PSE Midterm 2 - Fall25

Name: _____

75 min.
You must show your work, and indicate final answer clearly. Please try to turn your pictures into one PDF file for the submission.

1. Let X be a random variable with mean of 5 and variance of 4.5. What is the probability that X will be less than 7?

2. Suppose r.v. X has the normal distribution with mean of 15 and standard deviation of σ. Probability that X is more than 20 is .05. Find the value of σ.

3. Suppose random variable X has normal distribution with mean μ and variance of 3. Probability that X is more than 12 is .05. Find the value of μ.

4. Suppose random variable X has normal distribution with mean 50 and SD of 7. What is the 90 th percentile of X?

5. Suppose waiting time until the next failure of oil pump system is exponentially distributed, with mean of 30 days. The pump is continuously in operation. What is the probability that the system does not fail for 30 days?

6. Suppose that random sample of size 100 are drawn from Normal distribution with mean five and variance four. Calculate the probability that the sample mean will be in between (5-.4) and (5+.4).

7. The manufacturer of a new fiberglass tire claims that its average life will be at least 40,000 miles. To verify this claim, sample of 78 tires are tested. Sample mean was 37.8 (in 1000 miles). Assume the measurements were random sample from a Normal distribution with standard deviation of 12.5.

   Using 95% one-sided confidence interval, choose the best answer from choices (a) - (e) below to the following question: Can this data be taken as a statistical evidence that the true mean life of tires is LESS THAN 40 (in 1000 miles)?

(a) Yes, because the CI does not include 40. We have the evidence that the mean is less than 40.

(b) Yes, because the CI includes 40. We have the evidence that the mean is less than 40.

(c) No, because the CI does not include 40.

(d) No, because the CI includes 40. The analysis is inconclusive regarding the mean being less than 40.

(e) No, because the CI includes 40. This is an evidence that the mean is not below 40.

8. What is the numerical value of 97.5th percentile of t-distribution with degrees of freedom 11 (i.e. $t_{.025,11}$)?

9. Random sample of size 50 are drawn from a lot containing new tires from brand T. Under a controlled condition, life of each tire was measured in the unit of 1000 miles. The sample mean was 32.5, and sample SD was 5.83. We want to answer the question: is the true mean of brand T tires greater than 30,000 miles? Assuming the random sample were drawn from from a Normal distribution, calculate appropriate 95% one-sided confidence interval for the true mean life of the tire. You must write your answer in a form of interval (,).

10. Suppose you and your friend are playing the following game. First, your friend must pay 2 dollars to you. Then you roll a die and calls the resulting number X. If X is 1 or 2, you pay back to the friend \$1. If X is 6, you pay \$7. Otherwise, you pay nothing. You rolled the die 15 times, and here's the result:

$$\{1,4,3,4,1,2,3,2,6,2,3,5,2,5,3\}$$

In the long-run, what is your profit/loss will be? Answer in per game basis.

11. b). For the above problem, what is the theoretical variance of the profit from one roll?

12. c). For the above probelm, if your friend played this game 100 times, what is the probability that profit will be positive? (i.e. $\bar{X} > 0$). Use the central limit theorem to approximate.