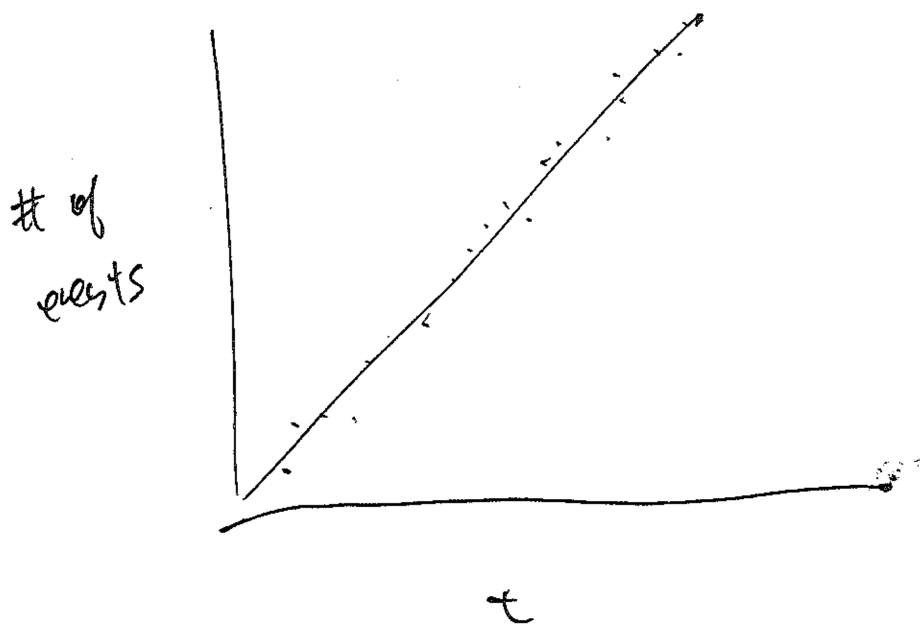
Non homogeneous Poisson Process

$$N(t_2) - N(t_1) \sim \text{PoiProb.}\left(\lambda(t_2 - t_1)\right).$$

Intensity function.

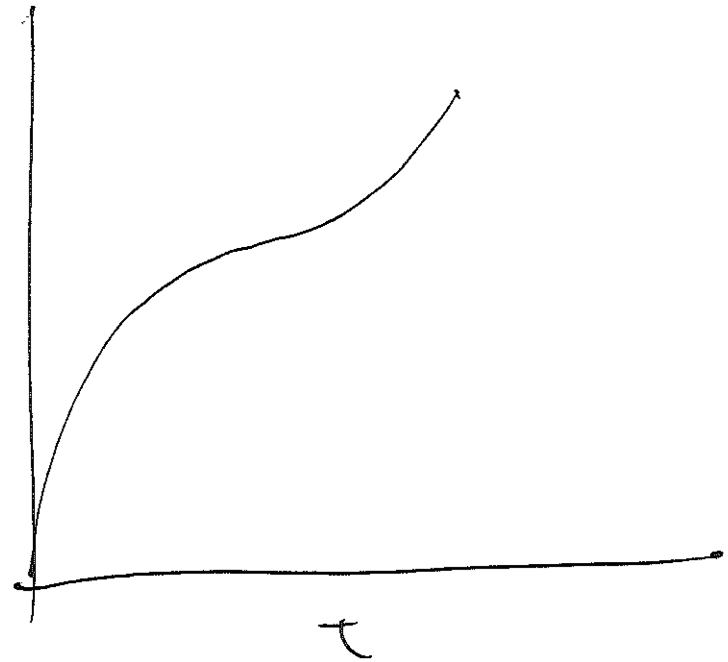
$$E\left[N(t_2) - N(t_1)\right] = \int_{t_1}^{t_2} \lambda(u)\, du$$

