471 - HW3

due Fri, Oct 14th

Name: Solution

1. Jimmy took a loan of amount T, which is amortized with monthly payments over five years at $i^{12} = 9\%$. The first payment is coming up in a month and is 1000. Calculate the outstanding loan balance immediately after the 40th payment is made.

Find T

N	I/Y	PV	PMT	FV
60	9/12	(T)	1000	0

→ -48173.37

Retrospective

N	I/Y	PV	PMT	FV
40	9/12	-48173.37	1000	(OB_{40})

→ 18508.02

— or —

Prospective

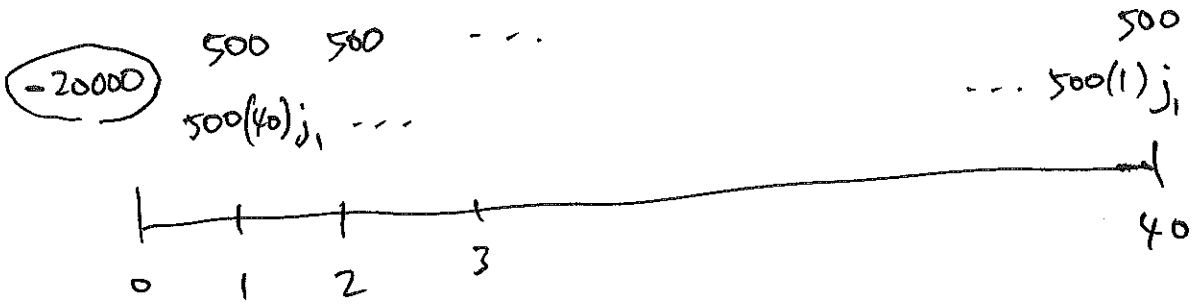
N	I/Y	PV	PMT	FV
20	9/12	(OB_{40})	1000	0

→ -18508.02

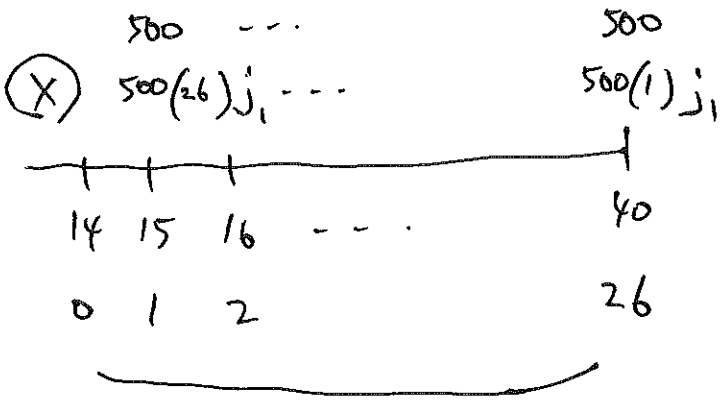
2. Johnny took a loan of 20,000 from a Bank A, with 40 level payments of principal at the end of each month. He also pays interest on the unpaid balance each month at a nominal rate of 10%, compounded monthly. Immediately after the 14th payment is made, Bank sells the rights to futme payments to another Bank B. Bank B wishes to yield a nominal rate of 12%, compounded semi-mnually, on its investment. What price does Bank A receive?

$i_1 = \frac{.1}{12} = .00833$ 1 mo.

$\frac{20{,}000}{40} = 500$



After the sale



Bank B yield 6% 6 mo.

$(1.06)^{1/6} - 1 = .0098$ 1 mo.

$j_2 = .0098$

$\Rightarrow$

$$X = 500\, a_{\overline{26}|j_2} + 500\,(j_1)\,(Da)_{\overline{26}|j_2}$$

$j_1 = .00833$

$j_2 = .0098$

$$= 11426.86 + (4,1650)\left(\frac{26 - a_{\overline{26}|j_2}}{j_2}\right)$$

1334.6754

$$= \boxed{}$$

$$\boxed{12761.54}$$

3000

3. A loan of ~~3500~~ will be repaid with 15 annual payments at the end of each year. You have following two options:

(a) Equal annual payments at an effective annual rate of 7%.

