Fall 2016

3470:651 - Practice Final Exam

Name: ________________________________

Final answer must be clearly indicated, and all pdf and cdf must be accompanied by support.

You are encouraged to EXPLAIN your thought process for partial credit.

1. Suppose r.v. X has cdf,

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{5} & \text{if } 0 \leq x < 1 \\ \frac{x^2}{5} & \text{if } 1 \leq x < 2 \\ 1 & \text{if } 2 \leq x \end{cases}$$

   Calculate $E(X)$. (No need to simplify the expression.)

2. Let $X_1 \sim Exp(2)$ with cdf

$$F(x) = 1 - e^{-x/2} \qquad x > 0.$$

and $X_2 \sim U(0,5)$ with cdf

$$F(x) = \frac{x}{5} \qquad 0 < x < 5.$$

X_3 has same cdf as X_2. Assuming X_1 and X_2 are independent, obtain expression for $E(Y)$ where $Y = \min(X_1, X_2)$. (No need to simplify the expression.)

3. Let X_1, X_2 be independent Unif(0,1) r.v. Obtain joint pdf of r.v. Y_1, Y_2, where

$$Y_1 = X_1/X_2^2 \qquad Y_2 = X_2^2.$$

(No need to simplify the expression.)

4. Let r.v. X and Y have conditional pdf $f(x|y) = c_1 x^2/y^2, 0 < y < x < 1$, zero elsewhere. Obtain

$$P\Big\{(3X < 2) \cap (2X - 2Y < 1)\Big| Y = y\Big\}$$

as a function of y. Set up integral(s). (No need to solve the integral.)

5. Below is a two-way table for joint pmf of discrete r.v. X and Y. Compute $V(X|Y \geq 3)$.

	Y		
	2	3	4
X=1	.07	.2	.1
X=2	.03	.23	.05
X=3	.15	.12	.05

(No need to simplify the expression.)

6. Suppose that a hospital has large stock of vaccination. 50% of stock are from company A, 30% of stock are from company B, and rest is from company C. Defect rate of vaccines from each companies are as follows:

$$A : .1 \qquad B : .15 \qquad C : .05$$

Suppose a batch of 10 vaccine is randomly chosen. It is known that all of 10 vaccine came from same company. When all 10 vaccine were tested, there were 2 defectives.

Given this test result, what is the probability that the chosen batch came from company A? (No need to simplify the expression.)

7. Compute the integral

$$\int_{-\infty}^{\infty} e^{3^2/4} e^{-(x^2+4x+4)/2} dx.$$

Obtain the actual constant. Simplify as much as possible.

8. There are three cards of "A", five cards of "B", four cards of "C". When all twelve cards are randomly put into sequence, what is the probability that the sequence contains string "AAA"?

9. Prove

$$E(e^X) \geq e^{E(X)},$$

using one of Markov, Chebychev or Jensen's inequality.

10. Let X have a binomial distribution with parameters n and p, and let the conditional distribution of Y given $X = x$ be Poisson with mean x. What is the variance of Y?