ARIMA forecasting

Suppose $\nabla X_t = Y_t \sim ARMA(p, q)$

We know how to get $\hat{Y}_n(h) = a_1 Y_n + \cdots + a_n Y_1$

How can we calculate $\hat{X}_n(h)$ such that

$$E\left\{\left(X_{n+h} - \hat{X}_n(h)\right)^2\right\}$$ is minimized?

$\{1, X_0, X_1, \ldots X_n\}$ span the same space as

$\{1, X_0, Y_1, \ldots, Y_n\}$

because

$$\begin{aligned} X_1 - X_0 &= Y_1 \\ X_2 - X_1 &= Y_2 \\ X_3 - X_2 &= Y_3 \\ &\vdots \\ X_n - X_{n-1} &= Y_n \end{aligned} \quad \Rightarrow \quad \begin{aligned} X_1 &= X_0 + Y_1 \\ X_2 &= X_1 + Y_2 \\ X_3 &= X_2 + Y_3 \\ &\vdots \\ X_n &= X_{n-1} + Y_n \end{aligned}$$

minimize $E\{(X_{n+1} - \hat{X}_n(1))^2\}$ where

$$\hat{X}_n(1) = a_0' + a_1' X_n + \cdots + a_n' X_1$$

$$= a_0 + a_1 Y_n + \cdots + a_n Y_1 + b_0 X_0$$

$$= E\{(\underbrace{Y_{n+1} + X_n}_{X_{n+1}} - \hat{X}_n(1))^2\} \quad \text{——} \quad ☆$$

X_n given.

If we let $\hat{X}_n(1) = \hat{Y}_n(1) + X_n$, then

$$☆ = E\{(Y_{n+1} - \hat{Y}_n(1))^2\}$$

minimizer of $E\{(Y_{n+1} - \hat{Y}_n(1))^2\}$ with

$$\hat{Y}_n(1) = a_0 + a_1 Y_n + \cdots a_n Y_1 + b_0 X_0 .$$

$b_0 = 0$ if X_0 is uncorrelated with $\{Y_i\}$.

$$\hat{X}_n(h) = X_n + \hat{Y}_n(h)$$

"Unit - Root" test

Dickey - Fuller test.

ARMA(p, q)

$\Phi(B) Y_t = \Theta(B) e_t$,

If $\Phi(x)$ has root outside of the unit circle ,

$\rightarrow$ Causal.

Y_t can be written using past e_t .

$\sum_{j=0}^{\infty} \psi_j e_{t-j}$

~~unique~~ stationary .

ψ_j unique .

If $\Phi(x)$ has root inside of the unit circle

→ Not Causal.

If you try to write Y_t using past e_t,

the expression is 'divergent'.

non-stationary.

→ You can write Y_t using future e_t

$$\sum_{j=0}^{\infty} \psi_j e_{t+j}$$

stationary

unique

but does not make sense applying to data.

→ You can reparametrize ~~non-causal~~ stationary non-causal AR ~~model~~

using different e_t.

To model stationary data, assume AR

with causal parameters.

→ If $\Phi(x)$ has root <u>on</u> the unit circle.

Y_t non-stationary.

nothing you can do.

(linear)
Time Series is non-stationary.

$\Downarrow$

$\Phi(x)$ has root on the Unit-circle.

Is series non-stationary?

$\Downarrow$

Does Series have root on unit circle

"unit root"..

Dickey - Fullar test

$$\begin{cases} H_0 : Y_t \text{ has unit root.} \\ H_A : H_0 \text{ is false.} \end{cases}$$

Ex. For AR(1) $\begin{cases} H_0 : \phi_1 = 1 \\ H_A : \phi_1 \neq 1 \end{cases}$

