Interhemispheric exchange is a simple metric that we can use to measure the mixing of the atmosphere for various constituents. Visualize the globe as two boxes each representing a hemisphere as seen in 1. In Figure 1, we see that the mean concentration noted as C, where 1 or 2 represents

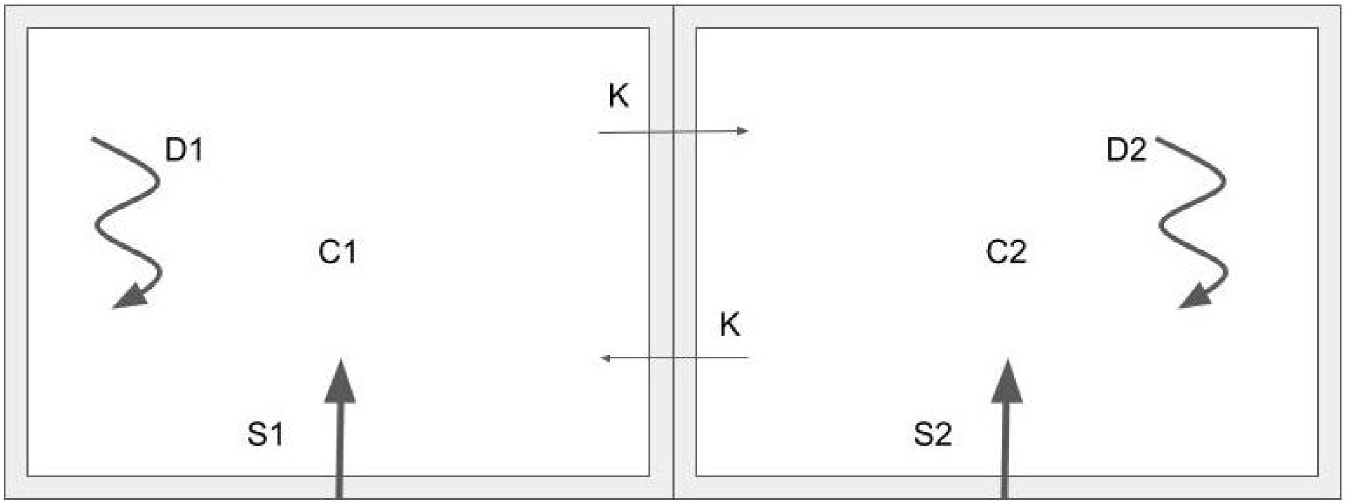

Figure 1: Cartoon of 2 hemisphere model

the hemisphere of choice. These mean concentration are changed by a surface emission, S, a deposition or atmospheric decay term, D, and an exchange with the other hemisphere, K. There is no advective term since these are means and this mean is integrated around the entirety of the hemispheric surface.

Given the explanation above, we can write a mass balance equation for each hemisphere as the following:

$$\frac{dC_1}{dt} = S_1 - K - D1$$
$$\frac{dC_2}{dt} = S_2 + K - D2$$

The source terms for a particular constituent can be thought of as the following:

$$S_1 = \frac{E_1}{\alpha}$$
$$S_2 = \frac{E_2}{\alpha}$$

Where E represents the total hemispheric emission rate and α is the conversion factor from emission to mixing ratio. The deposition/decay term can be thought of as the following:

$$D_1 = \frac{C_1}{\tau_a}$$
$$D_2 = \frac{C_2}{\tau_a}$$

Where τ_a is the atmospheric lifetime of a constituent. The interhemispheric exchange can be thought of as rate of change based on the mixing ratio gradient between the two hemispheres:

$$K = \frac{\Delta C_{12}}{\tau_{ex}}$$

Where τ_{ex} is the interhemispheric exchange rate and ΔC_{12} is the hemispheric mixing ratio gradient. With these assumptions, we can formally write the mass balance equation:

$$\frac{dC_1}{dt} = \frac{E_1}{\alpha} - \frac{\Delta C_{12}}{\tau_{ex}} - \frac{C_1}{\tau_a} \tag{1}$$

$$\frac{dC_2}{dt} = \frac{E_2}{\alpha} + \frac{\Delta C_{12}}{\tau_{ex}} - \frac{C_2}{\tau_a} \tag{2}$$

1 Determining τ_{ex}

The solution for τ_{ex} is straightforward for the instaneous interhemispheric exchange. First, we can isolate α, the emission to mixing ratio conversion term, since it is the same between both hemispheres. For the first hemisphere, we can isolate this in the following manner.

