Chapter 5 Rate of Return of an Investment

Invest $100,000 → End up with K ~~\$___~~ annual payments for 20 years

Present Value

$$100{,}000 = K\, a_{\overline{20}|i}$$

solve for i.

i = effective ann. rate of return of investment.

"yield rate" "internal rate of return"

Internal Rate of Return (IRR).

future payments from investment $\quad K_1, K_2, \ldots, K_n$.

Present Value.

$$L = K_1 v + K_2 v^2 + \cdots + K_n v^n.$$

Solve for i.

if $L < \sum_{i=1}^{n} K_i$, solution for i is unique and positive.

if all $K_i > 0$, unique solution for $i > -1$.

Internal Rate of Return

Suppose transaction has net cashflow of

amounts $C_0, C_1, C_2, \ldots C_n$ at

times $t_0, t_1, t_k, \ldots t_n$, then

IRR for the transaction is rate i satisfying eq'n

$$\sum_{k=0}^{n} C_k v^{t_k} = 0$$

Example 5.1

- Buys 1000 shares of stock at 5.00 per share.
  Pay commission of 2 %.

- Six-month later, receive dividend of .20 per share.
  immediately reinvests with 4.00 per share. Commission free.

- Six-more month later buy another 500 shares at 4.50.
  pay 2% Com.

- Six-mo. later, receive divided of .25 per share.
  sell all shares @ 5.00 paying 2 % Comm.

Find $i^{(2)}$

$t=0$

$\rightarrow \quad \$5 \times 1000 \text{ shares } \underbrace{(1.02)}_{2\% \text{ commission}} = 5100 \qquad C_0 = -5100$

$t=1 \quad \underbrace{\$.20 \times 1000}_{\text{dividend}} = \$200 \quad \rightarrow \quad \frac{200}{4} = 50 \text{ shares.} \qquad C_1 = 0$

$t=2 \quad \$4.50 \times 500 \, (1.02) = 2295 \qquad C_2 = -2295$

$t=3 \quad \left.\begin{array}{l} \$.25 \times 1550 = 387.50 \\ \$5 \times 1550 \, (.98) = 7595 \end{array}\right\} \qquad C_3 = 7982.50$

$$5100 + 0 + 2295v^2 + 7982.50v^3 = 0$$

Solve for $j \Rightarrow j = 3.246\%$ $\qquad i^{(2)} = 2j = \boxed{6.49\%}$.

Example 5.2 Uniqueness of IRR

Line of Credit. balance can be + or −.

earns interest rate i regardless,

balance 0 at time = 2.

time	0	1	2
a)	+1	−2.3	+1.33
b)	+1	−2.3	+1.32
c)	+1	−2.3	+1.3125
d)	+1	−2.3	+1.2825

a) $1(1+i)^2 - 2.3(1+i) + 1.33$

no real solution for $(1+i)$.

b) $1(1+i)^2 - 2.3(1+i) + 1.32$

$(1+i) = 1.1$ or 1.2

c) $(1+i)^2 - 2.3(1+i) + 1.3125$

$(1+i) = 1.05$ or 1.25

d) $(1+i)^2 - 2.3(1+i) + 1.2825$

$(1+i) = .95$ or 1.35.

- IRR for given ~~transaction~~ sequence of cash flow may not be unique.

- If $C_0 > 0$ and $C_k < 0$ for $k = 1, \ldots, n$ then IRR is unique and > -1.

- Furthermore, if $\sum_{k=0}^{n} C_k < 0$ then $i > 0$.

- needs other method of comparing transactions,

NET Present Value. (NPV)

