#1

$$E(Y) = 1(.2) + 2(.1) + 3(.4) + 4(.3)$$

$$= \boxed{2.8}$$

$$E\left(\frac{1}{Y}\right) = \frac{1}{1}(.2) + \frac{1}{2}(.1) + \frac{1}{3}(.4) + \frac{1}{4}(.3)$$

$$= \boxed{.4583}$$

$$E(Y^2) = 1^2(.2) + 2^2(.1) + 3^2(.4) + 4^2(.3)$$

$$= \boxed{}\ 9$$

$$E(Y^2-1) = E(Y^2) - 1 = \boxed{5.06}$$

$$V(Y) = E(Y^2) - [E(Y)]^2 = 9 - (2.8)^2$$

$$= \boxed{1.16}$$

#2

$P(\text{drop out}) = .2$

$P(\geq 9 \text{ complete in 1 group})$

$= P(X \geq 9)$ when $X \sim BIN(10, .8)$

$= 1 - P(X = 10) = 1 - \binom{10}{10}(.8)^{10}(.2)^0$

$= .8926$

$P(\geq 9 \text{ Comp in 1 of 2 group but not in both})$

$= P(\geq 9 \text{ in group A}) + P(\geq 9 \text{ in group B})$

$- P(\geq 9 \text{ in both})$

by independence

$= P(\geq 9 \text{ in A}) \cdot P(\geq 9 \text{ in B})$

$= (.8926) + (.8926) - (.8926)^2 = \boxed{.9885}$

#3

X = # of International Student in the Council.

$X \sim Bin(10, .2)$

$P(X \leq 1)$ = pbinom(1, 10, .2)

$= P(X=0) + P(X=1)$

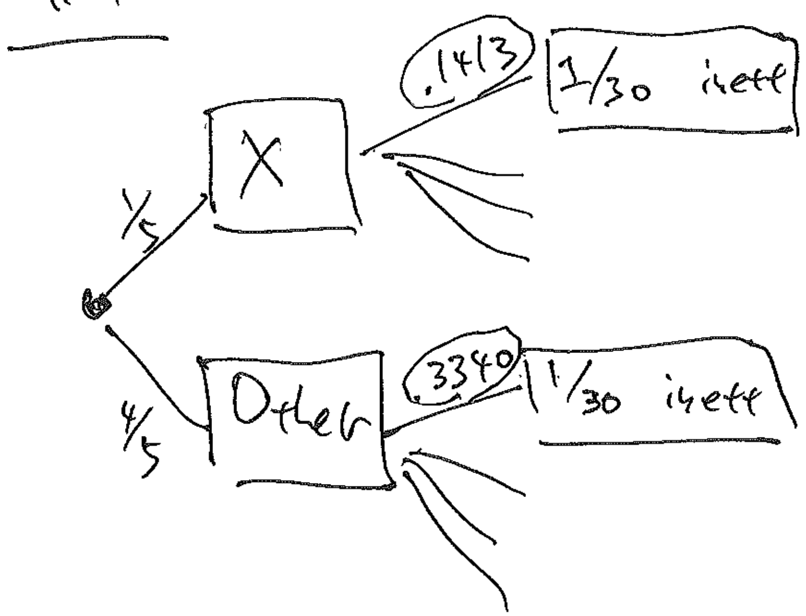
$= \binom{10}{0}(.2)^0(.8)^{10} + \binom{10}{1}(.2)^1(.8)^9$

$= .1074 + .2684$

$= \boxed{.3758}$

Prob. of Int'l Student under-represented in student council.

#4



$-\ ① = \left(\frac{1}{5}\right)(.1413)$

$-\ ② = \left(\frac{4}{5}\right)(.3340)$

$$P(\text{1 out of 30 ineff} \mid X)$$

$$= \binom{30}{1}(.1)(.9)^{29} = .1413$$

$$P(\text{1 out of 30 ineff} \mid \text{Other}) = \binom{30}{1}(.02)(.98)^{29} = .3340$$

$$P(X \mid \text{1/30 ineff}) = \frac{①}{① + ②} \quad \text{by Bayes' Formula.}$$

$$= \frac{\left(\frac{1}{5}\right)(.1413)}{\left(\frac{1}{5}\right)(.1413) + \left(\frac{4}{5}\right)(.3340)} = \boxed{.0956}$$

#5

$P(\text{Def}) = .1$

$P(\cancel{\text{}}\text{ line stopped within 8 hrs})$

$8 \cdot 60_{min} / 7_{min} = 68.57$

$= P(X < 68.57 \text{ products})$

$X =$ # of products before 10th defective.

$X \sim NB(10, .1)$

$= P(X \le 68)$

$=$ ~~[illegible]~~ pnbinom$(\cancel{68}\ 58, 10, .1)$

because in R, $X =$ # of failure before 10th success.

$= \boxed{.1385}$

#6

$$\left(\begin{array}{l} 10 \text{ Real} \\ 25 \text{ Fake} \end{array} \right.$$

$P($ Exactly 2 Fakes before 2nd Real. $)$

$$\underbrace{R\,F\,F}\ \underbrace{R}$$

$$\underbrace{F\,R\,F}\ \underbrace{R}$$

⋮

(1 R ~~&~~ 2F) . 1R out of 32 left.

$$\boxed{\frac{\binom{10}{1}\binom{25}{2}}{\binom{35}{3}} \cdot \frac{9}{32}}$$

#7

$\text{Poi}(1)$ per 12h.

$\Rightarrow \text{Poi}(2)$ per 24h.

$P(X \leq 1)$ in 24h

$= P(X=0) + P(X=1)$

$= e^{-2} + e^{-2} 2$

$= \boxed{.4060}$

7. Let X be a random variable with moment generating function

$$M(t) = (\frac{2+e^t}{3})^9.$$

Calculate the variance of X.

① $M'(t) = 9\left(\frac{2+e^t}{3}\right)^8 \cdot \frac{1}{3}e^t$

$M'(0) = 9 \cdot \frac{1}{3} = E[X]$

$M''(t) = 9 \cdot 8\left(\frac{2+e^t}{3}\right)^7\left(\frac{1}{3}e^t\right)^2 + 9\left(\frac{2+e^t}{3}\right)^8 \cdot \frac{1}{3}e^t$

$M''(0) = 9 \cdot 8\left(\frac{1}{3}\right)^2 + 9 \cdot \frac{1}{3} = E[X^2]$

$V[X] = E[X^2] - (E[X])^2 = 9 \cdot 8 \cdot \left(\frac{1}{3}\right)^2 + 9\frac{1}{3} - \left(9 \cdot \frac{1}{3}\right)^2$

$= -9\left(\frac{1}{3}\right)^2[+9\frac{1}{3}] = \boxed{9\frac{1}{3}[1-\frac{1}{3}]}$

② Notice that $M(t)$ is MGF of $BIN(n=9, p=\frac{1}{3})$

$V[X] = n \cdot p \cdot (1-p) = \boxed{9 \cdot \frac{1}{3}[1-\frac{1}{3}]}$