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Time Series

Tesla Stock Volatility Forecasting

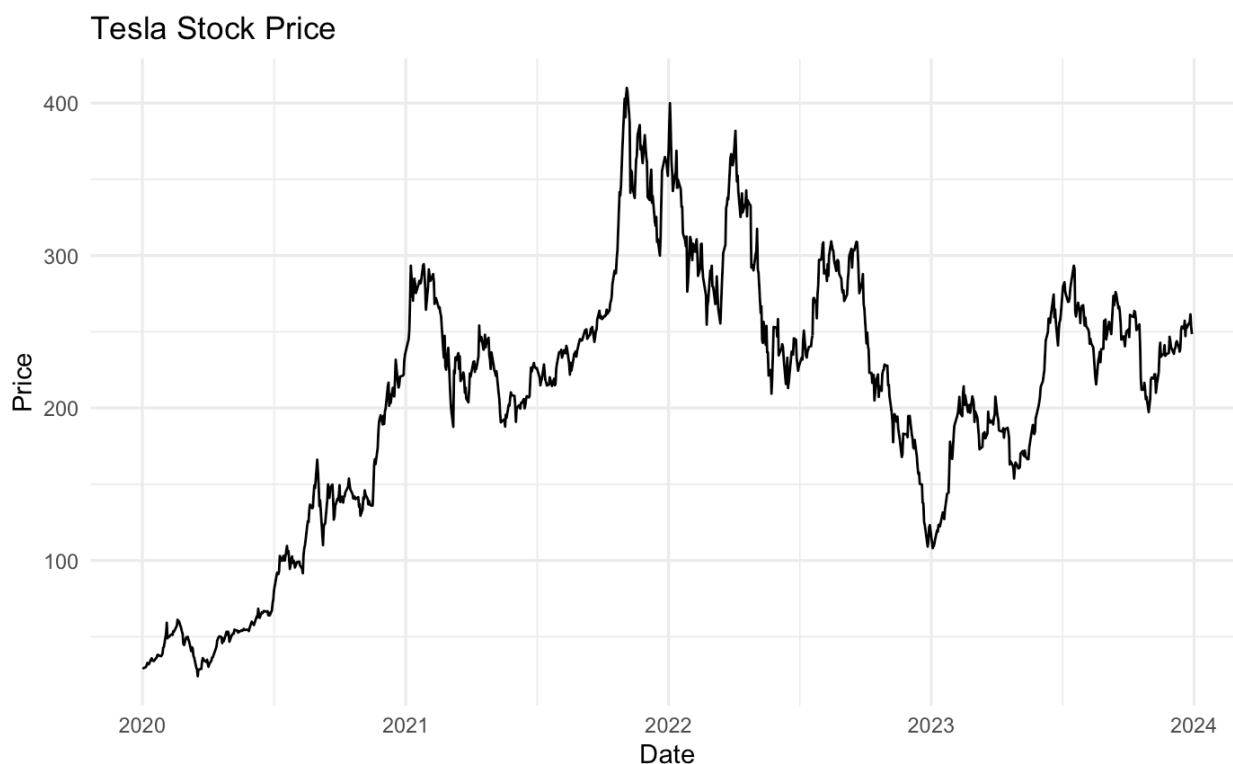
Introduction

Following the same theme as my previous project, I wanted to stay on stock market data for this project as well, and analyze specifically Tesla stock data. I chose Tesla specific data because my dad has tried buying options for Tesla in the past, and I thought it would be cool to use this project as an opportunity to try and garner some sort of real life application. Options have two main types, with a call option giving the holder the right to buy an asset at a predetermined price within a given amount of time or days. Essentially, this is when buyers think the asset or stock will increase in value. A put option gives the holder of the stock the right to sell the asset at a certain price within a certain amount of time. These are usually purchased when buyers are worried about potential decrease in the stock price, and by giving the holder of the stock the right to sell at a certain price, they can potentially set a floor level on the value of this asset ensuring the stock will sell at least at the price they purchased the put option for, regardless of how far in value the stock price actually drops. Of course, if the stock doesn't drop to this strike price, the put option will expire basically worthless. This is why being able to forecast the volatility of a stock price is valuable, as if you can forecast when a stock may rise or fall, you can place smarter option purchases and potentially outsmart other buyers. The price of options are also increased when there is an increase in volatility as well, which is why I will be trying to fit a GARCH model for Tesla stock prices, as this would help for forecasting when and where to buy Tesla options. Of course, Tesla has been a very volatile stock in recent years due to various reasons, which is another reason why I wanted to forecast volatility for Tesla stock specifically.

