4

separate	Joint
$N_1 \sim Poi(\lambda)$ husband. $N_2 \sim Poi(\lambda)$ wife 1st accident for each is covered 100%.	$N = N_1 + N_2 \sim Poi(2\lambda)$ First two acci. are covered.
$P(N_1 \le 1 \text{ and } N_2 \le 1)$ $= P(N_1 \le 1)^2$ $= (e^{-\lambda} + e^{-\lambda}\lambda)^2$ $= e^{-2\lambda}[1+\lambda]^2$	$P(N \le 2)$ $= e^{-2\lambda} + e^{-2\lambda} 2\lambda + \frac{e^{-2\lambda}(2\lambda)^2}{2!}$ $= e^{-2\lambda}[1 + 2\lambda + 2\lambda^2]$

Compare.

#4-2

$$e^{-2\lambda}[1+\lambda]^2 \quad vs \quad e^{-2\lambda}[1+2\lambda+2\lambda^2]$$

$$= e^{-2\lambda}[(1+\lambda)^2+\lambda^2]$$

always bigger.

Joint insurance is better.

N_1	N_2
0	0
1	0
0	1
1	1
2	0
0	2

Covered by sep.

Covered by Joint.

[illegible] [illegible]

#6.

$n = 702$

a) $\lambda_1 = (231)(.01) = 2.31$

$\lambda_2 = (124)(.05) = 6.2$

$\lambda_3 = (347)(.03) = 10.41$

$\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 18.92$

$\bar{P}_4 = \frac{\lambda}{n} = \frac{18.92}{702} = \boxed{.027}$

$P_1 = 2.31/18.92 = .122$

$P_2 = 6.2/18.92 = .328$

$P_3 = 10.41/18.92 = .550$

#6-2

b)

$$N \approx Poi(\lambda = 18.92)$$

$$P(N = 15) = .0661$$

$$P(N \leq 15) = .220$$

(c) 22% prob. of Less than 16 per year.

$\approx$ Once in every five years.

#21

$$S = X_1 + \cdots + X_N$$

$$E(N) = 50. \qquad SD(N) = 10.$$

$$E(X) = 4 \qquad SD(X) = \sqrt{2}$$

$$E(S) = E(N)E(X) = 200.$$

$$V(S) = V(N)E(X)^2 + V(X)E(N) = 100(4^2) + 2(50) = 1700.$$

$$S \sim GAM(\alpha, \beta) \qquad \left.\begin{array}{l} E(S) = \alpha\beta = 200 \\ V(S) = \alpha\beta^2 = 1700 \end{array}\right\} = \begin{array}{l} \beta = 8.5 \\ \alpha = \frac{200}{8.5} = 23.53. \end{array}$$

$$P(S - 50 < 200) = P(S < 250) =$$

32.

$$M_S = \exp\left\{200\left[\left(\frac{1}{1-2z}\right)-1\right]\right\} \qquad z < \frac{1}{2}.$$

$$\downarrow$$

$$\ln(M_X(z))$$

$$\downarrow$$

$$M_X(z) = \frac{1}{1-2z}$$

MGF of $\mathrm{Exp}\left(a = \frac{1}{2}\right)$

$E(X) = 2$

$V(X) = 4$

$$M_N(z) = \exp(200\, e^{z} - 1)$$

$$\Rightarrow \quad N \sim \mathrm{Poi}(200)$$

$E(N) = 200$

$V(N) = 200$

$$E(S) = 2\,(200) = 400$$

$$V(S) = 200\,(2^2) + 4\,(200) = 1600$$

33.1

$E(N_1) = 10 \qquad N_1 \sim Poi(10)$

$E(N_2) = 40 \qquad N_2 \sim Poi(40)$

a) $E(N) = 50$ b/c $N \sim Poi(50)$

$V(N) = 50$

$P(N \le 50) = F_{Poi}(50)$

b) $N_1 | N=50 \sim BIN(50, \frac{10}{50})$

$P(N_1 < 11 | N=50) =$

#33-2

(c)

$X_1 = 100$

$X_2 = 300$

$N = N_1 + N_2 \sim \text{poi}(50)$

$$S = \underbrace{100 N_1}_{*\text{poi}} + \underbrace{300 N_2}_{*\text{poi}}$$

$$= X_1 + \cdots + X_N$$

$$X = \begin{cases} 100 & \text{w.p. } 1/5 \\ 300 & 4/5 \end{cases}$$

Compound Poisson.

$$E(X) = \frac{1}{5}100 + \frac{4}{5}300$$

$$= \frac{1300}{5} = 260$$

$$E(X^2) = 74000$$

#33-3

(d)

$$E(S) = 260 \cdot 50 = 13000$$

$$V(S) = \lambda\, E(X^2) = 3700000$$

$$M_S(z) = \exp\{50(\cancel{\ } e^{\ln(M_X(z))} - 1)\}$$

$$= \exp\{50(M_X(z) - 1)\}$$

$$M_X(z) = \left(e^{100z} \cdot \frac{1}{5} + e^{300X} \cdot \frac{4}{5}\right)$$

$$= E(e^{zX})$$