Chapter 4 Bond Valuation

Bond: interest-bearing certificate of public or private indebtedness.

Interest Rate : nominal annual rate
(Coupon rate) compounded semi annually.

Yield Rate : "

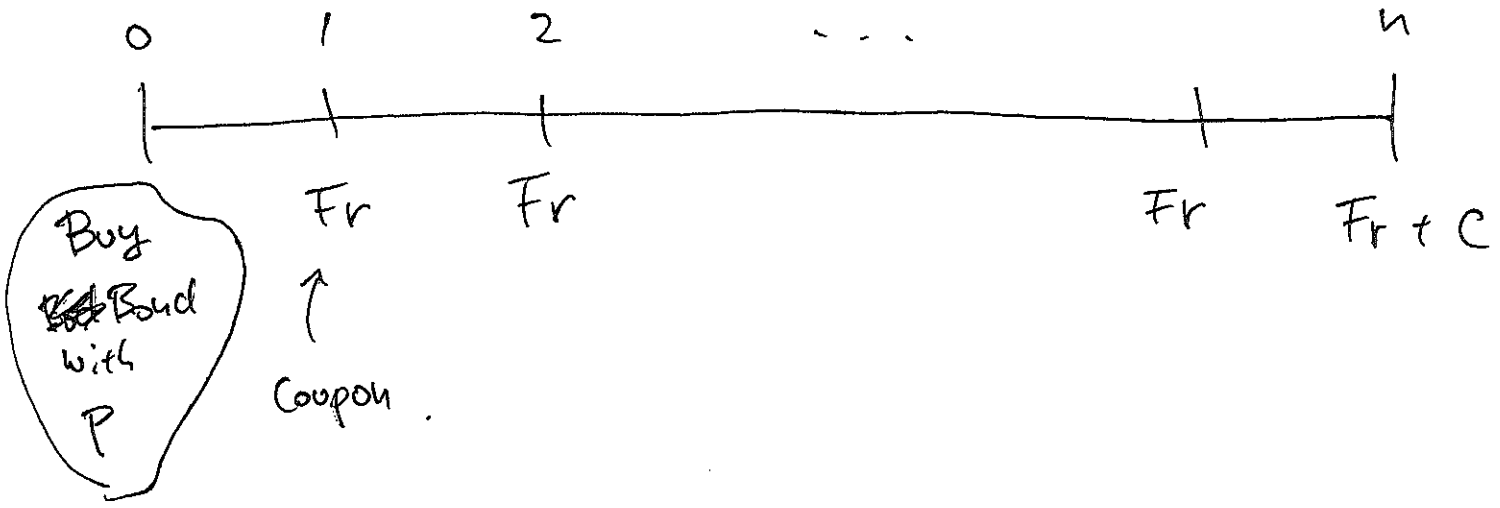
(used to calculate price of bond).

F = Face value

r = Coupon rate per period. (semiannual)

C = redemption value. ($= F$)

n = # of coupon periods until maturity.



$$P = C v^n + Fr\, a_{\overline{n}|j}$$

j = effective yield rate per period

on issue date.

Ex 4.1

10% bond with Face amount of 100,000,000.

Matures in 20 years.

a) Find price of the bond on issue date using yield rate of 5%.

b) Find price of bond just after the payment of 20th Coupon.

a)

$$P = F v^{40} + Fr\, a_{\overline{40}|j}$$

$$j = \frac{.05}{2}$$

$$r = \frac{.1}{2}$$

$$v = \frac{1}{1+j}$$

$$F = 100{,}000{,}000$$

$$= 162{,}756{,}938.$$

b)

$$P = F v^{20} + Fr\, a_{\overline{20}|j}$$

$$= 138{,}972{,}906.$$

Premium or Discount,

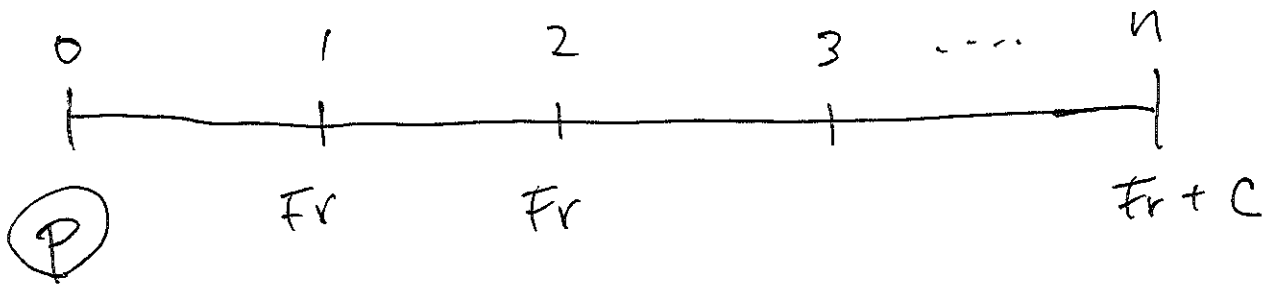
price Face value

$P > F \iff r > j$ at premium.

