CLIMATE BASICS 1: TOP OF ATMOSPHERE RADIATION

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1 Solar Radiation

All energy provided to the Earth originates from the yellow ball of nuclear fusion known as the Sun. In a nutshell, nuclear fusion is the process in which lighter elements are collided together to form heavier elements. This occurs at the Sun's core and the chemical reaction creates electromagnetic energy (i.e., energy) as a biproduct. We find that the Sun's "surface" or Photosphere is generally where the electromagnetic radiation is emitted from the Sun. This radiation is not a singular beam of energy, but rather a complex series of radiative waves vibrating at different frequencies and interacting with nearly everything that gets in the way of the radiation. The complexity of these radiative waves can be summed up by the ELECTROMAGNETIC SPECTRUM. All objects emit radiation over different sections of this classification. It is why when we step outside on a sunny day and see a bright yellow ball in the sky as well as feel the warmth. Both colored light and thermal radiation are part of this spectrum. At the photosphere, the Sun's temperature is nearly 6000 K and it emits largely in the visible, infrared (e.g., thermal), and ultraviolet portions of the electromagnetic spectrum. A singular radiative wavelength is thus, insufficient to fully categorize the total energy emitted by the Sun. What one must do to characterize the total energy released by the Sun, or luminosity (i.e., total intensity) is to integrate the emitted radiation over the all wavelengths of the electromagnetic spectrum. If this is done, we find on average the luminosity of the Sun is about $3.9 \times 10^{26} \frac{J}{s}$.

The luminosity is the amount of energy the Sun produces at any given time. The amount of energy passing leaving the Sun, on average, is known as the flux density. The flux density is simply the amount of energy passing through an area at a given time. Thus, the average flux density at the surface of the Sun is the following:

$$\begin{aligned} Flux\,Density_{Solar\,Surface} &= \frac{Luminosity}{Area\,of\,Surface} = \frac{L_o}{4\pi r_{sfc}^2} = \frac{3.9 \times 10^{26} \frac{J}{s}}{4\pi(6.96 \times 10^8 m)^2} \\ &= 6.4 \times 10^7 \frac{J}{m^2 s} \end{aligned}$$

2 From Sun to Earth

The previous section suggests a certain amount of flux (i.e., luminosity) originating at the surface of the Sun. Once emitted the radiation ventures into the cosmos in all directions (i.e., omnidirectionally). The first law of thermodynamics states that all energy must be conserved. That is true for the heat coming off your skin as well as sunshine. Since space is nearly a complete vacuum, there is almost neglible amounts of medium (e.g., air, liquids, solids) for Solar radiation to interact with before being intercepted by the Earth. This implies that the amount of energy passing through an area with the Sun as its center should be the same as the energy passing at the surface of the Sun. If we assume that radiation is uniform around the sphere (i.e., isotropic), the flux at a distance is as follow:

$$\frac{S}{S_d} = \frac{r_d^2}{r_{sfc}^2}$$
$$Sr_{sfc}^2 = S_d r_d^2$$
$$\frac{L_o}{4\pi r_{sfc}^2} r_{sfc}^2 = S_d r_d^2$$
$$L_o = S_d 4\pi r_d^2$$
$$S_d = \frac{L_o}{4\pi r_d^2}$$

where L_o is the solar luminosity, r_d is the distance from the sun to an object, and S_d is the average flux density average at that distance.

Placing this into context with the Earth, the average Sun-Earth distance is roughly $1.5 \times 10^{11} m$. This results in an average flux density of $1367 \frac{J}{m^2 s}$. This is known as the "Solar constant" and can also be rewritten as $1367 \frac{W}{m^2}$.

3 Distribution of Solar Radiation at the Top of the Atmosphere

The solar constant is the flux of solar radiation impinging at the top of Earth's atmosphere at any given time. The amount of this energy that is able to interact with the air and surface is depenedent on characteristics of the planet, such as shape, orbital location, declination, and rotation rate.

The first thing to consider is the shape of the planet and its effects. The solar constant is formulated for a surface that is is perpendicular to the solar beam. That means, it is only valid for when the surface is directly perpendicular to the Suns's rays, which is not true for the vast majority of the Earth since it is spheroid. The angle at which the solar beam makes with the axis normal to the surface of the Earth is known as the solar zenith angle. [INSERT A DRAWING]

Utilizing the conservation of energy again and the inverse square law, the instantaneous insolation per unit area beam have the following balance:

$$Q 4\pi d^2 = S_o 4\pi d_m^2 \cos\theta_s$$
$$Q = S_o \frac{d_m^2}{d^2} \cos\theta_s$$

As can be seen, the solar zenith angle and orbital distance determine the amount of the incoming average solar insolation that will be emparted on the surface.

