Fall 2016 451/551 - Midterm 1

Name: Solution

1. An advertising agency notices that approximately 1 in 50 potential buyers of a product sees a given magazine ad, and 1 in 5 sees a corresponding ad on television. One in 100 sees both. One in 3 actually purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product?

$$P(\text{Seeing ad}) = P(\text{Mag} \cup TV)$$

$$= P(M) + P(TV) - P(M \cap TV)$$

$$= \frac{1}{50} + \frac{1}{5} - \frac{1}{100}$$

$$= \frac{21}{100} = .21$$

$\Longrightarrow$

#1-2

$$P(\text{no ad}) = 1 - P(\text{see ad}) = .79$$

Law of total prob.

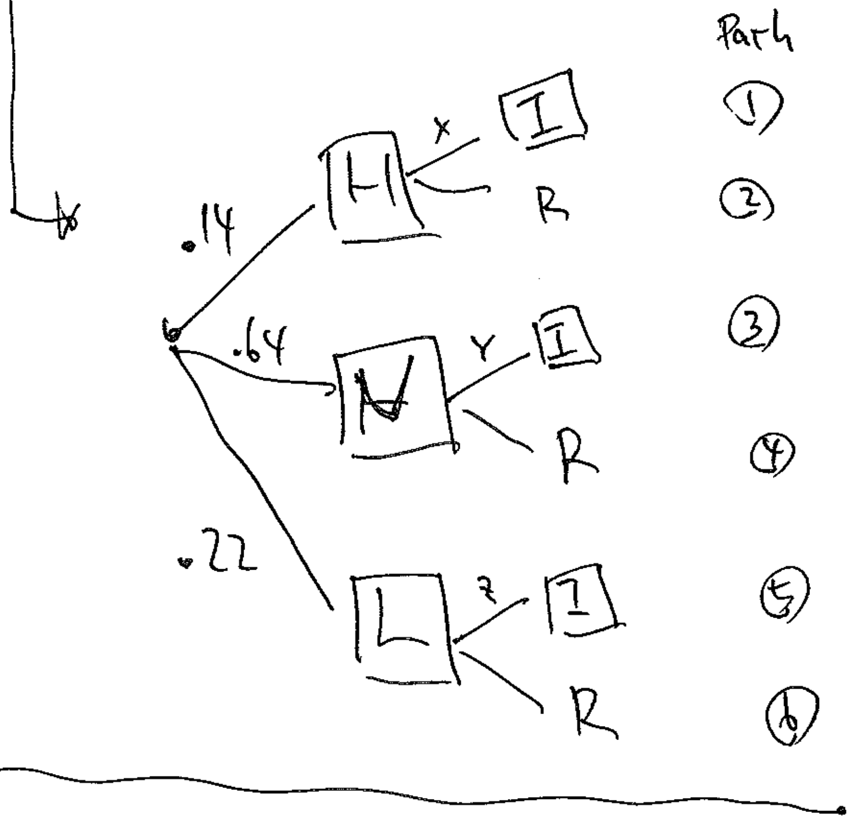
$$P(\text{Purchace}) = P(\text{purchace} \mid \text{see ad}) \cdot P(\text{see ad}) + P(\text{purchace} \mid \text{no ad}) \cdot P(\text{no ad})$$

$$= \boxed{\tfrac{1}{3}(.21) + \tfrac{1}{10}(.79)}$$

$$= \boxed{.149}$$

2. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.
(ii) 22% have low blood pressure.
(iii) 15% have an irregular heartbeat.
(iv) Of those with an irregular heartbeat, one-third have high blood pressure.
(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.
What portion of the patients selected have a regular heartbeat and low blood pressure?