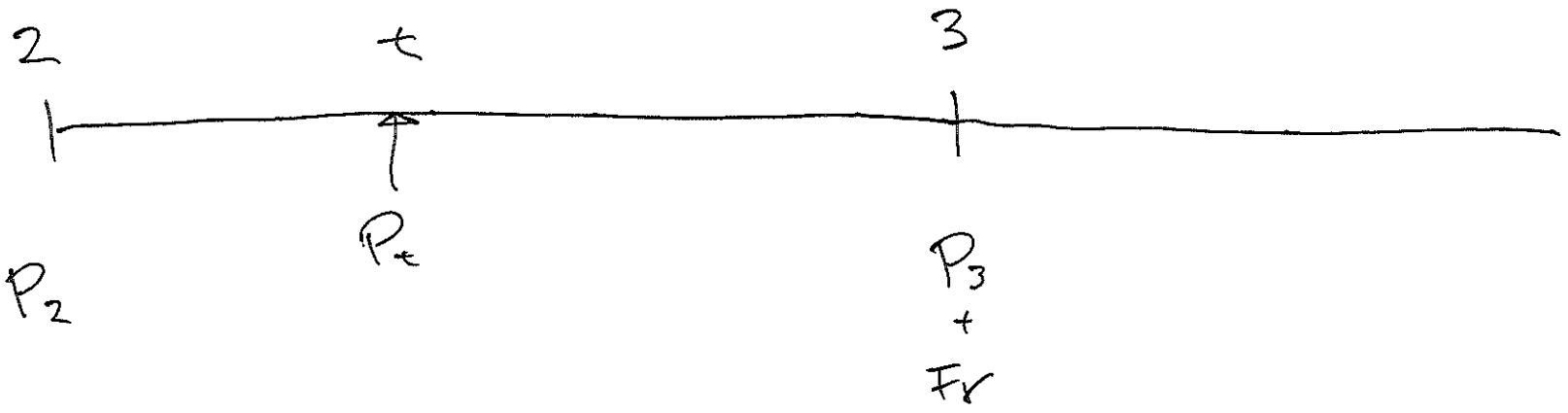
$P = F \iff r = j$ at par

$P < F \iff r < j$ at discount

Bond Price b/w Coupon dates.



P: present value of the bond
with 1st coupon starting in 1 period.



P_2 = present value of the bond with $n-2$ coupons starting in 1 period.

P_3 = " with $n-3$ coupons starting in 1 period.

$$P_t = P_2 (1+j)^t$$

$$= [P_3 + Fr] v^t$$

price-plus-accrued
"dirty price"
"flat price".

$$P_t = P_2 (1+j)^t$$

j = Yield rate for 1 period.

$$t = \frac{\#\text{ of days since last coupon paid}}{\#\text{ of days in the coupon period}}$$

$0 < t < 1$.

dirty price

$\times$

Quoted in Newspaper { clean price / price / Market price } $=$ [dirty price] $-$ $(Fr)\,t$.

actual/actual method.

Single interest approximation

Instead of $P_t = P_2 (1+j)^t$

do $P_t = P_2 (1 + jt)$ simple interest.

Ex 4.2

10% bond

Face amount of $100.

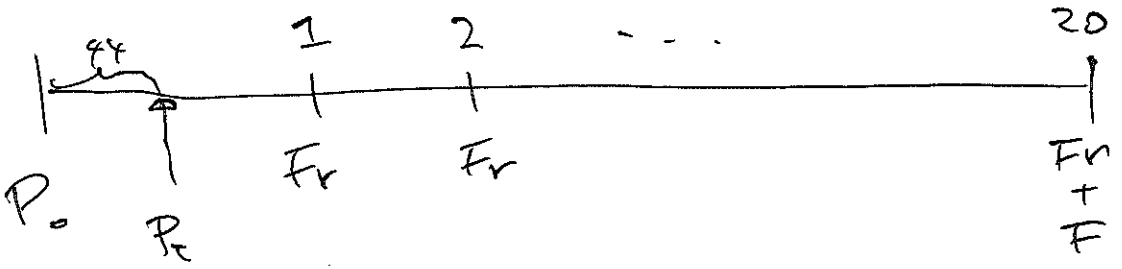
44 days since last coupon payment.

Last payment was on June 15. Next is Dec 15.

183 days

20 coupon payments left.

