9. The probability of a fire in a certain structure during a given period is 0.03. If a fire occurs, the damage is uniformly distributed on the interval $[0, 100]$ (say, the unit of money is \$10,000).

   (a) Assume that the insurance company covers the total damage and denote by Y the (random) amount the company will pay. Write $E\{Y\}$ and $Var\{Y\}$. Graph the distribution function of Y.

   (b) Do the same for the case when the insurance contract provides coverage above a deductible of 5.

   (c) Do the same for the case when the insurance contract provides coverage above a deductible of 5, and the maximum (limit) payment is 90 units.

Rotar

#2-9,

(a)
$$Y = \begin{cases} 0 & \text{w.p. } .97 \\ \xi & \text{w.p. } .03 \end{cases} \qquad \xi \sim U(0,100)$$

$$E[Y] = q\,E(\xi) = (.03)(50) = 1.5$$

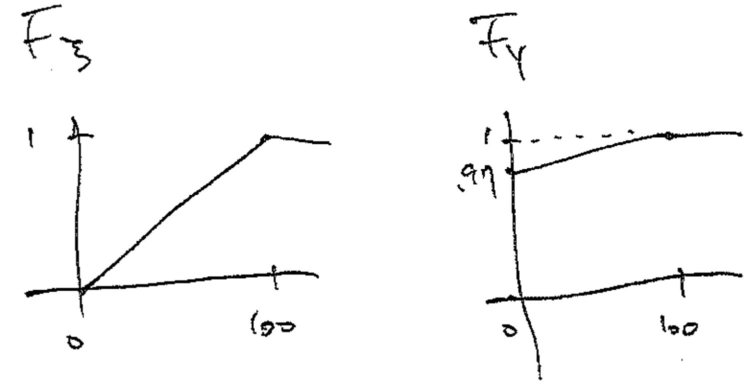
$$E[Y^2] = q\,E[\xi^2] = q\left(V(\xi) + E(\xi)^2\right) = (.03)\left(\frac{100^2}{12} + 50^2\right)$$

$$= 100$$

$$V[Y] = E[Y^2] - E[Y]^2$$

$$= q\,E(\xi^2) - q^2 E(\xi)^2 = 97.75$$

#2-9_2



(b)

$$r(Y) = \begin{cases} 0 & Y \le 5 \\ Y-5 & Y > 5 \end{cases}$$

$$E[r(Y)] = q\int_5^{100} (y-5)\cdot\left(\tfrac{1}{100}\right) dy = 1.354$$

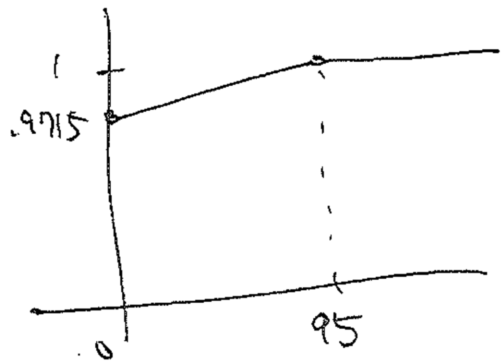
$$E[r(Y)^2] = q\int_5^{100} (y-5)^2 \left(\tfrac{1}{100}\right) dy = 85.74$$

$$V[r(Y)] = 85.74 - 1.354^2 = 83.90$$

2-a_3

$$F_{r(Y)}(t) = (1-q) + q\,F_Y(t+5)$$

$$F_{r(Y)}(0) = .97 + (.03)\left(\frac{0+5}{100}\right) = .9715$$



(c)

$$r(Y) = \begin{cases} 0 & Y \le 5 \\ Y-5 & 5 < Y \le 95 \\ 90 & 95 < Y \end{cases}$$

$$E[r(Y)] = q\int_{5}^{95} (y-5)\left(\frac{1}{100}\right) dy + q\int_{95}^{100} 90\left(\frac{1}{100}\right) dy$$

$$= 1.35$$

2-94

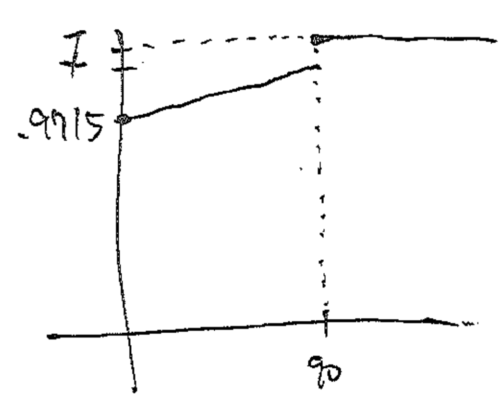
(c)

$$E[r(Y)^2] = q \cdot \int_{5}^{95} (y-5)^2 \left(\frac{1}{100}\right) dy + q \int_{95}^{100} 90^2 \left(\frac{1}{100}\right) dy$$

$$= 85.05$$

$$V[r(Y)] = 85.05 - 1.35^2 = 83.23$$

$$F_Y(y) = \begin{cases} (1-q) + q\, F_S(t+5) & t \le 90 \\ 1 & t > 90 \end{cases}$$



48. An insurance company has a portfolio of 2000 cars insured for a single period of one year. The probability that a particular car will be involved in an accident is 0.05. The losses for different cars are independent. The distribution of the damage, *if an accident occurs*, may be approximated by the exponential distribution with a mean of \$1000.