Methods

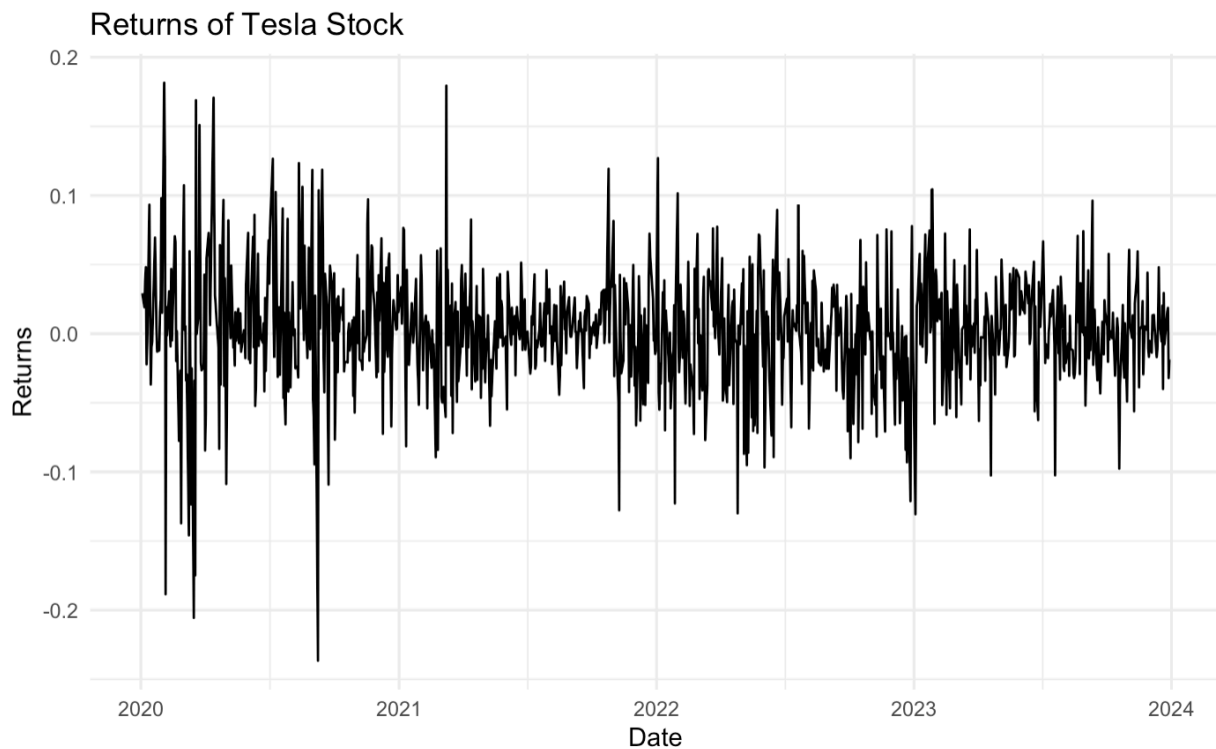
The dataset I used is called `tesla_stock_data`, which has data starting at the beginning of 2020 and continues until the end of 2023. The dataset only contained weekdays as those are the trading days the stock market is open for. The dataset had a column for the date, which made plotting much easier than my previous project, as well as the opening price, closing price, and the high and low price everyday. I looked at specifically the closing price for the data I used in my analysis, and the data was collected by taking the reported prices for Tesla everyday over this four year stretch, which was reported in US dollars.

Data Analysis



Above is the raw data of Tesla's stock price over the four year period. We can immediately see the price was steadily increasing until 2022, where it continued to decrease for the 2022 year. In 2023 the price began to rise again, but it never reached the highest price the

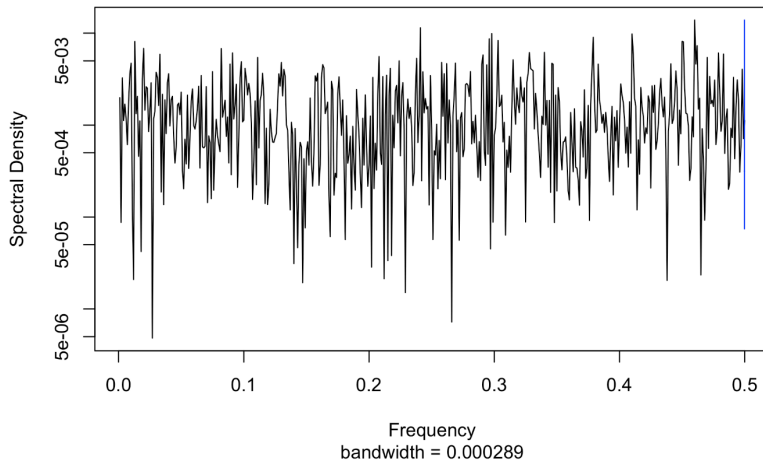
stock reached at the end of 2021 and beginning of 2022. In addition, this plot gives us some insight on how volatile Tesla's stock price can be.



