Chapter 3

Loan Repayment

Example 3.1 (Amortization Table)

- Loan of amount $1000 at a nominal ann. interest rate of 12% Comp. monthly.
- Repaid by 6-monthly payments, starting after one month.
- 1st three payments of X each, find three payments of 2X each.

Construct Amortization Schedule.

Present value

$$1000 = X\, a_{\overline{3}|.01} + (2X)\, v^3\, a_{\overline{3}|.01}$$

$$= X\,\frac{(1-v)^4}{i} + 2X\, v^3 \cdot \frac{(1-v)^4}{i}$$

$i = .01$ $\quad n = 3.$

$v = \frac{1}{1.01}$

$$X = 115.61$$

Outstanding Balance (Principal)

$$OB_1 = OB_0(1+i) - K_1$$

- OB_1: How much left
- OB_0: amount borrowed
- K_1: 1st payment

$$OB_1 = OB_0 - \left(K_1 - \underbrace{OB_0 \cdot i}_{\text{Interest paid in 1st period}}\right)$$

$K_1 - OB_0 \cdot i$: Principal Repaid

For each $i = 0, \ldots, n-1$

$$OB_{i+1} = OB_i - \left(K_{i+1} - \underbrace{OB_i \cdot i}_{I_{i+1}}\right)$$

$K_{i+1} - OB_i \cdot i$: PR_{i+1}

$$OB_n = 0$$

t	Payment	Interest Due	Principal Repaid	Outstanding Balance
0	–	–	–	$L = OB_0 = 1000$
1	$K_1 = 115.61$	$I_1 = OB_0 \times i$ $= 10$	$PR_1 = K_1 - I_1$ $= 105.61$	$OB_1 = OB_0 - PR_1$ $= 894.39$
2	$K_2 = 115.61$	$I_2 = OB_1 \times i$ $= 8.94$	$PR_2 = K_2 - I_2$ $= 106.67$	$OB_2 = OB_1 - PR_2$ $= 787.72$
3	$K_3 = 115.61$	$I_3 = OB_2 \times i$ $= 7.88$	$PR_3 = K_3 - I_3$ $= 107.73$	$OB_3 = OB_2 - PR_3$ $= 679.99$
4	$K_4 = 231.21$	$I_4 = OB_3 \times i$ $= 6.80$	$PR_4 = K_4 - I_4$ $= 224.41$	$OB_4 = OB_3 - PR_4$ $= 455.58$
5	$K_5 = 231.21$	$I_5 = OB_4 \times i$ $= 4.56$	$PR_5 = K_5 - I_5$ $= 226.65$	$OB_5 = OB_4 - PR_5$ $= 228.93$
6	$K_6 = 231.21$	$I_6 = OB_5 \times i$ $= 2.29$	$PR_6 = K_6 - I_6$ $= 228.92$	$OB_6 = OB_5 - PR_6$ $= .01$
Totals	**1040.46**	**40.47**	**999.99**	

Amortization Table

Retrospective Form of OB_3:

$$OB_3 = OB_0(1+i)^3 - K_1(1+i)^2 - K_2(1+i) - K_3$$

$$= OB_0(1+i)^3 - K\,s_{\overline{3}|i} \quad \text{(if } K \text{ and } i \text{ are the same)}$$

Prospective Form of OB_3:

$$OB_3 = K_{3+1}v + K_{3+2}v^2 + \cdots + K_n v^{n-3}$$

$$= K\,a_{\overline{n-3}|i} \quad \text{(if } K, i \text{ are the same)}$$

$$OB_t = OB_0(1+i)^t - K_1(1+i)^{t-1} - \cdots - K_{t-1}(1+i) - K_t \qquad (3.9)$$

In Ex 3.1 . $OB_0 = 1000$

$k_1 = k_2 = k_3 = 115.61$

$$OB_3 = 1000(1.01)^3 - 115.61(1.01)^2 - 115.61(1.01) - 115.61$$

$$= 1000(1.01)^3 - 115.61\, s_{\overline{3}|1.01}$$

$$= 679.99$$

$$OB_t = k_{t+1} v + k_{t+2} v^2 + \cdots + k_n v^{n-t}$$

prospective form.

In Ex 3.1

$$OB_3 = 231.21\, v + 231.21\, v^2 + 231.21\, v^3$$

$$= 231.21\, a_{\overline{3}|.01}$$

$$= 679.88$$

Example 3.3 — Capitalization of interest.

Loan of 100,000

payments at the end of each month for 12 yrs.

2 /mo. for 6 yrs.

22 /mo. for last 6 yrs.

nominal annual rate 12%, compounded monthly.

→ Find Outstanding Balance at the end of 1st year.