    (a) Graph the distribution function of a payment of the company *for a separate policy.*

    (b) Write the expected value and the variance of the payment for a separate policy, and for the total payment of the company.

    (c) Graph the distribution function of a payment of the company *for a separate car* in the case of a deductible of *\$200.*

    (d) Considering the insurance *without deductible,* assuming all claims to be independent, and using normal approximation, compute the security loading coefficient for which the probability that the company would not lose money by the end of the year is 0.95. Use the heuristic approach.

    (e) * Do the same using the rigorous estimation approach.

    (f) Not providing any calculations, write the answer for Exercise 48d for the case when the number of cars is equal to 4000.

12. Losses are modeled by the exponential distribution with a mean of 300. An insurance plan includes an ordinary deductible of 100 and pays 50% of the costs above 100 until the insured is paid 600. Then the plan pays 20% of the remaining costs. Find the expected payment in the case of loss event.

Rotan
#2-12

$\xi \sim Exp(1/300)$ $E(\xi) = 300$ $d = 100$.

$$r(X) \text{ given loss} = \begin{cases} 0 & \xi \le 100 \\ \dfrac{\xi - 100}{2} & 100 < \xi \le 1300 \\ 600 + \dfrac{\xi - 1300}{5} & 1300 < \xi \end{cases}$$

$$E[r(X) \mid \text{loss}] = 0 + \int_{100}^{1300} \frac{x-100}{2} \cdot \left(\frac{1}{300} e^{-x/300}\right) dx$$

$$+ \int_{1300}^{\infty} \left(600 + \frac{\xi - 1300}{5}\right)\left(\frac{1}{300} e^{-x/300}\right) dx$$

$= 106.3$

47. Consider two portfolios of 2000 and 3000 cars, respectively, insured for a single period of one year with \$1000 deductible. The damage per car (per year) is distributed as follows:

The first portfolio		**The second portfolio**	
Damage (in \$1000)	probability	Damage (in \$1000)	probability
0	0.78	0	0.8
< 1 ($\times$ \$1000)	0.12	< 1 ($\times$ \$1000)	0.1
6	0.05	6	0.08
11	0.05	11	0.02

(a) Assuming all claims to be independent, compute the expectation and the variance of the total amount of claims from the two portfolios.

Use the heuristic approach to normal approximation in order to estimate the security loading coefficient θ such that the probability that the insurance company will not suffer a loss by the end of the year is 0.99.

(b) Assume that the number of cars (clients) in both portfolios became twice as large. Will θ in this case be larger or smaller? Determine how θ will change.

(c) * Estimate θ for the case (a), using rigorous calculations.

#2-47

$n=2000$

X = 1st portfolio

$d=1$

$m=3000$

Y = 2nd portfolio.

a)

$E(X) = 5(.05) + 10(.05)$

$= .75$

$E(X^2) = 5^2(.05) + 10^2(.05)$

$= 6.25$

$V(X) = 5.6875$

$SD(X) = \sqrt{5.6875} = 2.38$

$E(Y) = 5(.08) + 10(.02) = .6$

$E(Y^2) = 5^2(.08) + 10^2(.02) = 4$

$V(Y) = 4 - .6^2 = 3.64$

$SD(Y) = \sqrt{3.64} = 1.91$

2-49$_2$

a)

$$E(S) = 2000\,E(X) + 3000\,E(Y) = \boxed{3300}$$

$$V(S) = V\left(X_1 + \cdots + X_{2000} + Y_1 + \cdots + Y_{3000}\right)$$

$$= 2000\,V(X) + 3000\,V(Y) = \boxed{22295}$$

Loading Coef. $Z_{.01} = 2.33$

$$\theta = 2.33\,\frac{\sqrt{V(S_4)}}{E(S_4)} = 2.33\,\frac{\sqrt{22295}}{3300} = .1054$$

b) $$\theta_{new} = \theta_{old}\,\frac{1}{\sqrt{2}} = .1054/\sqrt{2} = .0745$$

48. An insurance company has a portfolio of 2000 cars insured for a single period of one year. The probability that a particular car will be involved in an accident is 0.05. The losses for different cars are independent. The distribution of the damage, *if an accident occurs*, may be approximated by the exponential distribution with a mean of \$1000.

(a) Graph the distribution function of a payment of the company *for a separate policy.*

(b) Write the expected value and the variance of the payment for a separate policy, and for the total payment of the company.

