Poisson Process with multiple type of events

Let.

$N(t)$: Poisson process w/ λ.

each event can be $\begin{cases} \text{type I} & \text{with prob. } p \\ \text{type II} & \text{with prob } (1-p) \end{cases}$

Then.

Type I events

$N_1(t)$: poi proc. w/ λp,

independent.

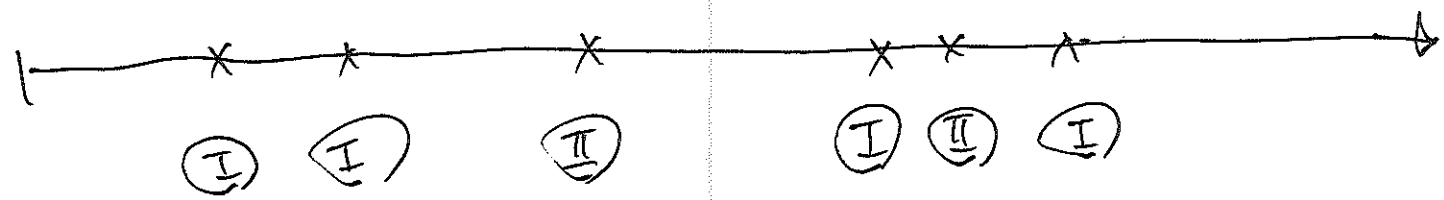
Type II events

$N_2(t)$: poi proc w/ $\lambda(1-p)$

$N(t) = N_1(t) + N_2(t)$

View 1

rate λ. $N(t)$

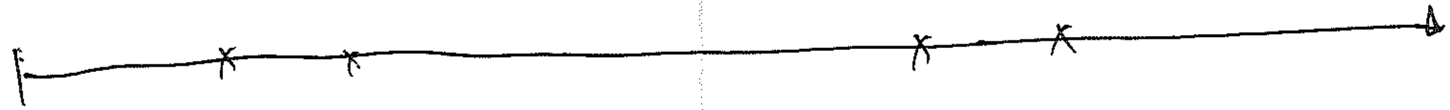


with prob. p and $(1-p)$,

View 2

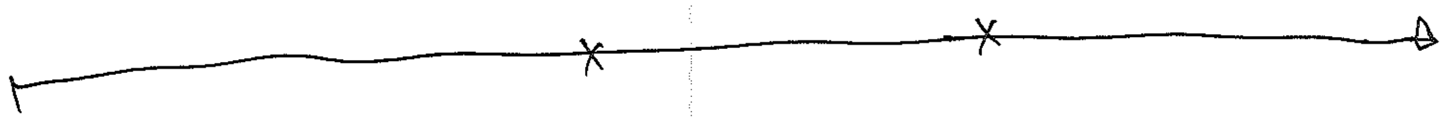
rate λp $N_1(t)$

I



independent.

rate $\lambda(1-p)$ $N_2(t)$.



II

Ex 5.14 Ross.

~~Bird~~ Immigration occurs at rate 3 per day.

60% Male,
40% Female.

1. P(10th female imigrates > 4 days)

2. P(more than 5 male immigration in 2 days)

~~3. P(Next Bird is Male).~~

$\lambda = 3$

① female ~ Poi Proc, $\lambda(.4)$

Time until 10th female

$P(S_{10} > 4)$.

$$S_{10} \sim GAM\left(\underset{\alpha}{\underset{\shortparallel}{10}}, \underset{\beta}{\underset{\shortparallel}{\frac{1}{\lambda(.4)}}}\right)$$

② male ~ Poi Proc $\lambda(.6)$

\# of Immigration in 2 days ~ Poi $(2\lambda(.6))$

$N(2)$

$P(N(2) > 5)$.

Prob. 5.24

n jobs in pool.

2 workers. $\sim \text{Exp}(\lambda_i)$

T = time until all jobs are done.

