

# Numerical Methods for Finance and Insurance Workshop (NMFIW) 2025

Sala Lauree, Via del Conservatorio 7  
Università degli Studi di Milano



UNIVERSITÀ DEGLI STUDI DI MILANO  
DIPARTIMENTO DI ECONOMIA,  
MANAGEMENT E METODI QUANTITATIVI



Workshop Schedule and Book of abstracts

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# Workshop Schedule

February, 27 2025

09:30-10:30    Session 1

- Anna **Battaaz** “Optimal Liquidation Policies of Redeemable Shares”
- Emanuele **Guidotti** “Efficient estimation of bid-ask spreads from open, high, low, and close prices”

Chair: Lorenzo Mercuri

10:30-11:00    Coffee break

11:00-12:30    Session 2

- Carole **Bernard** “Coskewness under dependence uncertainty”
- Roger J. A. **Laeven** “Saddlepoint Approximations for Hawkes Jump-Diffusion Processes with an Application to Risk Management”
- Paul **Schneider** “Joint Estimation of Conditional Mean and Covariance for Unbalanced Panels”

Chair: Edit Rroji

12:30-14:00    Lunch break

14:00-15:30    Session 3

- Damir **Filipović** “Kernel Density Machines”
- Gianluca **Fusai** “Monotonic transformation, implied stock price process and market consistent pricing of calendar spread options”
- Donatien **Hainaut** “Option pricing with model constrained Gaussian process regressions”

Chair: Lorenzo Mercuri

15:30-16:00    Coffee break

16:00-17:00    Session 4

- Viviana **Fanelli** “Investments in Renewable Energy Sources: Structural Changes in the Production and Consumption of Energy”
- Michele **Azzone** “The puzzle of carbon allowance spread”

Chair: Andrea Perchiazzo

## February, 28 2025

09:30-10:30    Session 5

- An **Chen** “The Role of Ambiguity Aversion in Consumer Purchasing Decision”
- Francesco **Della Corte** “Market-consistent valuation and Capital Assessment for Demographic Risk in Life Insurance: A Cohort Approach”

Chair: Edit Rroji

10:30-11:00    Coffee break

11:30-12:30    Session 6

- Giacomo **Bormetti** “Deep calibration with random smiles”
- Lech **Grzelack** “On Volatility Parametrizations with Random Coefficients”
- Andrea **Perchiazzo** “Pricing European Options using the Gauss-Laguerre Quadrature”

Chair: Andrea Perchiazzo

12:30-14:30    Lunch break

# Abstracts

## **The puzzle of carbon allowance spread**

**Michele Azzone** and Roberto Baviera

A growing number of contributions in the literature have identified a puzzle in the European carbon allowance (EUA) market. Specifically, a persistent cost-of-carry spread (C-spread) over the risk-free rate has been observed. We are the first to explain the anomalous C-spread with the credit spread of the corporates involved in the emission trading scheme. We obtain statistical evidence that the C-spread is cointegrated with both this credit spread and the risk-free interest rate. This finding has a relevant policy implication: the most effective solution to solve the market anomaly is including the EUA in the list of European Central Bank eligible collateral for refinancing operations. This change in the ECB monetary policy operations would greatly benefit the carbon market and the EU green transition.

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## **Optimal Liquidation Policies of Redeemable Shares**

**Anna Battauz** and Francesco Rotondi

In this paper we explore the optimal issuance and liquidation of redeemable shares. Redeemable shares are those that the issuer can repurchase, or redeem, at a predetermined price, known as the call price, as soon as a given barrier event is triggered. We first determine the optimal call price for the issuer by stating and solving a stylized earning per share maximization problem from the point of view of a company. Once the call price is determined, we focus on the valuation of both perpetual and finite-maturity redeemable shares and we examine the problem of their optimal liquidation from the point of view of a shareholder holding them. Along with the few closed-form results that can be obtained in a lognormal continuous-time framework, we propose an intuitive and flexible method to retrieve the optimal liquidation policy in the form of a liquidation boundary, thanks to a parsimonious Markovianization of the evaluation problem in a binomial framework. Numerical tests using alternative market models and different dividend formulations confirm the robustness of our results.

