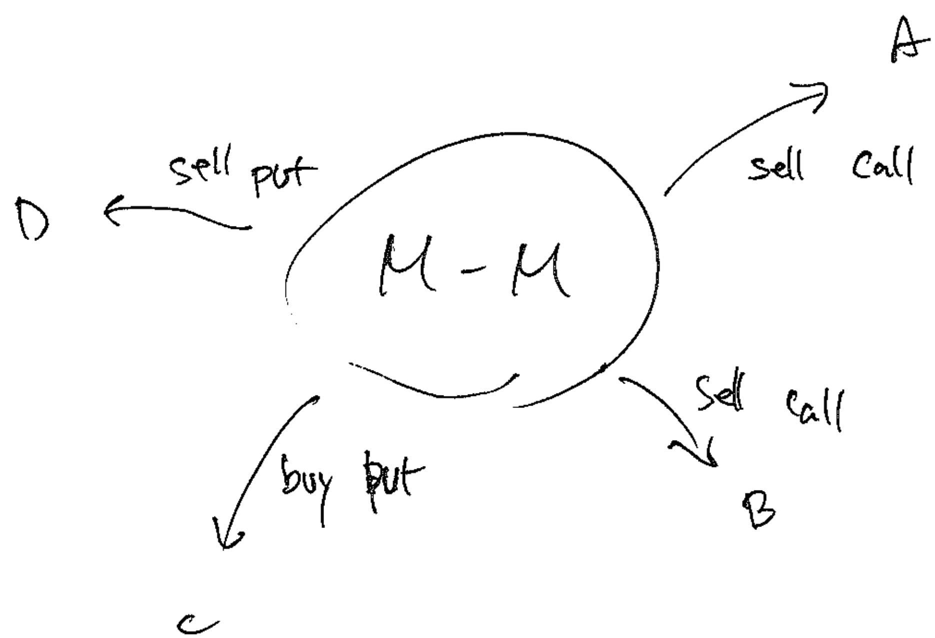
13. Market-Making and Delta-Hedging

Market-Maker



M-M makes profit by bid-ask spread.

Problem : Market-Maker is left with position determined by market demand

→ Delta-headging.

on 100 shares

→ Sold $K=40$ - Call
@ \$2.7804

$$\left\{\begin{array}{l} S=40 \\ \sigma=.3 \\ r=.08 \\ T=91/365 \\ \delta=0 \end{array}\right.$$

$\Delta =_{\alpha} 0.5824$

$\Gamma = .0652$

$\theta = -.0173$

Delta - Hedging

Day 0. Sold 40-strike Call @ \$2.7804 on 100 shares.

$\rightarrow$ \$278.04

$\Delta = .5824$

$\rightarrow$ buy 58.24 shares @ $S = 40$

$= -2329.6$

$278.04 - 2329.6 = \boxed{-2051.56}$ net investment

Borrow 2051.56
with $r = 8\%$.

$$2051.56 \left(e^{.08\left(\frac{1}{365}\right)} - 1 \right) = .45$$

overnight
interest charge.

Day 1 Marking-to-market

$S = 40 \rightarrow 40.50$

$T = \frac{91}{365} \rightarrow \frac{90}{365}$

New Option price (by B-S) $= 3.0621$

$-306.21 + 278.04 = \boxed{-28.17}$ loss on written Call

58.24 shares of stock

$58.24\,(40.50 - 40) = \boxed{29.12}$ Gain on stock.

2051.51 borrowing.

$-\left(e^{.08\left(\frac{1}{365}\right)} - 1\right) 2051.51 = \boxed{-.45}$ Interest

$\boxed{.50}$ over night profit.

Day 1 Rebalancing the portfolio

New $\Delta = 0.6142$

$61.42 - 58.24 = 3.18$ Additional stock ~~#~~ needed

$\$40.50 \times (3.18) = \boxed{\$128.79}$ Additional ~~stock~~ investment needed

Borrowing Capacity

$\underbrace{2051.56}_{\text{Day 0}} + \overset{\text{Int}}{.45} + \underbrace{128.79}_{\text{need}}$

$= 2180.80$

$\underbrace{61.42}_{\text{Stock}} (\$40.50) = 2181.30$

borrow this anyway.

