deret Taylor** untuk orde rendah saja (HOT tidak disertakan)

$$X = \mathbb{C} + \frac{\partial X}{\partial u}u + \frac{\partial X}{\partial w}\alpha + \frac{\partial X}{\partial \beta}\beta + \frac{\partial X}{\partial p}p + \frac{\partial X}{\partial q}q + \frac{\partial X}{\partial \dot{w}}\dot{\alpha} + \frac{\partial X}{\partial \dot{\beta}}\dot{\beta} + \frac{\partial X}{\partial i_H}i_H + \frac{\partial X}{\partial \delta_f}\delta_f + \cdots$$

$$\frac{1}{2}\frac{\partial^2 X}{\partial u^2}u^2 + \frac{1}{2}\frac{\partial^2 X}{\partial \alpha^2}\alpha^2 + \frac{1}{2}\frac{\partial^2 X}{\partial \beta^2}\beta^2 + \frac{1}{2}\frac{\partial^2 X}{\partial p^2}p^2 + \frac{1}{2}\frac{\partial^2 X}{\partial q^2}q^2 + \cdots + \cdots$$

$$\frac{\partial^2 X}{\partial u \partial \alpha}u\alpha + \frac{\partial^2 X}{\partial u \partial \beta}u\beta + \frac{\partial^2 X}{\partial u \partial p}up + \cdots + \frac{\partial^2 X}{\partial \alpha \partial \beta}\alpha\beta + \frac{\partial^2 X}{\partial \alpha \partial p}\alpha p + \cdots + \cdots$$

$$\frac{1}{6}\frac{\partial^2 X}{\partial u^3}u^3 + \cdots + \cdots$$

$$\cdots$$

HOT (High Order Taylor-series)

Source: Handout Dinamika Terbang

[Review Koefisien Aerodinamika]

Persamaan gaya akan menjadi:

$$X = X_o + X_u u + X_\alpha \alpha + X_\beta \beta + X_p p + X_q q + X_{\dot{\alpha}} \dot{\alpha} + X_{\dot{\beta}} \dot{\beta} + X_{iH} i_H + X_{\delta f} \delta_f + \dots$$

Lalu dengan melakukan non-dimensionalisasi, dengan mengeluarkan $\boldsymbol{q\ S}$, maka akan didapat:

$$X = \frac{1}{2}\rho V^2 S \Big(C_{X_o} + C_{X_u} \hat{u} + C_{X_\alpha} \alpha + C_{X_\beta} \beta + C_{X_p} \hat{p} + C_{X_q} \hat{q} + \\ + C_{X_{\dot{\alpha}}} \hat{\dot{\alpha}} + C_{X_{\dot{\beta}}} \hat{\dot{\beta}} + C_{X_{i_H}} i_H + C_{X_{\delta_f}} \delta_f + \dots \Big)$$

Dan untuk kasus momen, dengan mengeluarkan $\boldsymbol{q\ S\ \bar{c}}$, didapat:

$$M = \frac{1}{2}\rho V^2 S\bar{c} \Big(C_{m_o} + C_{m_u} \hat{u} + C_{m_\alpha} \alpha + C_{m_\beta} \beta + C_{m_p} \hat{p} + C_{m_q} \hat{q} + \\ + C_{m_{\dot{\alpha}}} \hat{\dot{\alpha}} + C_{m_{\dot{\beta}}} \hat{\dot{\beta}} + C_{m_{i_H}} i_H + C_{m_{\delta_f}} \delta_f + \dots \Big)$$

Source: Handout Dinamika Terbang

[Review Terbang Lurus Stationer]

Pada modul ini, akan digunakan persamaan gerak (EoM) yang sifat nya **linear** dan **dalam gangguan kecil.** Sering disebut juga ***principal trim condition***.

