Inverse Method

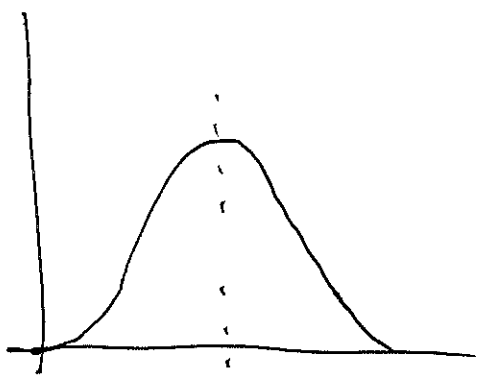
$X \sim F_X(x)$

CDF

Normal

pdf

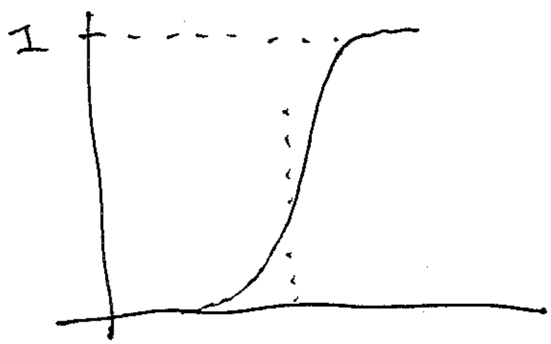
$f_X(x)$



$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

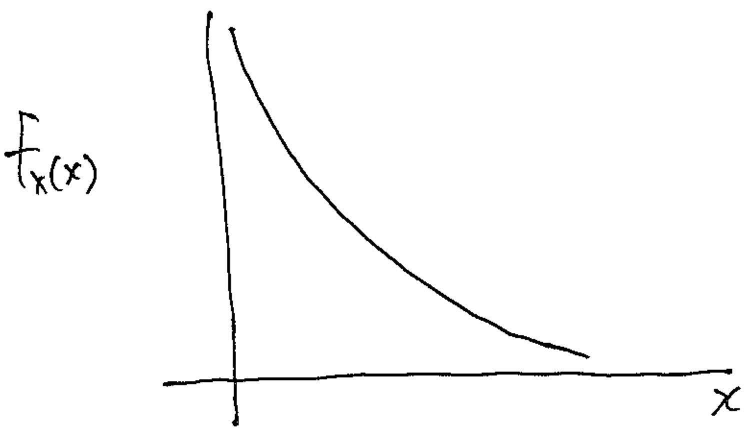
CDF

$F_X(x)$

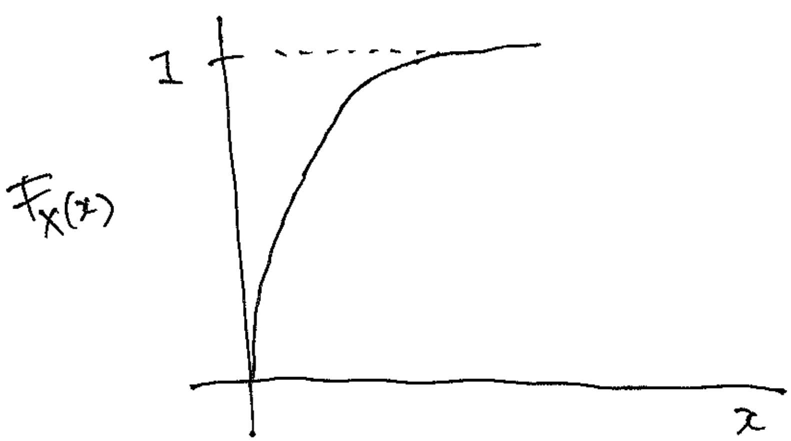


$$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$$

$\boxed{\text{Exp}}$



$$f_X(x) = \frac{1}{\lambda} e^{-x/\lambda} \qquad x \geq 0$$



$$F_X(x) = \int_0^x \frac{1}{\lambda} e^{-t/\lambda}\, dt \qquad x \geq 0.$$

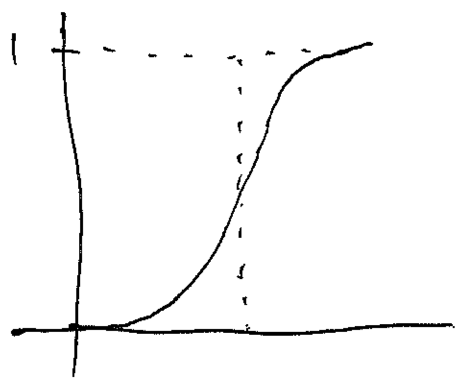
$$= 1 - e^{-x/\lambda}$$

Thm : what ever the distribution is ;

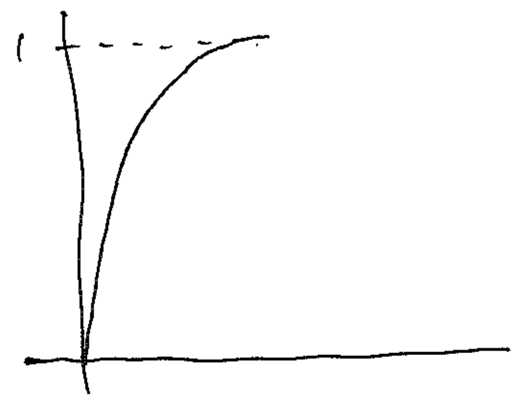
