B3. What is the empirical mean excess loss at $d = 4$, given the following sample of total lifetimes:

3   2   5   8   10   1   6   9

A. < 1.5 B. ≥ 1.5 but < 2.5 C. ≥ 2.5 but < 3.5 D. ≥ 3.5
E. It cannot be determined from the information given   (88–4–60–1)

B4. A random loss (X) has the following probability function:

x	0	1	2	3	4	5	6	7	8	9
f(x)	.05	.06	.25	.22	.10	.05	.05	.05	.05	.12

You are given that $E[X] = 4$ and $E[X] - E[X \wedge d] = 2$. Determine d.

A. 1/4 B. 5/4 C. 7/4 D. 9/4 E. 11/4 (89S–151–16)

B5. For aggregate claims (S), you are given:

$$P(10 < S < 20) = 0 \qquad E[S] - E[S \wedge 10] = .6 \qquad E[S] - E[S \wedge 20] = .2$$

Determine $F_S(10)$.

A. .88 B. .90 C. .92 D. .94 E. .96 (89F–151–16)

B6. Klugman et al. define two functions:

i) The limited expected value function ($E[X \wedge d]$)
ii) The mean excess loss function [$e_X(d)$]

If $F(d) = Pr\{X \le d\}$ and the expected value of X is denoted by E[X], then which of the following equations expresses the relationship between $E[X \wedge d]$ and e(x)?

A. $E[X \wedge d] = E[X] - e_X(d)/[1 - F(d)]$ B. $E[X \wedge d] = E[X] - e_X(d)$
C. $E[X \wedge d] = E[X] - e_X(d)[1 - F(d)]$ D. $E[X \wedge d] = E[X][1 - F(d)] - e_X(d)$
E. None of these equations express that relationship. (90–4–53–2)

B7. The probability that an individual admitted to the hospital will stay k days or less is $1 - .8^k$ for $k = 0, 1, 2, \ldots$. A hospital indemnity policy provides a fixed amount per day for the fourth day through the tenth day (i.e., for a maximum of 7 days). Determine the percentage increase in the expected cost per admission if the maximum number of days paid is increased from 7 to 14.

A. 13 B. 15 C. 17 D. 19 E. 21 (90F–151–18)

B8. Given the following, determine the probability that a claim exceeds \$3,000:

i) Based on observed data truncated from above at \$10,000, the probability of a claim exceeding \$3,000 is .30.

ii) Based on the underlying distribution of losses, the probability of a claim exceeding \$10,000 is .02.

A. $< .28$ B. $\geq .28$ but $< .3$ C. $\geq .3$ but $< .32$ D. $\geq .32$ but $< .34$ E. $\geq .34$ (92F–4B–3–1)

B9. The following random sample has been observed:

2.0 10.3 4.8 16.4 21.6 3.7 21.4 34.4

Calculate the value of the empirical mean excess loss [$e_X(d)$] for x = 8.

A. < 7 B. ≥ 7 but < 9 C. ≥ 9 but < 11 D. ≥ 11 but < 13 E. ≥ 13 (93S–4B–25–2)

B10. The limited expected value function evaluated at any point $d \geq 0$ equals

$$E[X \wedge d] = \int_0^d x\, f_X(x)\, dx + d[1 - F_X(d)]$$

where $f_X(x)$ and $F_X(x)$ are the probability density and distribution functions, respectively, of the loss random variable X. (93F–4B–16–MC)

B11. A random sample of auto glass claims has yielded the following five observed claim amounts:

100 125 200 250 300

What is the value of the empirical mean excess loss at $x = 150$?

A. 75 B. 100 C. 200 D. 225 E. 250 (94F–4B–16–1)

C2. The mean excess loss at d [$e_X(d)$] is linear in x for the Pareto distribution. (87–4–59–MC)

C3. If $e_X(d)$ increases as d increases, this suggests that a Pareto model may be appropriate. (92S–4B–14–MC)

C4. Losses follow a Pareto distribution. Determine the ratio of the mean excess loss at $d = 2\lambda$ to the mean excess loss at $d = \lambda$.

