UCI

Brain-Inspired Learning Machines

Neural Models of Associative Memory and Working Memory

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Types of Declarative Memory

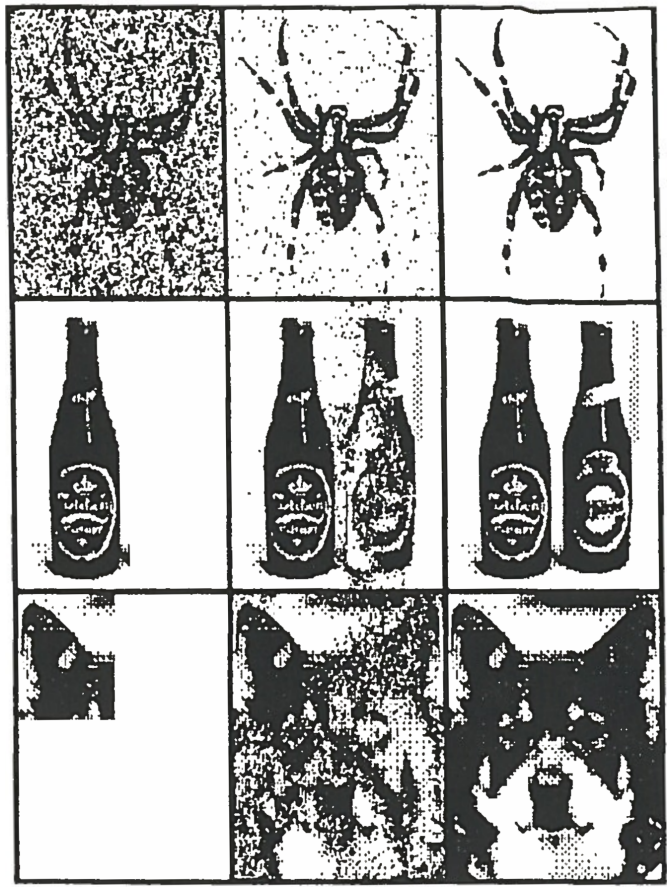
- **Working Memory** (<1m) - Prefrontal lobe, *Example:* Remembering digits of a phone number
- **Long-term Memory** (years) - Hippocampus, Inferior Temporal (IT) lobe, *Example:* Recognition Memory, Autoassociative memory

Associative Memory

Associative Memory

"In associative memory, a partial or approximate representation of a stored item is used to recall the full item"

Dayan and Abbott,, 2001



Hertz, Krogh, and Palmer,, 1991

In computer science, autoassociative memory is called Content-Addressable Memory (CAM)

Address-based Memory vs. Content-based Memory

Address-based memory *e.g.* Random Access Memory (RAM):

- High capacity
- Need to know the “address” to retrieve memory
- Recall is not robust/fault-tolerant

Content-based memory *e.g.* Cache:

- Low capacity
- Ability to recall a memory for partial cues
- Errors in recall cues can be corrected
- Hardware can be fault tolerant

Biological memory mechanisms for long-term storage are CAMs

(a) Stored Memories

moscow------russia
lima----------peru
london-----england
tokyo--------japan
edinburgh-scotland
ottawa------canada
oslo--------norway
stockholm---sweden
paris-------france

(b) Recall

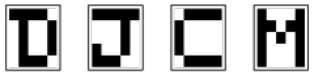
moscow---::::::::: $\Longrightarrow$ moscow------russia

::::::::::--canada $\Longrightarrow$ ottawa------canada

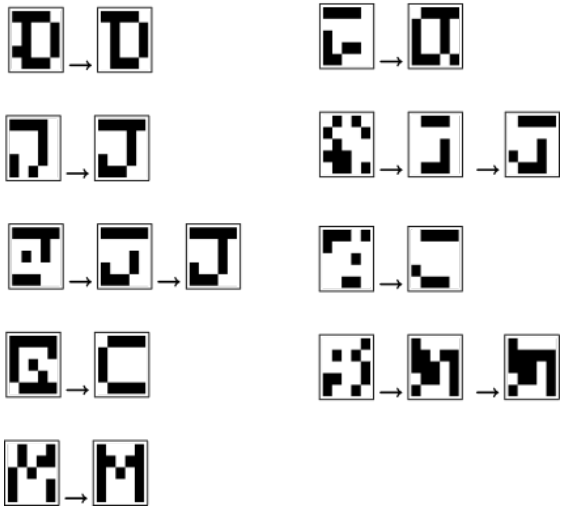
otowa-------canada $\Longrightarrow$ ottawa------canada

