451/551 - HW on Ch. 5 Joint Distributions

Due Mon Oct. 31st

The final answer must be clealy indicated.

Name: _____

1. A company is reviewing tornado damage claims under a farm insurance policy. Let X be the portion of a claim representing damage to the house and let Y be the portion of the same claim representing damage to the rest of the property. The joint density function of X and Y is

$$f(x,y) = 6(1-(x+y)) \qquad \text{for } x>0, y>0, x+y<1$$

and 0 otherwise. Determine the probability that the portion of a claim representing damage to the house is less than 0.5.

2. A joint density function is given by

$$f(x, y) = kxy^2 \qquad \text{for } 0 < x < 1, 0 < y < 2$$

and 0 otherwise, where k is a constant. What is Cov(X,y)? (hint: Are X, Y independent?)

3. The stock prices of two companies at the end of any given year are modeled with random variables X and Y that follow a distribution with joint density function

$$f(x,y) = 2x \quad for 0 < x < 1, x < y < x + 1,$$

and 0 otherwise. What is the conditional expectation Y given that X = x? (i.e. $E[Y|X]$)

4. Suppose a system is consisted by two components. The system needs both of the componens to operate properly. The joint density function of the lifetimes of the components, measured in hours, is $f(x, y) = k(x+y^2)$, where $0 < x < 2$ and $0 < y < 2$. What is the probability that the system fails within the first hour? Set up the integral with proper bounds, but do not evaluate the integral.

5. Claim amounts for wind damage to insured homes are modeled by random variable with CDF

$$F(x) = 1 - \frac{1}{x^2} \qquad \text{for } 1 \leq x,$$

and 0 for $x < 1$. Claim amounts are assumed to be independent from home to home. Suppose 3 such claims will be made. What is the expected value of the largest of the three claims? Write down the integral with proper bounds, but do not evaluate the integral.