Homog. PP



$$\int_0^t \lambda \, du = \lambda t$$

Non hom. PP



$$\boxed{\int_0^t \lambda(u) \, du}$$

Prob. 5.4 P 360 Ross

Two indep. NHPP

$$N(t) = N_1(t) + N_2(t) \sim NHPP(\lambda(t) + \mu(t))$$

$P($ event from N occured at t_1 is of type 1 $)$

$$= \frac{\lambda(t_1)}{\lambda(t_1) + \mu(t_1)}$$

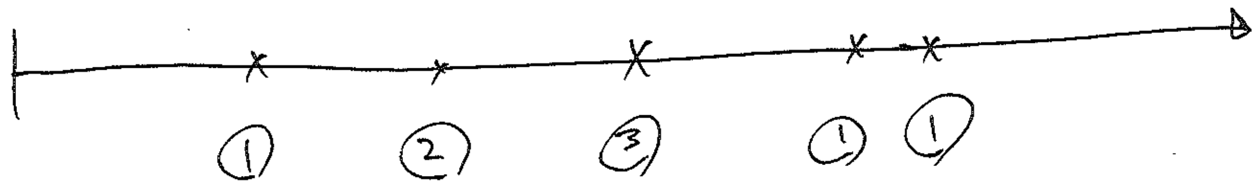
~~Results~~ Example of NHPP

Poi Proc. w/ multiple types of events.

k types.

$N(t)$ rate $= \lambda$.

View 1.

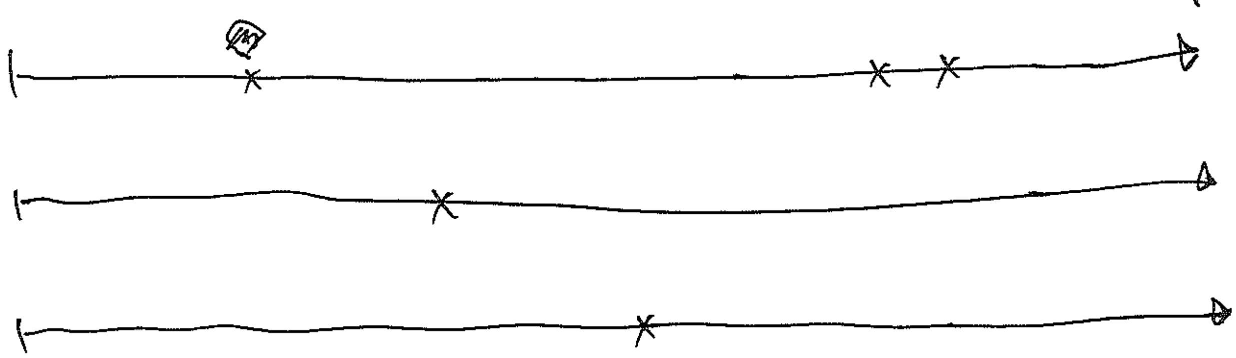


each event with prob. P_j of being type j.

rate

View 2

$N_1(t)$ 1



λP_1

$N_2(t)$ 2

λP_2

$N_3(t)$ 3

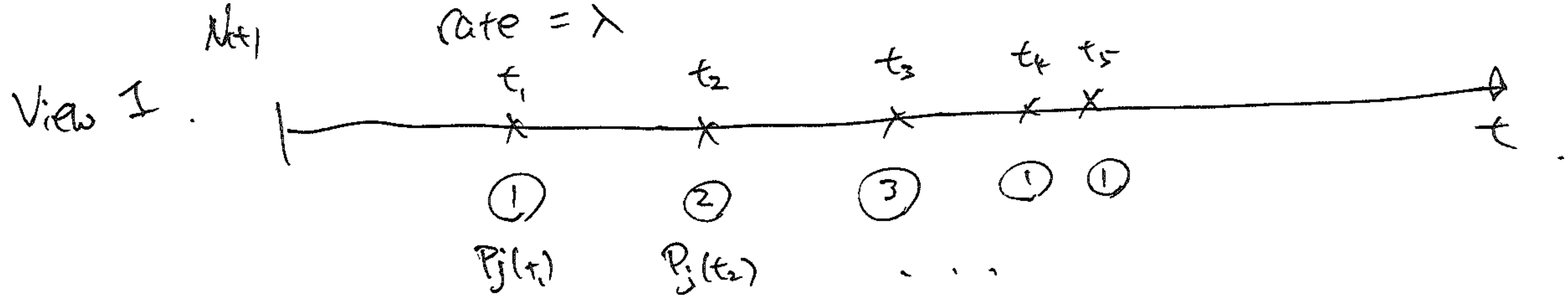
λP_3.

