Rodent - Relevant ODE identifier

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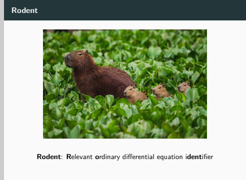
Rodent

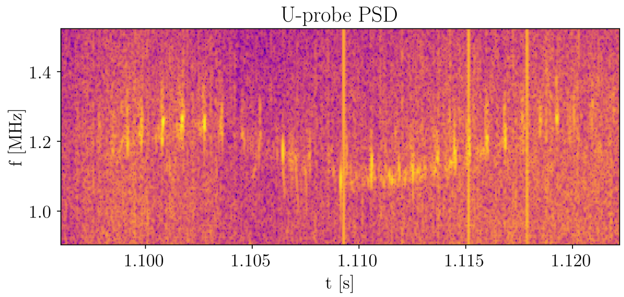
Rodent: **R**elevant **o**rdinary differential equation **ident**ifier



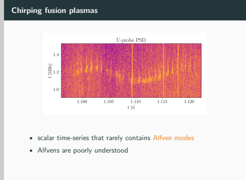
Today we will talk about rodents, which are a very investigative species, just like the framework we propose for model identification.

Also, they are very agile on land and feel equally at home in the water, so they are in many ways very similar to ODEs and their generality.

Chirping fusion plasmas

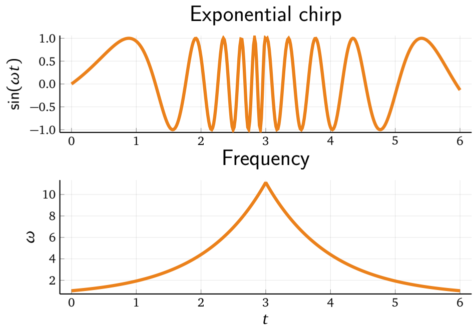


- scalar time-series that rarely contains Alfven modes
- Alfvens are poorly understood

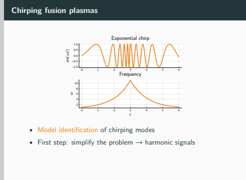


- Alfven modes in Tokamaks
- poorly understood, anomalous frequencies, in the plasma
- typical problem of physicists: loads of data, few labels
- Physicists are interested in finding more alfvens in their data, but even better would be an interpretable/explainable model

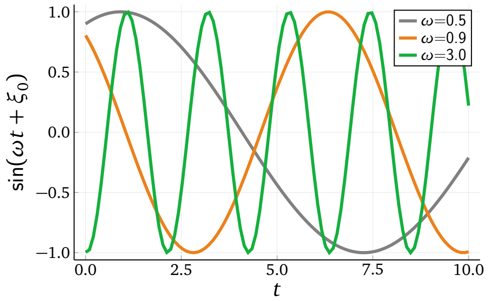
Chirping fusion plasmas



- Model identification of chirping modes
- First step: simplify the problem $\longrightarrow$ harmonic signals



Chirping fusion plasmas



The simplified problem

- Learn generating model of harmonic signals
- varying frequency ω and phase ξ_0

Chirping fusion plasmas
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Outline

1. Explainability via ODEs
2. Sparsity of the ODE via *Automatic Relevance Determination* (ARD)
3. Generative Models for manifold learning

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1. Explainability via ODEs
2. Sparsity of the ODE via *Automatic Relevance Determination* (ARD)
3. Generative Models for manifold learning

- ODE because physicists like them
- sparsity for simplicity (occam's razor)

Ordinary differential equations

$$\frac{\partial \boldsymbol{\xi}}{\partial t} = f(\boldsymbol{\xi}, \boldsymbol{\theta}, t) \approx \boldsymbol{W}\boldsymbol{\xi} + \boldsymbol{b}$$

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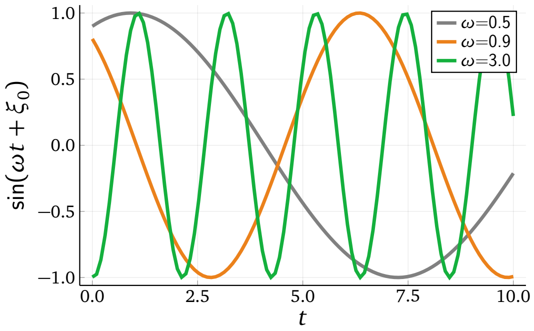
- ODE: diff eq. with one variable ξ
- we can use vectorized from to represent nth order ODE: $W\xi + b$
- parameters W, b are (almost) intuitively interpretable