Persamaan Longitudinal

$$
\begin{bmatrix} d\dot{\hat{u}} \\ d\dot{\alpha} \\ d\dot{\theta} \\ d\dot{\hat{q}} \end{bmatrix} =
\begin{bmatrix}
-\frac{V}{\bar{c}}\frac{\tilde{C}_{D_u}}{2\mu_c} & -\frac{V}{\bar{c}}\frac{C_{D_\alpha}}{2\mu_c} & -\frac{V}{\bar{c}}\frac{C_{L_1}\cos\gamma_s}{2\mu_c} & -\frac{V}{\bar{c}}\frac{C_{D_q}}{2\mu_c} \\
-\frac{V}{\bar{c}}\frac{C_{L_u}}{2\mu_c+C_{L_{\dot\alpha}}} & -\frac{V}{\bar{c}}\frac{C_{L_\alpha}}{2\mu_c+C_{L_{\dot\alpha}}} & -\frac{V}{\bar{c}}\frac{C_{L_1}\sin\gamma_s}{2\mu_c+C_{L_{\dot\alpha}}} & \frac{V}{\bar{c}}\frac{2\mu_c-C_{L_q}}{2\mu_c+C_{L_{\dot\alpha}}} \\
0 & 0 & 0 & \frac{V}{\bar{c}} \\
\frac{V}{\bar{c}}\frac{\tilde{C}_{m_u}-\frac{C_{L_u}}{2\mu_c+C_{L_{\dot\alpha}}}C_{m_{\dot\alpha}}}{2\mu_c K_Y^2} & \frac{V}{\bar{c}}\frac{C_{m_\alpha}-\frac{C_{L_\alpha}}{2\mu_c+C_{L_{\dot\alpha}}}C_{m_{\dot\alpha}}}{2\mu_c K_Y^2} & -\frac{V}{\bar{c}}\frac{\frac{C_{L_1}\sin\gamma_s}{2\mu_c+C_{L_{\dot\alpha}}}C_{m_{\dot\alpha}}}{2\mu_c K_Y^2} & \frac{V}{\bar{c}}\frac{C_{m_q}+\frac{2\mu_c-C_{L_q}}{2\mu_c+C_{L_{\dot\alpha}}}C_{m_{\dot\alpha}}}{2\mu_c K_Y^2}
\end{bmatrix}
\begin{bmatrix} d\hat{u} \\ d\alpha \\ d\theta \\ d\hat{q} \end{bmatrix} +
\begin{bmatrix}
-\frac{V}{\bar{c}}\frac{C_{D_{\delta e}}}{2\mu_c} & \frac{V}{\bar{c}}\frac{C_{T_{\delta t}}}{2\mu_c} \\
-\frac{V}{\bar{c}}\frac{C_{L_{\delta e}}}{2\mu_c+C_{L_{\dot\alpha}}} & 0 \\
0 & 0 \\
\frac{V}{c}\frac{C_{m_{\delta e}}-\frac{C_{L_{\delta e}}}{2\mu_c+C_{L_{\dot\alpha}}}C_{m_{\dot\alpha}}}{2\mu_c K_Y^2} & \frac{V}{c}\frac{C_{T_{\delta t}}}{2\mu_c K_Y^2}\frac{d_T}{c}
\end{bmatrix}
\begin{bmatrix} d\delta_e \\ d\delta_T \end{bmatrix}
$$

$$
\begin{bmatrix} \dot{\hat{u}} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{\hat{q}} \end{bmatrix} =
\begin{bmatrix}
x_u & x_\alpha & x_\theta & x_q \\
z_u & z_\alpha & z_\theta & z_q \\
0 & 0 & 0 & \frac{V}{\bar{c}} \\
m_u & m_\alpha & m_\theta & m_q
\end{bmatrix}
\begin{bmatrix} \hat{u} \\ \alpha \\ \theta \\ \hat{q} \end{bmatrix} +
\begin{bmatrix}
x_{\delta_e} & x_{\delta_t} \\
z_{\delta_e} & 0 \\
0 & 0 \\
m_{\delta_e} & m_{\delta_t}
\end{bmatrix}
\begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix}
$$

[Review Terbang Lurus Stationer]

Pada modul ini, akan digunakan persamaan gerak (EoM) yang sifat nya **linear** dan **dalam gangguan kecil.** Sering disebut juga ***principal trim condition***.