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## Coskewness under dependence uncertainty

Carole Bernard, Jinghui Chen, Ludger Ruschendorf, and Steven Vanduffel

Recent literature has identified the coskewness as one of the most important factors of portfolio optimization. In this paper, we study the impact of dependence uncertainty first on the expectation of a product  $\mathbf{E}(X_1, X_2, \dots, X_d)$  when  $X_i$  has cdf  $F_i$  for all  $i$  is studied and then on the coskewness typically used in portfolio optimization. Under some conditions on the  $F_i$ , explicit sharp bounds are obtained, and a numerical method is provided to approximate them for arbitrary choices of the  $F_i$ . The results are applied to assess the impact of dependence uncertainty on coskewness. A novel notion of “standardized rank coskewness” is introduced, which is invariant under strictly increasing transformations and takes values in  $[1, 1]$ .

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## Deep calibration with random smiles

Fabio Baschetti, **Giacomo Bormetti**, and Pietro Rossi

We propose a neural network-based approach to calibrating stochastic volatility models, which combines the pioneering grid approach by Horvath et al. (2021) with the pointwise two-stage calibration of Bayer et al. (2018) and Liu et al. (2019). Our methodology inherits robustness from the former while not suffering from the need for interpolation/extrapolation techniques, a clear advantage ensured by the pointwise approach. The crucial point to the entire procedure is the generation of implied volatility surfaces on random grids, which one dispenses to the network in the training phase. We support the validity of our calibration technique with several empirical and Monte Carlo experiments for the rough Bergomi and Heston models under a simple but effective parametrization of the forward variance curve. The approach paves the way for valuable applications in financial engineering - for instance, pricing under local stochastic volatility models - and extensions to the fast-growing field of path-dependent volatility models.

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# The Role of Ambiguity Aversion in Consumer Purchasing Decision

An Chen and Shihao Zhu

This paper studies a representative consumer's contingent consumption plan in an ambiguous environment in which the probability of a state occurring is uncertain and the utility is state-dependent. We characterize the conditions that ambiguity aversion leads to an increase or decrease in the consumption good in the  $\alpha$ -maxmin and smooth ambiguity models. Our main results show that comparative statics of ambiguity aversion are about levels of utility while the optimal consumption plan is about marginals, so the effects of ambiguity aversion are non-obvious in some settings. We apply our findings to life insurance and life annuity decisions, where individuals face uncertain mortality risks, as well as to insurance for irreplaceable commodities, where individuals confront uncertain health risks.

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## Market-Consistent Valuation and Capital Assessment for Demographic Risk in Life Insurance: A Cohort Approach

Gian Paolo Clemente, **Francesco Della Corte**, Nino Savelli, and Diego Zappa

We explore the quantification of demographic risk in accordance with the market-consistent actuarial valuation principles. Our contribution includes closed formulas for assessing the inflows and outflows of an insurance policy portfolio using a cohort approach. To maintain versatility, we address both traditional and equity-linked policies, providing a market-consistent valuation of liabilities. Subsequently, we compute the capital requirement for idiosyncratic risk (linked to accidental mortality) and systematic risk (trend risk), presenting an approach that allows to consider the risks at cohort level. Specifically, we evaluate the minimum capital using an annual time horizon, a 99.5% confidence level, and the Value-at-Risk as the risk measure, developing a framework aligned with the European Solvency II regulation for insurance and reinsurance companies. Moreover, the model can be easily adapted to accommodate other regulatory frameworks, incorporating specific rules and accounting principles relevant to diverse jurisdictions worldwide. The numerical analysis and the sensitivities reveal that the accidental volatility of policyholders' deaths is influenced by the inherent characteristics of the cohort's policies, the age of policyholders and the variability of sums insured. Furthermore, trend risk is contingent on both accidental volatility and the longevity forecasting model employed.