Example: Harmonic oscillator

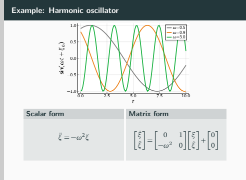


Scalar form

$$\ddot{\xi} = -\omega^2 \xi$$

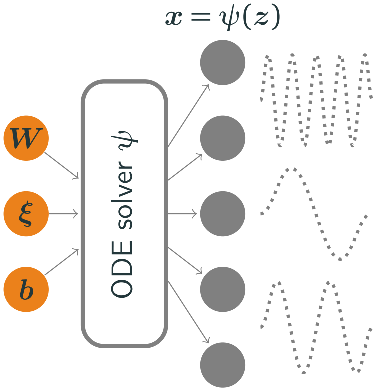
Matrix form

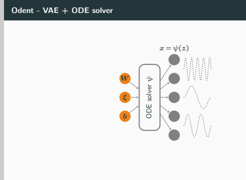
$$\begin{bmatrix} \dot{\xi} \\ \ddot{\xi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \dot{\xi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



- Attempt: learn right box only from **partial obs**.
- that means: sparsity, 1, ω, ξ, $\dot{\xi}$
- awesome toy problem! (const, spec, zeros)

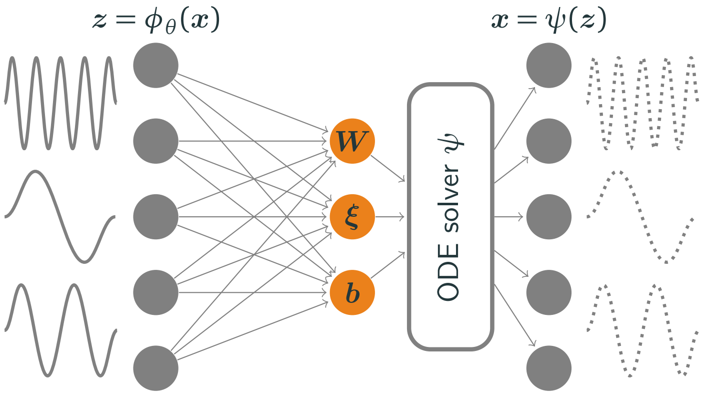
Odent - VAE + ODE solver

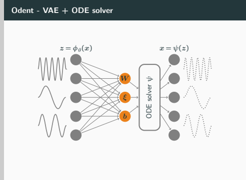




- we have harmonic samples
- need params → inverse mapping!
- normal AE + ODE decoder

Odent - VAE + ODE solver





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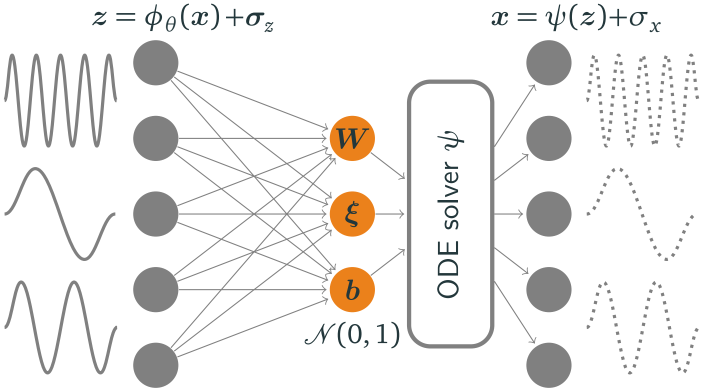
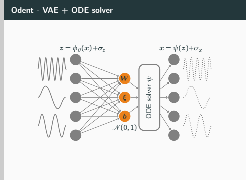
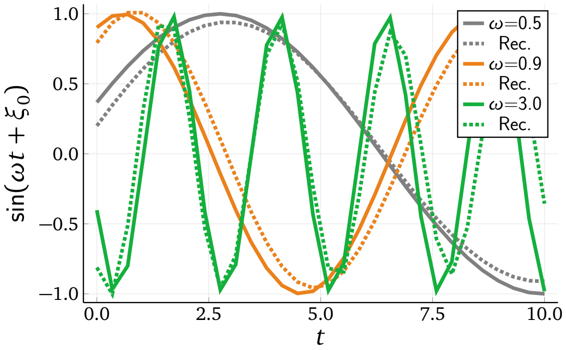
Odent - VAE + ODE solver