.50 difference

↳ cash in

Day 2

Marking - to - Market

$S = 40.50 \rightarrow 39.25$

$T = \frac{90}{365} \rightarrow \frac{89}{365}$

New option price $= 2.3282$

$-232.82 + 306.21 = \boxed{73.39}$ Gain on written Call

61.42 shares of Stock

$61.42\,(39.25 - 40.50) = \boxed{-76.78}$ loss on stock.

Borrowing 2181.30

$\left(e^{.08\left(\frac{1}{365}\right)} - 1\right) 2181.30 = \boxed{-.48}$ Interest.

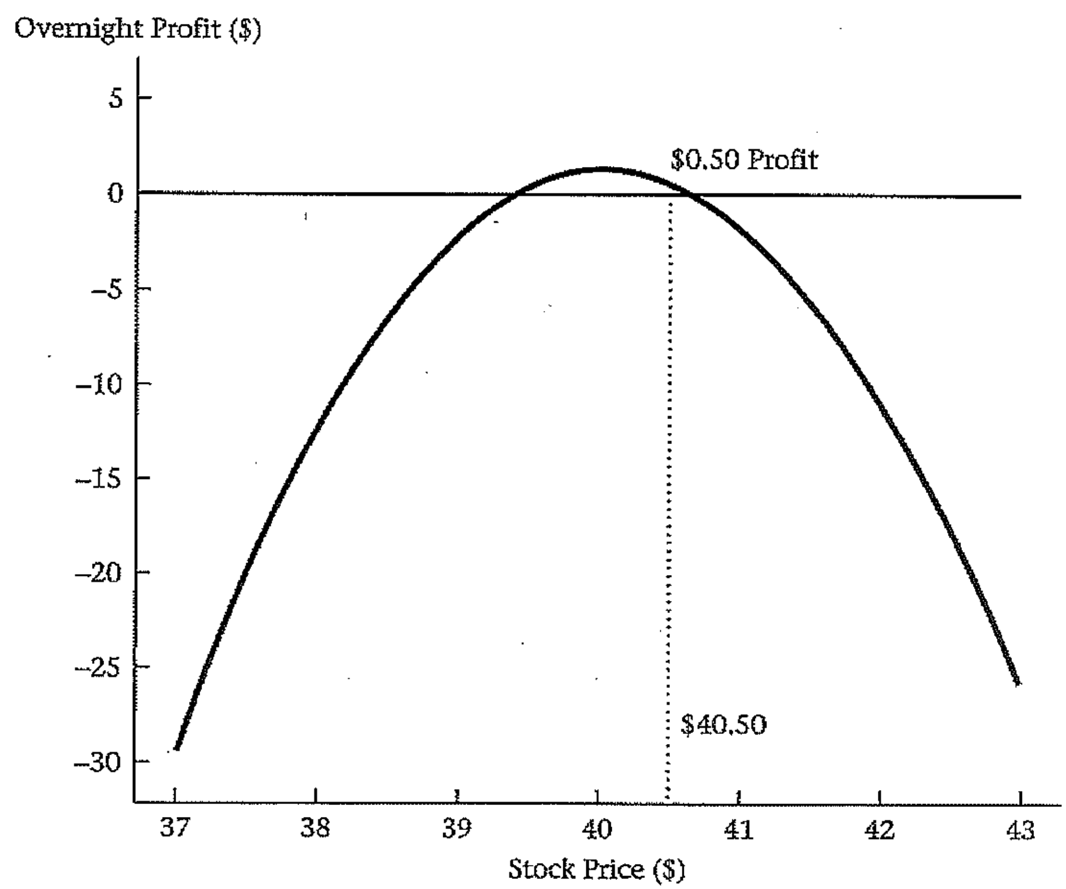
$-\$3.87$ overnight profit.

TABLE 13.2 Daily profit calculation over 5 days for a market-maker who delta-hedges.

	Day					
	0	1	2	3	4	5
Stock ($)	40.00	40.50	39.25	38.75	40.00	40.00
Call ($)	278.04	306.21	232.82	205.46	271.04	269.27
Option delta	0.5824	0.6142	0.5311	0.4956	0.5806	0.5801
Investment ($)	2,051.58	2,181.30	1,851.65	1,715.12	2,051.35	2,051.29
Interest ($)		−0.45	−0.48	−0.41	−0.38	−0.45
Capital gain ($)		0.95	−3.39	0.81	−3.62	1.77
Daily profit ($)		0.50	−3.87	0.40	−4.00	1.32