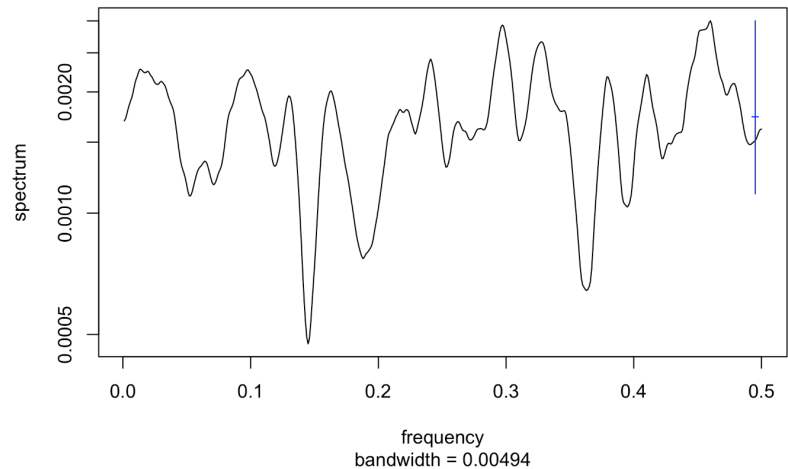
Above is the graph of the dataset after I took the log difference of the data. Obviously the original stock data was not stationary, and taking the logged difference gets us the stock returns, which is a stationary plot. This displays the approximate percent change, or returns for Tesla stock. This gives us a direct measure of volatility, which is the focus of the GARCH model we are going to try to fit and use to forecast how the volatility changes over time later in the paper. The returns plot is also very volatile in itself, with many dips and peaks in the beginning especially. The peaks in the returns plot tells us times where the market had a positive reaction to Tesla, whether it be because of a good earnings report or beneficial announcement, there are many different reasons for why these peaks could occur. They also signify strong investor confidence about the stock and its future. The dips in the returns plot are the opposite, or where

the market seems to be reacting negatively to Tesla. These dips signify investors are unsure about Tesla's future, leading to sell offs.

Spectral Density of Tesla Stock Returns



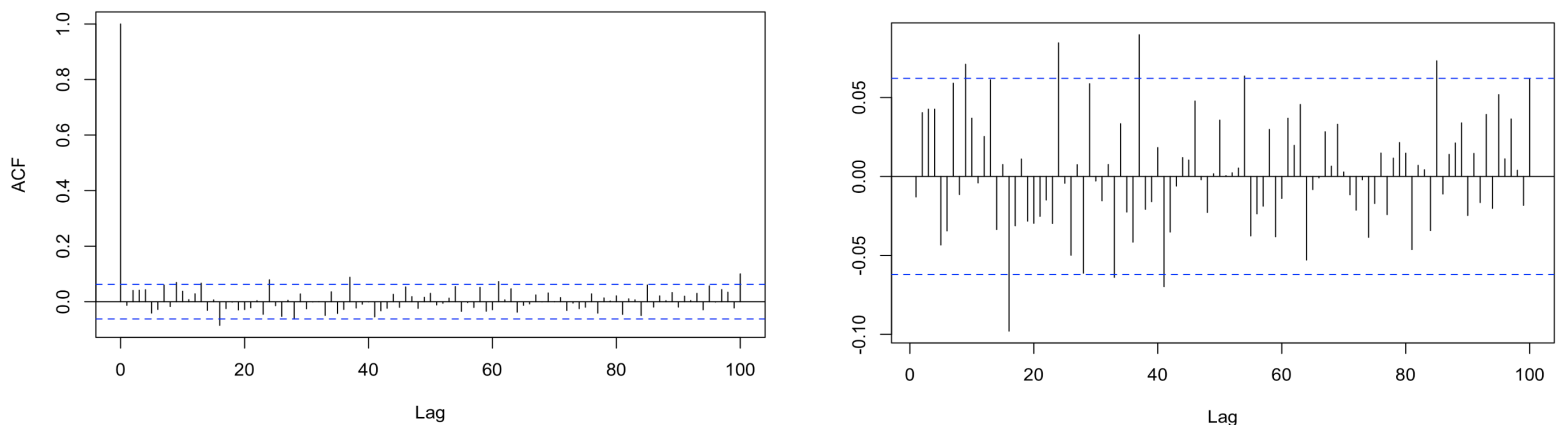
Smoothed Periodogram of Tesla Stock Returns



