Repeat Example w/ $\omega = 2$.

$\omega = 2$

$\xi \sim U(0,1)$

$U(x) = 2x - x^2$

$E[\xi] = \frac{1}{2}$ $\qquad E[\xi^2] = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}$

Customer:

With Ins

$$U(\omega - G)$$

$$= 2(\omega - G) - (\omega - G)^2$$

$$= 2G - G^2$$

Without Ins

$$E[U(\omega - \xi)]$$

$$= E\left[2(\omega - \xi) - (\omega - \xi)^2\right]$$

$$= 4 - 2E[\xi] - E\left[(4 - 4\xi + \xi^2)\right]$$

$$= -1 + 2 - \frac{1}{3}$$

$$= \frac{2}{3}$$

Setting both side equal to e.o.

$$2G - G^2 = \frac{2}{3}$$

$$\frac{2}{3} - 2G + G^2 = 0$$

$$1 - 3G + \frac{3}{2}G^2 = 0$$

$$G = .4227,\ 1.578$$

$$\boxed{G_{max} = 1.578}$$

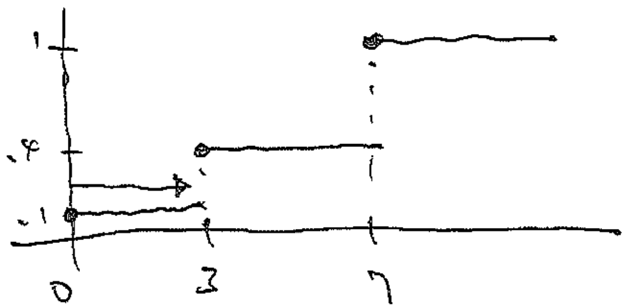
higer than the case of $\omega = 1$!

6 EXERCISES

Sections 1 and 2

1. Make sure that you indeed understand why if $E\{X\} = m$ and $Var\{X\} = \sigma^2 \neq 0$, then for the normalized r.v. $X^* = (X - m)/\sigma$, we have $E\{X^*\} = 0$, $Var\{X^*\} = 1$.
2. Find the 0.2-quantile of a r.v. X taking values $0, 3, 7$ with probabilities $0.1, 0.3, 0.6$, respectively.
3. This exercise concerns the VaR criterion with a parameter γ. R.v.'s X with or without indices correspond to an income.
   (a) Let X_1 take on values $0, 1, 2, 3$ with probabilities $0.1, 0.3, 0.4, 0.2$, respectively, and X_2 take on the same values with probabilities $0.1, 0.4, 0.2, 0.3$. Find all γ's for which $X_1 \succsim X_2$.
   (b) Let r.v. X_1 be uniform on $[0,1]$, and X_2 be exponential with $E\{\xi_2\} = m$. When is the relation $X_2 \succsim X_1$ true for all γ's? Let $m = 1/2$. Find all γ's for which $X_1 \succsim X_2$.
   (c) Let r.v.'s $X_1 = 1 - \xi_1$, $X_2 = 1 - \xi_2$, where the loss ξ_1 is uniform on $[0,1]$, and ξ_2 is exponential with $E\{\xi_2\} = m$. When is the relation $X_1 \succsim X_2$ true for all γ ? Let $m = 1/2$. Find all γ's for which $X_1 \succsim X_2$. (*Advice*: Compare just ξ's and observe that $q_\gamma(X) = 1 - q_{1-\gamma}(\xi)$.)
   (d) Let X_1 be uniform on $[0,3]$, and let X_2 be uniform on $[1,2]$. Find all γ's for which $X_1 \succsim X_2$.

