HW on Ch 3

Name: ______________________

Questions:

1. Let X be a discrete random variable with $V(X) = 8.6$, then $V(3X+5.6)$ is ______________.
   $V(3X + 5.6) = 3^2 V(X) = 9(8.6) = 77.4$.

2. Let X be a discrete random variable with $E(X^2) = 19.75$ and $V(X) = 16.3$, then $E(X)$ $=$ ______________.
   $E(X) = \sqrt{E(X^2) - V(X)} = \sqrt{19.75 - 16.3} = 1.86$.

3. If random variable X has distribution Bin (10,.75), $V(X)$ is ______________.
   For Bin (10,.75), $V(X) = np(1-p) = 10(.75)(.25) = 1.875$.

4. If random variable X has distribution Bin (10,.75), $P(X = 3)$ is ______________.
   For Bin (10,.75), $P(X = 3) = \binom{10}{3}.75^3(.25)^7 = .0031$.

5. If the expected value of a discrete random variable X is $E(X) = 5$, then $E(2X + 3)$ is ______________.
   If $E(X) = 5$, then $E(2X + 3) = 2E(X) + 3 = 13$.

6. The probability mass function of a discrete random variable X is defined as $p(x) = x/10$ for $x = 0, 1, 2, 3, 4$. Then, the value of the cumulative distribution function $F(x)$ at $x = 3$ is $F(3) = p(0) + p(1) + p(2) + p(3) = 0 + 1/10 + 2/10 + 3/10 = .6$.

7. If random variable X has distribution Bin(6, .3), $E(X)$ is ______________. For Bin(6, .3), $E(X) = np = 6(.3) = 1.8$.

8. The mean of the hypergeometric random variable X with parameters n=10, $M = 50$, and $N = 100$ is ______________. For HG(10, 50, 100) $E(X) = nM/N = 5$.

9. The cumulative distribution function $F(x)$ of a discrete random variable X is: $F(1) =$ .4, $F(2) = .7$, $F(3) =$.9, and $F(4) = 1$, then the value of the probability mass function $p(x)$ at $X = 3$ is ______________. $p(3) = F(3) - F(2) = .9 - .7 = .2$.

10. The expected value of the negative binomial random variable X with parameters $r =$ 5 and $p = .8$ is______________. For NB(5, .8), $E(X) = r(1-p)/p = 5(.2)/.8 = 5/4$.

11. Suppose pmf for X = the number of major defects on a randomly selected gas stove of a certain type is

x	0	1	2	3	4
P(x)	.10	.15	.45	.25	.05

Compute the following:

(a) $E(X)$
$E(X) = 0(.10) + 1(.15) + 2(.45) + 3(.25) + 4(.05) = 2$

(b) $V(X)$

$$\begin{aligned} E(X^2) &= 0(.10) + 1^2(.15) + 2^2(.45) + 3^2(.25) + 4^2(.05) = 5 \\ V(X) &= E(X^2) - [E(X)]^2 = 5 - 2^2 = 1. \end{aligned}$$

(c) The standard deviation of X
$SD(X) = \sqrt{V(X)} = 1.$

12. There are 20 steel manufacturer in the city, and 6 of them is actually violating the city's enviromental protection law. You are going to randomly select 10 manufacturer in the city and and inspect them. Let X be the number of violating manufacturer caught in violation.

(a) What is the distribution function of X?

$$X \sim HG(n = 10, M = 6, N = 20)$$

(b) What is the probability, that all of 6 violators will be caught?

$$P(X = 6) = \frac{\binom{6}{6}\binom{14}{4}}{\binom{20}{10}} = .0054$$

13. Suppose in a large pool of students, 10 % are internarional students. We are going to randomly select n students to conduct a survey. We are concerned about international students being either over-represented, or under-represented in the survey. Idealy, the proportion of the international students in the selected group should be 10 %.