(c) Graph the distribution function of a payment of the company *for a separate car* in the case of a deductible of *\$200.*

(d) Considering the insurance *without deductible,* assuming all claims to be independent, and using normal approximation, compute the security loading coefficient for which the probability that the company would not lose money by the end of the year is 0.95. Use the heuristic approach.

(e) * Do the same using the rigorous estimation approach.

(f) Not providing any calculations, write the answer for Exercise 48d for the case when the number of cars is equal to 4000.

2-48

$n = 2000$

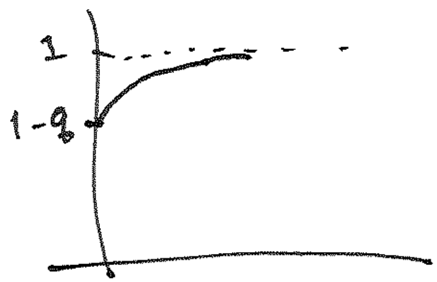
$q = .05$

$$X = \begin{cases} 0 & 1-q \\ \xi & q \end{cases}$$

$\xi \sim Exp(1/1000)$.

a)

F(x) of payment for each policy.



$$\boxed{F(x) = (1-q) + q\left(1 - e^{-x/1000}\right)}$$

b)

$$E(X) = q\,E(\xi) = (.05)(1000) = \boxed{50}$$

$$E(X^2) = q\,E(\xi^2) = (.05)(\underbrace{1000^2}_{V(\xi)} + \underbrace{1000^2}_{E(\xi)^2}) = 100{,}000$$

$$V(X) = 100{,}000 - 50^2 = \boxed{97500}$$

b) cont'd.

$$E(S) = 2000\,(50) = \boxed{100{,}000}$$

$$V(S) = 2000\,(97500) = \boxed{195{,}000{,}000}$$

c) If $d = 200$.

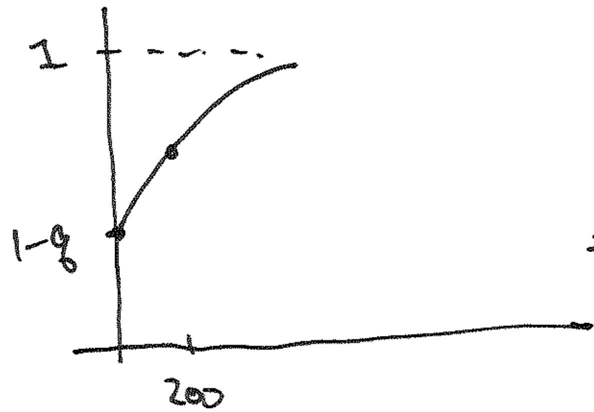
$d = 0$

$F_{old}(x) \qquad x \in [0, \infty)$

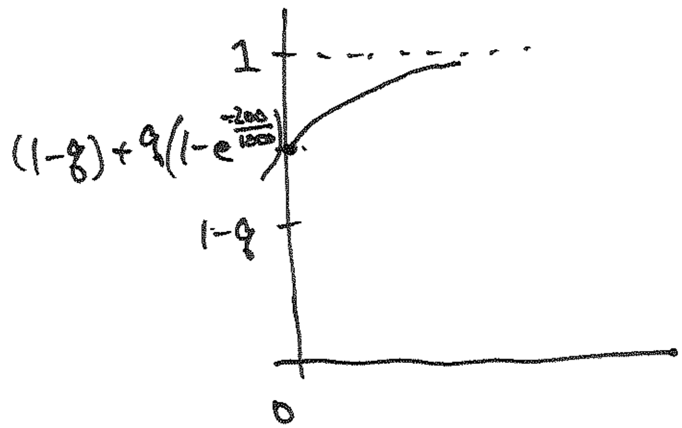
$d = 200$

$F_{new}(x) = F_{old}(x + 200)$

$x \in [0, \infty)$



$\Rightarrow$



$$\boxed{F_{new}(x) = (1-q) + q\left(1 - e^{-\frac{(x+200)}{1000}}\right)}$$

(d)

$$(1+\theta)\,E(S)$$

$$\theta = z_p \frac{\sqrt{Var(S)}}{E(S)} = (1.65)\frac{\sqrt{195{,}000{,}000}}{100{,}000}$$

$$= .23.$$

Total Prem. $(1+\theta)E(S) = 1.23\,(100{,}000)$

Individual Prem. $(1+\theta)E(X) = 1.23\,(50)$.

(e).

$$\theta = z_p \frac{\sqrt{Var(s)}}{E(s)} = z_p \frac{\sqrt{Var(x)}}{E(x)\sqrt{n}}.$$

If n becomes $2n$,

then θ becomes $\frac{\theta}{\sqrt{2}}$.

$$\theta = 0.23 \longrightarrow \frac{0.23}{\sqrt{2}} = .163$$

new θ with

$n = 4000$.