A. 1/2 B. 1 C. 3/2 D. 2 E. It cannot be determined from the given information.
(95S–4B–21–3)

C5. If it exists, the mean excess loss function of a Pareto distribution is decreasing. (97S–4B–13–MC)

C6. You are given the following:

i) Claim sizes follow a Pareto distribution with parameters α (unknown) and $\lambda = 10{,}000$.
ii) The null hypothesis (H_0), $\alpha = .5$, is tested against the alternative hypothesis (H_1), $\alpha < .5$.
iii) One claim of 9,600,000 is observed.

Determine the mean excess loss at 10,000 under the assumption that H_0 is true.

A. 5,000 B. 10,000 C. 20,000 D. 40,000 E. ∞ (98F–4B–6–2)

USE the following information for the next three questions. You are given:

i) The random variable X follows a Pareto distribution with parameters $\theta = 100$ and $\alpha = 2$.
ii) The mean residual life function, $e_X(k)$, is defined to be $E[X - k \mid X \geq k]$.

C7. Determine the range of $e_X(k)$ over its domain of $[0, \infty)$.

A. $[0, 100]$ B. $[0, \infty)$ C. 100 D. $[100, \infty)$ E. ∞ (99F–4B–25–2)

C8. $Y = 1.10X$. Determine the range of the function $e_Y(k)/e_X(k)$ over its domain of $[0, \infty)$.

A. $(1, 1.10]$ B. $(1, \infty)$ C. 1.10 D. $[1.10, \infty)$ E. ∞ (99F–4B–26–1)

C9. $Z = \min(X, 500)$. Determine the range of $e_Z(k)$ over its domain of $[0, 500]$.

A. $[0, 150]$ B. $[0, \infty)$ C. $[100, 150]$ D. $[100, \infty)$ E. $[150, \infty)$ (99F–4B–27–2)

D1. Let X be a random variable with the following density function:

$$f(x) = \begin{cases} ae^{-ax} \text{ for } x > 0 \\ 0 \text{ otherwise} \end{cases}$$

where $a > 0$. If $M(t)$ denotes the moment-generating function of X, what is $M(-3a)$?

A. e^{-3a} B. 1/30 C. 1/3 D. 1/4 E. $+\infty$ (81F–2–29)

D3. $S = X_1 + X_2 + \cdots + X_6$. The X_i's, $i = 1, 2, \ldots, 6$, are independent random variables each with a gamma distribution. $E[X_i] = Var(X_i) = i$ for $i = 1, 2, \ldots, 6$. Determine $E[S^3]$

A. 9,261 B. 9,606 C. 9,896 D. 9,996 E. 10,626 (86F–151–5)

D6. X_1, X_2, X_3, and X_4 are independent random variables for a gamma distribution $G(\alpha, \theta)$ with the parameter $\alpha = 2.2$ and the parameter $\theta = 1/5$. If $S = X_1 + X_2 + X_3 + X_4$, then what is the distribution function for S?

A. Gamma (8.8, 4/5) B. Gamma (8.8, 1/5) C. Gamma (2.2, 4/5) D. Gamma (2.2, 1/5)

E. None of these answers are correct. (94F–5A–23–1)

D10. The following information is available for a collective risk model:

i) X is a random variable representing the size of each loss.
ii) X follows a gamma distribution with $\alpha = 2$ and $\theta = 100$.
iii) N is a random variable representing the number of claims.
iv) S is a random variable representing aggregate losses.
v) $S = X_1 + \cdots + X_N$

Calculate the mode of S when $N = 5$.

A. < 950 B. ≥ 950 but < 1,050 C. ≥ 1,050 but < 1,150 D. ≥ 1,150 but < 1,250 E. ≥ 1,250
(06S–3–36–2)

D13. The random variables, $X_1, X_2, \ldots, X_n$ are independent and identically distributed with probability density function

$$f(x) = e^{-x/\theta} \quad x \geq 0$$

Determine $E[\bar{X}^2]$.

A. $\frac{(n+1)\theta^2}{n}$ B. $\frac{(n+1)\theta^2}{n^2}$ C. $\frac{\theta^2}{n}$ D. $\frac{\theta^2}{\sqrt{n}}$ E. θ^2 (06F–C–26)