Rotor 1-2

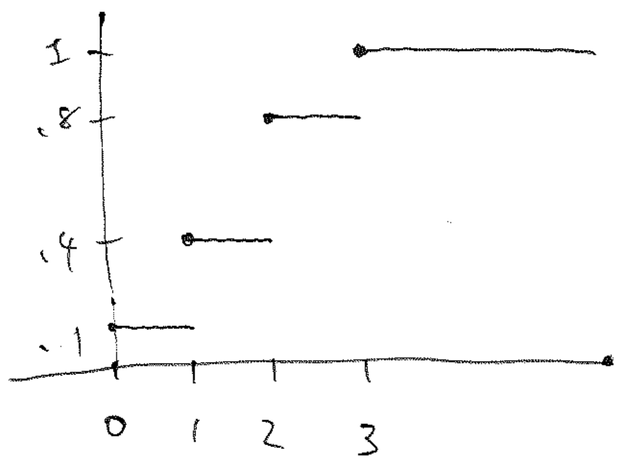


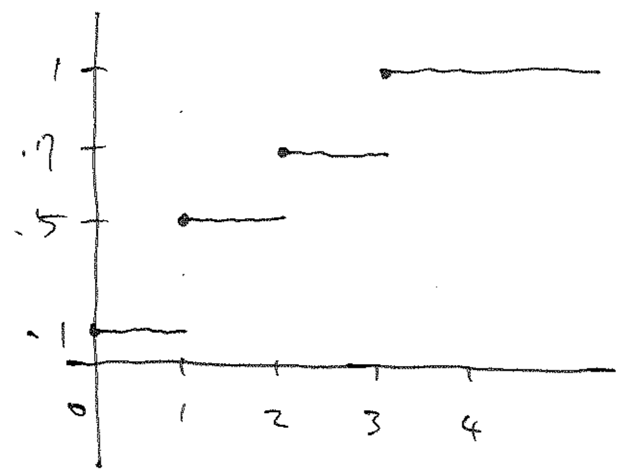
.2 quantile = 3

Rotar 1-3

VaR

a) $X_1 = \begin{cases} 0 & .1 \\ 1 & .3 \\ 2 & .4 \\ 3 & .2 \end{cases}$ $\qquad X_2 = \begin{cases} 0 & .1 \\ 1 & .4 \\ 2 & .2 \\ 3 & .3 \end{cases}$





a) cont'd

	X_1 VaR	X_2
γ = .9	3	3
.8	3	3
.7	2	3
.6	2	2
.5	2	2
.4	2	1
.3	1	1
.2	1	1
.1	1	1

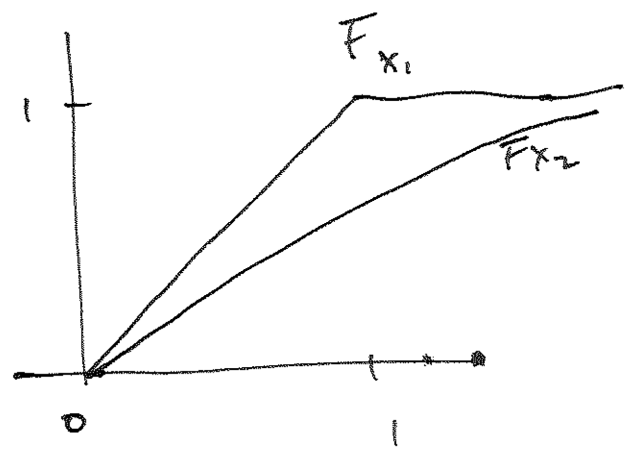
$X_1 \geq X_2$ when

$\gamma \in [0, .7)$ or $[.8, 1]$

b) $X_1 \sim U(0,1)$ $\qquad F_{X_1}(x) = x$

$X_2 \sim Exp(1/m) \qquad E(X_2) = m \qquad F_{X_2}(x) = 1 - e^{-\frac{x}{m}}$

Find m so that $X_2 \gtrsim X_1$ for all γ.



$$x \geq 1 - e^{-\frac{x}{m}} \quad \text{for } x \in (0,1)$$

$$1 - x \leq e^{-\frac{x}{m}}$$

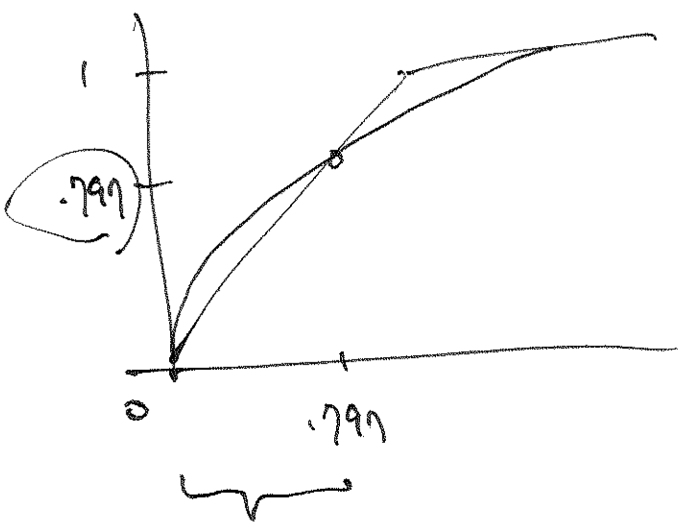
$$\log(1-x) \leq -\frac{x}{m}$$

$$m \geq \frac{x}{-\log(1-x)}$$

$$\boxed{m \geq 1}$$

b) cont'd

$m = \frac{1}{2}$. for what γ $x_1 \succsim x_2$?