$E(T) =$

$V(T) =$

W_1. $N_1(t) = \text{Poi Proc}(\lambda_1)$

W_2 $N_2(t) = \text{Poi Proc}(\lambda_2)$.

$$E(N_1(t)) = \lambda_1 t$$

$$E(N_2(t)) = \lambda_2 t.$$

$N_1 + N_2 = N(t) \sim \text{Poi Proc}(\lambda_1 + \lambda_2)$

with 2 types. $\rightarrow P_1 = \frac{\lambda_1}{\lambda_1+\lambda_2}$

$$P_2 = \frac{\lambda_2}{\lambda_1+\lambda_2}.$$

T = time of n th event.

$$S_n \sim GAM\left(n, \frac{1}{\lambda_1+\lambda_2}\right)$$

$$E(T) = E(S_n) = n\left(\frac{1}{\lambda_1+\lambda_2}\right)$$

$$V(T) = V(S_n) = n\left(\frac{1}{\lambda_1+\lambda_2}\right)^2$$

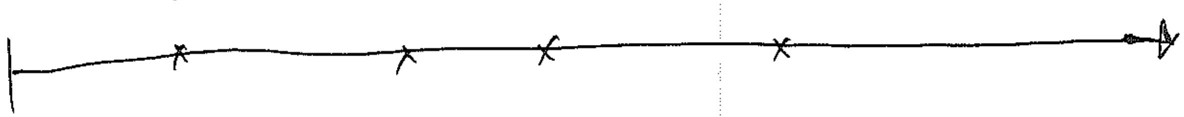
5.37 Repairman and Machine

Machine lifetime $X \sim \text{Exp}(\mu)$.

repair man arrives $\sim PP(\lambda)$ immediate fix.

$E(\text{time b/w repairs}) = ?$

repair man



$P(\text{fix}) = P(X < T_i) = \frac{\mu}{\lambda+\mu}$

Each time repairman comes in, $P(\text{fix}) = \frac{\mu}{\lambda+\mu}$.

$\rightarrow$ PP with 2 types $\begin{cases} \text{fix} \\ \text{not fix} \end{cases}$

5.37

rate of fix PP $\quad \lambda\left(\frac{\mu}{\lambda+\mu}\right)$

mean time b/w fix $\quad \frac{\lambda+\mu}{\lambda\mu} = \frac{1}{\lambda} + \frac{1}{\mu}$.

Prob. 5.53 Water Reservoir

drain 1000 /day.

rain $\sim$ Poi Proc ($\lambda = 0.2$)

2 types $\begin{cases} +8000 & \text{w.p. } .2 \\ +5000 & \text{w.p. } .8 \end{cases}$

Water level Currently at 5000.

a) P(reservoir empty after 5 days)

b) P(reservoir empty sometime within next 10 days)

a) $P(\text{No rain in 5 days})$

$= P(X = 0) \qquad X \sim Poi(\lambda 5)$

$= e^{-5\lambda}.$

b) $P($ No 8000 rain in 1st 5 days $)$

$+ P($ No more than 1 5000 rain in 1st 5 days$)$

$$= \boxed{e^{-5\lambda(.2)} + e^{-5\lambda(.8)} + e^{-5\lambda(.8)} 5\lambda(.8)}$$

Ross p 315 Nth event vs Mth event

Ex.

$$2 \text{ Poi. Proc} \begin{cases} N_1(t) & \lambda_1 \\ N_2(t) & \lambda_2 . \end{cases} \quad \text{indep.}$$

what is $P(S^1_n < S^2_m)$

= P (n th event in Proc. 1
is before m th event
in Proc 2.)

We saw before

$$P(S_1^1 < S_1^2) = \frac{\lambda_1}{\lambda_1 + \lambda_2}.$$

$S_1 \sim Exp(\lambda_1)$

$S_2 \sim Exp(\lambda_2)$.

