471/571 - HW4

due Mon, Nov 2nd

Name: _____

1. The problems in this section use the yield curve table. YearSpot Rate

Table 1:

year	spot rate
1	5.00%
2	4.50%
3	4.00%
4	4.00%
5	4.00%

(a) A three year annual \$1000 par bond has a coupon rate of 4%. Use the yield curve above to find the price P and then use this price to find the yield to maturity.

$$PV = \frac{40}{1.05} + \frac{40}{1.045^2} + \frac{40}{1.04^3} + \frac{1000}{1.04^3} = 999.28$$

Yield

N	I/Y	PV	PMT	FV
3	?	-999.28	40	1000

I/Y = 4.026 %

(b) Find the one year forward rate.

1yr Fwrd rate on Yr 2 $\quad \frac{1.045^2}{1.05} - 1 = .04$

Yr 3 $\quad \frac{1.04^3}{1.045^2} - 1 = .349$

Yr 4 $\quad \frac{1.04^4}{1.04^3} - 1 = .04$

Yr 5 $\quad \frac{1.04^5}{1.04^4} - 1 = .04$

2. An investment pays 2000 in three years and 3000 at the end of the fourth year. An investor has purchased it to yield the annual rate i = .075. Find the Macaulay duration and the modified duration.

$$PV = \frac{2000}{1.075^3} + \frac{3000}{1.075^4} = 1609.9 + 2246.4 = 3856.3$$

$$D = \frac{1609.9}{3856.3}(3) + \frac{2246.4}{3856.3}(4) = 3.58$$

$$DM = \frac{3.58}{1.075} = 3.33$$

3. An annual corporate bond is priced to yield 7% annually and has a price of 940.29 and a Macaulay duration of D= 6.5317. Estimate the change in price if rates increase by 0.10%.

$$\Delta P = (DM)(\Delta I) \cdot P$$

$$= (6.5317)(.001)(940.29)$$

$$= 5.74$$ price goes down

PV

(e) An investor has a portfolio containing \$1,000 worth of a three year bond with a modified duration of 2.7, \$4,000 worth of a five year bond with a modified duration of 4.6, and \$5,000 worth of a 6 year bond with a modified duration of 5.50. Find the modified duration of the entire portfolio.

$$DM = \frac{1000}{10000}(2.7) + \frac{4000}{10000}(4.6) + \frac{5000}{10000}(5.50)$$

$$= \boxed{4.86}$$

(f) You have a single liability of 200,000 payable at time 7. The valuation interest rate is i 0 = .06. You wish to attempt to immunize this portfolio by buying two zero coupon bonds with maturities at times 4 and 10. Find the amounts of the two bonds, and verify that the portfolio is immunized.

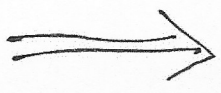
① $Xv^4 + Yv^{10} = 200000\,v^7$

② $4Xv^4 + 10Yv^{10} = 7(200000)v^7$

} solve for X, Y

③ $4^2Xv^4 + (10)^2Yv^{10} \geq 7^2(200,000)v^7$

If yes, then Portfolio is immunized.



X, Y in unit of \$1000.

① $X + Y v^6 = 200000 v^3$

② $4X + 10Y v^6 = 7(200) v^3$

$v = \frac{1}{1.06}$

① → $4X + 4Y v^6 = 4(200) v^3$ — ③

② − ③ → $6Y v^6 = 3(200) v^3$

$Y = \frac{1}{2}(200)\frac{1}{v^3} = \boxed{119.1016}$

① → $X = 200 v^3 - Y v^6 = \boxed{83.96193}$

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check $4^2 X + 10^2 Y v^6 = 9939.58$

$7^2(200) v^3 = 8828.27$

Yes, Immunized.