$$X = 1 - e^{-2x}$$

$$-\frac{1}{2}\log(1-x) = x$$

$$x = .797$$

$\rightarrow$ $x_1 \succsim x_2$ or $\boxed{\gamma \in (0, .797)}$

c) $X_1 = 1 - \xi_1$ $\quad \xi_1 \sim U(0,1)$

$X_2 = 1 - \xi_2$ $\quad \xi_2 \sim Exp(1/m)$ $\quad E[\xi_2] = m$

$X_1 \sim U(0,1)$

$F_{X_2}(x) = P(1-\xi_2 < x) = P(1-x < \xi_2) = 1 - F_{\xi_2}(1-x)$

$= e^{-\frac{1}{m}(1-x)}$ $\quad x \in (-\infty, 1)$

$F_{X_1}(x) = x$ $\quad x \in (0,1)$

$$\text{VaR}: \left\{ \begin{array}{ll} X_1 & q_\gamma(X_1) = \gamma \\ X_2 & q_\gamma(X_2) = 1 + m\ln(\gamma) \end{array} \right]$$

$\ln(\gamma) = -\frac{1}{m}(1-x)$

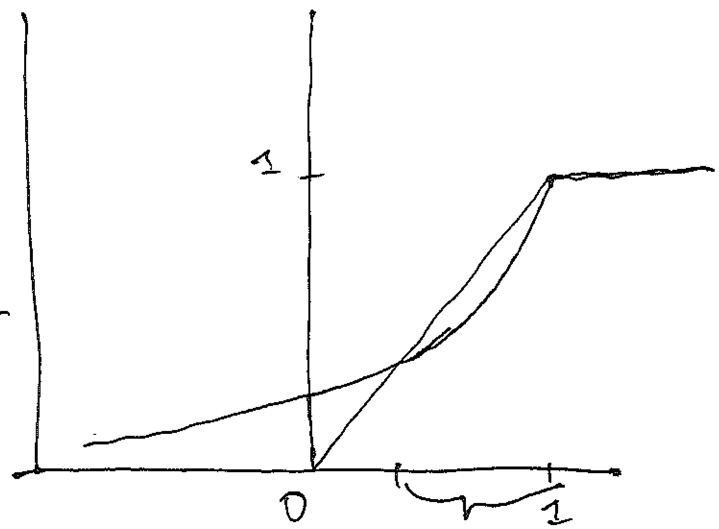
$-m\ln(\gamma) = 1 - x$

$x = 1 + m\ln(\gamma)$

$$\begin{cases} F_{X_1} = x & x \in (0, 1) \\ F_{X_2} = e^{-\frac{1}{m}(1-x)} & x \in (-\infty, 1) \end{cases}$$

$\boxed{X_1 \succeq X_2 \text{ If}}$

$m = \frac{1}{2}$



In this region, $X_2 \succeq X_1$.

$$X \leq e^{-\frac{1}{m}(1-x)} \qquad x \in (0,1)$$

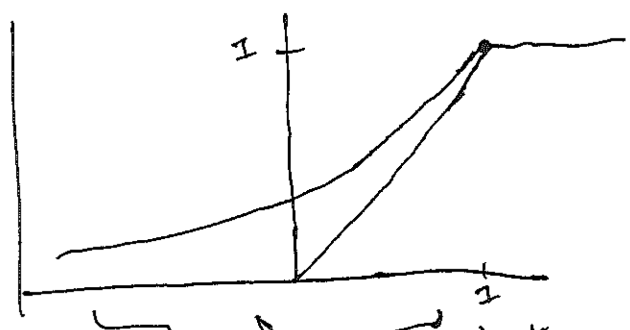
$$\ln X \leq \frac{X-1}{m}$$

$$m(\ln X) \leq X - 1$$

$$m \geq \frac{X-1}{\ln(X)} \qquad x \in (0,1).$$

$\ln(x)$ is neg.

