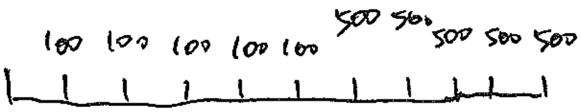
471 - HW2

due Fri, Sep. 30th

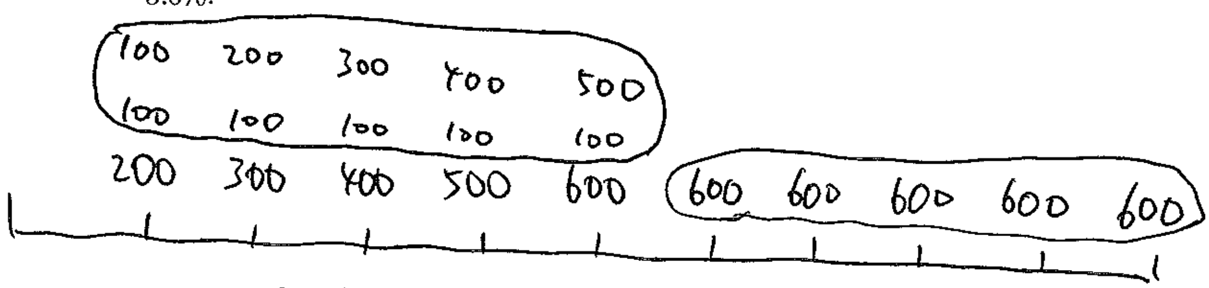
NAME: Solution

1. An annuity pays 100 at the end of each of the next 5 years and 300 at the end of each of the five following years. If i = .06, find the present value of the annuity.



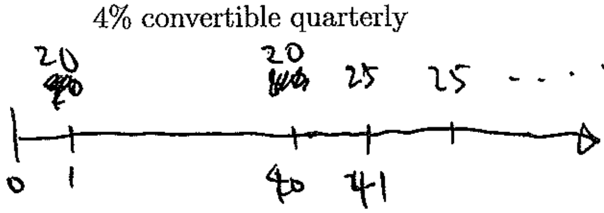
$$PV = 100\, a_{\overline{5}|.06} + 300\, a_{\overline{5}|.06}\, v^5$$
$$= 421.236 + 944.3169 = \boxed{1365.55}$$

2. An annuity immediate has a first payment of 200 and increases by 100 each year until payments reach 600. There are 5 further payments of 600. Find the present value at 5.5%.



$$PV = 100\,(Ia)_{\overline{5}|.055} + 100\, a_{\overline{5}|.055} + 600\, a_{\overline{5}|.055}\, v^5 = \boxed{3622.58}$$

3. An annuity immediate has 40 initial quarterly payments of 20 followed by a perpetuity of quarterly payments of 25 starting in the eleventh year. Find the present value at 4% convertible quarterly



$j = .01$

$$PV = 20\, a_{\overline{40}|.01} + 25\left(\frac{1}{.01}\right) v^{40}$$
$$= 656.694 + 1679.133 = \boxed{2335.83}$$

4 ~~4~~. Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an effective annual interest rate of i. The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of20 years. Calculate X.

4 yrs = 1 period $\qquad 1+j = (1+i)^4$

4yr rate

Annuity-Due

$$\underbrace{100\,\ddot{s}_{\overline{10}|j}}_{X} = 5 \cdot 100\,\ddot{s}_{\overline{5}|j}$$

Deposit at beggining
amount at the end } → annuity-due

$$\overbrace{100\left(\frac{(1+j)^{10}-1}{j}\right)(1+j)}^{X} = 5 \cdot 100\left(\frac{(1+j)^{5}-1}{j}\right)(1+j)$$

$$\cancel{100}\left[(1+j)^{10}-1\right] = 5 \cdot \cancel{100}\left[(1+j)^{5}-1\right]$$

$$\left[(1+j)^{5}-1\right]\left[(1+j)^{5}+1\right] = 5\left[(1+j)^{5}-1\right]$$

$$\left[(1+j)^{5}+1\right] = 5$$

$j = .3195$ $\qquad \boxed{X = 6195}$

5 ~~4~~. Sally lends 10,000 to Tim. Tim agrees to pay back the loan over 5 years with monthly payments at the end of each month. Sally can reinvest the payments from Tim in a savings account paying interest at 6%, compounded monthly. The yield rate earned on Sally's investment over the five-year period turned out to be 7.45%, compounded semi-annually. What nominal rate of interest, compounded monthly, did Sally charge Tim on the loan?