    (a) If n is 100, what is the probability that the international students are reprented in the survey by approximately correct proportion (8 to 12% of n) ?

$$\begin{aligned} P(8 \le X \le 12) &= P(X \le 12) - P(X \le 7) \\ &= B(12, 100, .1) - B(7, 100, .1) = .596. \end{aligned}$$

    (b) if n is 1000, what is the probability that the international students are reprented in the survey by approximately correct proportion (8 to 12% of n) ?

$$\begin{aligned} P(80 \le X \le 120) &= P(X \le 120) - P(X \le 79) \\ &= B(120, 1000, .02) - B(79, 1000, .02) = .97. \end{aligned}$$

14. The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a rate parameter of five per hour.

    (a) What is the probability that exactly four arrivals occur during a particular hour?

$$P(X = 4) = \frac{e^{-5}5^4}{4!} = .175$$

    (b) What is the probability that at least four people arrive during a particular hour?

$$\begin{aligned} P(X \ge 4) &= 1 - P(X \le 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) \\ &= 1 - e^{-5} - \frac{e^{-5}5}{1!} - \frac{e^{-5}5^2}{2!} - \frac{e^{-5}5^3}{3!} = .735 \end{aligned}$$

15. There are 5 red balls and 3 blue balls in a box. 3 balls are drawn from the box at once. If at least 1 blue ball is drawn, the draw is marked as "success." Let X denote the number of successful draws after 6 draws. What is the distrubution of X?

    If you let $Y =$ [# of blue balls in each draw],

$$\begin{aligned} Y &\sim HG(n = 3, M = 3, N = 8) \\ P(Y = 0) &= \frac{\binom{3}{0}\binom{5}{3}}{\binom{8}{3}} \\ P(\text{success for each throw}) = P(Y \geq 1) &= 1 - P(Y = 0) = .822 \end{aligned}$$

    Then

$$X \sim BIN(n = 6, p = .822).$$

16. You are playing a game that if you succesfully throw 4 bean bags in a bucket, you will win \$10. You must bay \$1 each time you throw a bean bag. Assume that each throw is independent, and probability that you can throw a bean bag into a bucket is .4. Let X be the number of throws you need to win that \$10.

    (a) What is the expected number of thows you need to win?

    You need to let $Y =$(number of failures untill 4th success). Then $Y \sim NB(4, .4)$.

$$E(X) = E(Y + 4) = E(Y) + 4 = \frac{4(1 - .4}{.4} + 4 = 10$$

    (b) You have \$6 in your pocket, and decided to play this game until either you win, or you go broke. What is your chance of winning?

$$\begin{aligned} P(Y \leq 2) &= P(Y = 0) + P(Y = 1) + P(Y = 2) \\ &= \sum_{x=0}^{2} \binom{x + 4 - 1}{4 - 1} (1 - .4)^x (.4)^4 = .1792 \end{aligned}$$

17. A hospital receives 1/5 of its flu vaccine shipments from Company X and the remainder of shipments from Company Y. Each shipment contains a very large number of vaccine vials. In Company X's shipments, it is known that 10% of the vials are ineffective. For Company Y, 2% of the vials are ineffective. The hospital just received a new shipment, but is not certain if it came from Company X or Y.

    The hospital tested 30 randomly selected vials from the new shipment and found only one vial is ineffective.

    (a) Suppose that if the shipment came from Company X. Then what is the probability of finding only 1 ineffective vial out of 30?

    If it was from company X, then number of ineffectives found in 30 random sample would have $Bin(30, .1)$ distribution. Therefore,

    $$P(\text{ 1/30 ineffective } |X) = P(\text{ 1 Head in 30 flips}) = \binom{30}{1}(.1)(.9)^{29} = .141.$$

    (b) Suppose that if the shipment came from Company Y. Then what is the probability of finding only 1 ineffective vial out of 30?

    If it was from company X, then number of ineffectives found in 30 random sample would have $Bin(30, .02)$ distribution. Therefore,

    $$P(\text{ 1/30 ineffective } |Y) = P(\text{ 1 Head in 30 flips}) = \binom{30}{1}(.02)(.98)^{29} = .334.$$

    (c) Use Bayes' theorem (Law of total probability) to calculate probability that this shipment came from Company X given the test result.

    $$\begin{aligned} P(X|\text{1/30 ineff}) &= \frac{P(\text{1/30 ineff}|X)P(X)}{P(\text{1/30 ineff}|X)P(X) + P(\text{1/30 ineff}|Y)P(Y)} \\ &= \frac{(.141)(1/5)}{(.141)(1/5) + (.334)(4/5)} = .0955. \end{aligned}$$

    (d) Use Bayes' theorem to calculate probability that this shipment came from Company Y given the test result.

    $$\begin{aligned} P(Y|\text{1/30 ineff}) &= \frac{P(\text{1/30 ineff}|Y)P(Y)}{P(\text{1/30 ineff}|X)P(X) + P(\text{1/30 ineff}|Y)P(Y)} \\ &= .9045. \end{aligned}$$