If $X \sim F_X$, then

$$F_X(X) \sim UNIF(0,1).$$

Normal



Exp



Inverse Method for Generating R.V.

Want to generate $X \sim F_X(x)$.

We know that

$$F_X(X) \underset{\text{in distribution}}{=} U .$$

That means,

$$X \underset{\text{in dist.}}{=} F_X^{-1}(U) .$$

where $U \sim UNIF(0,1)$.

We know how to generate U.

in R: runif(n)

Ex.

$X \sim Exp(5)$,

$$F_X(x) = 1 - e^{-x/5}.$$

let

$$U = 1 - e^{-x/5}.$$

sloving for x,

$$U - 1 = -e^{-x/5}$$

$$-\frac{x}{5} = \ln(1-u)$$

$$X = -5 \cdot \ln(1-u).$$

but, since

$1-u \sim Unif(0,1)$,

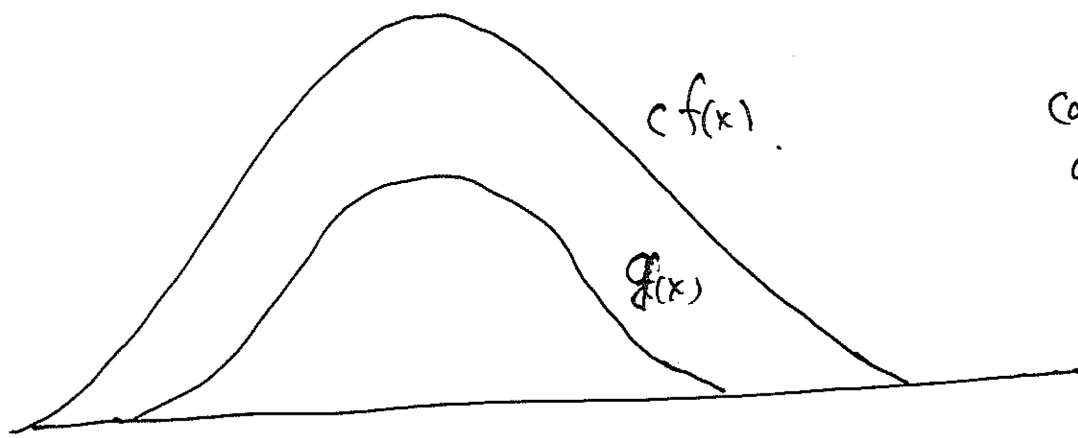
$$\boxed{X = -5\ln(u)}$$

① generate n #s from unif(0,1). call it u.

