Fall 2016 3470:651 - HW1

due Wed, Sep. 21

HW must be written only on one side of paper, and each problem must be started on new page. Clearly indicate your final answer.

Name: Solution

1. show that if A and B are disjoint, then

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}$$

2. If $P(A) = 1/3$ and $P(B^c) = 1/4$, can A and B be disjoint? Explain.

3. A person has purchased 10 of 1000 raffle tickets. To determine the five winner, five ticket numbers are to be randomly drawn out of 1000 ticket numbers. Calculate the probability that you will win at least 1 prize.

4. In Porker, what is the probability that you get exactly two pairs, excluding full house or four-of-a-kind? (e.g. A,A,5,5,K)

5. Suppose that 5% of men and 3% of women are color-blind. A person is chosen at random, and the person is color-blind. What is the probability that the person is male? (assume that male and female are equal in numbers).

6. Suppose that a perticular disease has infected 1 in 1000 population. Lab test to detect infection of this disease has 99% accuacy on infected patient, and 98% accuacy on non-infected patient. Suppose further, that a patient has taken the identical test twice, and both times the result was positive. Calculate the conditional probability that the patient is actually infected.

7. Suppose all drivers are divided into three categories A,B, and C. For each of the driver class, probabilities for number of accidents in a year are given below:

	0	1	2	3+
A	.7	.15	.1	.05
B	.5	.25	.15	.1
C	.3	.3	.25	.15

A driver had one accident during Year 1. Given this information at the end of Year 1, it was determined that probability distribution this driver is in class (A, B, C) is (.36, .36, .28) respectively.

Suppose in Year 2, the same driver had no accident. Given this information at the end of Year 2, use Bayes' formula to update the probability distribution that this driver is in class (A, B, C) respectively.

#1

$$P(A \mid A\cup B) = \frac{P(A\cap(A\cup B))}{P(A\cup B)}$$

since A is inside $A\cup B$,

$$= \frac{P(A)}{P(A)+P(B)-P(A\cap B)}$$

If A, B are disjoint, $A\cap B = \emptyset$.

$$= \frac{P(A)}{P(A)+P(B)}$$

//

#2

If A, B disjoint,

$$A \cap B = \phi, \text{ and } P(A \cap B) = 0.$$

Then,

$$P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}^{0}.$$

$$= \frac{1}{3} + \frac{3}{4} > 1.$$

Impossible.

$\therefore$ If $P(A) + P(B) > 1$, A, B

can't be disjoint. ///

#3

P(win at least one)

$= 1 - P(\text{don't win any})$

$= 1 - P\left(\begin{array}{l}\text{None of 5 tickets drawn from 1000} \\ \text{matches 10 you bought}\end{array}\right)$

$= 1 - \dfrac{\left(\begin{array}{l}\#\text{ of ways to draw} \\ \text{tickets you don't have}\end{array}\right)}{\left(\begin{array}{l}\text{total }\#\text{ of ways} \\ \text{to draw 5 from 1000}\end{array}\right)}$

$$= 1 - \frac{\binom{990}{5}}{\binom{1000}{5}}$$

#4

Exactly two pairs.

XX YY Z.

$$\begin{pmatrix}\text{Choose}\\ 3\ \#\\ \text{from}\\ 13\end{pmatrix}\cdot\begin{pmatrix}\text{choose}\\ \text{which of}\\ \text{the 3 will}\\ \text{be Z}\end{pmatrix}\cdot\begin{pmatrix}\text{Choose}\\ 2\ X\\ \text{from}\\ 4\end{pmatrix}\begin{pmatrix}\text{Choose}\\ 2\ Y\\ \text{from}\\ 4\end{pmatrix}\begin{pmatrix}\text{choose}\\ Z\\ \text{from}\\ 4\end{pmatrix}$$