$$\hat{\phi}_1 \sim N(\phi_1 \ \ SD(\hat{\phi}_1))$$

t-test with

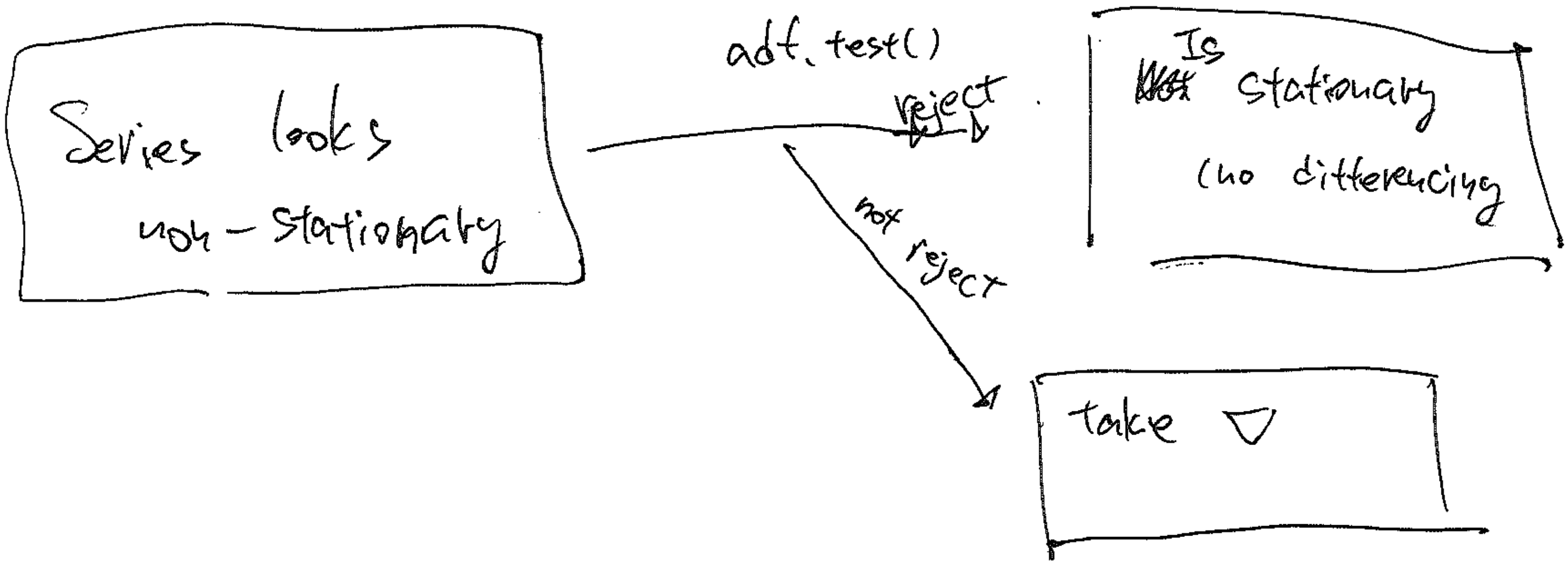
$$\frac{\hat{\phi}_1}{SE(\hat{\phi}_1)}$$

R code :

adf.test(x) in package "tseries"

augmented Dickey - Fuller .

Model Specification in ARIMA(p,d,q).



repeat.

~~check ACF of differenced~~

Over differencing

Suppose $\nabla X_t = Y_t$ iid.

$$Y_t = e_t.$$

Take ∇ again,

$$\nabla Y_t = \underbrace{e_t - e_{t-1}}_{\text{MA(1) with unit-root}},$$

$\to$ not invertible.

Overdifferencing

Suppose

$$\nabla X_t = Y_t \qquad Y_t \sim MA(1)$$

$$Y_t = e_t - \theta_1 e_{t-1}.$$

Take ∇ again,

$$\nabla Y_t = (e_t - \theta_1 e_{t-1}) - (e_{t-1} - \theta_1 e_{t-2})$$

$$= \underbrace{e_t - (1+\theta_1) e_{t-1} + \theta_1 e_{t-2}}_{MA(2)}.$$

∇Y_t is $MA(2)$ with

$$\Theta(x) = 1 - (1+\theta_1)x + \theta_1 x^2,$$

root is

$$1 - (1+\theta_1)x + \theta_1 x^2 = 0.$$

c b a

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{(1+\theta_1) \pm \sqrt{(1+\theta_1)^2 - 4\theta_1}}{2\theta_1}$$

$$= \frac{(1+\theta_1) \pm \sqrt{1 - 2\theta_1 + \theta_1^2}}{2\theta_1}$$

$$= \frac{(1+\theta_1) \pm \sqrt{(1-\theta_1)^2}}{2\theta_1}$$

$$= \frac{2}{2\theta_1} \quad \text{or} \quad \frac{2\theta_1}{2\theta_1}$$

unit root!

~~Te~~ Test unit-root in $MA(q)$ polynomials.

→ not fully resolved.

Test unit root in $MA(1)$

→ see if $\hat{\theta}_1$ is significantly different from ~~the~~ 1.

→ If not, the series may be over-differenced.