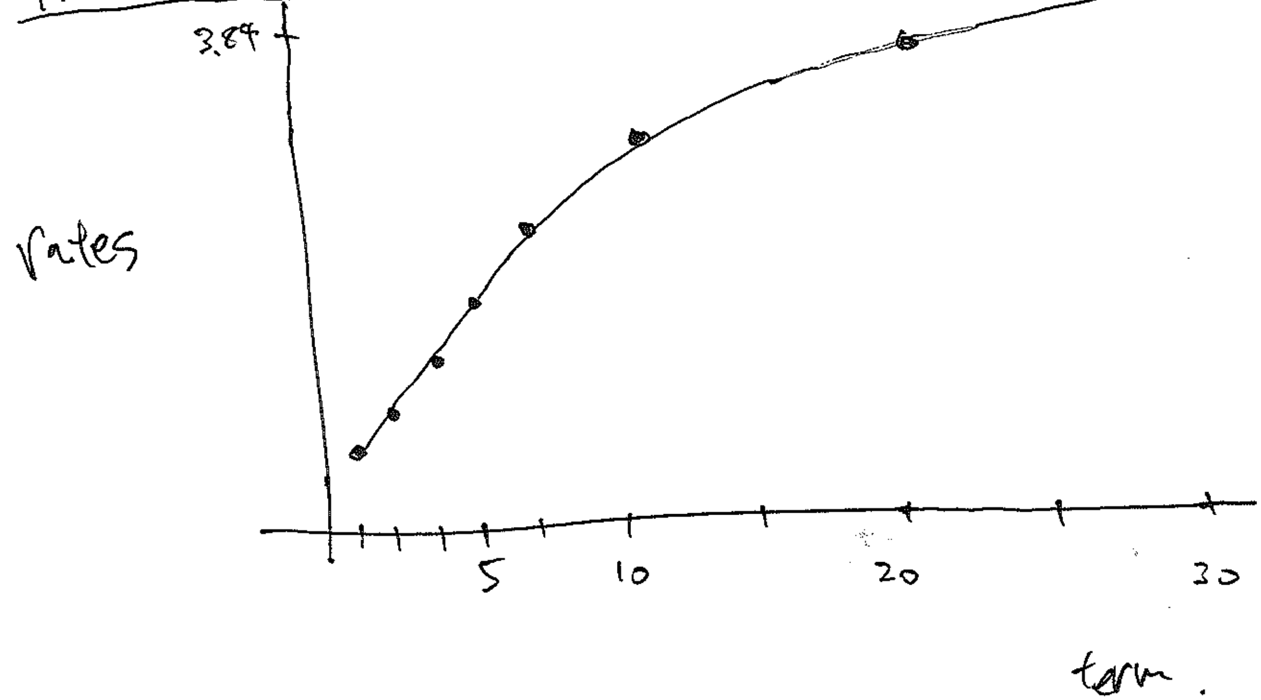
Chapter 6 Term Structure of Interest Rates

Yield rates



STRIPS

10%
5-year bond with F=100
(10 coupon left)

Single Yield rate to maturity

$$PV = 100v^{10} + 5a_{\overline{10}|j}$$

Separate trading of
Registered Interest and Principal
of Secrities.

1st Coupon in 6 mo. + Yield rate
2nd Coupon in 12 mo, + Yield rate
⋮
10th Coupon in 60 mo. +
F=100 payment in 60 mo. +

Zero-coupon Bond

Sell them separately.

Term Structure of Interest Rates

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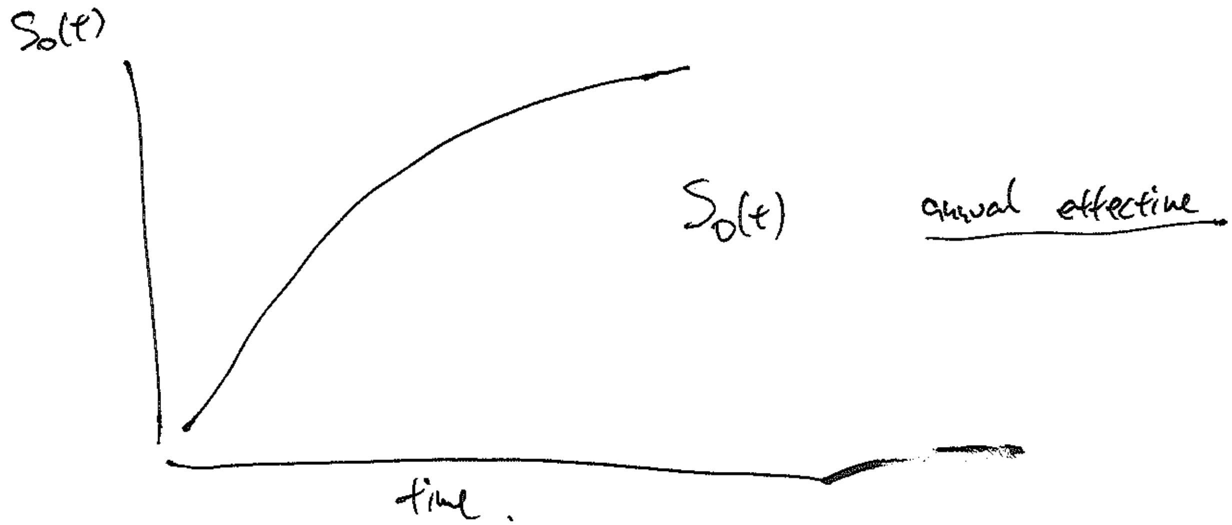
Yield Curve



Market's expectation of economy

Law of One Price

Price of a bond calculated with Yield to maturity rate.