Persamaan Lateral-Directional

$$
\begin{bmatrix} \dot{\beta} \\ \dot{\varphi} \\ \dot{\hat{p}} \\ \dot{\hat{r}} \end{bmatrix} =
\begin{bmatrix}
\frac{V}{b}\frac{C_{Y_\beta}}{2\mu_b} & \frac{V}{b}\frac{C_{L_1}}{2\mu_b} & \frac{V}{b}\frac{C_{Y_p}}{2\mu_b} & \frac{V}{b}\frac{\left(-4\mu_b + C_{Y_r}\right)}{2\mu_b} \\
0 & 0 & \frac{2V}{b} & 0 \\
\frac{V}{b}\frac{C_{\ell_\beta}K_Z^2 + C_{n_\beta}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} & 0 & \frac{V}{b}\frac{C_{\ell_p}K_Z^2 + C_{n_p}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} & \frac{V}{b}\frac{C_{\ell_r}K_Z^2 + C_{n_r}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} \\
\frac{V}{b}\frac{C_{n_\beta}K_X^2 + C_{\ell_\beta}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} & 0 & \frac{V}{b}\frac{C_{n_p}K_X^2 + C_{\ell_p}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} & \frac{V}{b}\frac{C_{n_r}K_X^2 + C_{\ell_r}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)}
\end{bmatrix}
\begin{bmatrix} \beta \\ \varphi \\ \hat{p} \\ \hat{r} \end{bmatrix} +
\begin{bmatrix}
\frac{V}{b}\frac{C_{Y_{\delta_a}}}{2\mu_b} & \frac{V}{b}\frac{C_{Y_{\delta_r}}}{2\mu_b} \\
0 & 0 \\
\frac{V}{b}\frac{C_{\ell_{\delta_a}}K_Z^2 + C_{n_{\delta_a}}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} & \frac{V}{b}\frac{C_{\ell_{\delta_r}}K_Z^2 + C_{n_{\delta_r}}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} \\
\frac{V}{b}\frac{C_{n_{\delta_a}}K_X^2 + C_{\ell_{\delta_a}}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)} & \frac{V}{b}\frac{C_{n_{\delta_r}}K_X^2 + C_{\ell_{\delta_r}}K_{XZ}}{4\mu_b\left(K_X^2K_Z^2 - K_{XZ}^2\right)}
\end{bmatrix}
\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}
$$

$$
\begin{bmatrix} \dot{\beta} \\ \dot{\varphi} \\ \dot{\hat{p}} \\ \dot{\hat{r}} \end{bmatrix} =
\begin{bmatrix}
y_\beta & y_\varphi & y_p & y_r \\
0 & 0 & \frac{2V}{b} & 0 \\
\ell_\beta & 0 & \ell_p & \ell_r \\
n_\beta & 0 & n_p & n_r
\end{bmatrix}
\begin{bmatrix} \beta \\ \varphi \\ \hat{p} \\ \hat{r} \end{bmatrix} +
\begin{bmatrix}
y_{\delta_a} & y_{\delta_r} \\
0 & 0 \\
\ell_{\delta_a} & \ell_{\delta_r} \\
n_{\delta_a} & n_{\delta_r}
\end{bmatrix}
\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}
$$

[Pengolahan Data Koefisien Aerodinamika]