Yield rate is 5%.



$F = 100$.

$$P_0 = F v^{20} + Fr\, a_{\overline{20}|j}$$

$$= 138,972,906$$

$r = .1/2$

$j = .05/2$

$v = \frac{1}{1+j}$.

$$P_t = P_0 (1+j)^{44/183} = 139.800$$

Dirty price

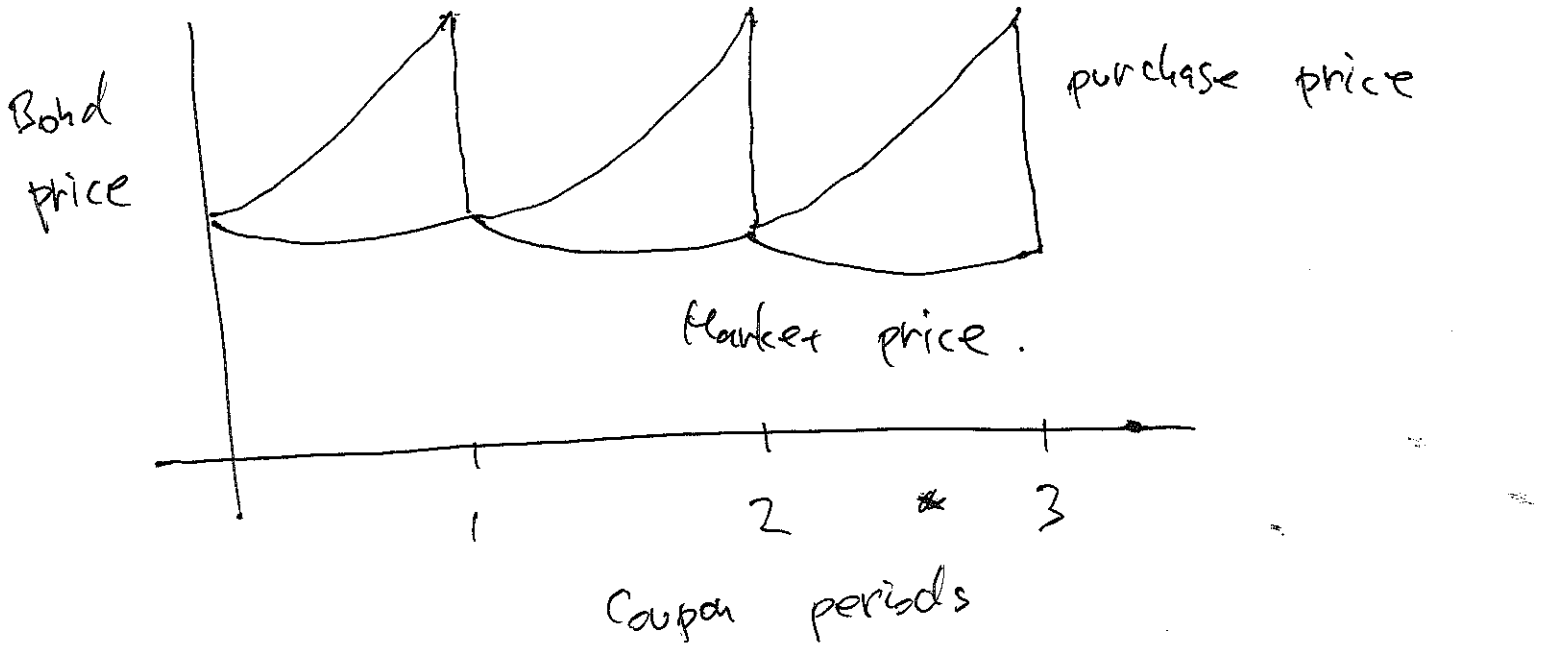
Market price

$$P = P_t - Fr \cdot \frac{44}{183} = 138.598.$$

Book value of a Bond

Book value : assigned value of a bond at time for reporting pourposes.

usually, original yield rate (at which the bond is ~~used~~ purchased) is used.



Finding Yield Rate

Ex 4.3

20 year 8% bond $F = 100$

purchase price = 70.400

a) what is the yield rate?

a) if semi-annual Yield rate $= j$.

$$70.4 = 100\, v^{40} + (100)(.04)\, a_{\overline{40}|j}$$

$$j = .059565$$

(nominal) Yield rate $= 11.913\%$
annual

5)

Bond was bought by new purchaser for price of 112.225 just after 10th Coupon payment.

i) Find Yield rate for new purchaser

ii) Find Yield rate for original holder.

b)

i) 30 coupon remaining.

$$112.225 = 100\,v^{30} + 100\left(\frac{.04}{\quad}\right) a_{\overline{30}|j}$$

$$j = .033479$$

ii)

$$70.400 = 112.225\,v^{10} + 100\left(\frac{.04}{\quad}\right) a_{\overline{10}|j}$$

$$j = .09500$$

iii) Suppose original holder deposit all coupons into an account with $i^{(2)} = 6\%$.