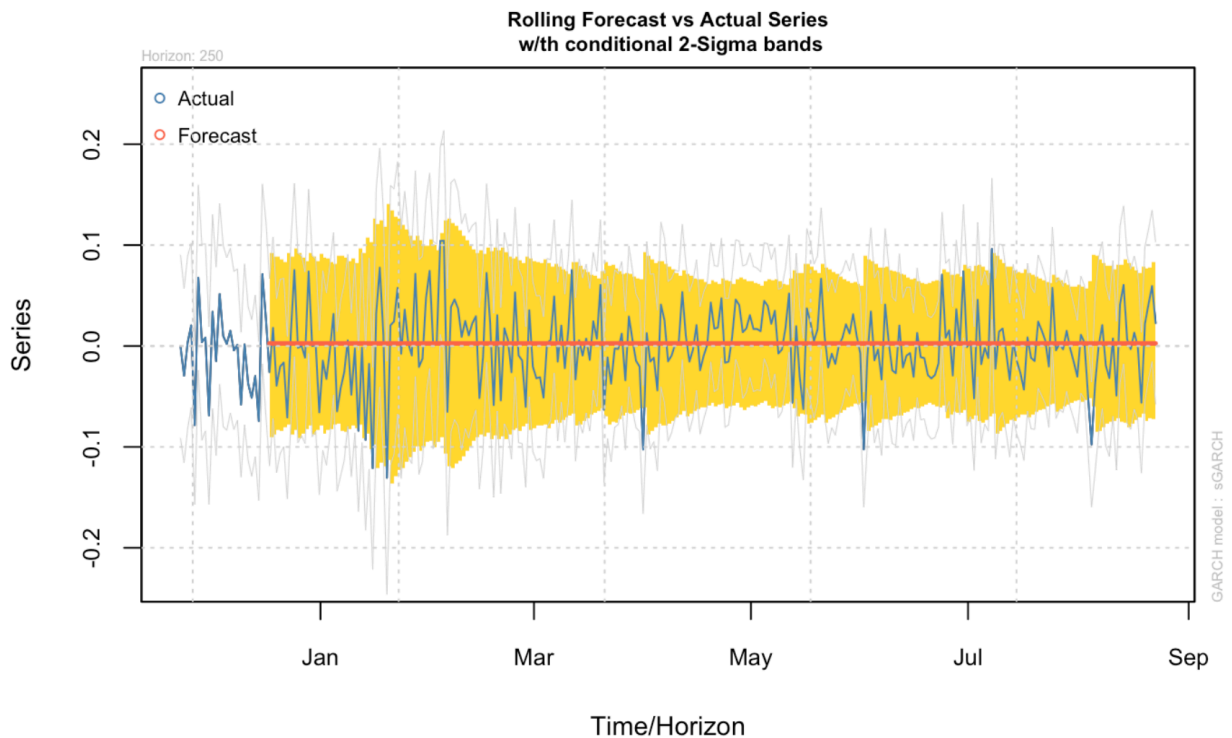
Above is the spectral density and smoothed periodogram for the Tesla returns data, which can help us see where the dominant frequencies are and aid us in fitting our GARCH model. The spectral density plot shows an estimation of how much the signal's power is distributed at each frequency, while the smoothed periodogram is an estimation of the spectral density. We can see in the spectral density plot a volatility cluster at about 0.15 frequency, which explains the big dip at the same frequency in the smoothed periodogram. This is due to the periodogram telling us the distribution of power among the different frequency components. We can see the highest peak on the periodogram at about 0.29 frequency, which is where the spectral density also shows a volatility cluster resulting in high frequency. A frequency of 0.29 means that the period of the cycle is $1/0.29$ which is about 3.5. This translates to a cycle that repeats about every 3 and a half days. This short cycle furthers the idea of Tesla's high volatility, and suggests there are rapid changes or effects that influence the stock's performance every few days. In regards to our

GARCH model, this is important as it indicates potential volatility clustering at a periodic rate, which could be caused by recurring market events or announcements.

