Forwards and Futures

Types of Forwards

	Pay at	Receive Asset	Price.
Outright Rupdhase	t=0	← = 0	S,
Fully leveraged Purchase	t=T	t= 0	Soett
Prepaid forward	t=0	t=T	?
Forward	t= T	t= T	(?).erT

Prepaid Forwards

Pay at t=0. reeile at t=T.

- could miss dividendes.

It there are no dividerals to be paid,

 $\overline{t_{0,7}} = S_0$  (fall at t=0)

Present Value 
$$A(0) = A(t) e^{\int_{t}^{t} r dt} = A(t) e^{-rT}.$$

# IF to + So, then there's an arbitrage

Suppose For \$ So, say Ao. then you can Ao  $\gtrsim S_o$  t = TSell forward at  $A_o$ . t = TSell forward t = TMay t = TMay t = TBuy saine stock at -So. - D ST A. < 5 ... Short Stock So

### with Dividends Prepaid Forward=

Discrete dividends

$$F_{0,T}^{PV} = S_0 - \sum_{i=1}^{N} PV(P_{x_i})$$

Dt: = dividend paid at t = t;

Ex 5, 1

50= 100.

\$1.25 dividades quarterly. Ist one in 3-40.

annual risk free rate 10%. Continuously compounded.

1-year phepaid forward price ?

Review.

nominal annual rate 
$$i^{(m)} = 8\%$$
.

effective annual rate  $1+i = (1+\frac{im}{m})^m$ .

 $m=2$   $1+i = 1.0824$ 

$$M = W$$
  $1+i = \lim_{N \to W} (1+\frac{iw}{w})^{N} = e^{-iw} = e^{-iw} = (.0833)$ 

Force of 
$$y = \ln(1+i) = \ln(e^{im}) = i^{(m)} = \lim_{n \to \infty} \frac{1}{n} = \frac{1}{n}$$

Ex 5. Confinously compounded (nominal) annual varie

(M) = 10%

Continuously compounded hominal quartery rate

(1) (1) (1) (1)

quarterly

= 1+j = (1+i)/4 ett ective

quarterly

Continuously Compounded

autimously Compounded

autimously Compounded

autimously Compounded

e = (+)

quarterly rate

e = (1+i)4

annval

 $r = \frac{10\%}{4} = 2.5\%$ 

and rate

$$F_{0,7}^{P} = 1000 - \sum_{i=1}^{4} \frac{-(.1)(i_{4})}{1.75}$$

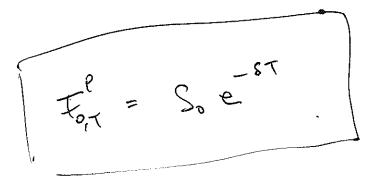
quarterly rate

$$t_{0,7}^{\ell} = (000 - \sum_{i=1}^{4} 1.25 e^{-(.025)(i)}$$

# Price of Prepaid Find with continuous dividends suppreximation suppreximatio

It we reinblest all the dividends,

$$A(t) = A(0) \left(1 + \frac{8}{365}\right)$$
 $A(0) = A(0) = A(0)$ 
 $A(0) = A(0)$ 



my Cour. dividends

## Forward Contract

pay at t=T, receive at t=T.

when there's no dividends.

For = FV(So) = Soet

r = risk-free vate Continuously compounded.

It Forward price is u	of Set
	there's alitrage
Say For \$ = Ao \$	Set
A. > S.e t=0	t=T
sell forward o	Ao
borrow at rate r	
buy asset So	-> St say by torward contract.
•	Ao - SoerT

Ao < S, e<sup>th</sup>

teo t=T.

buy forward o -Ao

lend at rate  $t=S_0 \rightarrow S_0 e^{tT}$ short asset  $t=S_0 \rightarrow S_0 e^{tT}$ short asset  $t=S_0 \rightarrow S_0 e^{tT}$ and  $t=S_0 \rightarrow S_0 e^{tT}$ short asset  $t=S_0 \rightarrow S_0 e^{tT}$ 

Torvard Discrete Dividends

Continuous Dividends

PV or stock.

Wo dividerds

Futures Coutracts

### Futures

- almost like forwards.

Agreeweht to buy/sell asset at given price in future dates.

AS

### Ditterences from torwards

- standardized dares, locations, procedures, quantity.
- Settled daily
- There's daily price limits in future markets
  - eg, if S&P500 & more than 5 90, then trading stops tor some period.

Example | Chicago Mercantile Exchange

Size \$250 x Sp500 index \* Mosson

Cash settled based on such spsoo price on 3-rd Friday.

- A Suppose you want to long \$500 Futures in for \$2.2 million

SP500 today = 1100

1 contract = \$250, 1100 = \$.275 mil.

2.2 mil = 8 contracts.

Broker finds somebody to sell this to YOV. (short)

Broker will require 'Margin' from both buyer and sellar.

It there went day price = 1099,

8 couperacts = 8 x 250 x 1100 = 2,200,000 must pay for sprop

but SP500 only worth

8 x 250 x 1099 = 2,198,000

- 2000 .

Margin will acquire interest.