1. Koefisien Aerodinamika Turunan terhadap $\boldsymbol{u}$

$$C_{D_u} = 2C_{D_s}, \qquad C_{L_u} = 2C_{L_s}, \qquad C_{m_u} = 2C_{m_s},$$

Koefisien turunan terhadap $\boldsymbol{u}$ dapat didekati dengan persamaan di atas untuk **aliran udara** masih dianggap **non-compressible**

$$C_{D_s} = C_{D_o} + C_{D_\alpha}\alpha_s + C_{D_{\delta e}}\delta_{e-s} = \underline{C_T - C_{L_1}\sin\gamma}$$

$$C_{L_s} = C_{L_o} + C_{L_\alpha}\alpha_s + C_{L_{\delta e}}\delta_{e-s} = \underline{C_{L_1}\cos\gamma}$$

$$C_{m_s} = C_{m_o} + C_{m_\alpha}\alpha_s + C_{m_{\delta e}}\delta_{e-s} = \underline{-C_T\frac{d_t}{\bar{c}}}$$

Bisa dicari nilai $\boldsymbol{\alpha_s}$, $\boldsymbol{\delta_{e_s}}$, dan $\boldsymbol{\gamma_s}$ untuk **trim point powered** yang dipilih (**Cek modul 3!**).

[Pengolahan Data Koefisien Aerodinamika]

2. Koefisien Propulsi Turunan terhadap $\boldsymbol{\delta_T}$

untuk **Pesawat Propeller**

$$C_{T_{\delta t}} = {C_T}/{0.75}, \qquad C_{m_{\delta t}} = C_{T_{\delta t}} \frac{d_t}{\bar{c}}$$

untuk **Pesawat Jet (Gas Turbine)**

$$C_{T_{\delta t}} = \frac{C_T}{(0.75 - 0.5)}, \qquad C_{m_{\delta t}} = C_{T_{\delta t}} \frac{d_t}{\bar{c}}$$

Gunakan nilai C_T sesuai dengan data yang dimiliki dan dihitung di modul sebelumnya!

$$C_T = \frac{T}{{}^1\!/_2\, \rho V^2 S} \qquad \tilde{C}_{m_o} = C_{m_o} + \underbrace{C_T \frac{d_T}{\bar{c}}}_{C_{m_T}}$$

3. Koefisien Aerodinamika Turunan pada DATCOM - Statik

- CLA - Derivative of lift coefficient with respect to alpha. If CLA is output versus angle of attack, these values correspond to numerical derivatives of the lift curve. When a single value of CLA is output at the first angle of attack, this output is the linear-lift-region derivative. CLA is based on the reference area. C_{L_α}
- CMA - Derivative of the pitching-moment coefficient with respect to alpha. If CMA is output versus angle of attack, the values correspond to numerical derivatives of the pitching-moment curve. When a single value of CMA is output at the first angle of attack, this output is the linear-lift-region derivative. CMA is based on the reference area and longitudinal reference length. C_{m_α}
- CYB - Derivative of side-force coefficient with respect to sideslip angle. When CYB in defined independent of the angle of attack, output is printed at the first angle of attack. CYB is based on the reference area. C_{y_β}
- CNB - Derivative of yawing-moment coefficient with respect to sideslip angle. When CNB is defined independent of angle of attack, output is printed at the first angle of attack. CNB is based on the reference area and lateral reference length. C_{n_β}
- CLB - Derivative of rolling-moment coefficient with respect to sideslip angle presented as a function of angle of attack. CLB is based on the reference area and lateral reference length. C_{l_β}

[Pengolahan Data Koefisien Aerodinamika]

4. Koefisien Aerodinamika Turunan pada DATCOM - Dinamik

- CLQ - Vehicle pitching derivative based on the product of reference area and longitudinal reference length. C_{L_q}
- CMQ - Vehicle pitching derivative based on the product of reference area and the square of the longitudinal reference length. C_{m_q}
- CLAD - Vehicle acceleration derivative based on the product of reference area and longitudinal reference length. $C_{L_{\alpha-dot}}$
- CMAD - Vehicle acceleration derivative based on the product of reference area and the square of the longitudinal reference length. $C_{m_{\alpha-dot}}$
- CLP - Vehicle rolling derivative based on the product of reference area and the square of the lateral reference length. C_{l_p}
- CYP - Vehicle rolling derivative based an the product of reference area and lateral reference length. C_{y_p}
- CNP - Vehicle rolling derivative based on the product of reference area and the square of the lateral reference length. C_{n_p}
- CNR - Vehicle yawing derivative based on the product of reference area and the square of the lateral reference length. C_{n_r}
- CLR - Vehicle rolling derivative based on the product of reference area and the square of the lateral reference length. C_{l_r}