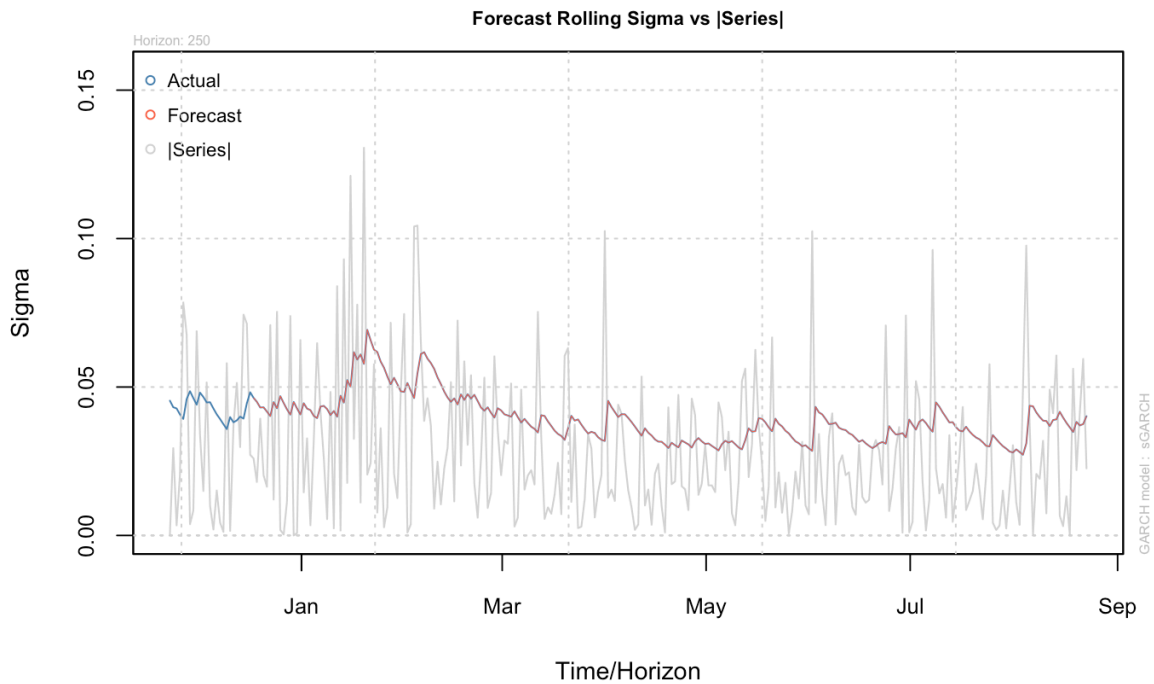
Now to actually fit our GARCH model to try and forecast volatility to help know when to buy options, we took a look at the ACF and PACF for the residuals to know what the parameters for the ARMA model would be.



These plots ended up not really giving us much structure, so after fitting a few different models I landed on an ARMA(1,1) to use in our GARCH model, which the parameters for GARCH model parameters come from the ARMA parameters. So we fit a GARCH(1,1) to forecast the volatility for the Tesla stock returns. After adding the ARMA(1,1) and our fourier term that came from the periodic cycle we saw in the periodogram, I saw that the AR1 estimate was -0.34 and the MA1 estimate was 0.30. These basically told us our ARMA model was not doing anything in our forecast as these two basically canceled each other out, so I got rid of the ARMA structure in the GARCH model.



