## Motte Carlo Intestation

$$= \int_{\alpha | x} x f(x) dx$$

It Xi - fix) is easy to generate, then we can use

 $\overline{X}$  as estimate for E(X).

It X1, ..., Xn are R.S. from f(x).

95% CI

F(X) X = 1.96Vour(X)

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we can use sample variance from  $\{X_1, ..., X_n\}$  as estimate for var(x)

$$E(e) = \int_{e}^{-x} e^{-x} \cdot 1 \, dx$$

when Unifon.

Then we can use Morte (and Simulation of to estimate 
$$E(\bar{e}^{U})$$
,  $U \sim U(0,1)$ 

## MC Integration

Generate  $U_1, ..., U_n \sim U(0,1)$ 2) let  $X_i = e^{-U_i}$ 3) 95 % CI for E(X) is  $X + 1.96 \sqrt{\frac{Var(X)}{n}}$ 

var(K) = sample variable of {x, --, xx}

$$5 \int_{0}^{5} e^{-x} \cdot \frac{1}{5} \cdot dx = 5 = \sqrt{e}$$

$$U \sim Unif(0, 5)$$

Method to accelerate

MC simulation of E()

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## Autithetic Variates

$$E(X) = E(X) = E(\frac{X+Y}{2})$$

$$|9590$$
 CT for  $E(x)$ .  
 $|X \pm 196| \frac{V(x)}{u}$ .

What if we use near 
$$\left(\frac{x_i + y_i}{2}\right)$$
 to restimate  $E\left(\frac{x_i + y_i}{2}\right) \in ?$ 

If each 
$$X_i + Y_i$$
 one independent for different i,

 $A_i = \frac{1}{2} \left( \frac{X_i + Y_i}{2} \right) + \frac{1}{2} \left( \frac{X_i + Y_i}{2} \right)^{\frac{1}{2}}$ 

Saul as before, but

$$Var\left(\frac{X_i + Y_i}{2}\right)$$
 is estimating  $Var\left(\frac{X+Y}{2}\right)$ 

$$V\left(\frac{X+Y}{2}\right) = \frac{1}{4}V(X+Y)$$

$$= \frac{1}{4} \left\{ V(X) + V(Y) + 2 G V(X,Y) \right\},$$

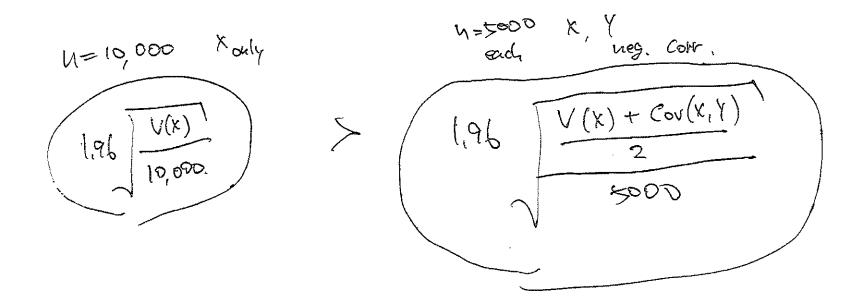
$$= \frac{\sqrt{(x)}}{2} + \frac{1}{2} C_{0}(x, y),$$

If, 
$$X_i, Y_i$$
 are independent (for same i),  
 $Gu(X,Y) = 0$ ,  
 $V(\frac{X+Y}{2}) = \frac{V(X)}{2}$ 

$$N = 10000$$
 Yard Y each.  $11,400$   $10,000$   $10,000$   $10,000$   $10,000$   $10,000$   $10,000$   $10,000$   $10,000$   $10,000$ 

Sque Margin of Error.

If  $X_i$  and  $Y_i$  are negatively correlated  $V(X_i, Y_i) = \frac{V(X_i)}{2} + \frac{1}{2} \frac{Cov(X_i, Y_i)}{vegative}$ 



Transformation of Weggethers
Correlated C.V. X, Y are correlated, then g(x), g(x) are also heg. correlated if g(·) is honotone function

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what is neg. correlated with U~UNIF(0,1) and still have UNIF(0,1) distribution?  $\frac{\text{let}}{V} = 1 - U \qquad V \sim U(0,1).$ Corr. (U, V) = Corr (U, 1-U) = Corr (U, 1) + Corr (U, -U) = Corr (U, U) = -1.

$$\frac{E_{X}}{\int_{0}^{\infty} e^{-X} dX} = E\left[e^{-U}\right] \text{ where } U \sim UNIF(0,1)$$

Depende 
$$U_1, ..., U_n \sim U(0,1)$$
.  $\sum_{i=1-U_i}^{i} Convelated}$  Correlated.

Then let 
$$X_i = e^{-V_i}$$
 hegafizely

$$Y_i = e^{-V_i}$$
 hegafizely

Correlated.

The mean  $\left(\frac{X_i + Y_i}{2}\right)$  to  $\left(\frac{Y_i + Y_i}{2}\right)$