451/551 - Final Review Problems

1 Probability by Sample Points

1. If four dice are rolled, what is the probability of obtaining two identical odd numbers and two identical even numbers?

2. A box contains 5 balls numbered 1,2,3,4, and 5. Three balls are drawn at random and without replacement from the box. If X is the median of the numbers on the 3 chosen balls, then what is the probability function for X, where nonzero?

2 Probability by Event Composition

1. Let X_1, X_2, X_3 be random variables satisfying the constraints $P(X_1 > X_2) = 0.7$ and $P(X_2 > X_3) = 0.6$. What is the minimum possible value of $P(X_1 > X_2 > X_3)$?

2. Workplace accidents are categorized into three groups: minor, moderate and severe. The probability that a given accident is minor is 0.5, that it is moderate is 0.4, and that it is severe is 0.1. Suppose exactly two accidents occur independently in one month. Calculate the probability that neither accident is severe and at most one is moderate.

3. A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house. Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

3 Conditional Proability and Bayes' Formula

1. An insurance policy covers the two employees of ABC Company. The policy will reimburse ABC for no more than one loss per employee in a year. It reimburses the full amount of the loss up to an annual company-wide maximum of 8000. The probability of an employee incurring a loss in a year is 40%. The probability that an employee incurs a loss is independent of the other employee's losses. The amount of each loss is uniformly distributed on [1000,5000]. Given that one of the employees has incurred a loss in excess of 2000, determine the probability that losses will exceed reimbursements.

2. Micro Insurance Company issued insurance policies to 32 independent risks. For each policy, the probability of a claim is 1/6. The benefit amount given that there is a claim

has probability density function

$$f(y) = 2(1 - y), \quad 0 < y < 1,$$

and 0 otherwise. Calculate the expected value of total benefit paid.

3. The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the conditional probability that a male has a circulation problem, given that he is a smoker?

4. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during anyone-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Prob of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

4 Popular Distributions

1. A box contains 35 gems, of which 10 are real diamonds and 25 are fake diamonds. Gems are randomly taken out of the box, one at a time without replacement. (a)What is the probability that exactly 2 fakes are selected? (b)What is the probability that exactly 2 fakes are selected before the second real diamond is selected?

2. As part of the underwriting process for insurance, each prospective policyholder is tested for high blood pressure. Let X represent the number of tests completed when the first person with high blood pressure is found. The expected value of X is 12.5. Calculate the probability that the sixth person tested is the first one with high blood pressure.

3. Let X be a random variable with a uniform distribution on the interval $(1, a)$ where $a > 1$. If $E(X) = 6 \cdot V(X)$, then $a =$

4. An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100. Losses incurred follow an exponential distribution with mean 300. What is the 95-th percentile of actual losses that exceed the deductible?

5 Continuous RV

1. An actuary determines that the claim size for a certain class of accidents is a random variable, X, with moment generating function $M(t) = (1 - 2500t)^{-4}$. Determine the standard deviation of the claim size for this class of accidents.

2. A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the fIrst year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made. At what level must x be set if the expected payment made under this insurance is to be 1000?

3. A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the fIrst such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5. What is the expected amount paid to the company under this policy during a one-year period?

4. An insurance policy is written to cover a loss, X, where X has a uniform distribution on [0,1000]. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

6 Joint Distribution

1. Let X and Ybe random losses with joint density function $f(x, y) = e^{-(X+Y)}$ for $x > 0$ and $y > 0$. An insurance policy is written to reimburse $X+Y$. Calculate the probability that the reimbursement is less than 1.

2. Let X be a random variable with mean 3 and variance 2, and let Y be a random variable such that for every x, the conditional distribution of Y given $X = x$ has mean x and variance x^2. What is the variance of the marginal distribution of Y?

3. Let X and Y be discrete random variables with joint probability function

$$p(x, y) = \frac{y}{24x} \quad x = 1, 2, 4, \quad y = 2, 4, 8 \text{ and } x \le y,$$

and 0 otherwise. What is $P\big(X + \frac{Y}{2} \le 5\big)$?

4. Let X and Y be continuous random variables with joint density function

$$f(x, y) = \frac{3}{4}x \quad 0 < x < 2 \text{ and } 0 < y < 2 - x,$$

and 0 otherwise. What is $P(X > 1)$?

5. Let X and Y be continuous random variables with joint density function

$$f(x, y) = 8xy \qquad \text{for } 0 \le x \le y \le 1,$$

and 0 otherwise. Let $W = XY$. What is the density function of W?

6. Let X and Y be continuous randomvariables with joint density function

$$f(x, y) = x + y \quad 0 < x < 1, \quad 0 < y < 1,$$

and 0 otherwIse. What is the marginal density function for X, where nonzero?

7. Let X and Y be discrete random variables with joint probability function

$$f(x, y) = \frac{(x + 1)(y + 2)}{54} \qquad \text{for } x = 0, 1, 2, \text{ and } y = 0, 1, 2,$$

and 0 otherwise. What is $E(Y|X = 1)$?

8. Let X and Y be continuous random variables with joint density function

$$f(x, y) = 6x \quad 0 < x < y < 1,$$

Note that $E(X) = 1/2$ and $E(Y) = 3/4$. What is $Cov(X, Y)$?

7 MGF of Multiple RVs

1. Let X and Y be two independent random variables with moment generating function

$$M_X(t) = e^{t^2+2t} \quad M_Y(t) = e^{3t^2+t}$$

Determine the MGF of $3X + 2Y$.

2. The random variables X_1, X_2, X_3, X_4 and X_5 are independent and identically distributed. The random variable $Y = X_1 + X_2 + X_3 + X_4 + X_5$ has moment generating function $M_Y(t) = e^{15e^t-15}$. Find the variance of X_1.

8 CLT and other topics

1. Let $X_1, \ldots, X_{100}$ and $Y_1, \ldots, Y_{100}$ be independent random samples from uniform distributions on the intervals $[-10\sqrt{3}, 10\sqrt{3}]$ and $[-30, 30]$, respectively. According to the Central Limit Theorem, what is the approximate value of $Pr(\bar{Y} - \bar{X} < 1)$?

2. Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000. What is the probability that the average of 25 randomly selected claims exceeds 20,000?

3. The number of serious crimes reported daily in a certain city is a random variable X with mean 2 and variance 4. According to Chebyshev's inequality, $P(X \geq 10)$ is less than or equal to which of the following?