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# Investments in Renewable Energy Sources: Structural Changes in the Production and Consumption of Energy.

Rosella Castellano, Roy Cerqueti, **Viviana Fanelli**, and Carme Frau

In this paper we propose a stochastic dynamic optimization model to derive optimal portfolios consisting of a number of power plant technologies given the uncertainty governing the dynamics of fuel prices. The model aims at deriving the optimal fuel mix over a long-term time horizon by including either fossil and renewable fuels. Indeed, based on the commonly accepted evidence regarding the occurrence of wide fluctuations for fossil fuel prices, the use of renewable energy may offer, along with environmental benefits, greater stabilization of electricity costs. We explore the potential for renewable energy to serve as a financial “hedge”, reducing exposure to fuel price risk. Renewable energy generation brings with it the price stability benefits of free-fuel generation from emerging technologies (i.e.: solar, wind, small hydro, and geothermal sources) and, consequently, costs tend to be stable or decreasing over time, compared to fluctuating costs for fossil fuels.

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## Kernel Density Machines

**Damir Filipović** and Paul Schneider

We introduce Kernel Density Machines (KDM), a nonparametric framework for estimating the Radon-Nikodym derivative between two probability measures from which samples can be drawn. This flexible and powerful approach enables a broad range of applications, including independence testing and the estimation of multivariate conditional distributions. Leveraging a low-rank kernel approximation with sharply controlled approximation error, KDM is both computationally efficient and scalable to very large datasets. We provide a comprehensive set of theoretical results, including asymptotic consistency and finite-sample guarantees, offering rigorous performance bounds and insights into the framework’s effectiveness across diverse applications.

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# Monotonic transformation, implied stock price process and market consistent pricing of calendar spread options.

Gianluca Fusai and Giovanni Longo

The paper builds on an idea by Erio Castagnoli and expanded in Fusai [1] to identify a diffusion process for the stock price that perfectly aligns with observed option prices at specific maturities. This is achieved by modeling stock returns as a monotonic continuous transformation  $g$  of a standard Brownian motion (SBM). With the function  $g$  defined, deriving the dynamics of log-returns becomes straightforward using Ito's lemma, an improvement over the approach by Dupire [2]. This paper further develops this framework by demonstrating its full implementation in pricing popular energy products, such as calendar spread options traded at Chicago Mercantile Exchange.

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## On Volatility Parametrizations with Random Coefficients

Lech Grzelak

It is a market practice to express market-implied volatilities in some parametric form. The most popular parametrizations are based on or inspired by an underlying stochastic model, like the Heston model (SVI method) or the SABR model (SABR-parametrization). Their popularity is often driven by a closed-form representation enabling efficient calibration. However, these representations indirectly impose a model-specific volatility structure on observable market quotes. When the market's volatility does not follow the parametric model regime, the calibration procedure will fail or lead to extreme parameters, indicating inconsistency. This article addresses this critical limitation - we propose an arbitrage-free framework for letting the parameters from the parametric implied volatility formula be random. The method enables a significant widening of the spectrum of permissible shapes of implied volatilities while preserving analyticity and, therefore, computation efficiency. We demonstrate the effectiveness of the novel method on real data from short-term index and equity options, where traditional parametrizations fail to capture market dynamics. Our results show that the proposed method accurately reflects market behavior, offering a robust extension of existing models.

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# **Efficient estimation of bid-ask spreads from open, high, low, and close prices**

David Ardia, **Emanuele Guidotti**, and Tim A. Kroencke

Popular bid-ask spread estimators are downward biased when trading is infrequent. Moreover, they consider only a subset of open, high, low, and close prices and neglect potentially useful information to improve the spread estimate. By accounting for discretely observed prices, this paper derives asymptotically unbiased estimators of the effective bid-ask spread. Moreover, we combine them optimally to minimize the estimation variance and obtain an efficient estimator. Through theoretical analyses, numerical simulations, and empirical evaluations, we show that our efficient estimator dominates other estimators from transaction prices, yields novel insights for measuring bid-ask spreads, and has broad applicability in empirical finance.

