

Swaps

Swap

- like forwards, but in series.

Example

100,000 barrels

IP Inc. - need to buy oil in 1 yr and 2 yr

Forward price \$20 in 1 yr

\$21 in 2 yr

6% risk-free rate for 1 yr

6.5% " 2 yr

If we enter long in 2 forwards, we need to
come up with cash

$$\begin{cases} 2 \text{ M in 1 yr.} \\ 2.1 \text{ M in 2 yr.} \end{cases}$$

$$\begin{aligned}
 PV &= \cancel{2.1m} \cdot 2m \left(\frac{1}{1.06} \right) + 2.1m \left(\frac{1}{1.65} \right)^2 \\
 &= \$3,738,300
 \end{aligned}$$

Prepaid Swap : pay \$3,738,300 ~~to~~ today, get
 100,000 barrel in 1yr and in 2yr.

Swap

Pay X in 1yr for 100,000 barrel of oil.
X in 2yr for 100,000 barrel of oil.

where

$$\frac{X}{1.06} + \frac{X}{1.065^2} = \frac{20}{1.06} + \frac{21}{1.065^2} = 37,38,300$$

$$X = \$20.483 \times 100,000$$

↑
2yr swap price.

This is physical
Settlement.

Financial Settlement.

pay \$20.483 \times 100,000 in 1 yr + 2 yr.

seller of this swap compensates the difference

	1 yr	2 yr	
Swap Price	\$20	\$21	
Spot price	\$20.5	\$20.5	
	\$21		notional amount.
<hr/>			↙
	\$1	-\$0.5	\times 100,000

Market Value of a Swap

Agreed to pay $\begin{matrix} 1y \\ \$20.483 \\ \$20 \end{matrix}$ $\begin{matrix} 2y \\ \$20.483 \\ \$21 \end{matrix}$ is stead of

Value of Swap = 0, until 1st swap is struck,

What if ^{right}~~the~~ after the agreement, Spot price of oil rises, and forwards are available at

$\left\{ \begin{array}{ll} \$22 & 1y \\ \$23 & 2y \end{array} \right.$

New swap

$$\frac{22}{1.06} + \frac{23}{1.065^2} = \frac{X}{1.06} + \frac{X}{1.065^2}$$

$$X = 22.483$$

Difference

$$\frac{2}{1.06} + \frac{2}{1.065^2} = 3.650$$

$$\text{Value of old swap} = 3.650$$

Interest Rate Swaps

XYZ corp. \$200m debt at floating-rate. LIBOR

London Inter-bank
offer rate

want fixed rate.

- ① Refinance with fixed-rate. (transaction cost)
- ② ^{may} forward rate agreements (cost will vary each year)
- ③ Interest rate swaps.

Prob

5 ways to present risk-free rates

Years to maturity	① Zero Coupon Yield	② Zero Coupon Bond Price	③ 1yr Implied Forward rate	④ Par Coupon (annual)	⑤ Continuously Compounded
1	6 %	.943396	6 %	6 %	5.82689 %
2	6.5 %	.881659	7.00236 %	6.48423 %	6.29748 %
3	7 %	.816298	8.00705 %	6.95483 %	6.76586 %
4 4					

<u>Year n</u>	① <u>Zero-Coupon Yield</u> <div>Yield Curve</div> Spot rate	② <u>Zero-Coupon Bond Price</u> PV of \$1 in year n	③ <u>1yr Implied Forward Rate</u>
1	1.06	$\frac{1}{1.06}$	1.06
2	1.065	$\frac{1}{1.065^2}$	$\frac{1.065^2}{1.06}$
3	1.07	$\frac{1}{1.07^3}$	$\frac{1.07^3}{1.065^2}$

④

Par

Coupon (annual)

Bond with $PV=1$

Face = 1

Coupon = C

$$1 = C \left(\frac{1}{1.06} \right) + 1 \left(\frac{1}{1.06} \right)$$

$$\ln(1.06)$$

$$1 = C \left(\frac{1}{1.06} \right) + C \left(\frac{1}{1.065^2} \right) + 1 \left(\frac{1}{1.065^3} \right)$$

$$\ln(1.065)$$

$$1 = C \left(\frac{1}{1.06} \right) + C \left(\frac{1}{1.065^2} \right) + C \left(\frac{1}{1.07^3} \right) + 1 \left(\frac{1}{1.07^3} \right)$$

$$\ln(1.07)$$

⑤

Continuously
Compounded

Interest Rate Swaps

1yr Forward rate — we can lock in today.

use Fwd rate

① Zero-Coupon Yield	③ Par Fwd rate Coupon
6%	6%
6.5%	7.00236%
7%	8.00705%

$$\frac{R - 6\%}{1.06} - \frac{R - 7.0024}{1.065^2} - \frac{R - 8.007\frac{1}{2}}{1.07^3} = 0$$

$$\Rightarrow R = 6.9548\%$$

Swap rate = Par coupon rate.

Lock in rate for 3 yrs.

Swap rate = par bond coupon.

Zero-coupon
bond Yld.

6.00

6.50

7.00

One-Year
implied Fw

6.00

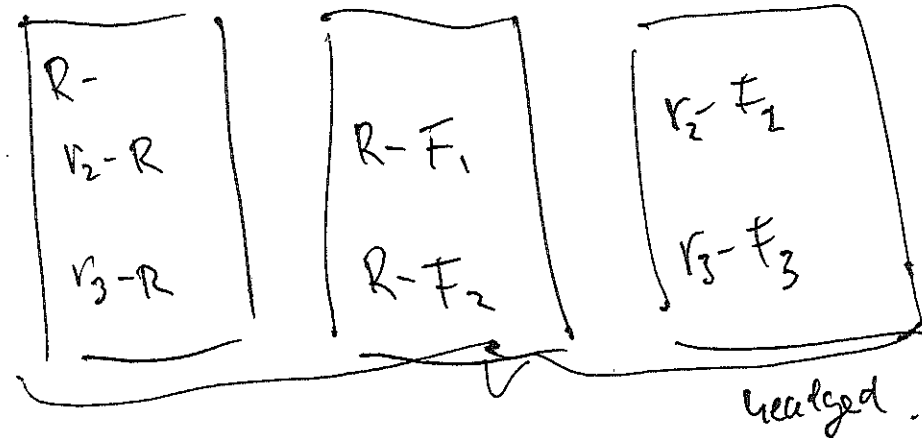
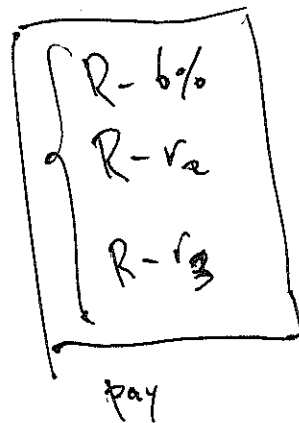
$$\frac{(6.50)^2}{6.00} = 7.00236$$

$$\frac{7.00^3}{(1.065)^2} = 8.0905$$

per Coupon
6.00

$$1 = \frac{c}{1.06} + \frac{c}{1.065^2} + \frac{1}{1.065^2} \quad C = .06484$$

$$1 = \frac{c}{1.06} + \frac{c}{1.065^2} + \frac{c+1}{1.065^3} \quad C = .06454$$



You

counter party.