Find average eff. rate of return in 1st 5yrs.

Time of 10th deposit.

$$4 \cdot s_{\overline{10}|.03} = 45.86.$$

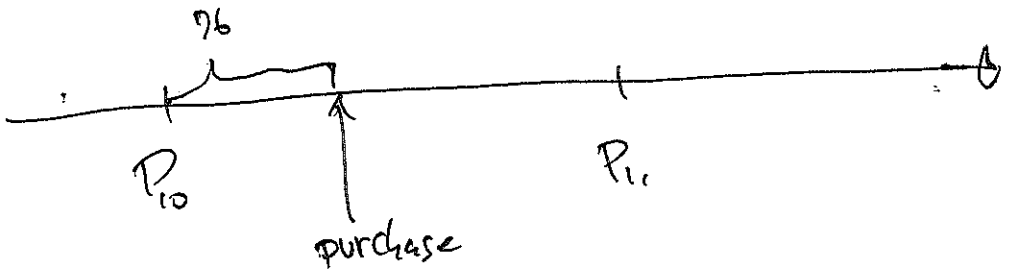
From the sales of the bond he has 112.225

total 158.08.

$$158.08 - 70.40 = (1+i)^5 \qquad i = 17.6\%$$

c) Suppose the new purchaser bought the bond 76/181 days after 10th coupon payment. (Jan 15 ~ July 15) ($112.225 market price)

(i) Find yield rate for new purchaser.



$$P_{10} = \left(100\, v_j^{30} + 4\, a_{\overline{30}|j}\right)$$ dirty price.

$$\text{dirty price} = P_{10}(1+j)^{76/181}$$

$$\text{market price} = P_{10}(1+j)^{76/181} - 4 \cdot \frac{76}{181} = 112.225.$$

Solve for $j = .033421$ 6.684% nominal ann. yield rate compounded semi-ann.

(ii) Find Yield rate for original holder.

(5 yrs + 76 days.)

(purchase price) (dirty price)

$112.225 + 4 \cdot \frac{76}{181} = 113.905$

~~113.905 = 100 j~~

$$70.40 = 113.905\, v_j^{10 + 76/181} + 4\, a_{\overline{10}|j}$$

$$j = .093054$$

4.2 Amortization of Bond

Loan:

$$OB_{t+1} = OB_t(1+i) - K_{t+1}.$$

(K_{t+1}: Loan repayment)

Bond.

$$BV_{t+1} = BV_t(1+j) - Fr$$

(j: yield rate; Fr: Coupon payment)

Ex. 4.4

10% with $F = 10{,}000$ matures in 4 years.

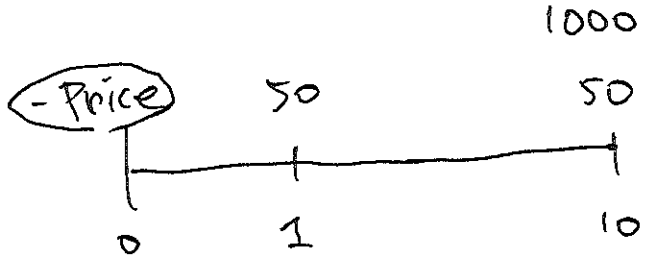
Construct amortization shedule for bond when annual yied rate is 8%.

k	Outstanding Balance	Payment	Interest Due	Principal Repaid
0	10,673.27	–	–	–
1	10,600.21	500	426.93	73.07
2	10,524.22	500	424.01	75.99
3	10,445.19	500	420.97	79.03
4	10,363.00	500	417.81	82.19
5	10,277.52	500	414.52	85.48
6	10,188.62	500	411.10	88.90
7	10,096.16	500	407.54	92.46
8	0	10,500	403.85	10,096.15

Callable Bonds

10 yr 10% semi-ann coupon callable at par after 5 yr. end of

Not called



Called at earliest.

	Price	Worst Case
At Discount :	< 1000	Not Called
At Premium :	> 1000	Called Earliest.

use worst case to calculate min yield.

Ex At discount.

Price
-900

N	I/Y	PV	PMT	FV
20	(?)	-900	50	1000

→ 5.8621

worst case

N	I/Y
10	(?)

→ 6.3835

At Premium

Price = 1100

N	I/Y	PV	PMT	FV
20	(?)	-1100	500	1000

→ 4.2479

N	I/Y
10	(?)

→ 3.7805

best case