egindurrh-sxotland $\Longrightarrow$ edinburgh-scotland

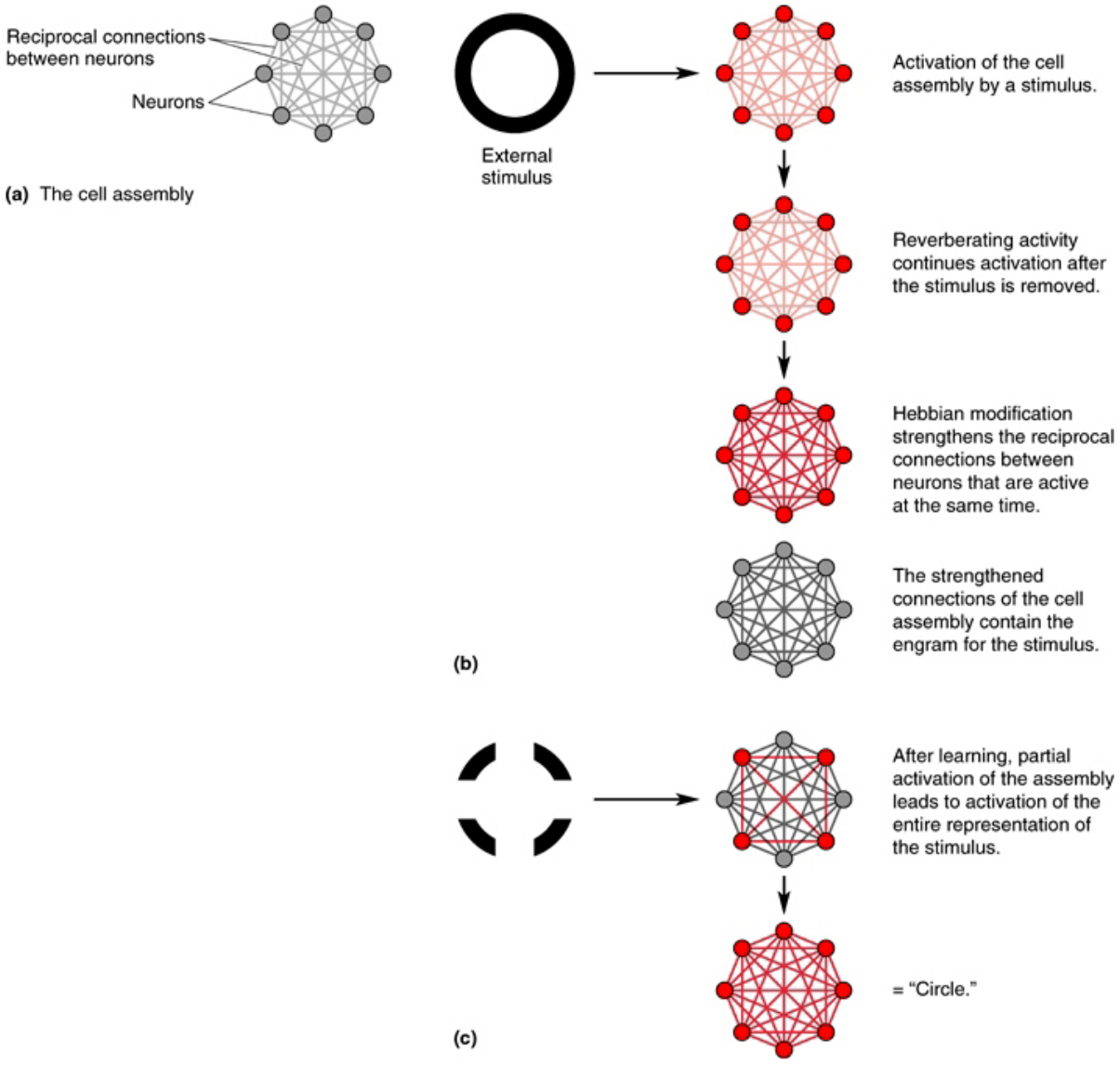
(a) Stored Memories

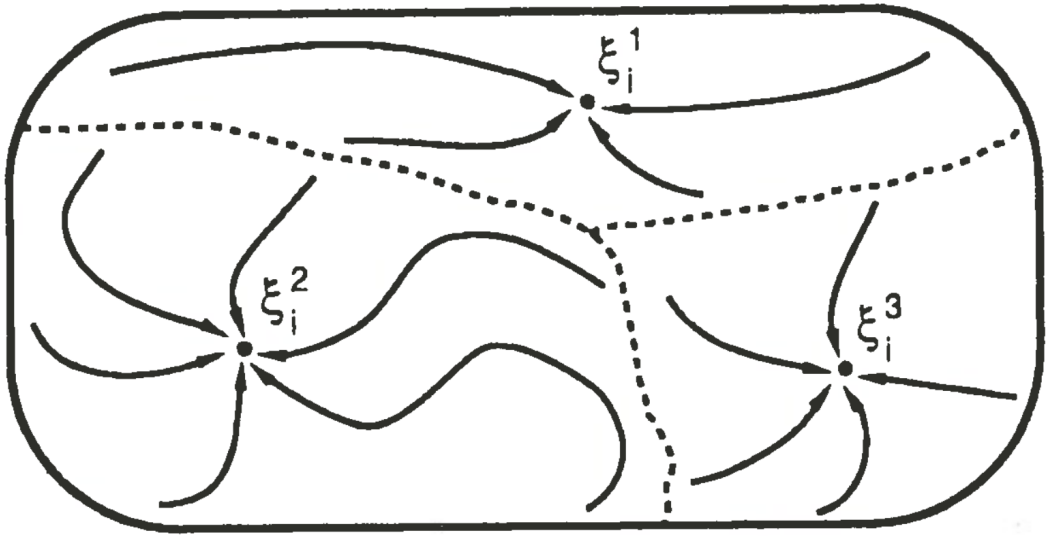


(b) Recall

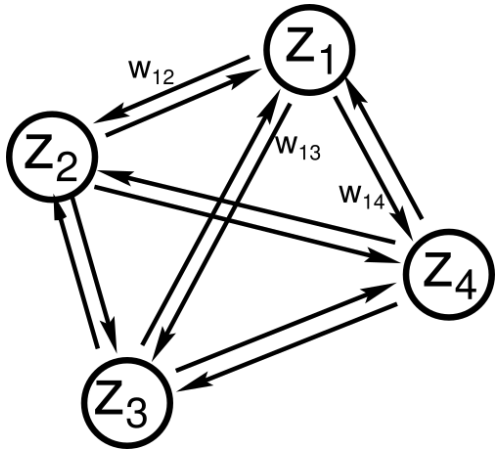


Hebb's Cell Assembly





- The rectangle is the **state space**.
- ξ_i^1, ξ_i^2, ξ_i^3 are **attactors**
- Regions bounded by dashed lines represent **basins of attraction.**



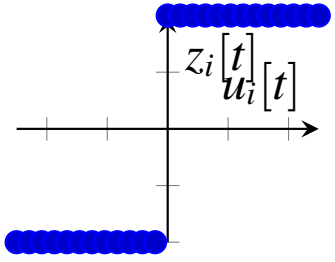
A Hopfield network is a recurrent neural net for content-addressable memory

Hopfield, *Proceedings of the national academy of sciences*, 1982

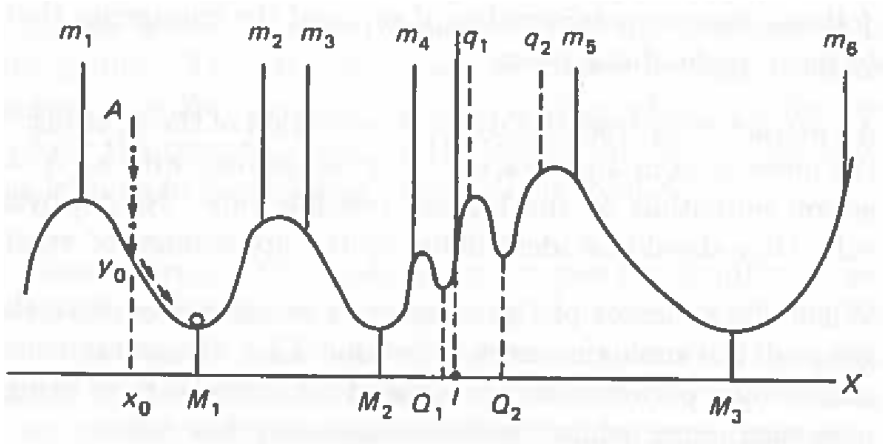
Neuron model:

$$u_i[t] = \sum_j w_{ij} z_i[t] + \theta_i$$

$$z_i[t] = \Theta(u_i[t]) = \begin{cases} 1 & \text{if } u_i[t] > 0 \\ -1 & \text{if } u_i[t] \leq 0 \end{cases}$$



Energy landscape metaphor and learning



Amit,, 1992

"Energy" of a Hopfield Network:

$$E(\mathbf{z}) = -\frac{1}{2}\sum_{i,j} w_{ij} z_i z_j - \sum_i \theta_i \, z_i$$

Learning of autoassociative memory = placing of attractors on or near regions of interest

Hopfield Learning Rule

$$w_{ij} = \frac{1}{N} \sum_{k \in data} z_i^k z_j^k$$

Example

code/ann_hopfieldnet.py

```
from npamlib import *

data = np.array([[1,1,-1,-1],[1,-1,1,-1]])

#"Train" Weight Matrix
W = np.zeros([len(data), 4, 4])
for i in range(len(data)):
    W[i, :, :] += np.outer(data[i], data[i].T)
W = np.mean(W, axis=0)
b = data.mean(axis=0)

out = FRN(N=4, b=b, W=W, rates_in = [1,1,1,-1], activation_function=hopfield, T=5)
```

Energy

$$E(\mathbf{z}) = -\frac{1}{2}\sum_{i,j} w_{ij} z_i z_j - \sum_i \theta_i \, z_i$$

z0	z1	z2	z3	Energy
-1	-1	-1	-1	-0.0
-1	-1	-1	1	-2.0
-1	-1	1	-1	-4.0
-1	-1	1	1	-6.0
-1	1	-1	-1	-4.0
-1	1	-1	1	-6.0
-1	1	1	-1	-0.0
-1	1	1	1	-2.0
1	-1	-1	-1	-6.0
1	-1	-1	1	-0.0
1	-1	1	-1	-10.0
1	-1	1	1	-4.0
1	1	-1	-1	-10.0
1	1	-1	1	-4.0
1	1	1	-1	-6.0
1	1	1	1	-0.0

Example Letters

code/ann_hopfieldnet_letter.py

```
from npamlib import *
data, labels = data_load_letters()
#Transform into +1/-1
data = (data>.5)*2-1

#"Train" Weight Matrix
W = np.zeros([len(data), 25, 25])
for i in range(len(data)):
    W[i, :, :] += np.outer(data[i], data[i].T)
W = np.mean(W, axis=0)-np.eye(25)
b = data.mean(axis=0)

res = np.zeros([len(data),25])
data_corrupt = np.empty_like(data)

#Number of pixels to corrupt
N_corrupt = 5
for i in range(len(data)):
    d = data[i,:]
    #Flip pixel value
    d[np.random.choice(range(25),N_corrupt)] *= -1
    data_corrupt[i,:] = d
    res[i] = FRN(N=25, b=b, W=W, rates_in = data[i], activation_function=hopfield, T=10)
```