- traditional VAE + ODE = Odent
- everything is learnt! (σ_z, σ_x, z, ϕ)
- like this we learn manifold (in W, ξ, b)
- OVERPARAMETRIZE

Results - Reconstructions



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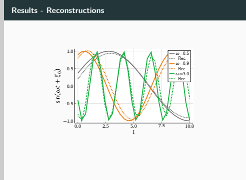
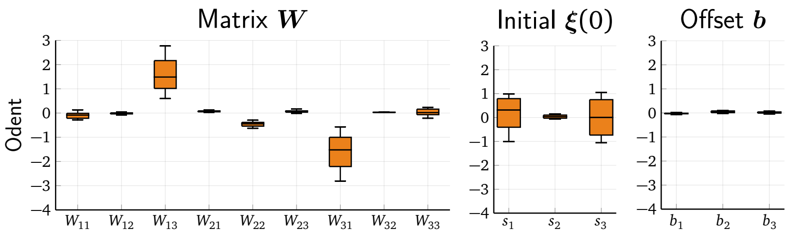
└Ordinary differential equations

└Results - Reconstructions





$$\begin{bmatrix} \dot{\xi} \\ \ddot{\xi} \\ \dddot{\xi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -\omega^2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ 0 \\ \dot{\xi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Successful model identification

But: 1-3 irrelevent parameters remain in $\boldsymbol{W}$

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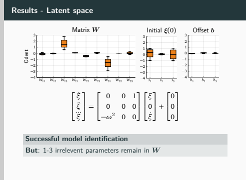
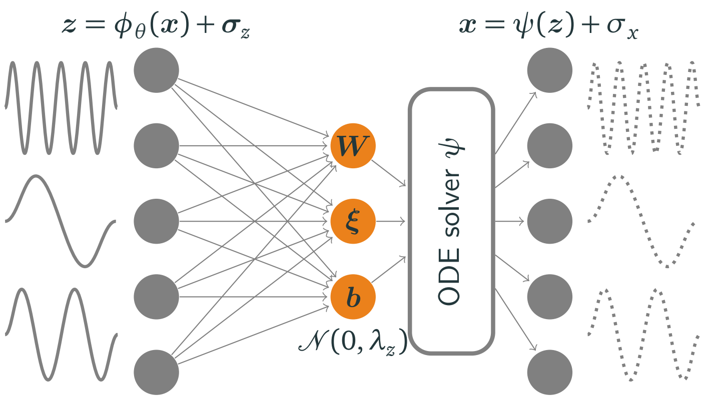
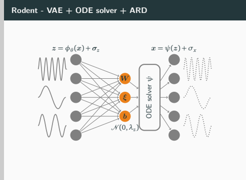
└─Ordinary differential equations
└─Results - Latent space



Rodent - VAE + ODE solver + ARD



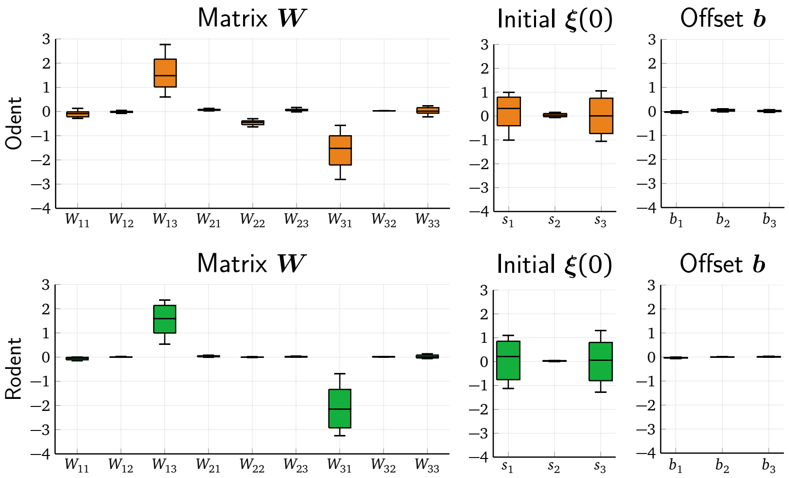


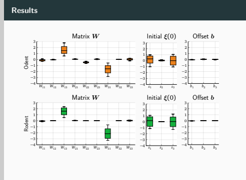
- hierarchical model
- enforces sparsity

I am not sure how deep to go here. is it confusing to just show the equations and say that ARD is our main workhorse?