We could compute as.

$$P(S_n^1 < S_m^2) = \frac{\square}{},$$

$S_n^1 \sim GAM(n, \frac{1}{\lambda_1})$

$S_m^2 \sim GAM(m, \frac{1}{\lambda_2})$.

$$P(S_n^1 < S_m^2) = P(S_n^1 - S_m^2 < 0)$$

Monte Carlo Simulation

$$\begin{bmatrix} X_1 \\ \vdots \\ X_{1000} \end{bmatrix} < \begin{bmatrix} Y_1 \\ \vdots \\ Y_{1000} \end{bmatrix} = \begin{bmatrix} \text{Yes} \\ \text{No} \\ \text{No} \\ \vdots \\ \text{Yes} \end{bmatrix} \leftarrow$$

See How many % are 'Yes'

$\Updownarrow$

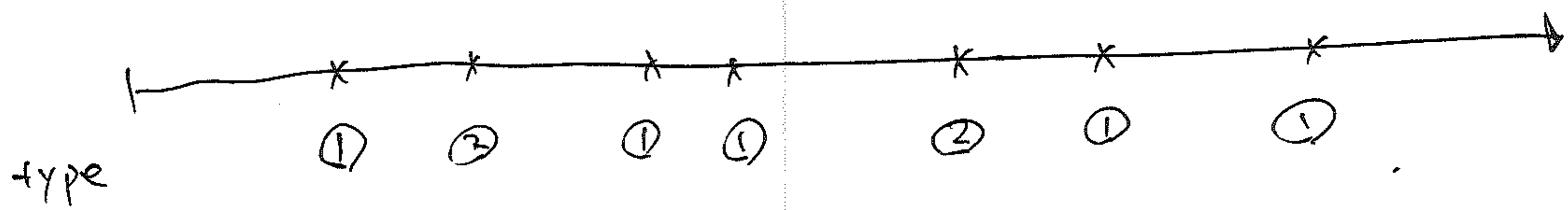
$P(S_n^1 - S_m^2 < 0)$

i.i.d. sample from $GAM(\underset{\alpha}{\underset{\parallel}{n}}, \underset{\beta}{\underset{\parallel}{\tfrac{1}{\lambda_1}}})$

i.i.d. sample from $GAM(m, \tfrac{1}{\lambda_2})$

<u>in R:</u> $rgamma(1000, \overset{\alpha}{\overset{\parallel}{n}}, \overset{1/\beta}{\overset{\parallel}{\lambda_1}})$

Alternatively, use <u>View 1</u> and see



Poi Proc. w. rate $(\lambda_1+\lambda_2)$.

events $\begin{cases} 1 & \text{w. prob. } \dfrac{\lambda_1}{\lambda_1+\lambda_2} \\ 2 & \text{w. p. } \dfrac{\lambda_2}{\lambda_1+\lambda_2} \end{cases}$.

$$(S^1_n < S^2_m) = n_{th} \text{ } 1 \text{ before } m_{th} \text{ } 2.$$

n th $\underline{\underline{\text{Head}}}$. before m th $\underline{\underline{\text{Tail}}}$

$$X \sim \text{Neg. Binomial}\left(\overset{r}{\overset{\shortparallel}{n}}, \overset{p}{\overset{\shortparallel}{\frac{\lambda_1}{\lambda_1+\lambda_2}}}\right).$$

$\boxed{X = \text{\# of throw until } n \text{ th Head}}$

$$P(n\text{th } H \text{ before } m\text{th } T)$$

$$= P(X < n+m)$$

$$= P(X \le n+m-1).$$

CDF of neg. bin.

$$P(S_n^1 < S_m^2)$$

$$= \text{CDF of } NB\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right) \text{ at } x = n+m-1.$$

$$= \sum_{k=n}^{n+m-1} \binom{n+m-1}{k} \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{k} \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n+m-1-k}.$$

$$= \text{pnbinom}\left(m-1, \; n, \; \frac{\lambda_1}{\lambda_1+\lambda_2}\right).$$

in R: X = # of Tails before n th Head.

Σ

Simulating Poisson Process

Monte Carlo Simulation

Kinetic Monte Carlo Simulation.