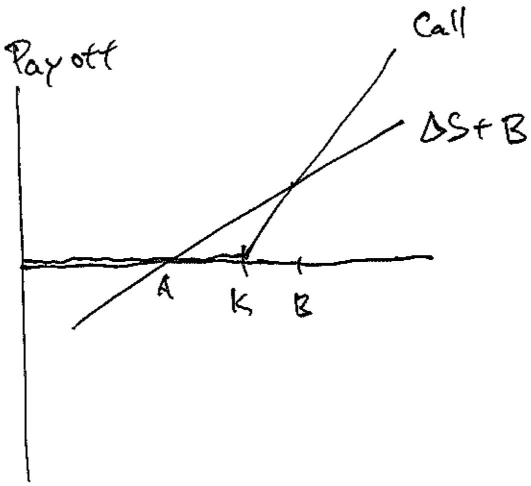
Self-financing portfolio

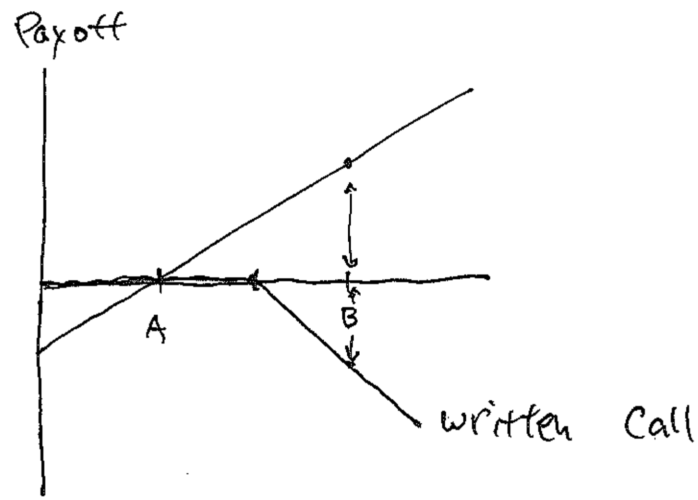
→ portfolio that does not require additional cash.

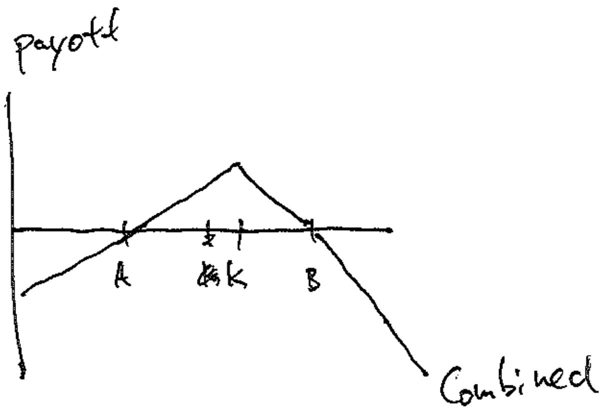


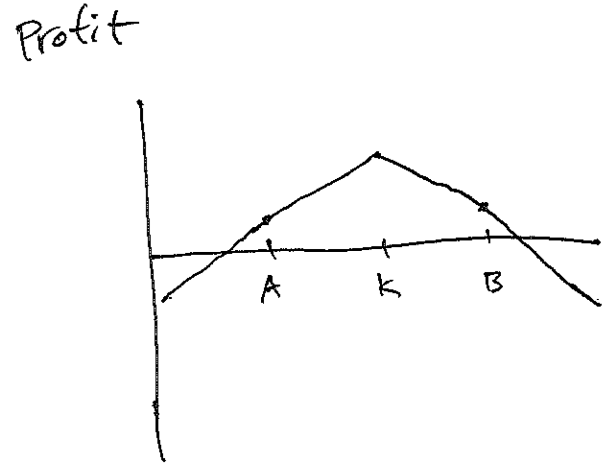
Overnight profit as a function of the stock price for a delta-hedged market-maker who has written a call.

Binomial pricing









Self-financing portfolio

Overnight profit $= 0$ if

Stock price move by $\pm \sigma$.

$$\boxed{\text{If stock move around by } \pm\sigma \text{ every day, delta-hedged portfolio will be self-financing.}}$$

delta-neutral portfolio : portfolio that is delta-headged

Delta - Gamma Approximation

Δ : change in C when $S \uparrow \$1$

Γ : change in Δ when $S \uparrow \$1$.