Example Letters

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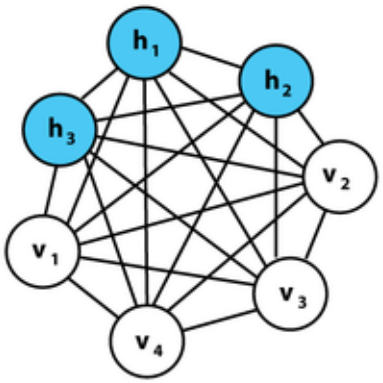
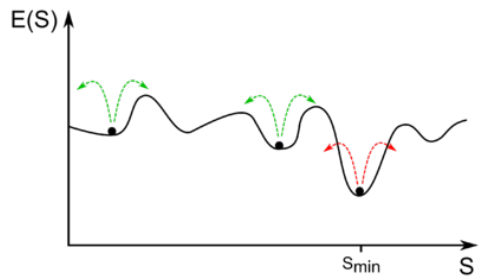
```

No labels were used for learning: Unsupervised Learning

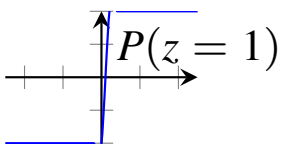


$$p^{BM}(\mathbf{z}) = \frac{1}{Z}\exp\big(-E(\mathbf{z})\big), \quad E(\mathbf{z}) = -\frac{1}{2}\sum_{i,j} w_{ij} z_i z_j - \sum_i \theta_i \, z_i$$

Intuition behind the BM: add stochasticity to a Hopfield network to escape local minima

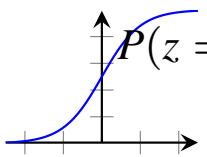


Deterministic unit (Hopfield Network)



$$\Delta w_{ij} \propto \langle z_i z_j \rangle_{data}$$

Stochastic unit (Boltzmann Machine)



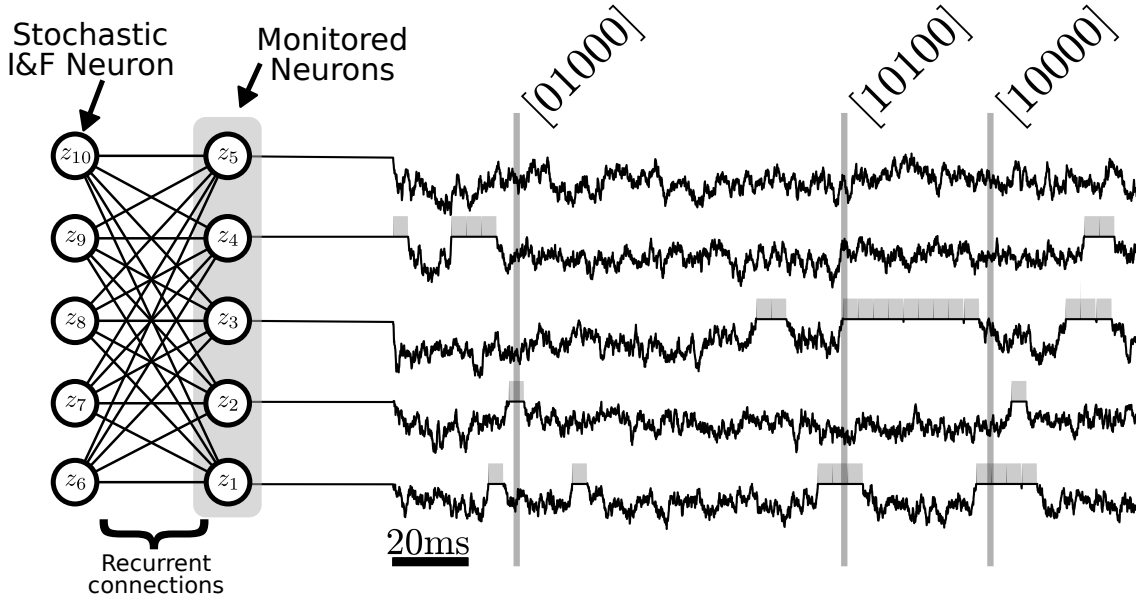
$$\Delta w_{ij} \propto \langle z_i z_j \rangle_{data} - \underbrace{\langle z_i z_j \rangle_{reconstruction}}_{\text{difficult to evaluate}}$$

Hinton and Sejnowski, 1986

The additional term is often estimated using Markov Chain Monte Carlo (MCMC) sampling

Neural sampling and learning with noisy Integrate & Fire (I&F) neurons

