3 ~~4~~ State Jensen's Inequality. Use the Inequality to show that when $X \sim Exp(2)$, we have $E(X^4) > 16$.

$$E(f(x)) \geq f(E(X))$$

when $f(x)$ is convex.

i.e. $f''(x) > 0$.

Let $f(x) = x^4$. then $f''(x) = 4 \cdot 3 \cdot x^2 \geq 0$

For $Exp(2)$, $E(x) = 2$. then since $f(x)$

~~[illegible]~~ is convex. by Jensen's ineq.

$$E(X^4) \geq [E(x)]^4 = 2^4 = 16.$$

///

4. Below is a two-way table for joint pmf of discrete r.v. X and Y. Compute $E(Y|X \geq 2)$.

	Y			
	0	1	2	3
X=1	.07	.15	.10	.08
X=2	.03	.11	.04	.06
X=3	.15	.05	.03	.13

adds to .6

(No need to simplify the expression.)

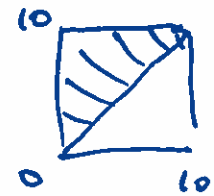
$Y | X \geq 2$

$Y=$	0	1	2	3
	$\frac{.18}{.6}$	$\frac{.16}{.6}$	$\frac{.07}{.6}$	$\frac{.19}{.6}$
	.3	.267	.117	.316

$E[Y | X \geq 2]$

$= 1(.267) + 2(.117) + 3(.316) = \cancel{.884}\ 1.449$

9. Let (X, Y) be random vector with joint pdf



$$f(x,y) = \begin{cases} 1/50 & \text{if } 0 < X < Y < 10 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P\Big\{\underbrace{(5 < 2X + Y < 10)}_{A} \cup \underbrace{(7X - 10Y > 30)}_{B}\Big\}$. [Obtain the numerical value for full credit].

$5 < 2X + Y$

$Y > -2X + 5$

and

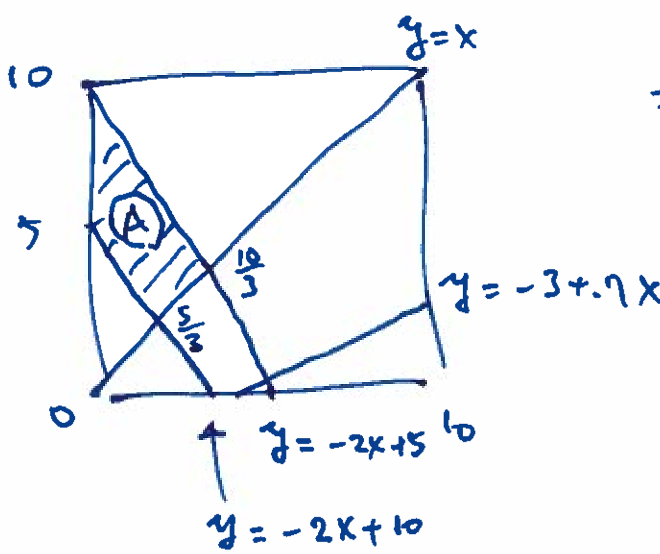
$Y < -2x + 10$

$-Y > 3 - .7X$

$Y < -3 + .7X$

$$P(A \cup B) = \iint_{A \text{ or } B} \frac{1}{50} \, dx \, dy$$

$$= \frac{1}{50} \iint_{A \text{ or } B} 1 \, dx \, dy$$



$$= \frac{1}{50} (\text{Area of } A)$$

$$= \frac{1}{50} \frac{\left[10\left(\frac{10}{3}\right) - 5\left(\frac{5}{3}\right)\right]}{2}$$

$$= \boxed{\frac{1}{4}}$$