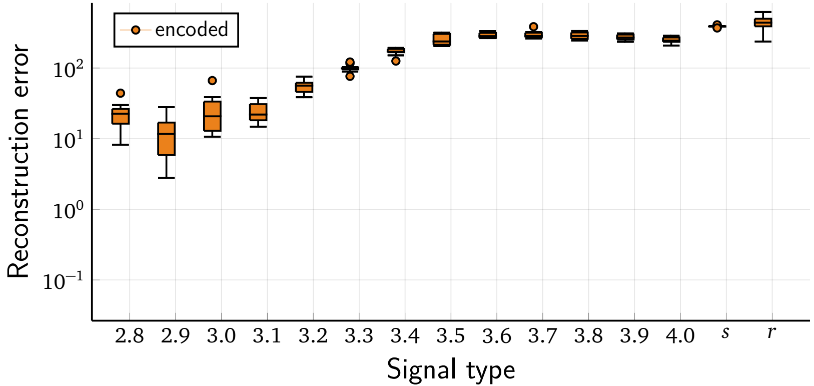
ODE adds explainability. we can now identify a physics motivated manifold of harmonic signals

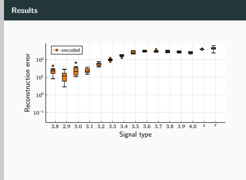
Results


Matrix $\boldsymbol{W}$
Initial $\xi(0)$
Offset $\boldsymbol{b}$
Odent
Rodent
3
2
1
0
−1
−2
−3
−4
W_{11} W_{12} W_{13} W_{21} W_{22} W_{23} W_{31} W_{32} W_{33}
s_1 s_2 s_3
b_1 b_2 b_3



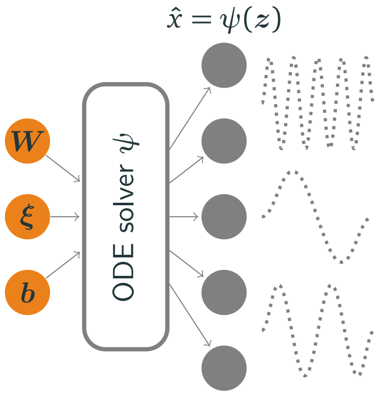
Results


encoded
Reconstruction error
10^{2}
10^{1}
10^{0}
10^{-1}
2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 4.0 s r
Signal type



Reidentification

- Starting point: $z = \phi(x)$
- Fix irrelevant parameters
- Continue optimizing $|\psi(z) - x|^2$



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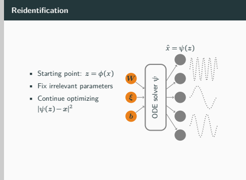
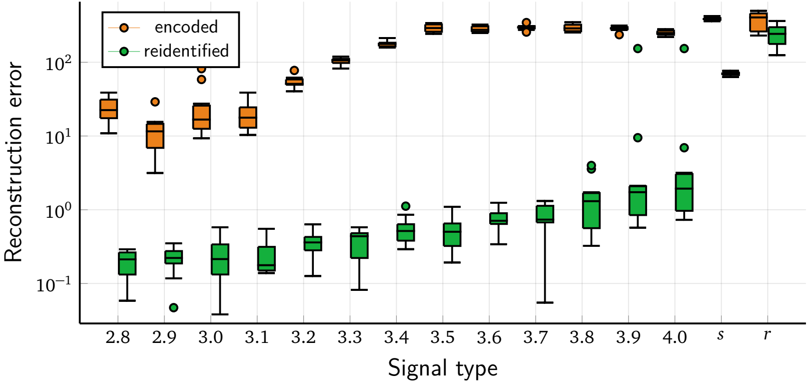
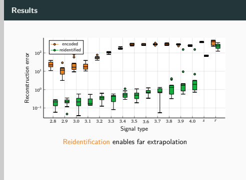


Results



Reidentification enables far extrapolation



Conclusion

Identification of partially observed system

Conclusion

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Conclusion

Identification of partially observed system

Sparse, explainable model

Conclusion

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- Identification of partially observed system
- Sparse, explainable model
- Good extrapolation

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Do you have data for us?

Conclusion

Do you have data for us?