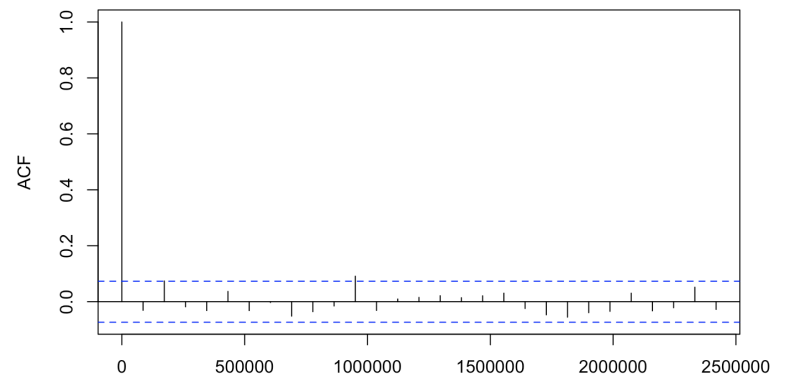
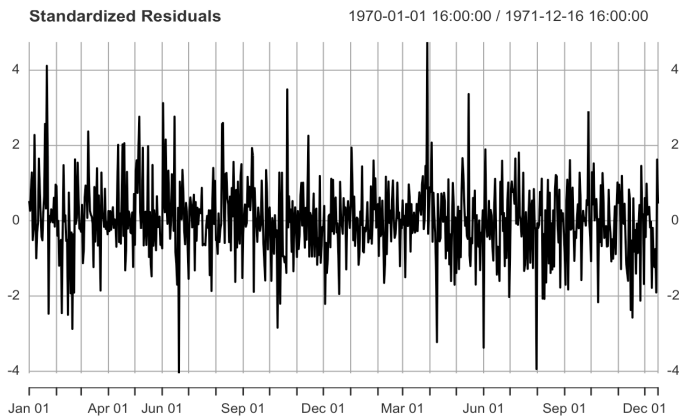
This plot comes from our GARCH model and shows the forecasted values, but more importantly the confidence intervals around the forecasts, calculated as two standard deviations (sigma) from the mean forecast of each point. We specifically tested it on the data from the first half of 2021, as that's where we saw the highest peak in the Tesla returns plot. The yellow bands illustrate the model's estimated volatility, with the sections with wider bands showing higher volatility and more uncertainty, while the narrower sections show lower volatility and more certainty in the forecasts. We can see how the estimated volatility changes after big dips or peaks in the actual series, as well as how besides a few large dips, the bands capture the volatility completely. Because almost all of the actual series falls in between the two bands, we can say that our GARCH model is pretty reliable and accurate.



The plot above shows the rolling forecasts of volatility (sigma) from the GARCH model we fit, plotted against the actual series of the Tesla returns data. The red line is the forecasted volatility from the GARCH model, while the gray line is the actual data. This plot is more focused on the trends relative to the actual volatility in itself, as the forecasted line clearly doesn't follow the same line as the actual series, but the forecasted structure of the volatility is similar. The rolling aspect of the model means the model is continually updated throughout based on the new data coming in. We can see how the forecasted volatility reacts to the previous data, and there are peaks at the right places where the actual series saw peaks as well. The forecasted line seems to be just a smoothed out version of the gray peaks and dips, which is typical in GARCH models, as they are designed to capture the general patterns of the volatility rather than mirroring every spike or drop. Overall, this plot shows again that our model did a pretty decent job fitting the

general volatility pattern of the stock, with room for improvement lying in doing a better job of capturing the magnitude of peaks in volatility.

To confirm our model did a good job of capturing the volatility, I looked at the ACF of the squared residuals from the model, as well as the overall standardized residuals plot.



The standardized residuals plot confirms our GARCH model does a good job fitting the noise and capturing the volatility clusters present in the data, as well as our ACF for the squared residuals showing no significant lags also confirms that. The scale of the ACF is really large, which is uncommon, however, the only way R could find any even potentially significant lags was to use a scale this large, which again suggests our model was good and fit the volatility rather well.

Conclusion

Tesla has continuously been a very volatile stock, especially in the early 2020s. While some of this is due to the constant fluctuation in Tesla's performance, a lot of the volatility can be attributed to the owner, Elon Musk. Elon is notorious for saying and tweeting some controversial things, which has historically had a major impact on the performance of Tesla's stock. While some of it can have a positive influence, usually his tweets and statements negatively affect the stock as a lot of potential buyers struggle with owning a piece of a company

where they do not agree with what the owner believes in or is saying politically. Another added layer to the volatility of Tesla specifically is how the company is often not viewed as a car company. While cars are their main product as of right now, a lot of people invest in Tesla for the potential of them as an AI company and potential new inventions in the future. Neurolink, something Elon has been publicly working on for a while now, is an example of the potential a lot of people are willing to invest in. This is why Tesla is often valued differently than other car companies, as well as way more volatile, as they are usually valued and traded as more than just a car company, but based on their potential in the AI world. Because of this volatility, it made sense to try and fit a GARCH model as they are meant to capture and estimate the volatility of usually financial markets. Due to option stocks being estimates on how high or low one thinks the Tesla stock price would strike, estimating volatility is the perfect model for that. Based on the GARCH model we produced, I was able to see a relatively good fit as a majority of the actual series was captured between the two sigma bands. In addition, the sigma forecasts for the most part copied the shape and structure of the series as well, but did not grasp the magnitude of some of the spikes in the raw data.