489/689 Assignment 3

Due: Tue Oct 8th.
This HW has been revised. Now have two questions.

Notation of this assignment follows that of Cryer. i.e. ARMA(p, q) parameters are defiend

$$\Phi(B)Y_t = \Theta(B)e_t,$$

where $\Theta(x) = 1 - \theta_1 x - \theta_2 x^2 - \cdots - \theta_q x^q$. There fore when you use R function, you muse use [θ_1 in R] $= -\theta_1$.

Submit printout of the R code as well as your answer. Class web page has sample code for Monte Carlo Simulation of ARMA.

Questions:

1. Consider an ARMA(1,1) model with $\phi = 0.5$ and $\theta = -0.45$, with mean of 3 and iid N(0,1) errors.

   (a) For n = 100, evaluate the variances and correlation of the maximum likelihood estimators of ϕ and θ using Equations (7.4.13) on page 161.

   (b) Repeat part (a) but now with n = 300.

   (c) By Monte Carlo Simulaiton, verify the convergence of variance and correlation of the MLE of ϕ and θ to asymptotic values obtained in part (a) and (b) above.

   Monte Carlo Simulation can be performes as following:

   i. Simulate ARMA process with given specification.
   ii. Obtain MLE for simulated series. Store your $\hat{\phi}$ and $\hat{\theta}$ in array.
   iii. Repeat (i) and (ii) many times (say 1000 times).
   iv. Look at the distribution of your 1000 realization of $\hat{\phi}$ and $\hat{\theta}$. Compute sample variance and correlation from the 1000 pairs.

   (d) What does above Monte Carlo Simulaiton, indicate about the bias in $\hat{\phi}$ and $\hat{\theta}$?

   (e) What does your simulation indicate about using asymptotic property of MLE?

2. Consider an ARMA(1,1) model with $\phi = 0.5$ and $\theta = 0.45$, with mean of 3 and iid N(0,1) errors.

   (a) For n = 1000, evaluate the variances and correlation of the maximum likelihood estimators of ϕ and θ using Equations (7.4.13) on page 161.

   (b) Repeat part (a) but now with n = 3000.

   (c) By Monte Carlo Simulaiton, verify the convergence of variance and correlation of the MLE of ϕ and θ to asymptotic values obtained in part (a) and (b) above.

   (d) What does above Monte Carlo Simulaiton, indicate about the bias in $\hat{\phi}$ and $\hat{\theta}$?

   (e) What does your simulation indicate about using asymptotic property of MLE?

3. Plot histgram of $\hat{\phi}$ and $\hat{\theta}$ for n=100, 300 from Problem 1, and for n=1000, n=3000 from Problem 2. Use layout() to plot 4 histogram in one page.

```
---------------------
X <- rnorm(100)

layout(matrix(1:4, 2,2))
hist(X)
hist(X)
hist(X)
hist(X)
layout(matrix(1,1,1))
```