Spring 2015 University of Akron, Dept. of Statistics

[3470 : 589/489] **Time Series Analysis**

Week 9 : Regression in Time Series

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Regressing Stationary TS

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9.1.1 OLS and GLS

From week 2 p.28,

OLS

- From model

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$$

- Minimizes sum of squares, $(\boldsymbol{Y} - \boldsymbol{X\beta})'(\boldsymbol{Y} - \boldsymbol{X\beta})$,

-

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y}.$$

GLS

- Minimizes weighted sum of squares,

$$(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta})'\boldsymbol{\Gamma}_n^{-1}(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta})$$

-

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{\Gamma}_n^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{\Gamma}_n^{-1}\boldsymbol{Y}.$$

If $\boldsymbol{X}$ is non-random, we have $E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$ and

$$V(\hat{\boldsymbol{\beta}}) = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{\Sigma}\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}.$$

For OLS, $\boldsymbol{\Sigma} = \sigma^2\boldsymbol{I}$, and

$$V(\hat{\boldsymbol{\beta}}) = \sigma^2(\boldsymbol{X}'\boldsymbol{X})^{-1}.$$

For GLS, $\boldsymbol{\Sigma} = \boldsymbol{\Gamma}_n$, and

$$V(\hat{\boldsymbol{\beta}}) = (\boldsymbol{X}'\boldsymbol{\Gamma}_n^{-1}\boldsymbol{X})^{-1}.$$

It can be shown that GLS estimator is the best linear unbiased estimator of $\boldsymbol{\beta}$. Therefore, GLS is better than OLS, but we need $\boldsymbol{\Gamma}_n$.

Iterative Regression Scheme

1. Compute OLS. Get residuals.

2. Fit ARMA model to the residuals by Gaussian Maximum Likelihood.

3. Using the fitted model, calculate $\mathbf{\Gamma}_n$. Go back to the original series, compute GLS.

4. Repeat from 2).

9.1.2 Regressing TS to TS

Sometimes, you want to model your TS lniarly dependaent on other time series.

- Sales depends on Global Economy
- Utility Demand depends on Temparature
- Level of toxin depends on local industory production

In that case, your Model is that the time series of interest X_t, has linear relstionship with your independent series B_t, plus stationary noise. In the vector notation,

$$\begin{aligned} X_t &= \boldsymbol{\beta} \boldsymbol{B}_t + Y_t \\ Y_t &\sim ARMA \end{aligned}$$

where x_t contains the "explanatory" TS.

In scalar notation,

$$X_t = a + bB_t + Y_t \qquad Y_t \sim ARMA$$

where x_t contains the "explanatory" TS.

Example

`See TS-09_R.txt`

9.1.3 ARMAX model

ARMAX(2,1) is the model

$$Y_t = \boldsymbol{\beta x}_t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t + \theta_1 e_{t-1}$$

Don't confuse this with additive model

$$\begin{aligned} K_t &= \boldsymbol{\beta x}_t + Y_t \\ Y_t &\sim ARMA(2,1) \end{aligned}$$

If we write ARMAX using backwards operator,

$$
\begin{aligned}
Y_t &= \boldsymbol{\beta}\boldsymbol{x}_t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t + \theta_1 e_{t-1} \\
\Phi(B) Y_t &= \boldsymbol{\beta}\boldsymbol{x}_t + \Theta(B) e_t \\
Y_t &= \frac{\boldsymbol{\beta}\boldsymbol{x}_t}{\Phi(B)} + \frac{\Theta(B)}{\Phi(B)} e_t
\end{aligned}
$$

which is very hard to interpret.

Currently, there is no package that directly deal with ARMAX model.

9.2

Cointegration

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9.2.1 Cointegration and Spurious correlation

Notation:

$$
\begin{aligned}
X_t \sim I(1) \quad &:= \quad X_t \text{ is integrated series of order 1} \\
&= \quad \nabla X_t \text{ is stationary}
\end{aligned}
$$

e.g. If X_t is ARIMA(2,1,2), then $X_t \sim I(1)$.

Cointegrated Relationship

Two $I(1)$ series have linear relationship?

$$
\begin{aligned}
W_t &\sim I(1) \\
U_t &= a + bW_t + Y_t
\end{aligned}
$$

where Y_t is a stationary series.

Cointegration

Many Economic Theory implies cointegrated relationship

- Money Demand Model
- Permanent Income Model
- Unbiased Forward Rates Hypothesis
- Fisher Equation

What to do?

$$
\begin{aligned}
W_t &\sim I(1) \\
U_t &= a + bW_t + Y_t
\end{aligned}
$$

Just regress W_t on U_t. It should give you $\hat{a}$ and $\hat{b}$.

Spurious correlation

Now imagine two $I(1)$ series

$$\begin{aligned} W_t &= W_{t-1} + e_t \qquad & e_t \sim WN(0,1) \\ U_t &= U_{t-1} + v_t \qquad & v_t \sim WN(0,1) \end{aligned}$$

These two RW have nothing to do with each other. So if you model relationship between these two as

$$U_t = a + bW_{t-1} + Y_t,$$

you should not find $\hat{a}$ and $\hat{b}$ significant, at least most of times, because in true relationship, $a = 0$ and $b = 0$.
However, if we regress W_t on U_t, we get what's called spurious correlation.

Example

`See TS-09_R.txt`