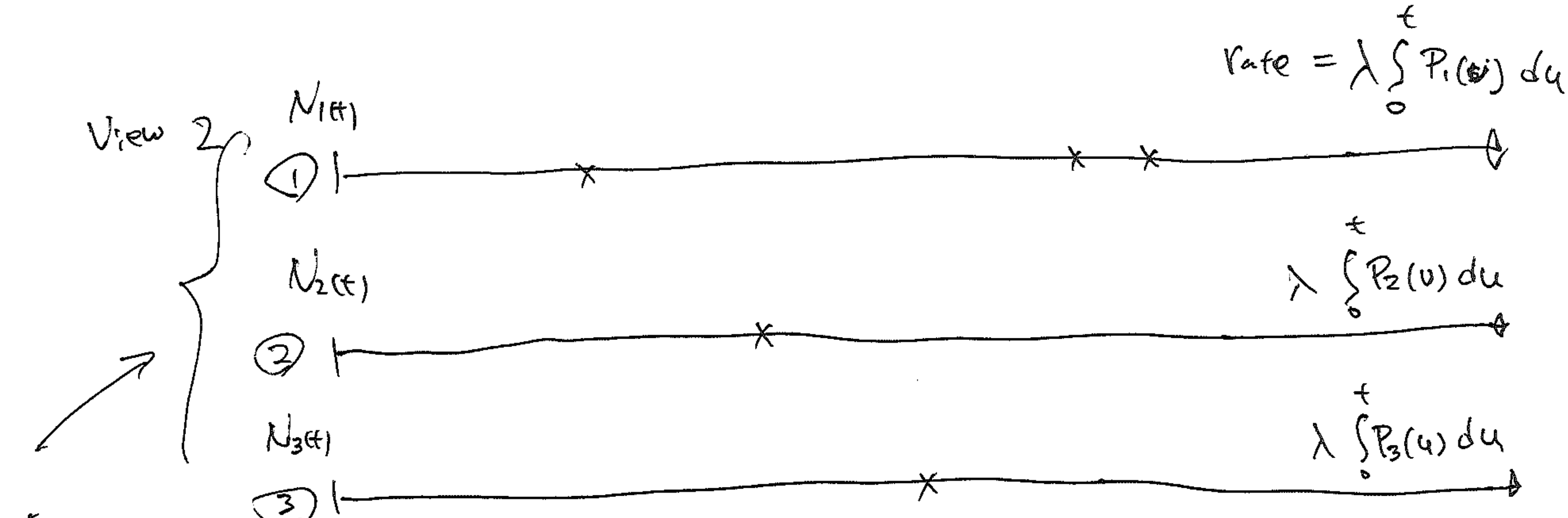
$\boxed{\text{What if } P_j \text{ depend on time?}}$ $P_j \to P_j(t)$.

Ross p 327

Proposition 5.3

Poi Proc w/ multiple types of events with time dependant $P_j(t)$.





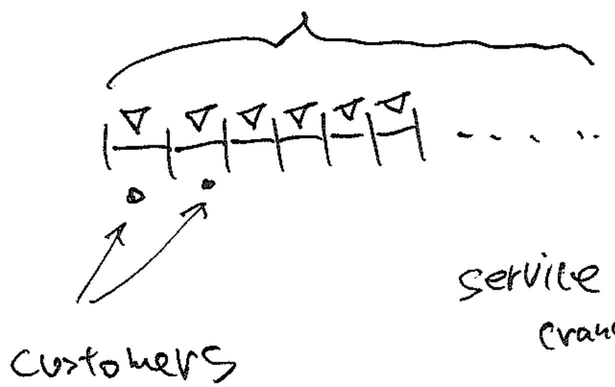
Inhomogeneous Poi Proc.

Ross p.304

Ex 5.18 (Infinite Server Queue).

Customers arriving $\sim$ Poi Proc. (λ).

Infinite Servers. $\rightarrow$ No waiting line



service time (random) $\sim G$ CDF

what are distributions of :

$Y(t)$:= # of customers being served, at time (t).

$X(t)$:= # of customers completed the service, by time (t).

Given $\underline{\underline{t}}$. let

$$\begin{cases} \text{type I} & : \text{entering customer who completes by time } t \\ \text{type II} & : \text{entering customer who is not done by time } t. \end{cases}$$

→ Customer enters at s. $s \le t$.

→ then he has $t-s$ to finish, and be type I.

→ $P(\text{type I}) = P(\text{Job} \le t-s) = G(t-s)$.

$\text{Job} \sim \underset{\text{CDF}}{G(x)}$.