$$= \binom{13}{3}\binom{3}{1}\binom{4}{2}\binom{4}{2}\binom{4}{1} = 123552$$

$$P(\text{Exactly two pair}) = \frac{123552}{\binom{52}{5}}$$

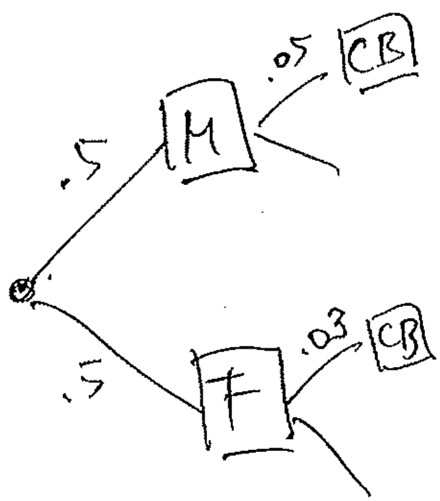
$$= \boxed{.047539}$$

#5

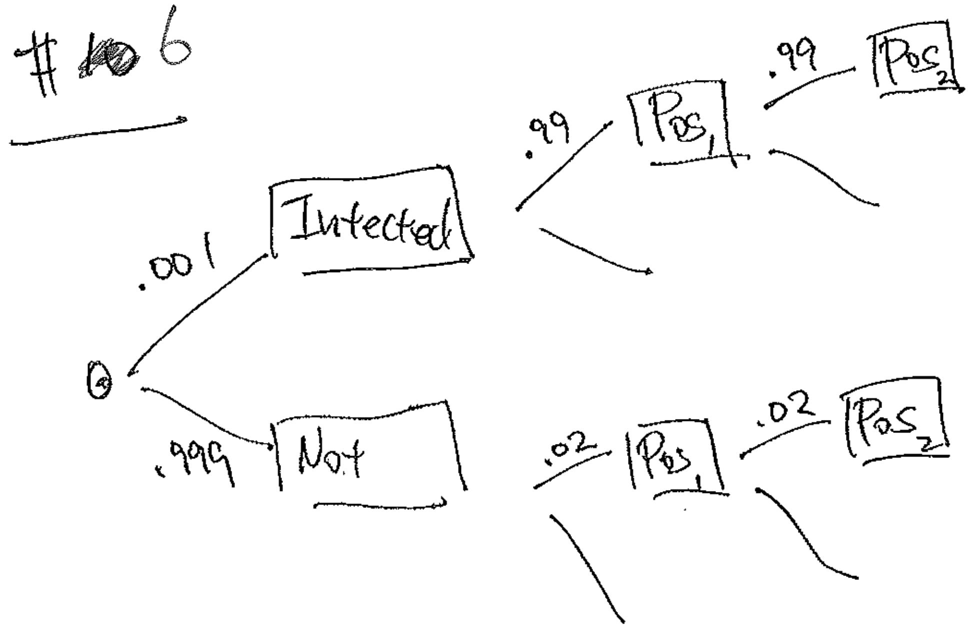
$$P(M \mid CB)$$

$$= \frac{P(CB \mid M)\,P(M)}{P(CB \mid M)\,P(M) + P(CB \mid F)\,P(F)}$$

$$= \frac{(.05)(\frac{1}{2})}{(.05)(\frac{1}{2}) + (.03)(\frac{1}{2})} = \boxed{.625}$$



#6



$$P(\text{Int} \mid P_1P_2)$$

$$= \frac{P(P_1P_2 \mid \text{Int}) \cdot P(\text{Int})}{P(P_1P_2 \mid \text{Int}) \cdot P(\text{Int}) + P(P_1P_2 \mid \text{Not}) \cdot P(\text{Not})}$$

$$= \frac{(.99)(.99)(.001)}{(.99)(.99)(.01) + (.02)(.02)(.999)}$$

$$= \boxed{.7104}$$

#7

$$P(A \mid O_{acc})$$

$$= \frac{P(O \mid A) \cdot P(A)}{P(O \mid A) \cdot P(A) + P(O \mid B) \cdot P(B) + P(O \mid C) P(C)}$$

$$= \frac{(.7)(.36)}{(.7)(.36) + (.5)(.36) + (.3)(.28)}$$

$$= \boxed{.488}$$

$$P(B \mid O) = \frac{(.5)(.36)}{''} = \boxed{.349}$$

$$P(C \mid O) = \frac{(.3)(.28)}{''} = \boxed{.163}$$