$m = 1$



all region $X_1 \succeq X_2$

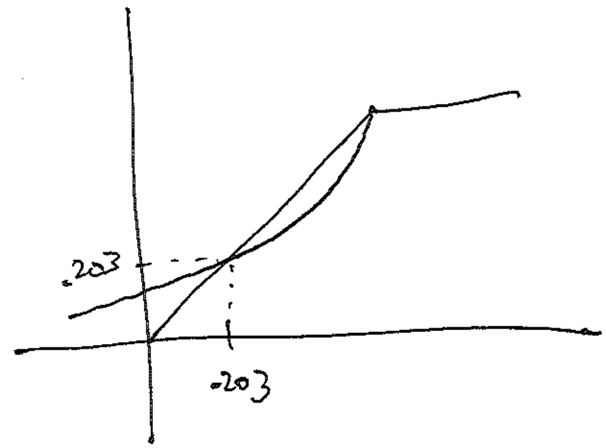
$$\boxed{m \geq 1}$$

c) cont'd   If $m=\frac{1}{2}$, then.

$$X = e^{-2(1-x)}$$

$$\frac{1}{2}\ln(x) = x-1$$

Solve for x using PC



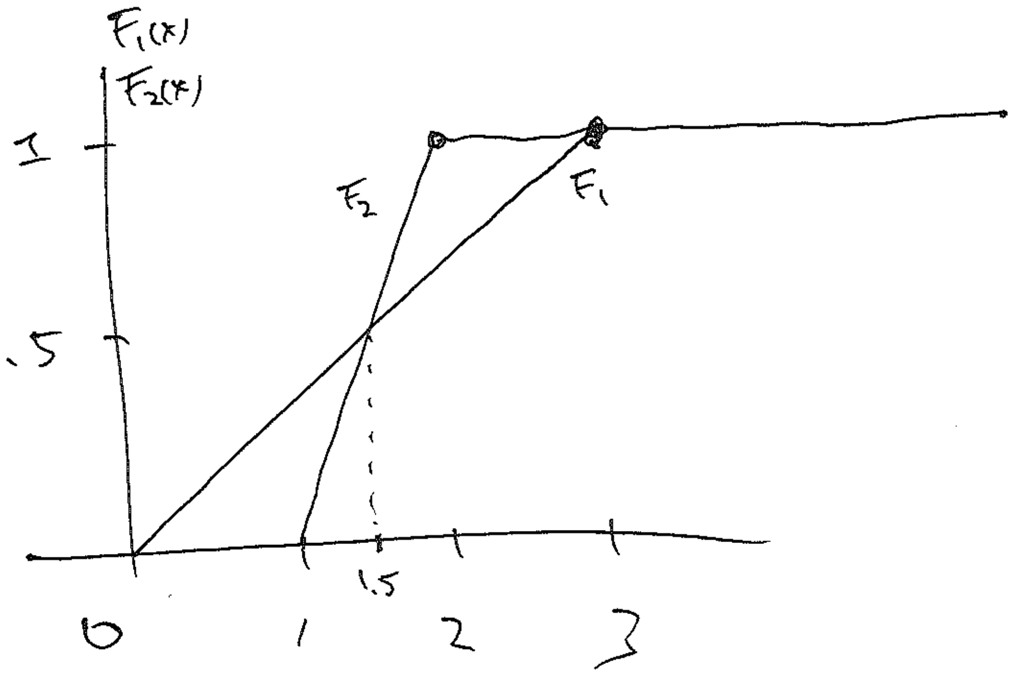
$$x \approx .203.$$

If $m=\frac{1}{2}$, $\boxed{\gamma < .203}$ then X_1 & X_2.

d)

$X_1 \sim U(0,3)$

$X_2 \sim U(\cancel{0}\,1,2)$



$X_1 \succsim X_2$ for

$\boxed{\gamma > .5}$

18. An EUM customer of an insurance company has a total wealth of 100 (in some units) and is facing a random loss ξ distributed as follows: $P(\xi = 0) = 0.9$, $P(\xi = 50) = 0.05$, $P(\xi = 100) = 0.05$.

(a) Let the utility function of the customer be $u(x) = x - 0.005x^2$ for $0 \leq x \leq 100$. Graph it. Is the customer a risk averter?

(b) What would you say in the case $u(x) = x + 0.005x^2$?