Neuron state associated with a binary random variable $z_k = 0, 1$



Neural sampling and learning with noisy I&F neurons

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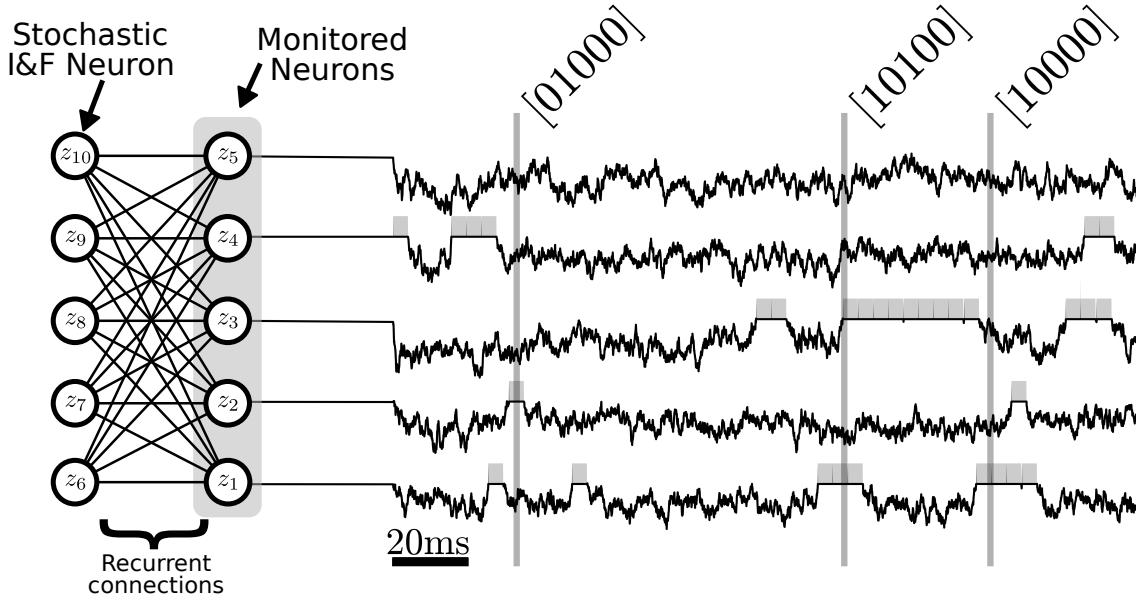
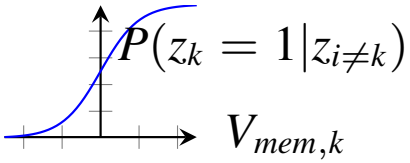


- Network states are Markov Chain Monte Carlo samples of the Boltzmann distribution if:



$$V_{mem,k}(t) = \sum_i W_{ki} z_i(t) + b_k$$

Linear sum of inputs

Buesing, Bill, Nessler, and Maass, *PLoS Computational Biology*, 2011

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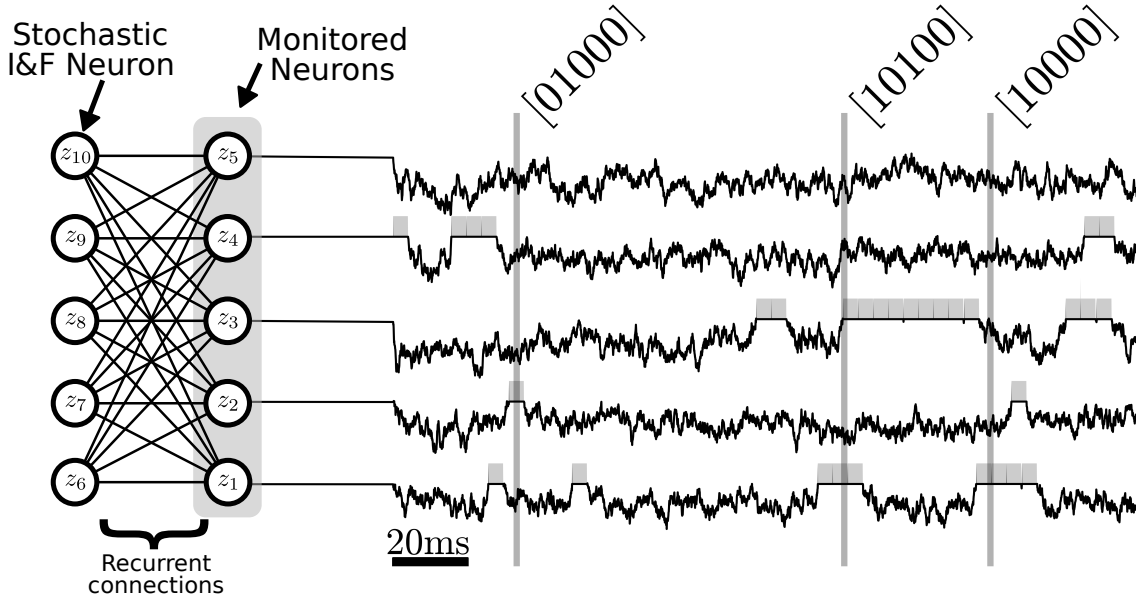

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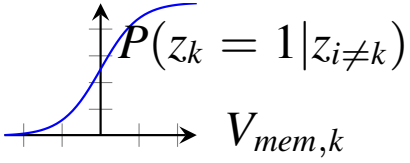

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Linear sum of inputs

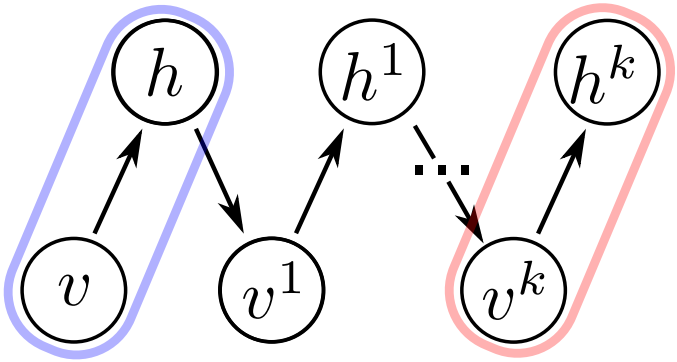
Buesing, Bill, Nessler, and Maass, *PLoS Computational Biology*, 2011

- Biophysical neuron models can meet these conditions
- Learning with STDP-based *Event-driven Contrastive Divergence*

Neftci, Posch, and Chicca, *Computational Intelligence*, 2014