(b) Payments of 200 each year plus interest on the unpaid balance at an effective annual rate of k.

The sum of the payments under option (a) equals the sum of the payments under option (b). Determine k.

(a)

N	I/Y	PV	PMT	FV
15	7	-3000	(k) → 329.38	0

total PMTs = 15 (329.38)
= 4940.76

(b)

200(15)(kj) + 200
+ 200(14) kj +
+ ⋮ ⋮
+ 200(1) kj + 200

} Net PV Calculation.

200 [(15+14+⋯+1) j + 15] = Total PMTs
120 = 4940.76

kj = .0809

8.09 %

4. A loan of 5,000 is to be repaid by monthly interest payments to the lender at rate at nominal annual rate of 10% converted monthly, and monthly installments of 100 to a sinking fund earning nominal annual rate of 12% converted monthly. Find the total amount paid by the borrower over the course of the loan.

$j = \frac{.1}{12} = .0083$ 1 mo.

Int : $5000\,j = 41.67$

Sinking :

N	I/Y	PV	PMT	FV
(?)	1	0	100	-5000

$\hookrightarrow 40.15 \to 41$

Total amount paid :

$[41.67 + 100]\,41$

$= \boxed{5808.47}$

5. Harry borrows 2000 for 15 years at an annual effective interest rate of 10%. He can repay this loan with annual level payments of K at the end of each year. Instead, Harry repays using a sinking fund that pays an annual effective rate of 14%. The deposits to the sinking fund are equal to K minus the interest on the loan and are made at the end of each year for 15 years. How much is left in the sinking fund immediately after the loan is repaid?

Find K :

N	I/Y	PV	PMT	FV
15	10	-2000	(K)	0

↳ 262.95

Interest

Sinking Fund Deposit $K - 200 = 62.95$

N	I/Y	PV	PMT	FV
15	14	0	62.95	(?)

↳ 2759.77

Saved up at end of 15 in sinking fund.

After paying back 2000, Fund has

759.77 left.

6. An n-year bond with coupon rate of 4.60% is selling with a price of 100. A Bond with same length with coupon rate of 6% would sell for 110.71 for the same yield. The face and redemption amount of the bond is 150. Find the yield rate.

j = 6 mo. yield

Bond 1 $\quad 100 = 3.45\, a_{\overline{24}|j} + 150\, v^{24} \quad -①$

Bond 2 $\quad 110.71 = 4.5\, a_{\overline{24}|j} + 150\, v^{24} \quad -②$

Multiply ① by 4.5, multiply ② by 3.45, subtract one from the other.

$(4.5)① - (3.45)②$

$\Rightarrow \quad (4.5)100 - (3.45)110.71$

$= (4.5)150\, v^{24} - (3.45)150\, v^{24}$

Solve for v^{24},

$v^{24} = .4321$

Take eqn (1).

$$100 = 3.45\left(\frac{1-v^{24}}{j}\right) + 150\,v^{24}$$

$$j = \frac{3.45(1-.4321)}{100 - 150(.4321)}$$

$$= \frac{1.9593}{35.18}$$

$$= .0557$$

5.57 % eff. 6 mo.

0.1114 nom. ann. conv. semi-ann.

7. A 10-year bond with face value fo 2000 and coupon rate of 8% was purchased with to give effective annual yield of 10% until maturity. After the 13th coupon, the bond was sold at the price to give a seller effective annual yield of 12%. What price was the bond sold?

Yield = 5% 6mo. 4% 6mo. Coupon

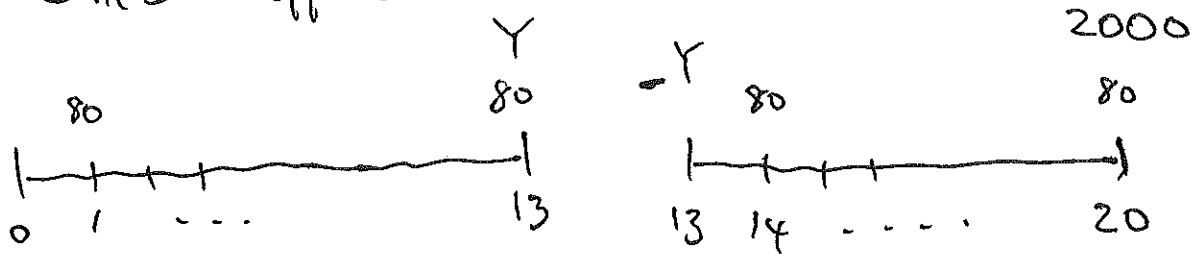
Find X (purchase price)