10 coupon + F

sold as one

=

Price of a bond calculated as 10 zero-coupon bond with 10 different Yield rates.

Sold as Ten.

If the market is efficient.

Example 6.2

Term	6-mo.	12-mo.	18 mo.	24 mo.
Yield Curve (Spot rate) ann. eff. rate	8%	9%	10%	11%

Find PV (and Yield to maturity) of $F = \$100$ bond maturing in 2-years.

i) zero-coupon

ii) 4 coupons with 5% rate

~~iii) 4 coupons with 10% rate.~~

i) $100\, v^2 = 100\left(\frac{1}{1.11}\right)^2 =$

ii) ~~[illegible]~~

Must convert spot rate (ann. eff) to

semi-ann. eff. rate. 6-mo.

$8\% \quad (1.08)^{1/2} - 1 = 3.92\%$

$9\% \quad (1.09)^{1/2} - 1 = 4.40\%$

$10\% \quad \to \quad (1.10)^{1/2} - 1 = 4.49\%$

$11\% \quad (1.11)^{1/2} - 1 = 5.36\%$.

ii) $Fr = 2.5$ Coupon amount.

$$2.5\left[\left(\frac{1}{1.0392}\right) + \left(\frac{1}{1.044}\right)^2 + \left(\frac{1}{\cancel{\$}1.0449}\right)^3 + \left(\frac{1}{1.0536}\right)^4\right]$$

$$+ 100\left(\frac{1}{1.0536}\right)^4$$

=

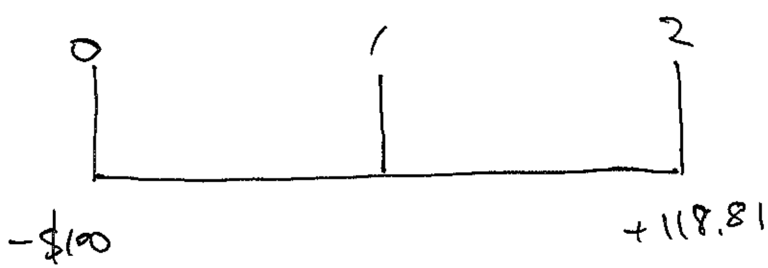
6.3 Foward Rates of Interest.

Suppose: (Yield Curve)

	1 yr	2 yr
Spot rate	.08	.09

Invest $100 in 2-year zero-coupon bond.

$\rightarrow$ get $100(1.09)^2 = 100(1.1881) = \$118.81$ in two years.



$\rightarrow$ 9% eff. ann. rate.

If we could borrow same rate as spot rate,

$S_0(t)$	1 yr	2 yr
	.08	.09

Borrow $100 for 1-year with rate 8% And invest in 2-year Zero-Coupon Bond → Pay $108 back to lender → Get $118.81 from Maturing bond

0	1	2
+100	− $108	+$118.81
−100		
= 0.		

$$\frac{118.81 - 108}{108} = .1001$$

10.01 %

Forward rate of Interest

'Borrowing' with Foward rate

Borrow \$100 for 2-year with 9% Spot rate

Invest in 1-year Zero-Coupon Bond

→ Get \$108 from Maturing Bond

→ Pay back \$118.81 to lender

0	1	2
+100	+108	−118.81
−100		
= 0		

- Borrowed with Future rate of 10.01 %.

Arbitrage

'free lunch'

Simultaneous purchase and sale of securities in different markets in order to profit from price discrepancies.

- risk - free.
- Arbitrage assumed not to exist.

Forward Rate : $i_0(n-1, n)$

Given Spot rate $S_0(t)$,

$$1 + i_0(n-1, n) = \frac{(1 + S_0(n))^n}{(1 + S_0(n-1))^{n-1}}.$$

+ Previous example.

$$1.1001 = \frac{(1+.09)^2}{(1+.08)^1}$$

$$1 + i_0(1,2) = \frac{(1 + S_0(2))^2}{1 + S_0(1)}.$$

Accumulated value using Foward rate

$$(1+S_0(n))^n = (1+S_0(n-1))^{n-1} \cdot (1+i_0(n-1,n))$$

$$\text{AV in 1 year} = 1+S_0(1) = [1+i_0(0,1)]$$

$$\text{AV in 2 years} = (1+S_0(2))^2 = [1+S_0(1)][1+i_0(1,2)]$$

$$= [1+i_0(0,1)][1+i_0(1,2)]$$

$$\text{AV in 3 years} = (1+S_0(3))^3$$

$$= [1+i_0(0,1)][1+i_0(1,2)][1+i_0(2,3)]$$

$$\vdots$$

Example 6.3

t	1	2	3	4
$S_0(t)$	.05	.1	.15	.2

Spot rate

Find Forward rate

$i_0(0,1)$

$i_0(1,2)$

$i_0(2,3)$

$i_0(3,4)$

$$i_0(0,1) = .05$$

$$i_0(1,2) = \frac{(1.10)^2}{1.05} - 1 = \cancel{.1524}\ .1524$$

$$i_0(2,3) = \frac{(1.15)^3}{(1.10)^2} - 1 = .2596$$

$$i_0(3,4) = \frac{(1.20)^4}{(1.15)^3} - 1 = .3634$$