Fall 2016 471 - HW 1

due Fri , Sep. 11th

NAME: Solution

1. 2500 is invested. Find the accumulated value of the investment 10 years after it is made for each of the following rates:

   (a) 4% annual simple interest (use simple interest for entire 10 years).

   $2500\,(1 + .04(10)) = \boxed{3500}$

   (b) 4% effective annual compound interest.

   $2500\,(1.04)^{10} = \boxed{3700.61}$

   (c) Nominal annual interest rate of 4% compounded semi-annually

   $2500\left(1 + \frac{.04}{2}\right)^{20} = \boxed{3714.868}$

   (d) Nominal annual interest rate of 4% compounded quarterly

   $2500\left(1 + \frac{.04}{4}\right)^{40} = \boxed{3722.159}$

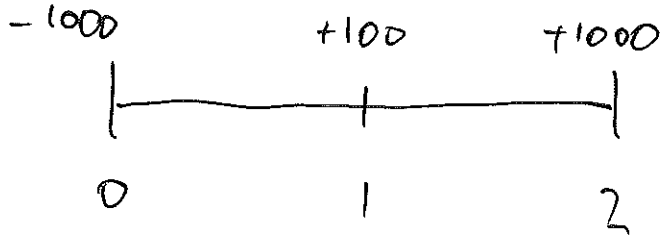
   (e) Find the rate of interest convertible quarterly that is equivalent to a nominal annual rate of interest of 6% convertible semiannually.

   $\left(1 + \frac{i^{(4)}}{4}\right)^{4} = \left(1 + \frac{.06}{2}\right)^{2} = 1.0609$

   $(1.0609)^{1/4} = 1.0149$ eff. 3mo rate

   $i^{(4)} = .0149(4) = \boxed{1.0596}$

2. Smith lends Jones 1000 on January 1, 2007 on the condition that Jones repay 100 on January 1, 2008, and 1000 on January 1, 2009. Let j be the 6-month rate earned on Smith's net transaction. Calculate j. (hint: PV of payments coming in must equal amount you lend.)



$$1060 = 100v + 1000v^2 .$$

$$1000v^2 + 100v - 1000 = 0$$

solve for v using quadratic eqn.

then $(1+i)^{1/2}$ to get bmo. rate

- OR -

	N	I/Y	PV	PMT	FV
BG	2	(?)	-1000	100	900

→ 5.12

5.12 % eff. ann.

$$(1.0512)^{1/2} = 1.0253$$

2.53 %

3. Smith receives income from his investments in Japanese currency (yen). Smith does not convert the yen to dollars, but invests the yen in a term deposit that pays interest in yen. He finds a bank that will issue such a term deposit, but it charges a 1% commission on each initial placement and on each rollover. The current interest rate on the yen deposits is a nominal annual rate of 3.25% convertible quarterly for a 3-month deposit. To keep his yen available, Smith decides to roll over the deposit every 3 months. What is the effective annual after-commission rate that Smith earns?

$i^{(4)} = 3.25\%$

$j = .008125$ 3 mo - rate.

effective 3 mo. rate $= .008125 - .01$
(commission)

$= -0.001875$ (lose money!)

$[1+j]^4 = [1-.001875]^4$

$= .992498$

$i = -.748\%$

Value should be .992521, and the answer should be i = - .748%

7. customer is offered an investment where interest is calculated according to the following force of interest:

$$\delta_t = \begin{cases} .02t & \text{if } 0 \le t \le 3 \\ .045 & \text{if } t > 3 \end{cases}$$

The customer invests 1000 at time $t = 0$. What nominal rate of interest, compounded quarterly, is earned over the first four-year period?

In 4 yrs,

$$A(4) = A(0)\, e^{\int_0^4 \delta_t \, dt}$$

$$\int_0^4 \delta_t \, dt = \int_0^3 .02t \, dt + \int_3^4 .045 \, dt$$

$$= .02\left(\frac{t^2}{2}\right)\Big|_0^3 + .045(1) = .135$$

$$A(4) = 1000 \cdot e^{.135}$$

$$e^{.135} = (1+i)^4 = (1+j)^{16}$$

in 4 yrs $\quad i$ = eff. ann. $\quad j$ = eff. 3mo. rate

$$1+j = \left(e^{.135}\right)^{1/16} = 1.008493$$

$$i^{(4)} = 4j = 4(.008493) = \boxed{.033893}$$

5. The newly independent nation of Falkvinas has a unit of currency called the Britarg. In the coming year inflation in Fallcvinas will be 100%, whereas Canada's inflation rate will be 14%. A Canadian investor can earn interest in Canada on Canadian dollars at an annual rate of 18%. What effective annual rate must an investor earn on Britmgs in Fallcvinas in order that his real rate of interest match the real rate earned by an investor in Canadian dollars?