(c) For the case 18a, find the maximal premium the customer would be willing to pay to insure his wealth against the loss mentioned. First, set the equation clearly and explain it, then solve. Is the premium you found greater or less than $E\{X\}$? Might you predict it from the beginning?

(d) Find the minimal premium which an insurance company would accept to cover the risk mentioned, if the company's preferences are characterized by the utility function $u_1(x) = \sqrt{x}$, and the company takes 300 as an initial wealth. Is the premium you found greater or less than $E\{\xi\}$. Might you predict it?

(e) Solve Exercise 18d for the case when the r.v. ξ is uniformly distributed on $[0, 100]$.

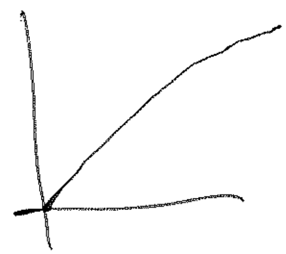
(f) Solve Exercise 18c for $u(x) = 200x - x^2 + 349$. (*Advice*: Look at this function attentively before starting calculations.)

Rotar #1.18

$W = 100$

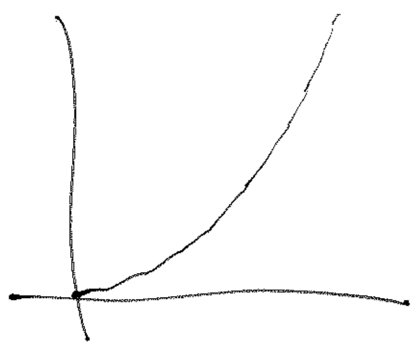
$$\xi = \begin{cases} 0 & \text{w.p. } .9 \\ 50 & .05 \\ 100 & .05 \end{cases}$$

a) Let $u(x) = x - .005x^2 \quad 0 \le x \le 100$



Risk - averter.

b) $u(x) = x + .005x^2 \quad 0 \le x \le 100$



Risk - seeker.

c) Customer premium : G

<u>With Ins</u>

$$U(w - G)$$

$$= 100 - G - .005(100 - G)^2$$

$$= 50 - .005G^2$$

<u>Without Ins</u>

$$E[U(w - \xi)]$$

$$= E[(w - \xi) - .005(w - \xi)^2]$$

$$= w - E[\xi] - .005\left(w^2 - 2wE(\xi) + E(\xi^2)\right)$$

$$= 46.875$$

$E(\xi) = 7.5$

$E(\xi^2) = 625$

$w = 100$

$$50 - .005G^2 = 46.875$$

$$\boxed{G_{max} = 25}$$

$> E(\xi)$ b/c he's risk-adverter.

(d) Company $\quad U(x) = \sqrt{x} \quad\quad w = 300$

C : Premium.

No Ins Sold	Ins Sold
$U(w)$	$E[U(w + c - \xi)]$
$= \sqrt{300}$	$= E\left[\sqrt{300 + C - \xi}\right]$

$$= \sqrt{300 + C}\,(.9) + \sqrt{300 + C - 50}\,(.05) + \sqrt{300 + C - 100}\,(.05)$$

Set equal to each other solve for C using PC.

$$\boxed{C_{min} = 8.05}$$

$> E(\xi)$ b/c company is risk-averter too.

e) Repeat (d) with $\xi \sim U(0, 100)$

<u>No Ins</u>

$$U(w) = \sqrt{300}$$

<u>Ins Sold</u>

$$E[U(w + c - \xi)]$$

$$= \int_0^{100} \sqrt{300 + c - x} \cdot \frac{1}{100} \, dx$$

$$= \frac{1}{100} \frac{2}{3} (300 + c - x)^{3/2} (-1) \Big|_0^{100}$$

$$= \frac{2}{300} \left[(300 + c)^{3/2} - (300 + c - 100)^{3/2} \right]$$

set equal, solve for c.

$$\boxed{c_{min} = 50.70} > E[\xi].$$

(f) $u_2(x) = 200x - x^2 + 349$

With Ins

$U(w - G)$

$= 200(w - G) - (w - G)^2 + 349$

Without Ins

$E[U(w - \tilde{z})]$

$= E[200(w - \tilde{z}) - (w - \tilde{z})^2 + 349]$

b/c new $U_2(x) = 200\,U(x) + 349$ using $U(x)$ of part (c), G_{max} will come out the same.

$\boxed{G_{max} = 25}$