Q-C5

$N \sim \text{Poi}(\lambda = 1)$

X	1	2
	$2/3$	$1/3$

$E(S \mid S \le 2)$.

$P(S=0) = P(N=0) = e^{-1}$

$P(S=1) = P(X=1, N=1) = (2/3) \cdot e^{-1}$

$P(S=2) = P(X=2, N=1) + P(X_1=1, X_2=1, N=2)$

$= \frac{1}{3} e^{-1} + \left(\frac{2}{3}\right)^2 \frac{e^{-1} 1^2}{2!} = \frac{5}{9} e^{-1}$

$$E(S \mid S \le 2) = \frac{1 \cdot (\frac{2}{3}) e^{-1} + 2 \cdot (\frac{5}{9}) e^{-1}}{e^{-1} + (\frac{2}{3}) e^{-1} + (\frac{5}{9}) e^{-1}} = \frac{16/9}{20/9} = \boxed{\frac{4}{5}}$$

Q-C10

$N \sim Pois(\lambda = 1)$

X	1	2	3
	a	b	1-(a+b)

find b

$P(S=1) = .5e^{-1}$ ⟶ $P(X=1, N=1) = a e^{-1}$ $\quad a = .5$

$P(S=2) = \frac{3}{8} e^{-1}$ ⟷ $P(X_1=1, X_2=1, N=2)$

$+ P(X_1=2, N=1) = a^2 e^{-1} + b \frac{e^{-1} 1^2}{2!}$

$= \left(a^2 + \frac{b}{2}\right) e^{-1}$

$\left(\frac{1}{2}\right)^2 + \frac{b}{2} = \frac{3}{8}$

$b = \boxed{\frac{1}{4}}$

9-C14

$N \sim Poi(.6)$

X	1	2	3
	.2	.3	.5

$$P(S \geq 3) = 1 - P(S < 3) = 1 - P(S \leq 2)$$

$$P(S=0) = e^{-.6}$$

$$P(S=1) = P(X=1, N=1) = .2\, e^{-.6}(.6)$$

$$P(S=2) = P(X_1=1, X_2=1, N=2) + P(X_1=2, N=1) = \frac{(.2)^2 e^{-.6}(.6)^2}{2!} + (.3)\, e^{-.6}(.6)$$

$$= 1 - \left(1 + (.2)(.6) + \frac{(.2)^2(.6)^2}{2} + (.3)(.6)\right) e^{-.6} = \boxed{.281}$$

Q-D8

$S = X_1 + \cdots + X_8$ $\quad M = \#\text{ of male} \sim BIN(8, .4)$

$= S_1 + S_2$ $\quad S_1 = X_1 + \cdots + X_M$ ↩ not independent

Want $E(S) + SD(S)$ $\quad S_2 = X_{M+1} + \cdots + X_8$

$$E(S) = E(M)E(X_m) + E(8-M)E(X_F)$$

$$= 3.2(6) + 4.8(3) = \boxed{33.6}$$

$$V(S) = V(S_1 + S_2) \neq V(S_1) + V(S_2)$$

not independent

$$= E[V(S|M)] + V[E(S|M)]$$

$$= E[V(\overbrace{X_1 + \cdots + X_M}^{\text{male}} + \underbrace{X_{M+1} + \cdots + X_8}_{\text{female}})] + V[M \cdot 6 + (8-M) \cdot 3]$$

Q-58 cont'd

$$= E\left[V(X_1) + \cdots + V(X_M) + V(X_{M+1}) + \cdots + V(X_8) \right] + V\left[24 + 6M - 3M \right]$$

$$= E\left[M\,V(X_{male}) + (8-M)\,V(X_{fem}) \right] + 3^2 V(M)$$

$$= V(X_m) \cdot E(M) + V(X_f)\,E(8-M) + 9V(M)$$

$$= 64(3.2) + 31(4.8) + 9(8(.4)(.6))$$

$$= \boxed{370.88}$$

$$E(S) + SD(S) = 33.6 + \sqrt{370.9} = \boxed{52.9}$$

Q-D11

M = # of male $\sim BIN(100, .4)$

$Q = E(S) + 2\,SD(S)$

$S = \underbrace{X_1 + \cdots + X_M}_{\text{male}} + \underbrace{X_{M+1} + \cdots + X_{100}}_{\text{female}}$

$$E(S) = E(E(S|M)) = E\left[M E(X_m) + (100-M) E(X_f)\right]$$

$$= E(M) E(X_m) + E(100-M) E(X_f)$$

$$= 40\,(2) \qquad 60\,(4)$$

$$= \boxed{320}$$

9-D11

$$V(S) = E[V(S|M)] + V[E(S|M)]$$

$$= E\left[\underbrace{V(X_1)+\cdots}_{\text{male}}\underbrace{+V(X_{100})}_{\text{female}}\right] + V[M\,E(X_m) + (100-M)E(X_f)]$$

$$= V(X_m)E(M) + V(X_f)E(100-M) + V[400-2M]$$

$$= (4)\left(\frac{40}{100}\right) + (10)(60) + 4V[M]$$

$$100(.4)(.6)$$

$$= 856$$

$$P = 320 + 2\cdot\sqrt{856} = 378.5$$

9-611

for Q 6, $M = 40 \quad F = 60$ non-random.

$$E(S) = 40E(X_m) + 60E(X_f) = 320.$$

$$V(S) = V(X_1 + \cdots + X_{40}) + V(X_{41} \cdots X_{60})$$

$$= 40V(X_m) + 60V(X_f)$$

$$= 40(4) + 60(10) = 760$$

$$Q = 320 + 2\sqrt{760} = \boxed{374.8}$$

$$\frac{P}{Q} = \boxed{1.00999}.$$