[Pengolahan Data Koefisien Aerodinamika]

DERIVATIVE (PER DEGREE)

CLA	CMA	CYB	CNB	CLB
8.305E-02	-5.905E-02	-1.774E-02	2.654E-03	-2.414E-03
1.019E-01	-5.560E-02			-2.527E-03
1.047E-01	-5.581E-02			-2.579E-03
1.044E-01	-5.397E-02			-2.628E-03
1.035E-01	-5.218E-02			-2.674E-03
1.034E-01	-5.068E-02			-2.714E-03
1.039E-01	-4.986E-02			-2.754E-03
1.040E-01	-5.064E-02			-2.794E-03
9.854E-02	-4.999E-02			-2.866E-03
9.406E-02	-4.303E-02			-2.877E-03
9.153E-02	-4.582E-02			-2.885E-03

DYNAMIC DERIVATIVES (PER DEGREE)

PITCHING		ACCELERATION		ROLLING			YAWING	
CLQ	CMQ	CLAD	CMAD	CLP	CYP	CNP	CNR	CLR
2.299E-01	-6.871E-01	4.829E-02	-1.837E-01	-7.168E-03	-5.613E-03	2.764E-03	-5.238E-03	-2.506E-03
		5.584E-02	-2.124E-01	-8.471E-03	-3.446E-03	1.412E-03	-5.665E-03	-3.555E-04
		5.848E-02	-2.225E-01	-8.620E-03	-2.857E-03	1.026E-03	-5.756E-03	2.525E-04
		5.978E-02	-2.274E-01	-8.538E-03	-2.264E-03	6.444E-04	-5.842E-03	8.543E-04
		6.080E-02	-2.313E-01	-8.425E-03	-1.668E-03	2.572E-04	-5.923E-03	1.444E-03
		6.218E-02	-2.366E-01	-8.397E-03	-1.069E-03	-1.344E-04	-5.997E-03	2.021E-03
		6.394E-02	-2.432E-01	-8.450E-03	-4.621E-04	-5.340E-04	-6.066E-03	2.597E-03
		6.392E-02	-2.432E-01	-8.439E-03	1.507E-04	-9.418E-04	-6.128E-03	3.171E-03
		5.905E-02	-2.246E-01	-8.007E-03	1.386E-03	-1.783E-03	-6.231E-03	4.281E-03

*Untuk nilai C_{y_p}, C_{n_p}, dan C_{l_r}, dapat dipilih untuk $\alpha = 0$!

5. Koreksi Koefisien Aerodinamika Turunan pada DATCOM

Pada DATCOM dan kuliah ini, terdapat perbedaan dimensionalisasi untuk parameter $\boldsymbol{q}$ dan $\dot{\boldsymbol{\alpha}}$ (kecepatan sudut serang). Nilai nya sebagai berikut:

$$\hat{q}\Big|_{\text{DATCOM}} = q\,\frac{\bar{c}}{2V}, \quad \text{sedangkan pada kuliah: } \hat{q}\Big|_{AE3220} = q\,\frac{\bar{c}}{V}$$

$$\hat{\dot{\alpha}}\Big|_{\text{DATCOM}} = \hat{\dot{\alpha}}\,\frac{\bar{c}}{2V}, \quad \text{sedangkan pada kuliah: } \hat{\dot{\alpha}}\Big|_{AE3220} = \hat{\dot{\alpha}}\,\frac{\bar{c}}{V}$$

Maka dari itu, **koefisien turunan aerodinamika** terkait dengan $\boldsymbol{q}$ **dan** $\dot{\boldsymbol{\alpha}}$ perlu **dibagi dengan 2** supaya sesuai dengan asumsi yang digunakan pada kuliah ini!