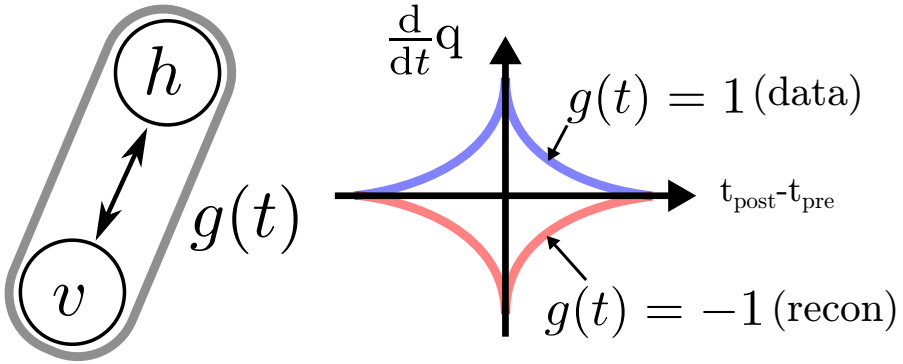
Event-driven Contrastive Divergence

Gibbs Sampling +
Standard CD



$$\Delta w \propto \langle vh \rangle_{\text{data}} - \langle v^k h^k \rangle_{\text{recon}}$$

Neural Sampling +
Event-driven CD

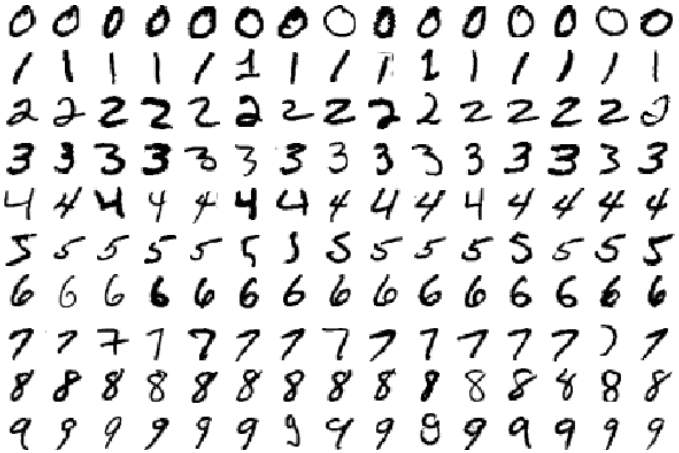


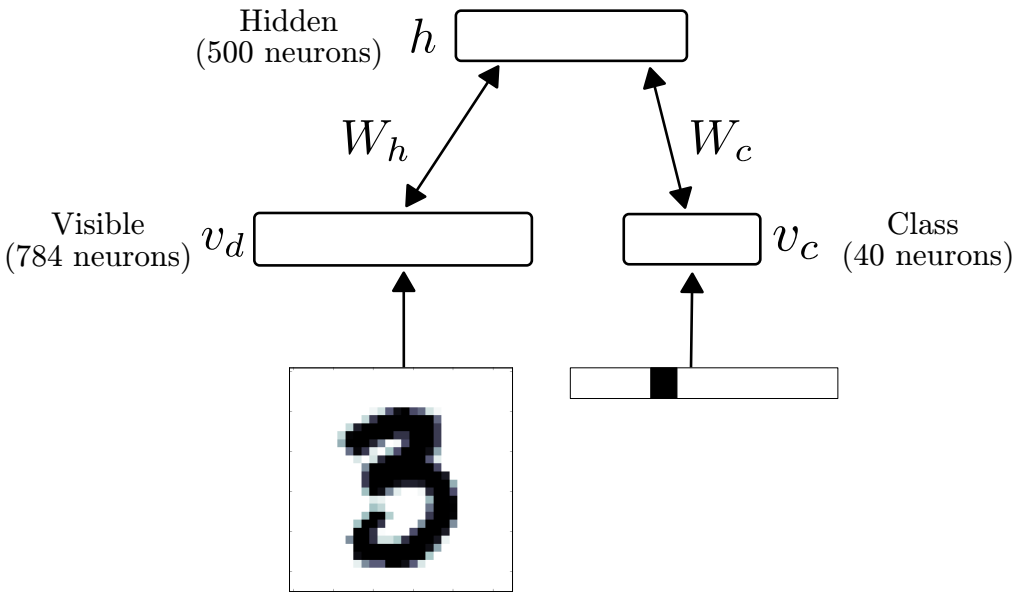
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{q} = g(t)\mathrm{STDP}(v(t), h(t))$$

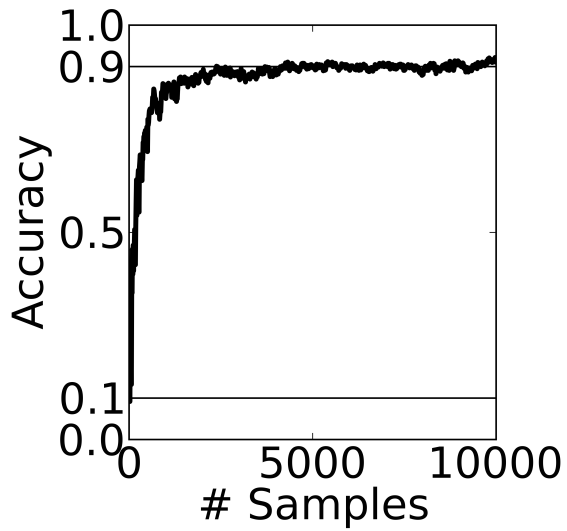
In average, eCD behaves like standard CD

Neftci_etal14 Neftci_etal14Neftci_etal14 Neftci_etal14

Learning a model of MNIST hand-written digits (software simulations)



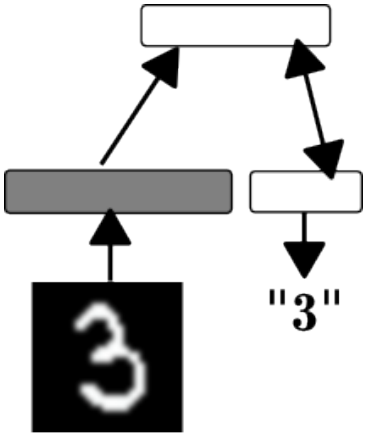




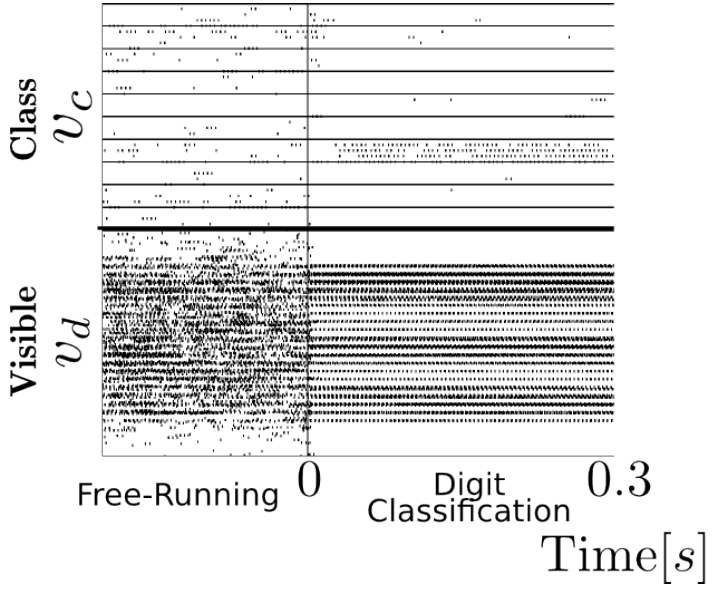
Recognition Accuracy: 91.9%, (*vs.* standard CD: 93.6%)

Neftci, Posch, and Chicca, 2014

The trained network performs Bayesian inference



Digit
Classification



Neftci, Posch, and Chicca, 2014

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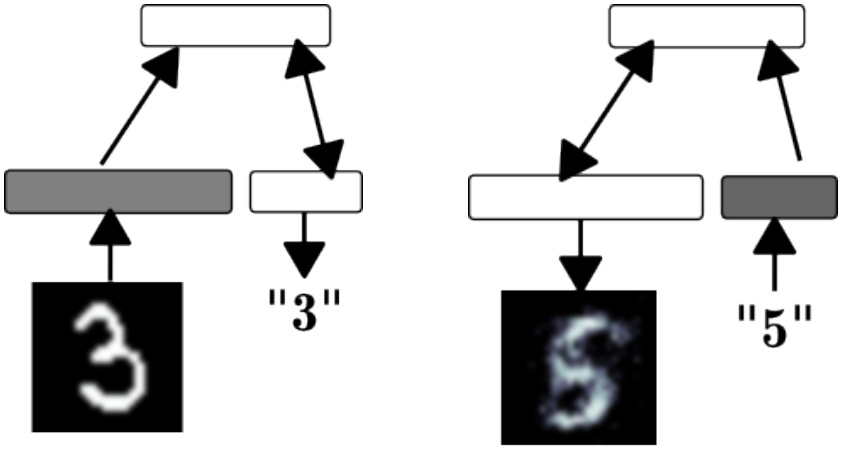