② let $X = -5\ln(u)$,

③ X is exponentially distributed with mean 5.

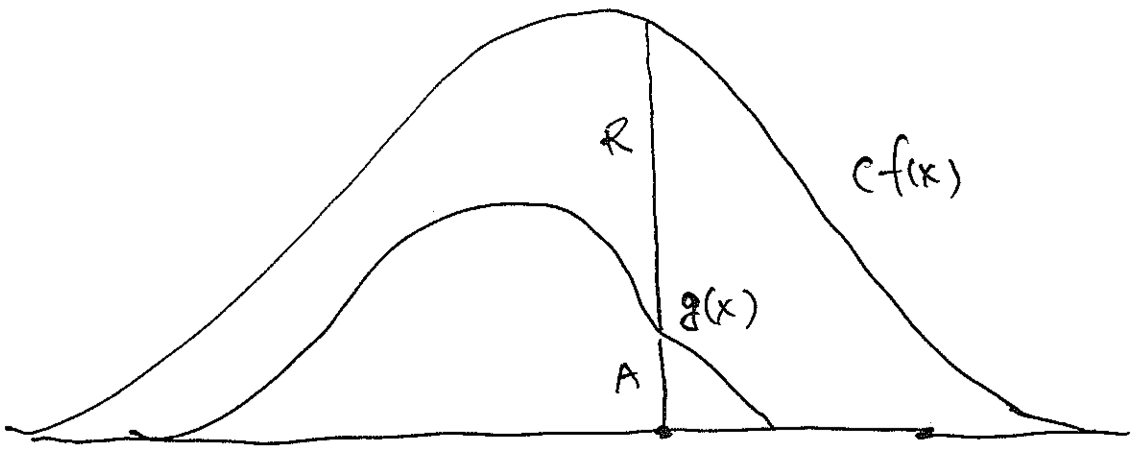
Acceptance - Rejection Method



$g(x) \leq c\,f(x)$

can't generate

Can generate



① generate X_1 from $f(x)$.

② ~~with~~ generate $U \sim UNIF(0, cf(X_1))$

③ if $U < g(X_1)$, then accept X_1 as generation from $g(x)$.

if not, disregard X_1.

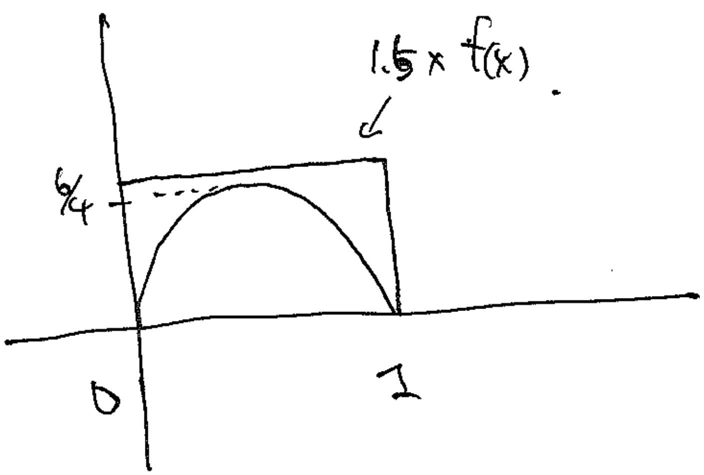
④ repeat.

Ex. A-R for Beta (2, 2)

Beta (2, 2)

$$g(x) = 6x(1-x).$$

pdf



① generate $X_1 \sim \text{UNIF}(0,1)$

② generate $U \sim \text{UNIF}(0, 1.5)$

③ if $U < 6X_1(1-X_1)$ accept.

④ repeat.

$f(x)$ = pdf of UNIF(0,1).