$10000 = X a_{\overline{60|}\, j_1}$

Sally lends to Tim.

j_1 = mo. rate for Tim.

$X s_{\overline{60|}\, j_2} = FV$ for Sally.

$j_2 = \frac{.06}{12} = .005$

FV for Sally $= 10000\left(1 + \frac{.0745}{2}\right)^{10}$

FV	N	I/Y	PV	PMT
FV for Sally	60	.5	0	X

$\longrightarrow X = -206.617$

FV	N	I/Y	PV	PMT
0	60	j_1	10000	X

$j_1 = .7334$

$i^{(12)} = 12(j_1) = \boxed{8.8\ \%}$

6~~8~~. At an effective annual interest rate of i, $i > 0$, the present value of a perpetuity paying 10 at the end of each 3-year period, with the first payment at the end of year 3, is 32. At the same effective annual rate of i, the present value of a perpetuity immediate paying 1 at the end of each 4-month period is X. Calculate X.

3 yr = 1 period $\qquad 1+j_1 = (1+i)^3$ $\qquad$ 3yr rate

$$\frac{10}{j_1} = 32 \quad - \text{(1)}$$

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4 mo. = 1 period $\qquad 1+j_2 = (1+i)^{1/3}$ $\qquad$ 4 mo rate

$$\frac{1}{j_2} = X \qquad\qquad j_2 = (1+i)^{1/3} - 1$$

From (1), $\quad j_1 = \frac{10}{32}$

$$(1+i)^3 = 1 + \frac{10}{32}$$

$$j_2 = \left(1+\frac{10}{32}\right)^{1/9} - 1$$

$$X = \frac{1}{j_2} = (1.307-1)^{-1} = \boxed{32.60}$$

7. 1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted. The annual withdrawals of interest and principal are deposited into FundY, which earns an annual effective rate of 9%. Determine the accumulated value of FundY at the end of year 10.

End of ~~Each~~ year $\quad i = .06$

1st	100	+	1000(i)	Deposit to Fund Y.
2nd year	~~2~~100	+	900(i)	
	~~3~~100	+	800(i)	
		⋮		
10th year	100	+	100(i)	

$$FV = 100\, s_{\overline{10|}.09} + (.06)(100)(Ds)_{\overline{10|}.09}$$

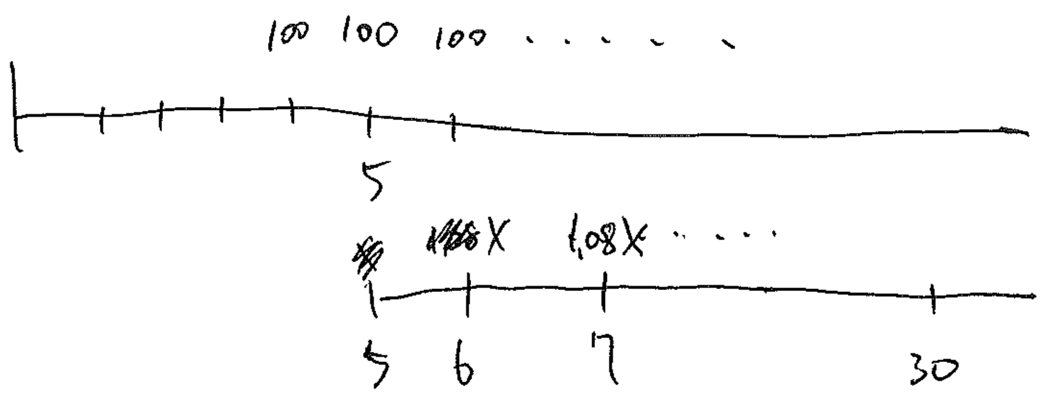
$$= 100\, s_{\overline{10|}.09} + 6\,(Da)_{\overline{10|}.09}(1+.09)^{10}$$

$$= 100\, s_{\overline{10|}.09} + 6\left(\frac{10 - a_{\overline{10|}.09}}{i}\right)(1.09)^{10}$$

$$= 1519.29 + 14.2042\,(39.8038)$$

$$= \boxed{2084.6712}$$

8. A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment. The annual effective rate of interest is 8%. Calculate X.



At E of Yr 5.