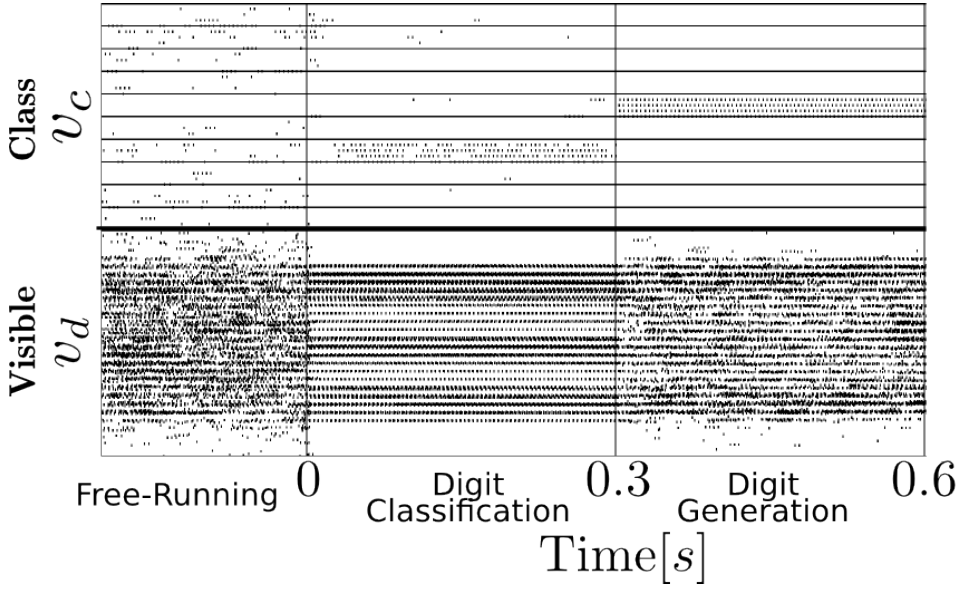


Digit Classification

Digit Generation



Neftci, Posch, and Chicca, 2014

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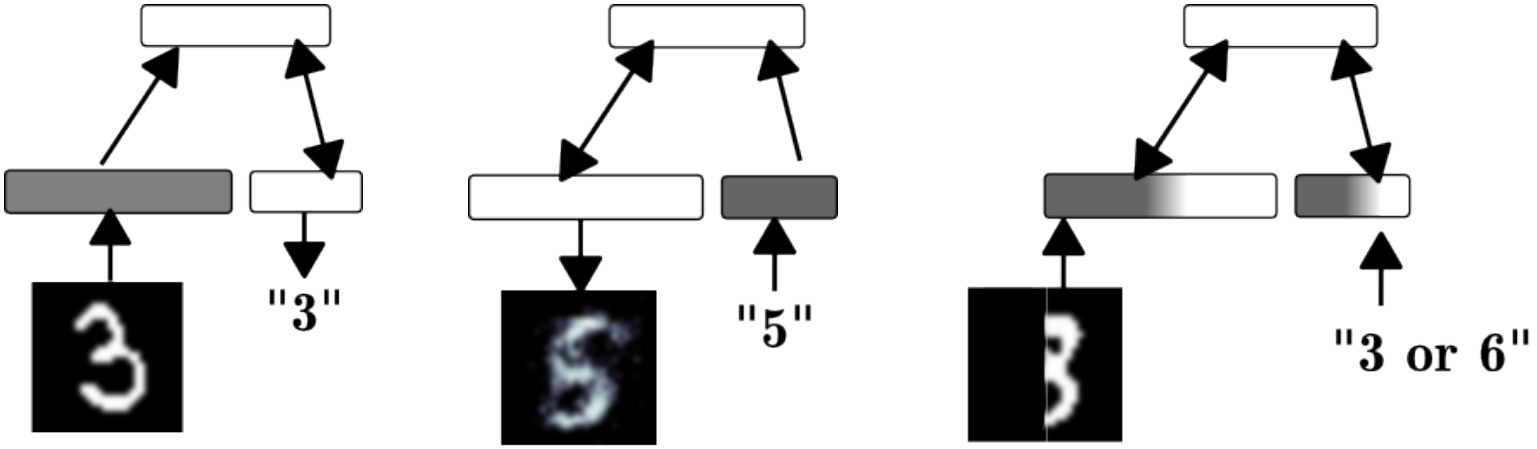
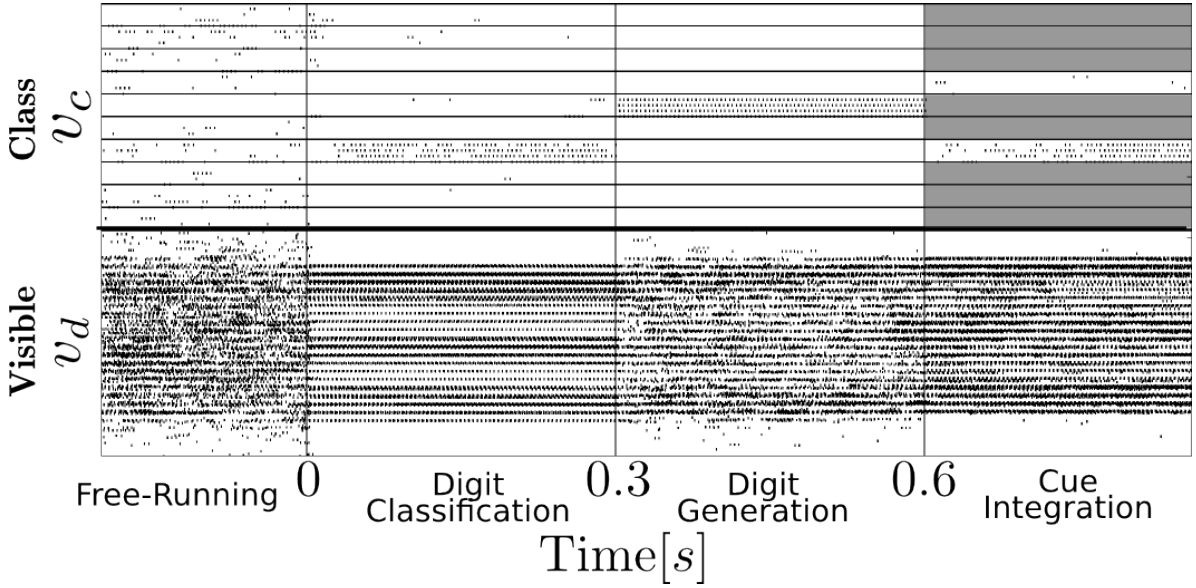


Digit Classification

Digit Generation

Cue Integration



Neftci, Posch, and Chicca, 2014

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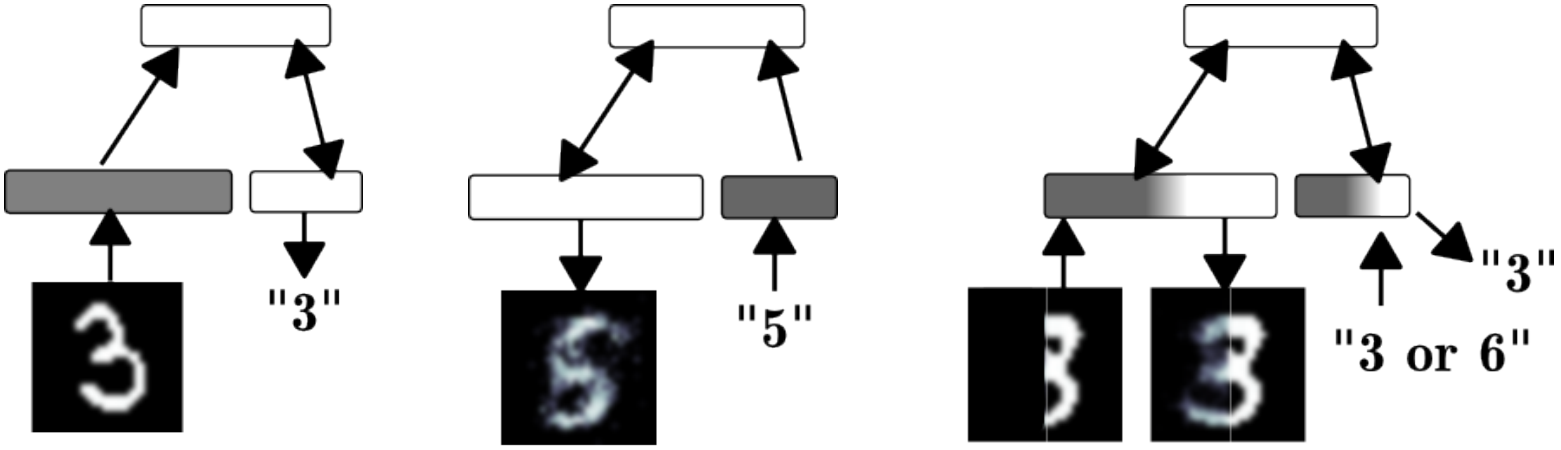

Digit Classification

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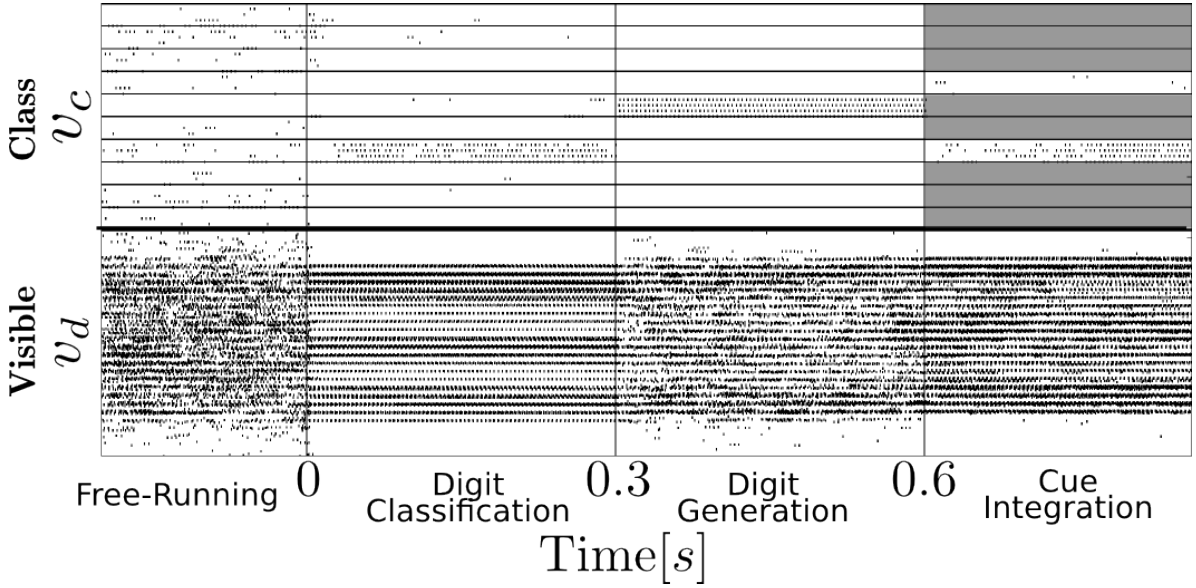
Cue Integration

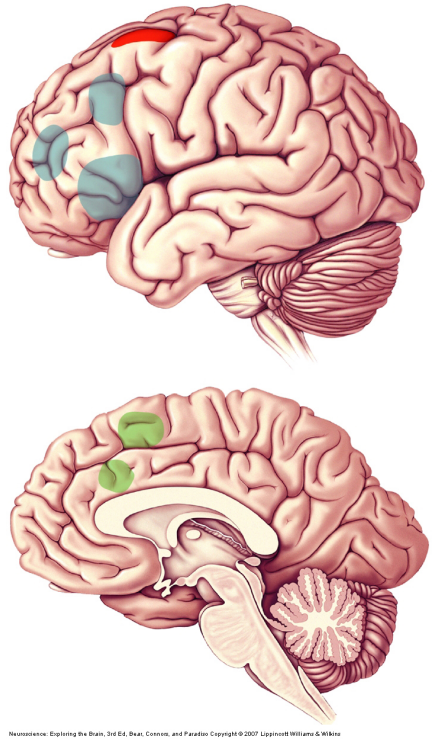


Neftci, Posch, and Chicca, 2014

Working Memory

Working Memory:

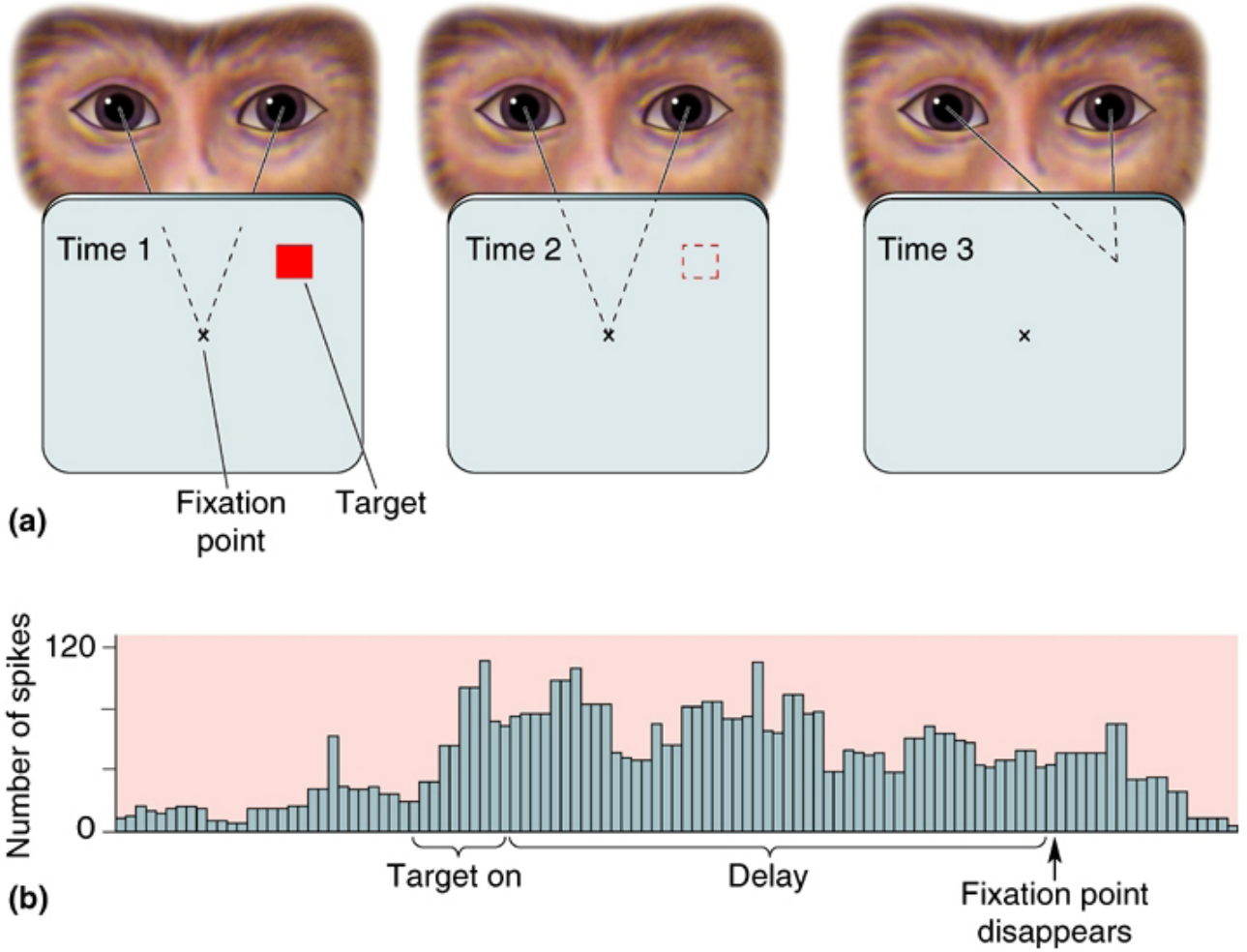
- A type of short-term memory
- Limited in capacity
- Task- and sensory modality-dependent
- Necessary for cognitive control

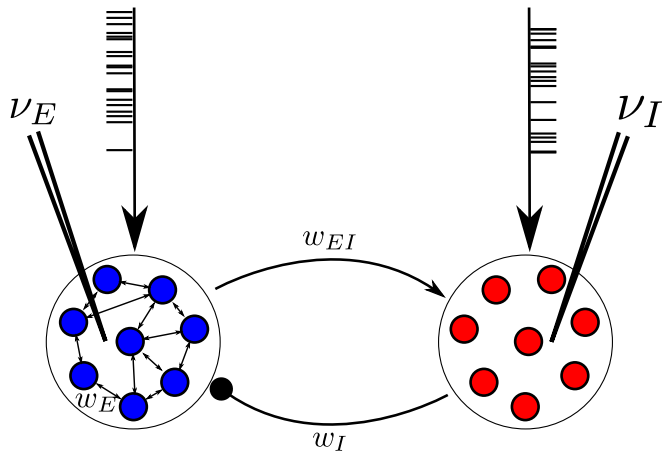


Human brain areas for working memory of face identity and location

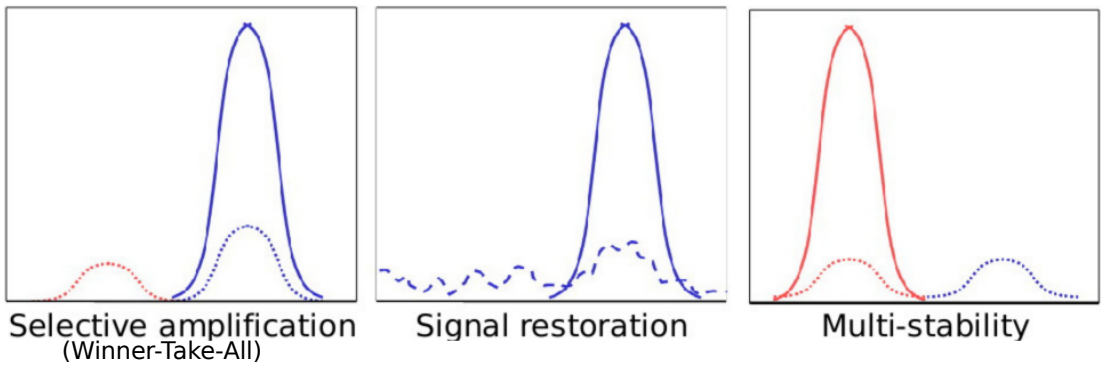
Working Memory is a "Mental Sketch Pad"