$S = 40 \rightarrow 40.75$

$C = 2.7804 \rightarrow 3.2352$

$\Delta = .5824$

$\Gamma = .0652$

by Δ only.

$$\frac{C(40.75) - C(40)}{1} = .75\,(.5824) = .4368$$

$$C(40.75) = \underset{\substack{\uparrow \\ 2.7804}}{C(40)} + .4368 = \boxed{3.2172}$$

Taylor Series Approximation

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2 + \dots$$

$$C(\text{new } s) = f(s) + \Delta(\text{change in } s)$$

1st order

$$C(\text{new } s) = f(s) + \Delta(\text{change in } s) + \frac{1}{2}\Gamma(\text{change in } s)^2$$

2nd order

Delta - Gamma Approximation

$$C(S_{t+h}) \approx f(S_t) + \Delta(S_{t+h} - S_t) + \frac{1}{2}\Gamma(S_{t+h} - S_t)^2$$

$$C(40.75) \approx C(40) + (.5824)(.75) + \frac{1}{2}(.0652)(.75)^2$$

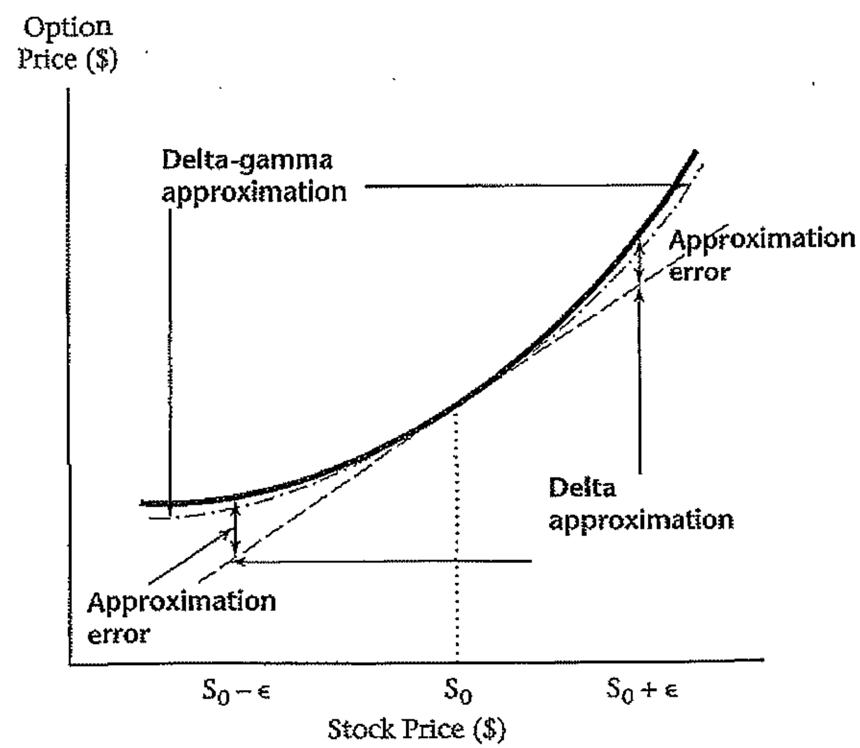
$$\approx 3.2355$$

true value

$$C(40.75) = 3.2352$$

FIGURE 13.3

Delta- and delta-gamma approximations of option price. The true option price is represented by the bold line, and approximations by dashed lines.



Use Θ as well

C (new S tomorrow)

$$= C(S) + \Delta\,(\text{change in } S) + \frac{1}{2}\Gamma\,(\text{change in } S)^2$$

$$+ \Theta\,(\underset{\substack{\uparrow \\ 1 \text{ day}}}{1})$$

TABLE 13.4 Predicted option price over a period of 1 day, assuming stock price move of \$0.75, using equation (13.6). Assumes that $\sigma = 0.3$, $r = 0.08$, $T - t = 91$ days, and $\delta = 0$, and the initial stock price is \$40.

	Starting Price	$\epsilon\Delta$	$\frac{1}{2}\epsilon^2\Gamma$	θh	Option Price 1 Day Later ($h = 1$ day) Predicted	Actual
$S_{t+h} = \$40.75$	\$2.7804	0.4368	0.0183	−0.0173	\$3.2182	\$3.2176
$S_{t+h} = \$39.25$	\$2.7804	−0.4368	0.0183	−0.0173	\$2.3446	\$2.3452