time t_0 t_1 t_2 ... t_n

Transaction A $(C_0, C_1, C_2, C_3, \ldots C_n)$

Transaction B $(D_0^{\$}, D_1, D_2, D_3, \ldots D_n)$

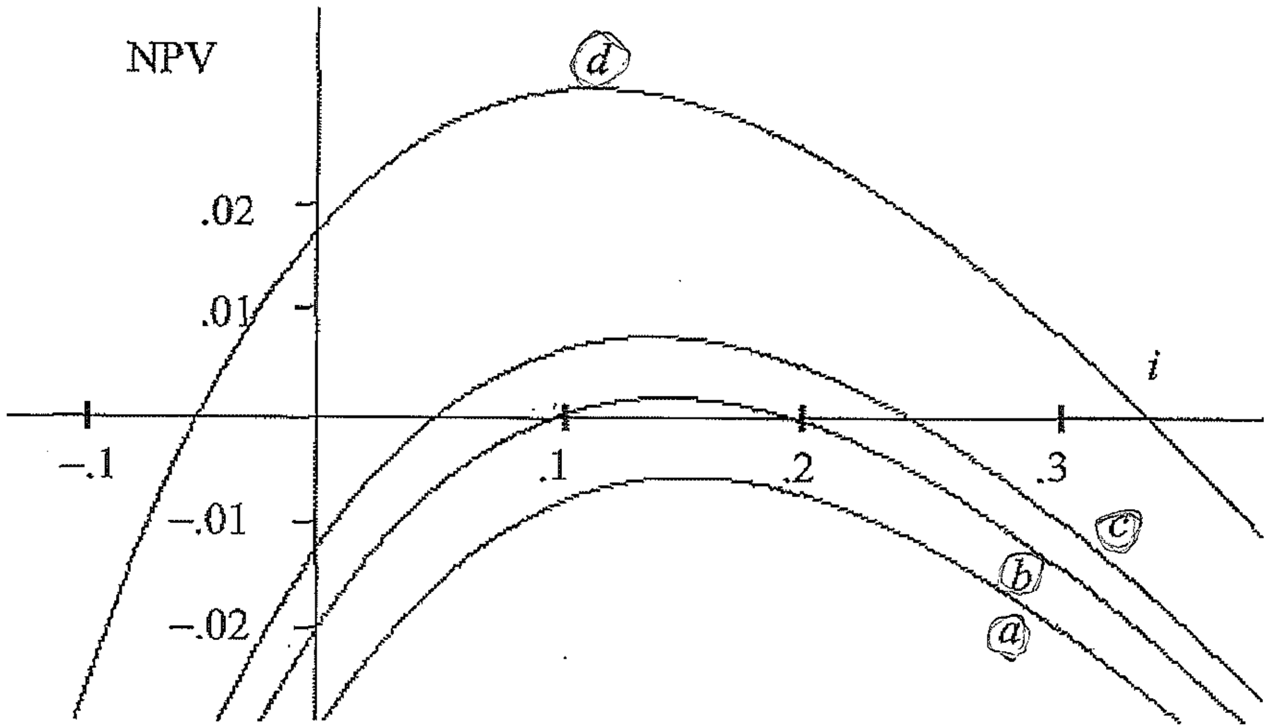
choose

i = "cost of capital"

"interest preference rate"

Compare by picking i, and

$$\left[\sum_{k=0}^{n} C_k \nu_i^{t_k} \right] \quad \text{vs} \quad \left[\sum_{k=0}^{n} D_k \nu_i^{t_k} \right]$$



NPV
.02
.01
−.01
−.02
−.1
.1
.2
.3
i
d
c
b
a

→ look at sign of NPV. + = profitable.

→ IRR = i such that NPV = 0.

5.2 Dollar-Weighted and Time-weighted Rate of Return

Dollar-Weighted RR

like IRR, but use simple interest.

solution i always exists.

Ex 5.3

2009	Feb	Aug	Oct	End
1M	+200,000	+200,000	-500,000	1,100,000

Dollar - weighted RR. assuming each mo. is exactly $\frac{1}{12}$.

$$1000{,}000\left(1+i\right)$$
$$+\,200{,}000\left(1+i\,\tfrac{10}{12}\right)$$
$$+\,200{,}000\left(1+i\,\tfrac{4}{12}\right) \quad \Rightarrow \quad i = .1739$$
$$-\,500{,}000\left(1+i\,\tfrac{2}{12}\right)$$
$$= 1{,}100{,}000$$

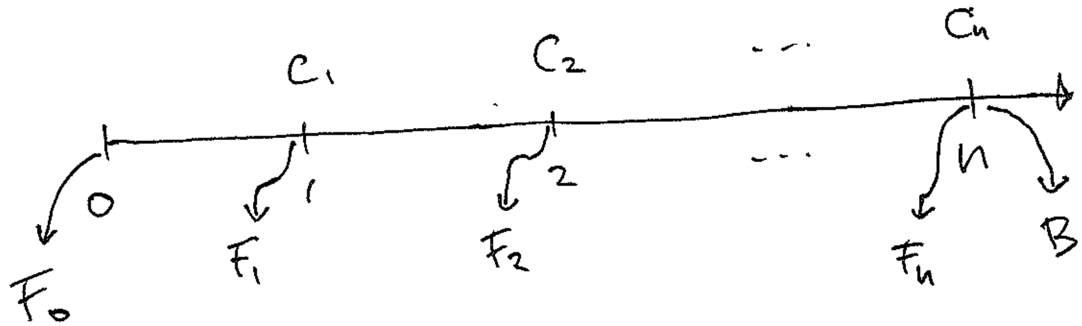
If IRR (compound interest) was used,

$$1{,}000{,}000\,(1+i) + 200{,}000\,(1+i)^{10/12} + 200{,}000\,(1+i)^{4/12} - 500{,}000\,(1+i)^{2/12} = 1{,}100{,}000$$

$$\Rightarrow \quad i = 1740$$

Time-Weighted RR

time interval may not be the same.



F_2 = balance in the fund just before the transaction C_2.

$$\left(\frac{F_1}{F_0}\right)\left(\frac{F_2}{F_1+C_1}\right)\left(\frac{F_3}{F_2+C_2}\right)\cdots\left(\frac{F_n}{F_{n-1}+C_{n-1}}\right)\left(\frac{B}{F_n+C_n}\right)-1$$

$$\left(\frac{1040{,}000}{1{,}000{,}000}\right)\left(\frac{1{,}400{,}000}{1240{,}000}\right)\left(\frac{1{,}580{,}000}{1{,}600{,}000}\right)\left(\frac{1{,}100{,}000}{1{,}080{,}000}\right)-1$$

$$= .1809$$

Portfolio Year rate

Interest rate earned by main fund.

Investment year rate

different interest rate for "new money"

Calendar Year of Original Investment	Investment Year Rates (in %)					Portfolio Rates (in %)
y	i_1^y	i_2^y	i_3^y	i_4^y	i_5^y	i^{y+5}
1992	8.25	8.25	8.40	8.50	8.50	8.35
1993	8.50	8.70	8.75	8.90	9.00	8.60
1994	9.00	9.00	9.10	9.10	9.20	8.85
1995	9.00	9.10	9.20	9.30	9.40	9.10
1996	9.25	9.35	9.50	9.55	9.60	9.35
1997	9.50	9.50	9.60	9.70	9.70	
1998	10.00	10.00	9.90	9.80		
1999	10.00	9.80	9.70			
2000	9.50	9.50				
2001	9.00					

Suppose that the amount in a fund is 1000 on January 1, 1997. Let the following be the accumulated value of the fund on January 1, 2000:

P: under the investment year method
Q: under the portfolio yield method
R: if the balance is withdrawn at the end of every year and is reinvested at the new money rate.

Determine the ranking of P, Q, and R.