4 Cosine of the Solar Zenith Angle

The cosine of the solar zenith angle can be found using either spherical trigonometry (i.e., yuck) or simple trigonometry. Both begin by understanding two different points on the planet. We begin by assuming a unit sphere, where all points sit along the sphere with a magnitude of 1. The two points on this sphere are the observer point, $\vec{X}$ and the subsolar point, $\vec{SS}$. The subsolar point is the location on the planet where the solar beam is normal to the planet. Please see the figure [INSERT FIGURE]. From here, we can decompose both of these points into their geometric cartesian coordinate vectors.

$$\vec{X} = \cos\phi\cos\lambda\hat{\imath} + \cos\phi\sin\lambda\hat{\jmath} + \sin\phi\hat{k}$$
$$\vec{SS} = \cos\phi_{ss}\cos\lambda_{ss}\hat{\imath} + \cos\phi_{ss}\sin\lambda_{ss}\hat{\jmath} + \sin\phi_{ss}\hat{k}$$

By definition the dot product of these two vectors will yield the angle between them. This angle is the cosine of the solar zenith angle.

$$\vec{X}\cdot\vec{SS} = |X||SS|\cos\theta_s$$
$$\vec{X}\cdot\vec{SS} = \cos\theta_s$$

Using subbing in and a sums and differences of angles identity, the cosine of the solar zenith angle becomes

$$\begin{aligned}\vec{X}\cdot\vec{SS} = \cos\theta_s &= \cos\phi\cos\lambda\cos\phi_{ss}\cos\lambda_{ss} + \cos\phi\sin\lambda\cos\phi_{ss}\sin\lambda_{ss} + \sin\phi\sin\phi_{ss}\\ &= \cos\phi\cos\phi_{ss}(\cos\lambda\cos\lambda_{ss} + \sin\lambda\sin\lambda_{ss}) + \sin\phi\sin\phi_{ss}\\ &= \cos\phi\cos\phi_{ss}\cos(\lambda-\lambda_{ss}) + \sin\phi\sin\phi_{ss}\end{aligned}$$

The difference in longitude between the observer point and the subsolar point is known as the hour angle, h. The longitude of the subsolar point is known as the declination angle, δ. Thus:

$$\cos\theta_s = \cos\phi\cos\delta\cos h + \sin\phi\sin\delta$$

5 Night and Day

To get at the solar insolation throughout the day we must think of the beam intercepting the Earth at different times. If the solar beam, is not directly incident on the surface in question, there is no solar insolation at that point. That means at night, the solar zenith angle is negative. Thus, at any point when this occurs Q = 0.

$$\cos\theta_s < 0\, then\, Q = 0$$

At the extremes of the day, we have sunrise and sunset. These are the absolute final points where the sun is incident on the surface. This occurs when the solar zenith angle is 90 degrees. Using this, we can solve for the hour angle at sunrise and sunset.

$$\cos h_o = -\tan\phi\tan\delta$$

At the poles, the latitude hits a limit. This is important because the hour angle is undetermined.

6 Daily Averaged Insolation

Daily average insolation is determined by taking the average of the instantaneous insolation throughout the day. This is done by integrating the instanteous solar insolation over the hour angles between sunrise and sunset. The mean value is determined over a 24 hour day or one revolution of the Earth (e.g., 2π radians)

$$\begin{aligned}
\overline{Q}^{day} &= \frac{1}{2\pi}\int_{-h_o}^{h_o} Q dh \\
&= \frac{1}{2\pi}\int_{-h_o}^{h_o} [S_o \frac{d_m^2}{d^2}\cos\theta_s] dh \\
&= \frac{S_o}{2\pi}\frac{d_m^2}{d^2}\int_{-h_o}^{h_o} \cos\theta_s dh \\
&= \frac{S_o}{2\pi}\frac{d_m^2}{d^2}\int_{-h_o}^{h_o} [\cos\phi\cos\delta\cos h + \sin\phi\sin\delta]\, dh
\end{aligned}$$

We can assume the ratio of earth sun distance and the solar constant do not vary greatly over a day.