6 yrs = 72 mo. $i = .01$

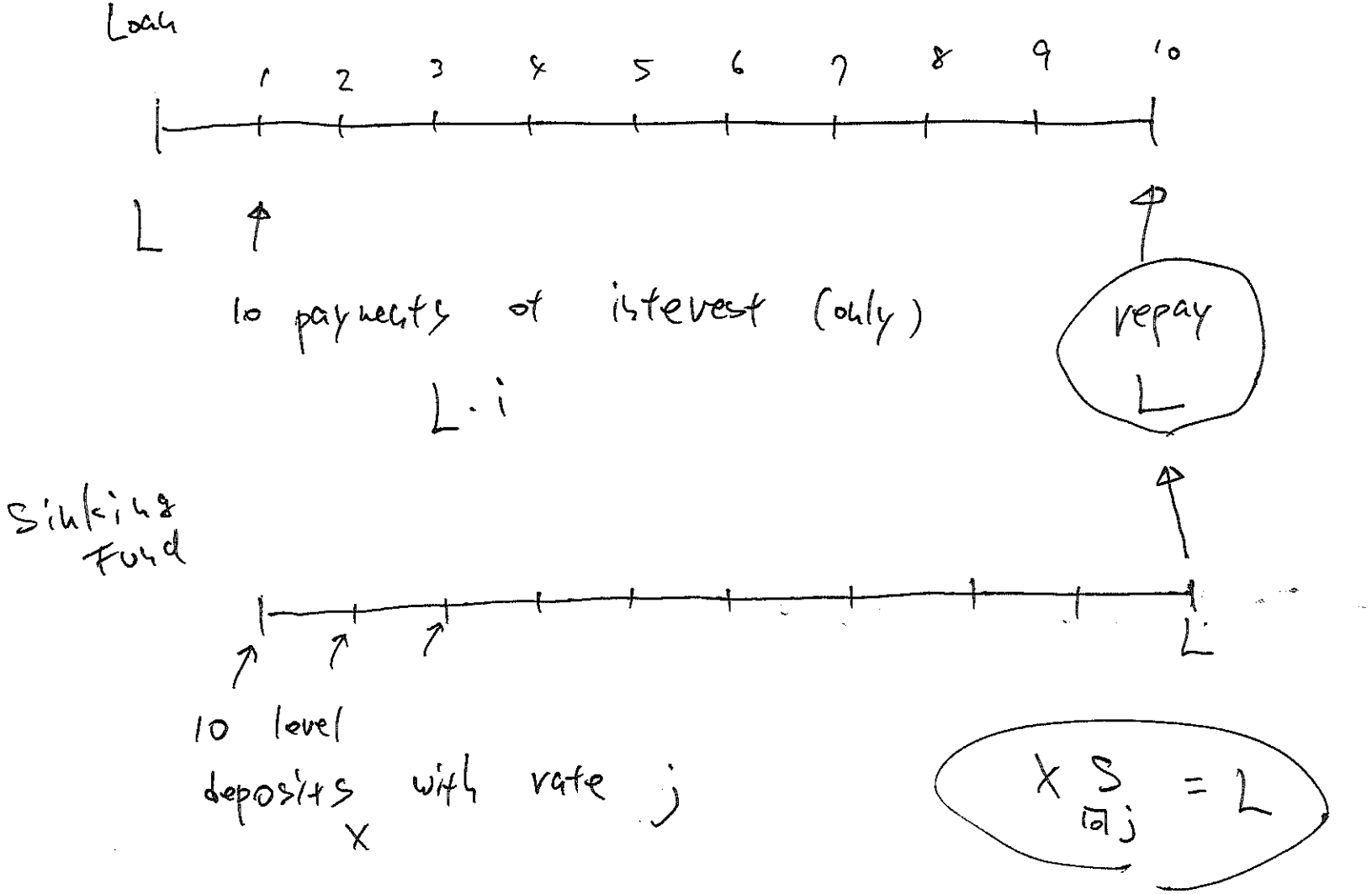
$$100{,}000 = 2\,a_{\overline{72}|.01} + 22\,v^{72}\,a_{\overline{72}|.01}$$

Solve for $2 = 988.89$. (note this is less than 1st mo. interest)

$$OB_{12} = 100{,}000\,(1.01)^{12} - 988.89\,s_{\overline{12}|.01}$$

end of 1st year $= 100{,}140.90$

3.3 The Sinking-Fund Method



Example 3.6 Sinking Fund.

100,000 Loan. payments start in one year.

Annual interest payments at $i = 10\%$.

Sinking Fund with $j = 8\%$. annual. starting in one year.

to level deposit.

a) Find borrower's total annual outlay, and compare to level annual payment by amortization method at 10%.

Interest payment $L \cdot i = 100{,}000 \cdot (.1) = 10{,}000$.

Sinking Fund Deposits $X = \dfrac{L}{s_{\overline{10}|.08}} = 6902.95$.

total annual outlay

16,902.95.

Amortization Method.

$$100{,}000 = X\, a_{\overline{10}|.01}$$

$$X = \frac{100{,}000}{a_{\overline{10}|.01}} = 16274.54$$

level annual payment.

a) Find rate i' so that payment in Amortization method will be the same as Sinking Fund Method.

$$16{,}902.95 = \frac{100{,}000}{a_{\overline{10}|i'}}$$

$$i' = .1089$$

b) Suppose that the lender's rate 8%
S-fund rate 10%.

Compare payments to amortization with 8%.

$$L\left[.08 + \frac{1}{s_{\overline{10}|.1}}\right] = 14{,}274.54$$

↑ interest ↑ s-fund

$$\frac{L}{a_{\overline{10}|.08}} = 14{,}902.95$$