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## **Option pricing with model constrained Gaussian process regressions**

**Donatien Hainaut**

In this talk, we propose a method for pricing European options based on Gaussian processes. We convert the problem of solving the Feynman-Kac (FK) partial differential equation (PDE) into a model-constrained regression. We form two training sets by sampling state variables from the PDEs inner domain and terminal boundary. The regression function is then estimated to fit the option payoffs on the boundary sample while satisfying the FK PDE on the inner sample. We adopt a Bayesian framework in which payoffs and the value of the FK PDE in the boundary and inner samples are noised. Assuming the regression function is a Gaussian process, we find a closed-form approximation for the option prices. We demonstrate the performance of the procedure on call options in the Heston model and basket call options in a Black-Scholes market. Next, we discuss the extension of this method to American options. The variational equation driving these options is converted into a penalized FK PDE that is solved by iterations to manage the non-linearity of the differential operator. The method is illustrated in the Heston model.

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# Saddlepoint Approximations for Hawkes Jump-Diffusion Processes with an Application to Risk Management

Yacine Aït-Sahalia and Roger J.A. Laeven

We propose a statistical model based on Hawkes processes in which large financial losses can arise in close succession serially as well as cross-sectionally. We derive in closed form saddlepoint approximations to the tails of profit and loss distributions, both marginal and joint, and use them to construct explicit risk measure formulae that account for the fact that a given financial institution's losses make it more likely that that institution will experience further losses, and that other financial institutions will experience losses as well. These closed-form risk measures can be used for comparative statics, parameter calibration, and setting capital requirements and potential systemic risk charges.

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## Pricing European Options using the Gauss-Laguerre Quadrature

Andrea Perchiazzo

In this work we propose a novel method for pricing European options numerically in an efficient manner using the Gauss-Laguerre quadrature. Instead of employing the Carr-Madan formula [1], which needs an appropriate choice for the damping factor, or the COS method of Fang and Oosterlee [2] in which three approximation errors are introduced (e.g., truncation of the integration range in the risk-neutral valuation formula) as discussed in [3, Chapter 6.2.3], the pricing of European options using the characteristic function is based on the Gauss-Laguerre quadrature. The approach does not necessitate the truncation of the integration range in the risk-neutral valuation formula and the approximation error term is controlled by the order of Laguerre polynomials. The new methodology is initially tested on the Black-Scholes model in order to confirm its efficacy and then applied to several models (e.g., the Merton, Kou, Variance Gamma, and Heston models). In addition, a comparison between the novel approach based on the Gauss-Laguerre quadrature, the COS method, and the Carr-Madan formula is performed.

- [1] Peter Carr and Dilip Madan. "Option valuation using the fast Fourier transform". In: *Journal of computational finance* 2.4 (1999), pp. 61–73.
  - [2] Fang Fang and Cornelis W Oosterlee. "A Novel Pricing Method for European Options Based on Fourier-Cosine Series Expansions". In: *SIAM Journal on Scientific Computing* 31.2 (2009), pp. 826–848. DOI: 10.1137/080718061.
  - [3] Cornelis W Oosterlee and Lech A Grzelak. *Mathematical modeling and computation in finance: with exercises and Python and MATLAB computer codes*. World Scientific, 2019.
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# Joint Estimation of Conditional Mean and Covariance for Unbalanced Panels

Damir Filipović and Paul Schneider

We propose a novel nonparametric kernel-based estimator of cross-sectional conditional mean and covariance matrices for large unbalanced panels. We show its consistency and provide finite-sample guarantees. In an empirical application, we estimate conditional mean and covariance matrices for a large unbalanced panel of monthly stock excess returns given macroeconomic and firm-specific covariates from 1962 to 2021. The estimator performs well with respect to statistical measures. It is informative for empirical asset pricing, generating conditional mean-variance efficient portfolios with substantial out-of-sample Sharpe ratios far beyond equal-weighted benchmarks.

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