Ross

Ex. 4.~~20~~23 Hardy-Weinberg Law in Genetics

2 types of gene : A , a

each subject can have a pair : AA , Aa , aa

with prob. : p_0 q_0 r_0 . $(p_0+q_0+r_0=1)$.

mating occurs homogeneously.

What is the prob. dist. of next generation?

c.f. law of total prob.

$P\{\text{give of } A\}$

$$= P\{A \mid AA\} \cdot p_0 + P\{A \mid aa\} q_0 + P\{A \mid Aa\} r_0$$

$$= \frac{2}{2} \cdot p_0 + 0 + \frac{1}{2} r_0$$

$$= p_0 + \frac{r_0}{2}.$$

$$P\{\text{give } a\}$$

$$= P\{a \mid AA\}p_0 + P\{a \mid aa\}q_0 + P\{a \mid Aa\}r_0$$

$$= 0 + q_0 + \frac{r_0}{2}$$

$P($ next child has $AA)$

$= P($ father gives A , and , mother gives $A)$

$= P(A) \cdot P(A)$ (by independence)

$= \left(p_0 + \frac{r_0}{2}\right)^2$

$P($ next child has $aa)$

$= \left(q_0 + \frac{r_0}{2}\right)^2$

$$P(Aa) = P(\text{father gives } A, \text{ mother gives } a) + P(\text{" } a, \text{ " } A)$$

$$= 2 \cdot P(A) \cdot P(a)$$

$$= 2\left(p_0 + \frac{r_0}{2}\right)\left(q_0 + \frac{r_0}{2}\right)$$

	Gen 0	Gen 1	Gen 2
AA	p_0	$\left(p_0 + \frac{r_0}{2}\right)^2 = P(AA) = p$	
aa	q_0	$\left(q_0 + \frac{r_0}{2}\right)^2 = P(aa) = q$	
Aa	r_0	$2\left(p_0 + \frac{r_0}{2}\right)\left(q_0 + \frac{r_0}{2}\right) = r = P(Aa)$	

	Gen 0	Gen 1	
AA	p_0	$p = \left(p_0 + \frac{r_0}{2}\right)^2$	$\left(p + \frac{r}{2}\right)^2$
aa	q_0	$q = \left(q_0 + \frac{r_0}{2}\right)^2$	$\left(q + \frac{r}{2}\right)^2$
Aa	r_0	$r = 2\left(p_0 + \frac{r_0}{2}\right)\left(q_0 + \frac{r_0}{2}\right)$	$2\left(p + \frac{r}{2}\right)\left(q + \frac{r}{2}\right)$

no sq.

$$\left(p + \frac{r}{2}\right) = \left[\left(p_0 + \frac{r_0}{2}\right)^2 + \frac{2\left(p_0 + \frac{r_0}{2}\right)\left(q_0 + \frac{r_0}{2}\right)}{2}\right]$$

$$= \left(p_0 + \frac{r_0}{2}\right)\underbrace{\left[p_0 + \frac{r_0}{2} + q_0 + \frac{r_0}{2}\right]}_{p_0 + r_0 + q_0 = 1}$$

$$= p_0 + \frac{r_0}{2} \quad = P(A)$$

$$= \sqrt{p}$$

$$\left(q+\frac{r}{2}\right)^2 = \left(q_0+\frac{r_0}{2}\right)^2 + \frac{2\left(P_0+\frac{r_0}{2}\right)\left(q_0+\frac{r_0}{2}\right)}{2}$$

$$= \left(q_0+\frac{r_0}{2}\right)\underbrace{\left[q_0+\frac{r_0}{2} + P_0+\frac{r_0}{2}\right]}_{1}$$

$$= q_0+\frac{r_0}{2} = p(a)$$

$$= \sqrt{q}.$$

	Gen 0	Gen 1	Gen 2
AA	p_0	$p = \left(p_0 + \frac{r_0}{2}\right)^2$	$\left(p + \frac{r}{2}\right)^2 = p^*$
aa	q_0	$q = \left(q_0 + \frac{r_0}{2}\right)^2$	$\left(q + \frac{r}{2}\right)^2 = q^*$
Aa	r_0	$r = 2\left(p_0 + \frac{r_0}{2}\right)\left(q_0 + \frac{r_0}{2}\right)$	$2\left(p + \frac{r}{2}\right)\left(q + \frac{r}{2}\right) = 2\sqrt{p}\sqrt{q}$ $= 2\left(p_0 + \frac{r_0}{2}\right)^{*}\left(q_0 + \frac{r_0}{2}\right)$

Hardy - Weinberg Law

Distribution of Gene-types stabilizes after $\underline{\underline{1}}$ generation.

As MC

Single subject – one sibling.

	AA	aa	Aa
AA	$P(A)$	0	$P(a)$
aa	0	$P(a)$	$P(A)$
Aa	①	②	③

① → $P\left(\text{get } A \text{ from parent}\right) \cdot P\left(\text{get } A \text{ from mating}\right) = \left(\frac{1}{2}\right) P(A)$.

② → $P\left(\text{get } a \text{ from parent}\right) \cdot P\left(\text{get } a \text{ by mating}\right) = \left(\frac{1}{2}\right) P(a)$

③ → $1 - (① + ②)$.

As MC

$$\mathbb{P} = \begin{array}{c|ccc} & AA & aa & Aa \\ \hline AA & p+\frac{r}{2} & 0 & q+\frac{r}{2} \\ aa & 0 & q+\frac{r}{2} & p+\frac{r}{2} \\ Aa & \frac{1}{2}\left(p+\frac{r}{2}\right) & \frac{1}{2}\left(q+\frac{r}{2}\right) & \frac{1}{2} \end{array}.$$

$$1-\left(\frac{1}{2}\left(p+\frac{r}{2}\right)+\frac{1}{2}\left(q+\frac{r}{2}\right)\right)$$

$$= 1-\frac{1}{2}(p+q+r) = \frac{1}{2}.$$

$$\underline{\pi} = \begin{bmatrix} p & q & r \end{bmatrix}$$

is a stable distribution for MC with

$$\mathbb{P}.$$