471 Formula Sheet

$$d = 1 - \nu \qquad \delta_t = \frac{A'(t)}{A(t)} \qquad A(n) = A(0)e^{\int_0^n \delta_t dt} \qquad i_0 = \frac{i-r}{1+r}$$

Annuities

$$a_{\overline{n}|i} = \frac{1-\nu^n}{i} \qquad \ddot{a}_{\overline{n}|i} = a_{\overline{n}|i}(1+i) \qquad a_{\overline{\infty}|i} = \frac{1}{i}$$

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i} \qquad \ddot{s}_{\overline{n}|i} = s_{\overline{n}|i}(1+i)$$

$$\bar{a}_{\overline{n}|i} = \frac{i}{\ln(1+i)} a_{\overline{n}|i} = \int_0^n e^{-\int_0^t \delta_r dr}\, dt$$

$$\bar{s}_{\overline{n}|i} = \frac{i}{\ln(1+i)} s_{\overline{n}|i} = \int_0^n e^{\int_t^n \delta_r dr}\, dt$$

Annuities with Progressions

$$(Ga)_{\overline{n}|i} = \frac{1-\nu_0^n}{i-r} \qquad (Ia)_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i} - n\nu^n}{i} \qquad (Da)_{\overline{n}|i} = \frac{n - a_{\overline{n}|i}}{i}$$

$$(Gs)_{\overline{n}|i} = (Ga)_{\overline{n}|i}(1+i)^n \qquad (Is)_{\overline{n}|i} = \frac{\ddot{s}_{\overline{n}|i} - n}{i} \qquad (Ds)_{\overline{n}|i} = (Da)_{\overline{n}|i}(1+i)^n$$

Loans

$$OB_{n+1} = OB_n - \underbrace{[K_{n+1} - \overbrace{(OB_n)\, i}^{\text{Interest paid}}]}_{\text{Principle paid}} \qquad OB_n = OB_0(1+i)^n - Ks_{\overline{n}|i}$$

$$OB_n = Ka_{\overline{N-n}|i}$$

$$Int_n = K(1-\nu^{N-n+1})$$

$$PR_n = K\nu^{N-n+1} \qquad PR_{n+k} = (1+i)^k PR_n$$

Bonds

$$BV_{t+1} = BV_t - [Fr - BV_t \cdot i] \qquad PV_0 = Fr\, a_{\overline{n}|i} + C\nu^n$$

$$\underbrace{PV_t}_{\text{Market Price}} = \underbrace{PV_0(1+j)^t}_{\text{Dirty Price}} - Fr \cdot t$$

IRR

$$\sum_{k=0}^{n} C_k \nu^{t_k} = 0 \quad \text{(IRR)} \qquad \sum_{k=0}^{n} C_k(1+it_k) = 0 \quad \text{(dollar-weighted)}$$

$$\left(\frac{F_1}{F_0}\right)\left(\frac{F_2}{F_1+C_1}\right)\cdots\left(\frac{B}{F_n+C_n}\right) - 1 \quad \text{(Time-weighted)}$$

$$\frac{D}{i-r} \qquad \text{Dividend growth model of Stock}$$

Duration

$$DM = \frac{-PV(i)'}{PV(i)} \qquad D = DM\,(1+i)$$

Forward Rate

$$F_{1,2} = \frac{i_2^2}{i_1}$$

Immunization

$$\sum_{t=0}^{n} L_t \nu^t = \sum_{t=0}^{n} A_t \nu^t, \qquad \sum_{t=0}^{n} t L_t \nu^t = \sum_{t=0}^{n} t A_t \nu^t, \qquad \sum_{t=0}^{n} t^2 L_t \nu^t \leq \sum_{t=0}^{n} t^2 A_t \nu^t$$

Series

$$\begin{aligned} 1 + \nu + \nu^2 + \nu^3 \cdots &= \frac{1}{1-x} \qquad \text{(Geometric Series)} \\ 1 + \nu + \nu^2 + \cdots + \nu^n &= \frac{1-\nu^n}{1-\nu} \\ \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n &= e^x \end{aligned}$$

Put-Call Parity

$$C - P = Se^{-\delta T} - Ke^{-rT}$$

Binomial Option Pricing

$$C_0 = \Delta S + B, \qquad \begin{cases} \Delta u S e^{\delta h} + B e^{rh} = C_u \\ \Delta d S e^{\delta h} + B e^{rh} = C_d \end{cases} \qquad \begin{cases} u = e^{(r-\delta)h + \sigma\sqrt{h}} \\ d = e^{(r-\delta)h - \sigma\sqrt{h}} \end{cases}$$

$$C_0 = e^{-rh}\big[p^* C_u + (1-p^*) C_d\big], \qquad p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$

Black-Scholes

$$\begin{aligned} C &= \quad Se^{-\delta T} N(d_1) \; - Ke^{-rT} N(d_2) \\ P &= -Se^{-\delta T} N(-d_1) + Ke^{-rT} N(-d_2) \end{aligned} \qquad \begin{cases} d_1 \\ d_2 \end{cases} = \frac{\ln(S/K) + (r - \delta \pm \sigma^2/2)T}{\sigma\sqrt{T}}$$