Neural Correlates of Working Memory

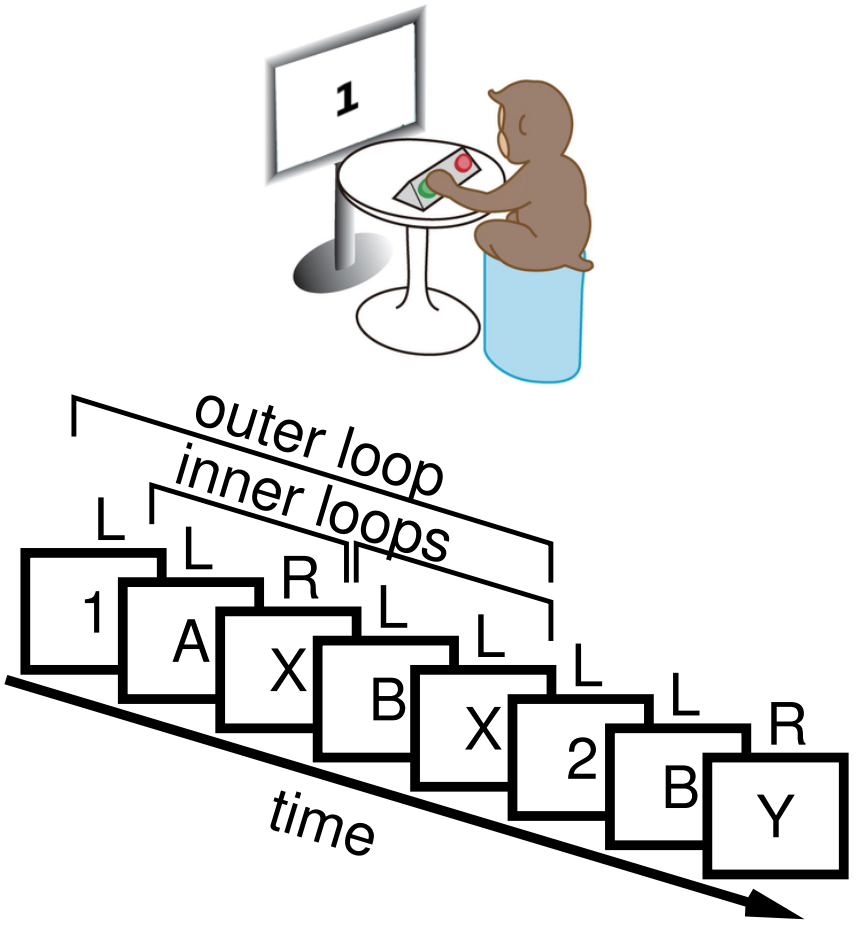



ν_E
ν_I
w_{EI}
w_E
w_I

Winner-Take-All (WTA) Properties



Douglas and Martin, *Annual Review of Neuroscience*, 2004



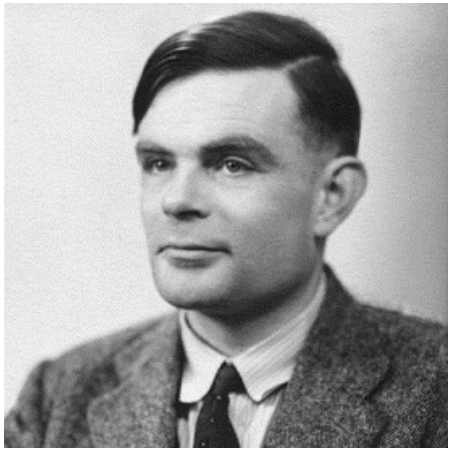
O'Reilly and Frank, 2006

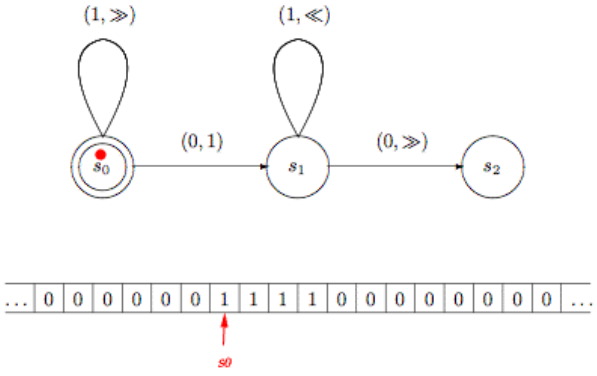
Finite State Machines (FSM)

State Machines are the Main Components of Turing Machines

A Turing machine is a theoretical computing machine invented by Alan Turing (1937) to serve as an idealized model for mathematical calculation.

Wolfram Mathworld





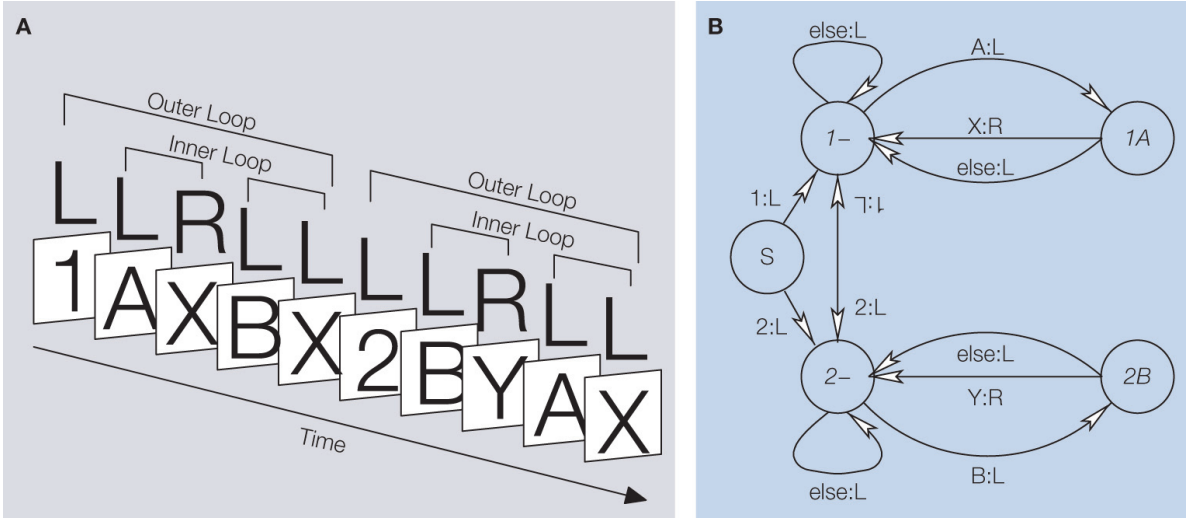
`http://www.google.com/doodles/alan-turings-100th-birthday`

Any real-world computation can be translated into an equivalent computation involving a Turing machine

The 12-AX Cognitive task

12AX Task

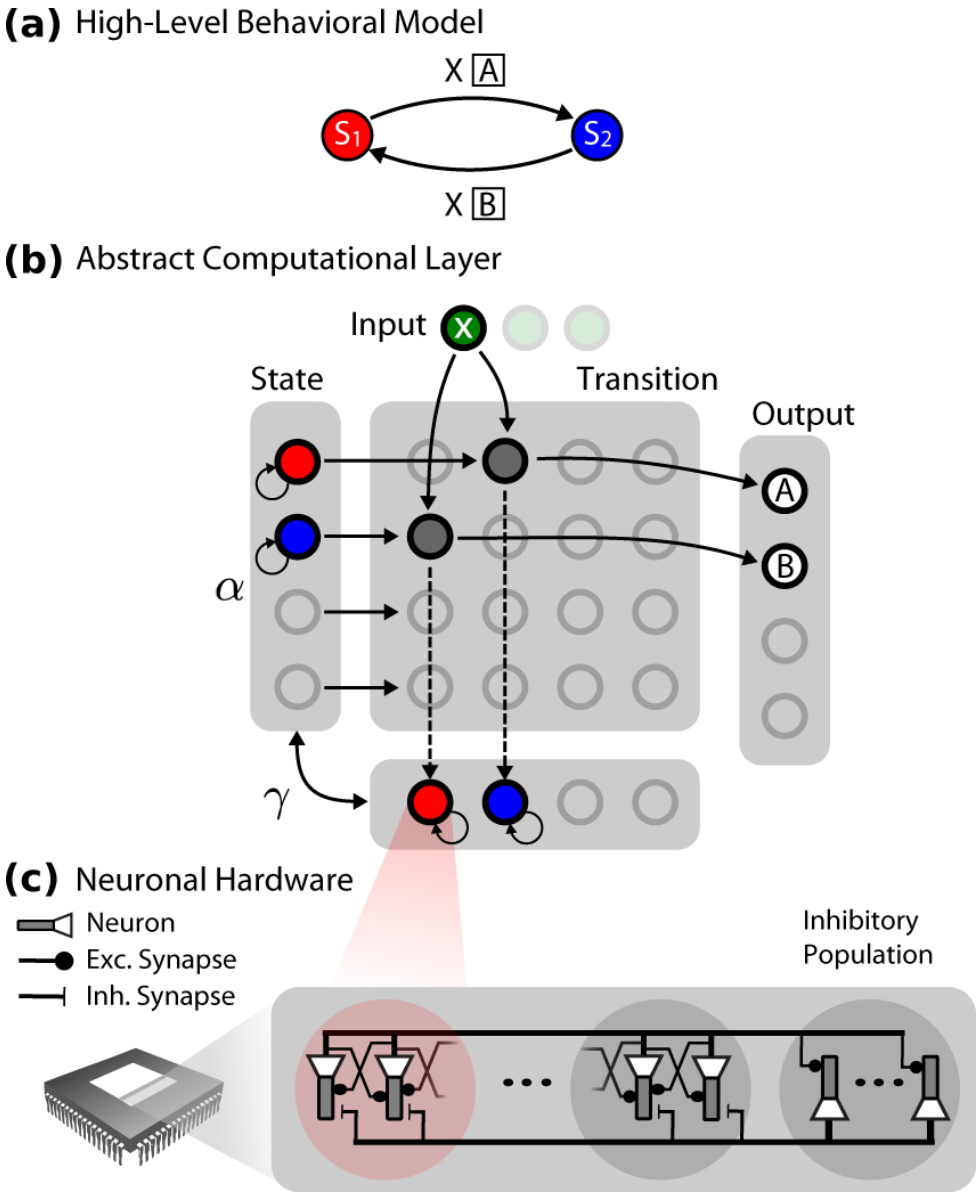
State machine solving 12AX



O'Reilly and Frank, Dayan, 2006, 2008

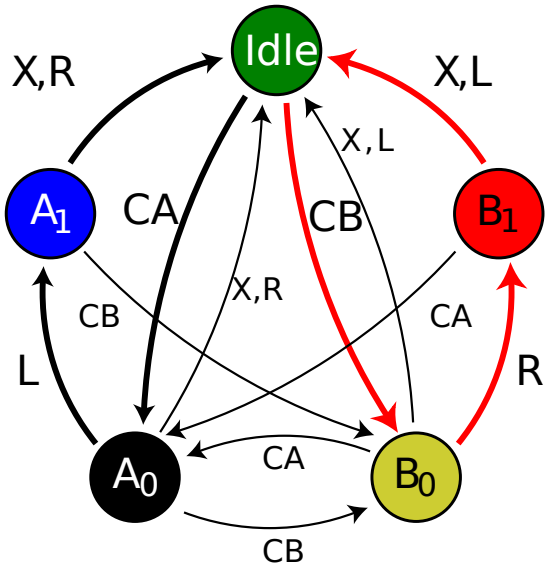
Can neurons implement state machines?

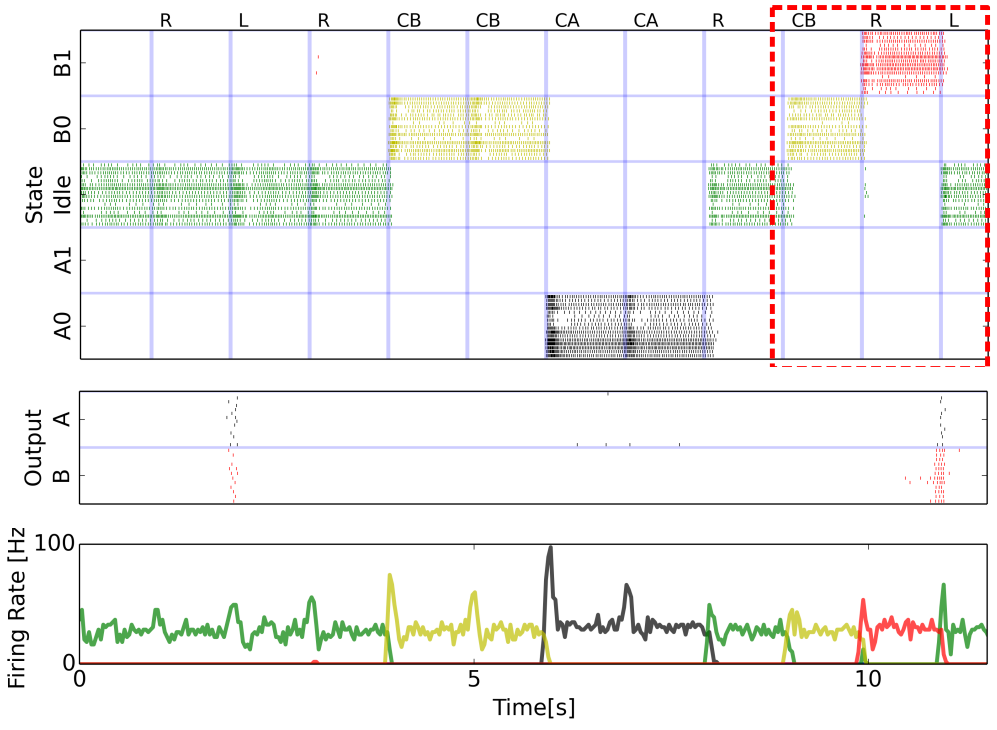