N	I/Y	PV	PMT	FV
20	5	(X)	80	2000

→ -1750.76

This shold have been (1.1)^(1/2) - 1 = 4.88%

When sales happens:



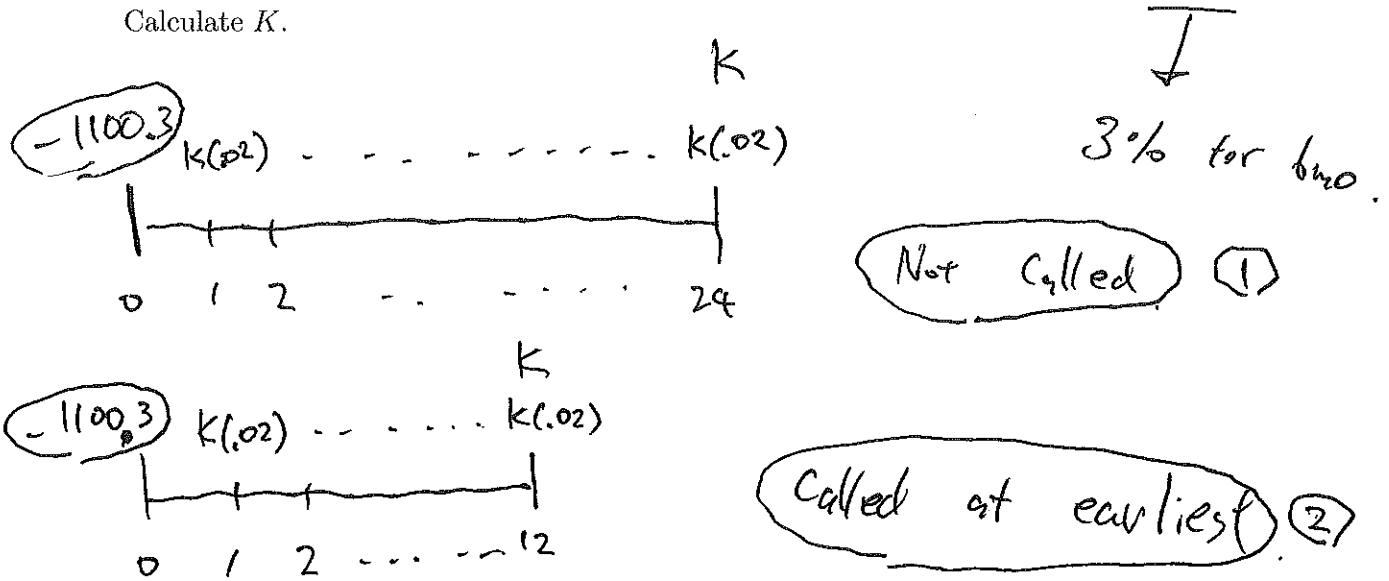
Seller Buyer

N	I/Y	PV	PMT	FV
13	6	-1750.76	80	(Y)

→ 2223.67

This should have been (1.12)^(1/2) - 1 = 5.83%

8. A 12-year par value bond was bought for 1100.30 with coupon rate of 4%. The bond is callable at par value K after the end of year 6. The price guarantees that the buyer will receive a nominal annual rate of yleid convertible semiannually of at least 6%. Calculate K.



This bond is bought at discount since

Yield % > Coupon rate. → use (1) to calculate K.

worst case

At Discount → Not Called

At Premium → Called Earliest

$$1100.3 = K(.02)\, a_{\overline{24}|.03} + K v^{24}$$

$$1100.3 = K\,[.3387 + .4919]$$

$$K = 1324.69$$