CANADA

$i = 18\%$ $\quad r = 14\%$

$$\frac{i-r}{1+r} = \frac{.18-.14}{1.14} = .035$$

Falkvinas

$i = ?$ $\quad r = 100\%$

$$\frac{i-r}{1+r} = \frac{i-1}{2} = .035$$

$$i = \boxed{1.0702}$$

6. Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest of 4% convertible semiannually. At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ. After 7.25 years, the value of each account is the same. Calculate δ.

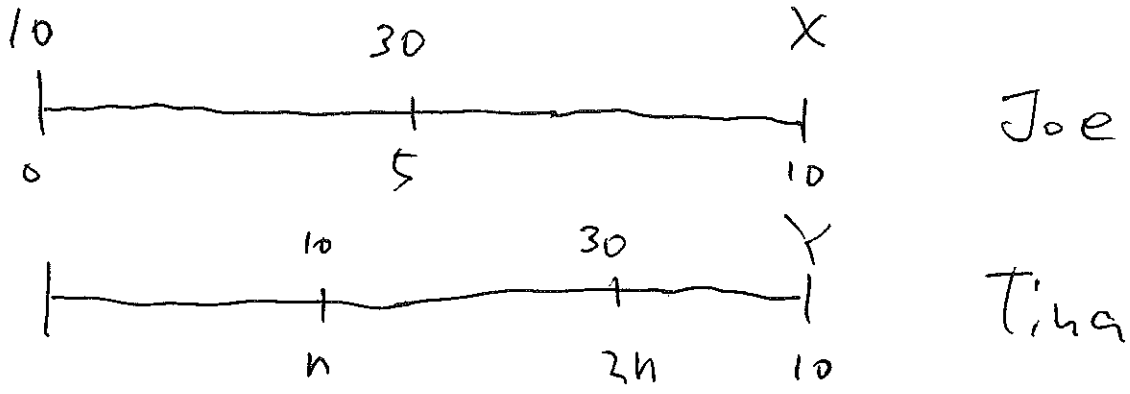
B: $100\left(1+\frac{.04}{2}\right)^{2(7.25)} = 133.26$

P: $100\, e^{\int_0^{7.25} \delta\, dt} = 100\, e^{\delta(7.25)} = 133.26$

$$e^{\delta(7.25)} = 1.3326$$

$$\delta = \boxed{.0396}$$

7. Joe deposits 10 today and another 30 in five years into a fund paying simple interest of 11 % per year. Tina will make the same two deposits, but the 10 will be deposited n years from today and the 30 will be deposited 2n years from today. Tina's deposits earn an effective annual rate of 9.15%. At the end of 10 years, the accumulated amount of Tina's deposits equals the accumulated amount of Joe's deposits. Calculate n.



$$X = 10(1+.11(10)) + 30(1+.11(5)) = 67.5$$

$$Y = 10(1.0915)^{10-n} + 30(1.0915)^{10-2n}$$

a

$$10a^{10-n} + 30a^{10-2n} = 67.5$$

multiply by a^{2n}

$$10a^{10+n} + 30a^{10} = 67.5a^{2n}$$

$$67.5a^{2n} - (10a^{10})a^{n} + 30a^{10} = 0$$

this is quadratic equation for $X = a^n$.

$$67.5x^2 - (10a^{10})x - 30a^{10} = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$67.5x^2 - 24.00145x - 92.00434 = 0$$

$$x = \frac{24.00145 \pm \sqrt{24.00145^2 - 4(67.5)(-92.00434)}}{2(67.5)}$$

$$= 1.2258 \text{ or } -.8702$$

$$1.2258 = (1.0915)^n$$

$$n = \frac{\ln(1.2258)}{\ln(1.0915)} = \boxed{2.33}$$