(iii) says

$① + ③ + ⑤ = .15$

(iv) says

$$\frac{①}{① + ③ + ⑤} = \frac{1}{3}$$

So

$① = \frac{1}{3}(.15) = .05$

$= .14(x)$

$x =$ ~~[illegible]~~

$$\frac{.05}{.14} = .2571$$

(V) says

$$Y = \frac{1}{8} = .1250$$

Since

$$① + ③ + ⑤ = .15$$

$$.14(\underset{.3571}{x}) + .64(\underset{.1250}{y}) + .22(z) = .15$$

$$z = \underline{.0909}$$

<u>Ans</u>

$$P(L \cap R) = .22(1-z)$$

$$= \boxed{.2}$$

path ⑥

#2 – Method 2

(i) $P(H) = .14$ $\quad$ $P(I) = .15$

(ii) $P(L) = .22$ $\quad$ $P(I^c) = \cancel{.85}$

$1 - .15 = .85$

(iii) $P(N) = 1 - (.14 + .22) = .64$

	H	L	N	
I	(iv)		(v)	.15
I^c				.85
	.14	.22	.64	

$$p(H|I)\cdot p(I)$$

iv) $P(H|I) = \frac{1}{3}$ so $P(I \cap H) = \frac{1}{3} \cdot .15 = .05$

$$p(I|N)\cdot p(N)$$

(v) $P(I|N) = \frac{1}{8}$ So $P(N \cap I) = \frac{1}{8} \cdot .14$

$= .08$

#9-2 Now we can fill the table.

	H	L	N	
I	.05	.02	.08	.15
I^c	.09	.20	.56	.85
	.14	.22	.64	

$P(I^c \cap L) = \boxed{.20}$ from table.

3. Suppose a study is monitoring the health of 30 independent groups over a one-year period. Each group contains 10 patients. Probability that a patient will drop out before the end of the study is 0.2 (independent of any other participants). What is the probability that there are more than 25 groups had no drop-out?

$P(\text{no drop-out for 1 group})$

$= (.8)^{10} = .1074$

$P(\text{more than 25 groups w/ no drop out})$

$= P(X > 25)$ $\quad X \sim BIN(30, .1074)$

$= 1 - P(X \leq 25)$

$= 1 - \left[\text{CDF of } BIN(30, .1074) \text{ at } x = 25\right]$

4. An insurance policy reimburses a loss with a deductible of 1. That is, if a loss is less than 1, policy will pay zero. If it is more than 1, then the policy will pay (loss - 1). The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} \frac{2}{y^3} & \text{for } y > 1 \\ 0 & \text{otherwise} \end{cases}$$

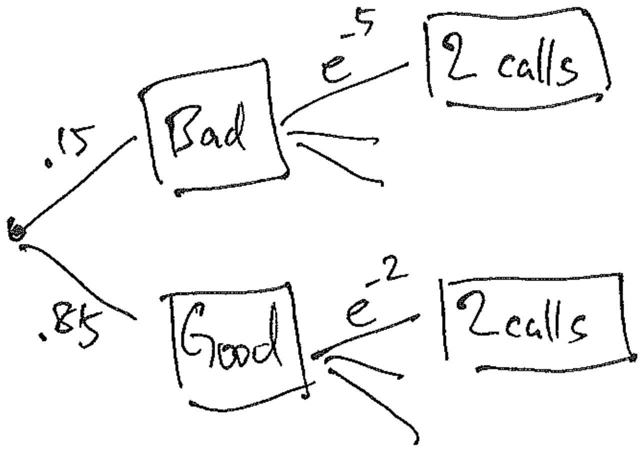
What is the expected value of the benefit paid under the insurance policy? (Set up integral(s) for full credit. Do not solve the integral)

$$\text{Payment} = \begin{cases} 0 & x < 1 \\ x - 1 & x > 1 \end{cases}$$

$$E[\text{Payment}]$$

$$= \boxed{\int_{1}^{\infty} (x-1)\frac{2}{x^3}\,dx}$$

5. Suppose 15% of products made in a factory is 'bad' product, and rest are 'good'. The number of customer service call made by a customer who bought 'bad' product in a year is modeled by Poisson r.v with mean 5. For customer with 'good' product, the number of call are modeled by Poisson with mean 1. Given the fact that a particular customer made 2 calls last year, calculate the probability he/she bought 'bad' product.



$① = .15\left(\frac{e^{-5}5^2}{2!}\right)$

$② = .85\left(\frac{e^{-2}2^2}{2!}\right)$

$X \sim Poi(5)$

$$P(2 \text{ calls} \mid \text{Bad}) = P(X=2) = e^{-5}5^2/2! = .0842$$

$$P(2 \text{ calls} \mid \text{Good}) = P(X=2) = e^{-2}2^2/2! = .2707$$

$X \sim Poi(2)$

$$P(\text{Bad} \mid 2 \text{ calls}) = \frac{.0842(.15)}{(.15).0842 + .2707(.85)}$$

$$= \boxed{.2373}$$