$$\begin{aligned}
\overline{Q}^{day} &= \frac{S_o}{2\pi}\frac{d_m^2}{d^2}\int_{-h_o}^{h_o} [\cos\phi\cos\delta\cos h + \sin\phi\sin\delta]\, dh \\
&= \frac{S_o}{2\pi}\frac{d_m^2}{d^2}\left[(\cos\phi\cos\delta\sin h|_{-h_o}^{h_o}) + \sin\phi\sin\delta h|_{-h_o}^{h_o})\right] \\
&= \frac{S_o}{2\pi}\frac{d_m^2}{d^2}[(\cos\phi\cos\delta(\sin h_o - \sin[-h_o])) + \sin\phi\sin\delta 2h_o)] \\
&= \frac{S_o}{2\pi}\frac{d_m^2}{d^2}[(\cos\phi\cos\delta 2\sin h_o) + \sin\phi\sin\delta 2h_o)] \\
\overline{Q}^{day} &= \frac{S_o}{\pi}\frac{d_m^2}{d^2}[(\cos\phi\cos\delta\sin h_o) + \sin\phi\sin\delta h_o)]
\end{aligned}$$

Using this, we can solve for the daily average insolation seen in figure 1:

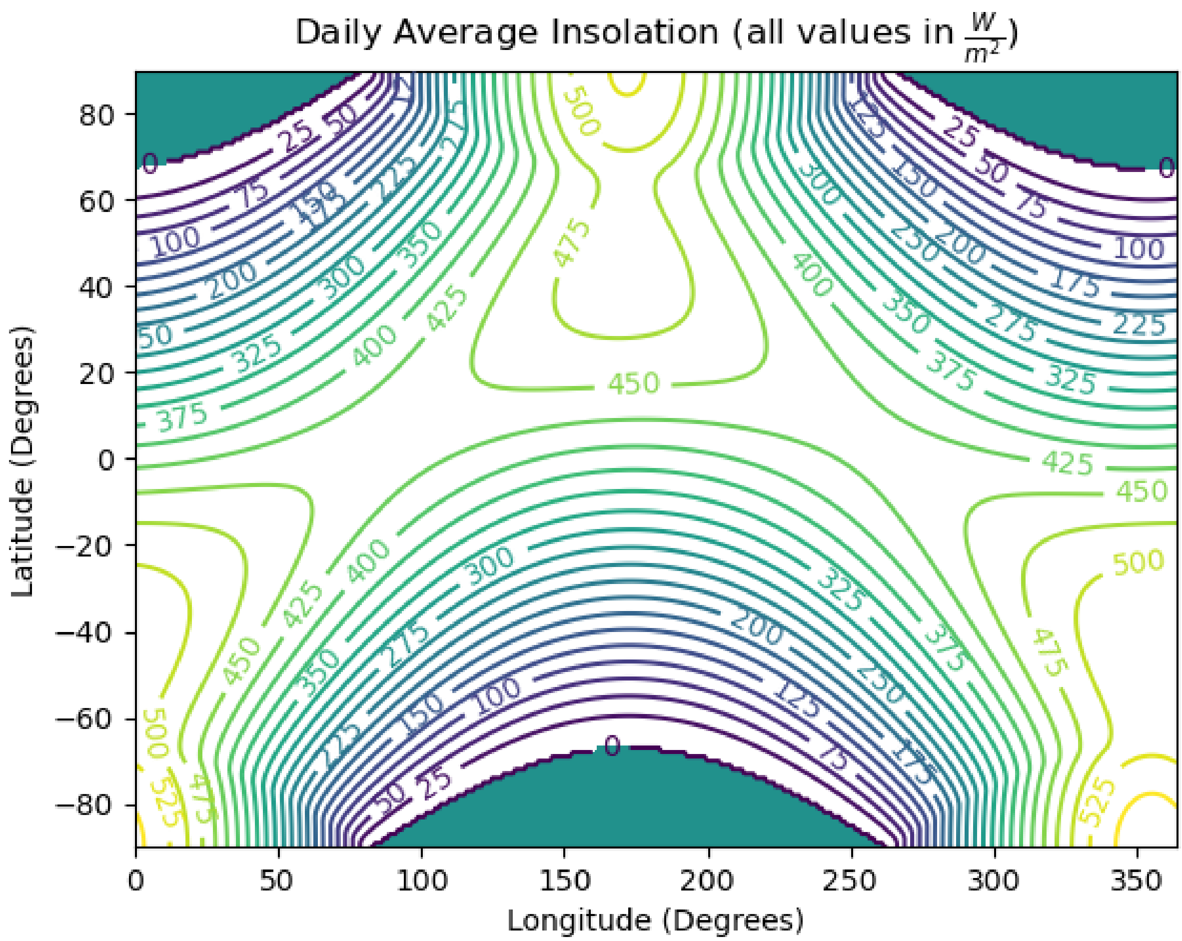

Figure 1: Daily Average Insolation