$$\begin{aligned}
\frac{dC_1}{dt} &= \frac{E_1}{\alpha} - \frac{\Delta C_{12}}{\tau_{ex}} - \frac{C_1}{\tau_a} \\
\frac{E_1}{\alpha} &= \frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_1}{\tau_a} + \frac{dC_1}{dt} \\
\alpha &= \frac{E_1}{\left[\frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_1}{\tau_a} + \frac{dC_1}{dt}\right]}
\end{aligned}$$

Similarly for the second hemisphere...

$$\alpha = \frac{E_2}{\left[-\frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_2}{\tau_a} + \frac{dC_2}{dt}\right]}$$

Set these equal to each other and solve for the interhemispheric exchange rate.

$$\begin{aligned}
\frac{E_1}{\left[\frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_1}{\tau_a} + \frac{dC_1}{dt}\right]} &= \frac{E_2}{\left[-\frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_2}{\tau_a} + \frac{dC_2}{dt}\right]} \\
\frac{E_1}{E_2}\left[-\frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_2}{\tau_a} + \frac{dC_2}{dt}\right] &= \left[\frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_1}{\tau_a} + \frac{dC_1}{dt}\right] \\
-\frac{E_1}{E_2}\frac{\Delta C_{12}}{\tau_{ex}} + \frac{E_1}{E_2}\left[\frac{C_2}{\tau_a} + \frac{dC_2}{dt}\right] &= \left[\frac{\Delta C_{12}}{\tau_{ex}} + \frac{C_1}{\tau_a} + \frac{dC_1}{dt}\right] \\
-\frac{E_1}{E_2}\frac{\Delta C_{12}}{\tau_{ex}} - \frac{\Delta C_{12}}{\tau_{ex}} &= \left[\frac{C_1}{\tau_a} + \frac{dC_1}{dt}\right] - \frac{E_1}{E_2}\left[\frac{C_2}{\tau_a} + \frac{dC_2}{dt}\right] \\
-\frac{\Delta C_{12}}{\tau_{ex}}\left[1 + \frac{E_1}{E_2}\right] &= \left[\frac{C_1}{\tau_a} + \frac{dC_1}{dt}\right] - \frac{E_1}{E_2}\left[\frac{C_2}{\tau_a} + \frac{dC_2}{dt}\right]
\end{aligned}$$

Thus, the instantaneous interhemispheric exchange rate is:

$$\tau_{ex} = \frac{\Delta C_{12}\left[1 + \frac{E_1}{E_2}\right]}{\frac{E_1}{E_2}\left[\frac{C_2}{\tau_a} + \frac{dC_2}{dt}\right] - \left[\frac{C_1}{\tau_a} + \frac{dC_1}{dt}\right]} \tag{3}$$

2 Determining τ_{ex}: Chemically Inert Molecules

For chemically inert situations, we can assume the lifetime of the constituent is very long:

$$\tau_a >> 0$$

This means 1 and 2 become:

$$\frac{dC_1}{dt} = \frac{E_1}{\alpha} - \frac{\Delta C_{12}}{\tau_{ex}}$$
$$\frac{dC_2}{dt} = \frac{E_2}{\alpha} + \frac{\Delta C_{12}}{\tau_{ex}}$$

Now, if we take the change in the gradient into account...

$$\frac{d(C_1 - C_2)}{dt} = \frac{E_1}{\alpha} - \frac{\Delta C_{12}}{\tau_{ex}} - \frac{E_2}{\alpha} - \frac{\Delta C_{12}}{\tau_{ex}}$$

Over long timescales or for constituents that do not vary greatly with time, we can assume a steady state solution. This requires:

$$\frac{dC_1}{dt} = \frac{dC_2}{dt} = 0$$

Now, solve for the interhemispheric exchange rate

$$0 = \frac{E_1}{\alpha} - \frac{\Delta C_{12}}{\tau_{ex}} - \frac{E_2}{\alpha} - \frac{\Delta C_{12}}{\tau_{ex}}$$
$$2\frac{\Delta C_{12}}{\tau_{ex}} = \frac{E_1}{\alpha} - \frac{E_2}{\alpha}$$

Thus:

$$\tau_{ex} = \frac{2\Delta C_{12}}{\frac{E_1}{\alpha} - \frac{E_2}{\alpha}} \tag{4}$$