$$\frac{100}{.08} = X\,(Ga)_{\overline{25}|\,.08,\ .08}$$

$$\frac{1-v_0^{25}}{i-r}$$

Can't use formula b/c $i = r$.

$$Xv + X(1.08)v^2 + X(1.08)^2 v^3 + \cdots + X(1.08)^{24} v^{25}$$

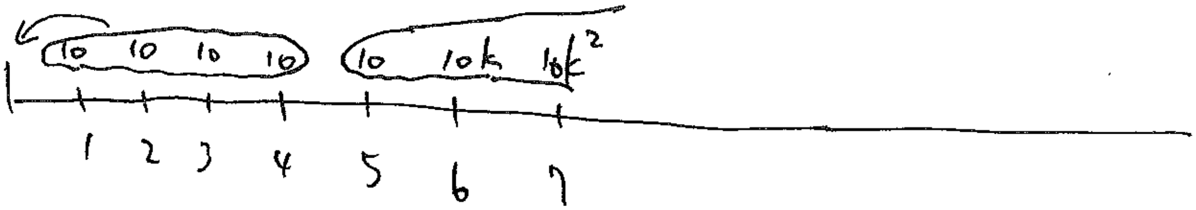
$$= Xv\left[1 + (1.08)v + \cdots + (1.08)^{24} v^{24}\right]$$

$$= Xv\,[25]$$

$$\frac{100}{.08} = X\left(\frac{1}{1.08}\right)25 \Rightarrow \boxed{X = 54}$$

9. Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the current year's payment is $K\%$ larger than the previous year's payment. At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50. Calculate K, given K ¡ 9.2.

$$PV = 10\,a_{\overline{4}|.092} + PV_2$$



$$PV_2 = v^5\, 10\,[v + Kv^2 + K^2v^3 + \cdots]$$

$$= v^6\, 10\,[1 + Kv + K^2v^2 + \cdots]$$

$$= v^6\, \frac{10}{1-Kv}$$

$$167.50 = 10\,a_{\overline{4}|.092} + 10\,\frac{v^6}{1-Kv}$$

$$167.50 - 10(3.2255) = \frac{10\,v^6}{1-Kv}$$

$$1-Kv = \frac{10\,v^6}{135.2450}$$

$$\boxed{\text{Increase by } 4.44\%}$$

$$K = \left[1 - \frac{10(.5897)}{135.2450}\right]1.092 = 1.0444$$

10. Olga buys a 5-year increasing annuity for X. Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2. The nominal interest rate is 9% convertible quarterly. Calculate X.

2 4 6 8 . . .

$$X = 2\left(Ia_{\overline{60}|}\right)_j$$

1.09

$$\left(1+\frac{.09}{4}\right)^{1/3} = 1.00744$$

$j = .00744$ 1 mo.

$$= 2\left(\frac{\ddot{a}_{\overline{60}|} - 60v^{60}}{.007444}\right)$$

$$= \boxed{2729.26}$$

12. An annuity-immediate pays 20 per year for 10 years, then decreases by 1 per year for 19 years. At an annual effective interest rate of 6%, the present value is equal to X. Calculate X.

$$\overbrace{20 \;-\; 20}^{10} \quad \overbrace{19 \quad 18 \quad 17 \quad \cdots \quad 1}^{19}$$

$$X = 20\, a_{\overline{10|}.06} + v^{10} (Da)_{\overline{19|}.06}$$

$$= 20\, a_{\overline{10|}.06} + v^{10} \left(\frac{19 - a_{\overline{19|}.06}}{i} \right)$$

$$= \boxed{220.18}$$