Synthesizing State-Dependent Behavior in WTA



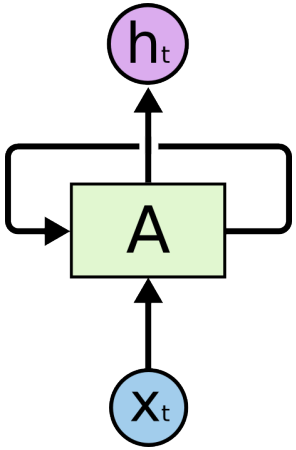
Neftci, Binas, Rutishauser, Chicca, Indiveri, and Douglas, *PNAS*, 2013

Example: Synthesis of 12-AX task state machine





Neftci, Binas, Rutishauser, Chicca, Indiveri, and Douglas, *PNAS*, 2013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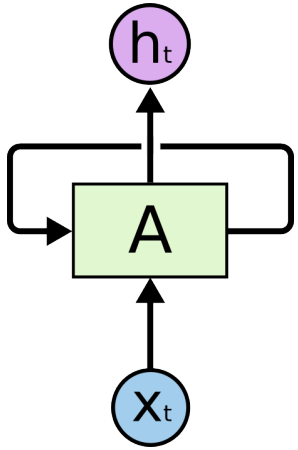
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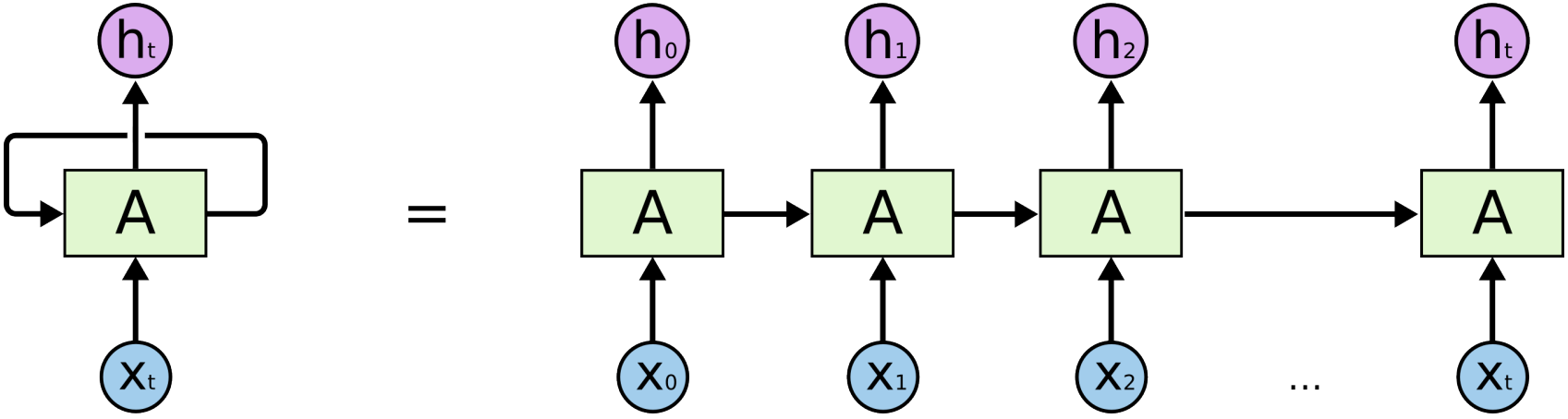
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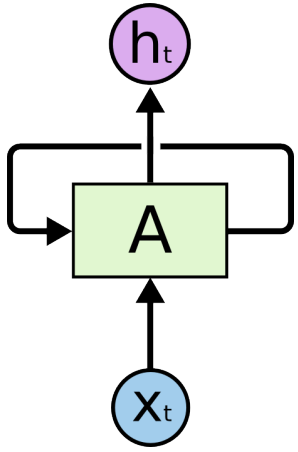

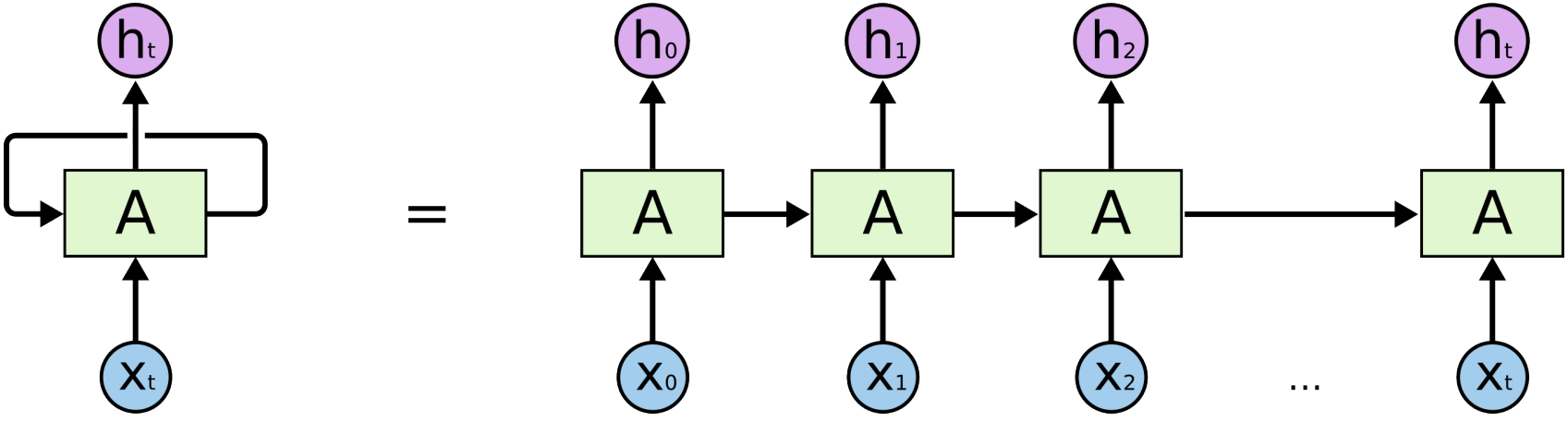

Learning with Back-Propagation on the “unrolled” network

Final Project and Presentations

- Topic in brain-inspired machine learning using what you have learned in class
- No class Nov. 25
- Dec 2: 10+5 min presentation
- Dec 7: Project report, using NIPS 2015 template `https://nips.cc/Conferences/2015/PaperInformation/StyleFiles`
- Discuss topic with instructor
- Groups of two OK, but delineate contributions