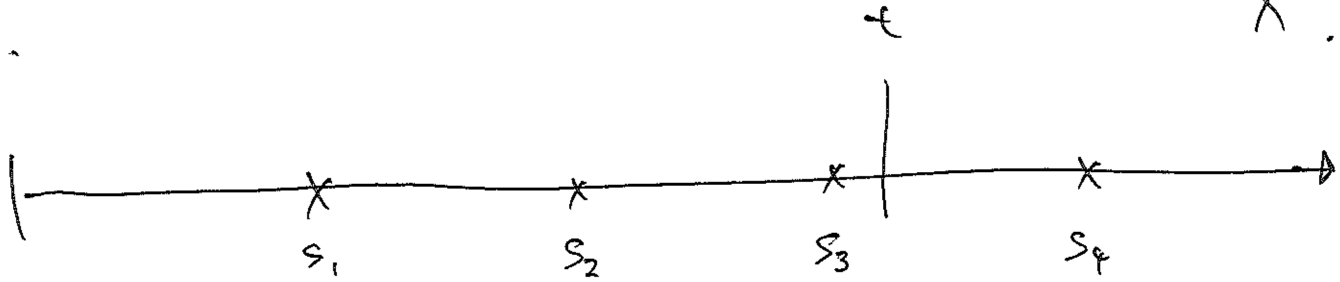
$= \underline{P_1(t-s)}$.

what is $P_2(t-s)$?

$= 1 - G(t-s)$

View I.

rate λ.

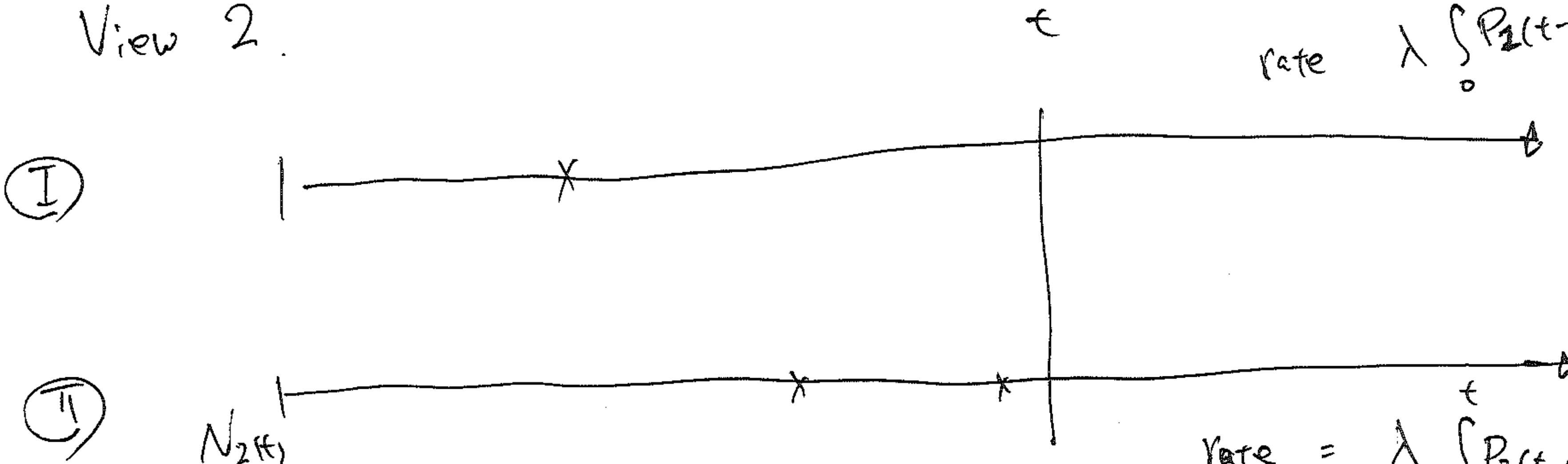


$P_1(t-s_1)$ $P_1(t-s_2)$ $P_1(t-s_3)$. $P_{1 \not t} = 0$

I II II

View 2.

rate $\lambda \int_0^t P_1(t-s)\,ds$.



rate $= \lambda \int_0^t P_2(t-s)\,ds$

$$E(X(t)) = E(\#\text{ complete by } t)$$

$$= E(\text{type I by } t)$$

$$= \text{rate of } N_1(t) \quad = \lambda \int_0^t P_1(t-s)\, ds$$

$$= \lambda \int_0^t G(t-s)\, ds$$

$$E(Y(t)) = E(\# \text{ incomplete by time } t)$$

$$= E(\text{type II by } t)$$

$$= \text{rate of } N_2(t),$$

$$= \lambda \int_0^t P_2(t-s)\,ds$$

$$= \lambda \int_0^t 1 - G(t-s)\,ds.$$