B3. What is the empirical mean excess loss at d = 4, given the following sample of total lifetimes:

3 2 5 8 10 1 6 9

A. < 1.5 B. ≥ 1.5 but < 2.5 C. ≥ 2.5 but < 3.5 D. ≥ 3.5
E. It cannot be determined from the information given (88–4–60–1)

Sample mean

$$E[X-d \mid X>d] \approx \frac{(5-4)+(8-4)+(10-4)+(6-4)+(9-4)}{5} = \boxed{3.6}$$

$d = 4$

B4. A random loss (X) has the following probability function:

x	0	1	2	3	4	5	6	7	8	9
f(x)	.05	.06	.25	.22	.10	.05	.05	.05	.05	.12

You are given that $E[X] = 4$ and $E[X] - E[X \wedge d] = 2$. Determine d.

A. 1/4 B. 5/4 C. 7/4 D. 9/4 E. 11/4 (89S–151–16)

$\rightarrow E[X \wedge d] = 2$

try with $d = 2$ $E[X \wedge d] = 1.84$

" $d = 3$ $E[X \wedge d] = 2.48$

try $d = \frac{9}{4} = 2.25$ $E[X \wedge d] = 2$

$\boxed{d = 2.25}$

B5. For aggregate claims (S), you are given:

$\boxed{P(10 < S < 20) = 0}$ $\quad E[S] - E[S \wedge 10] = .6 \quad E[S] - E[S \wedge 20] = .2$

Determine $F_S(10)$.

A. .88 B. .90 C. .92 D. .94 E. .96 (89F–151–16)

$$E[S \wedge 20] - E[S \wedge 10] = .6 - .2 = .4$$

$(1)\,p(1)$

$\vdots$

$(10)\,p(10)$

$\vdots$

$+ \cancel{(20)\,p(20)}$

$+ (20)(1 - F(20))$

$-$

$(1)\,p(1)$

$\vdots$

$(10)\,p(10)$

$(10)(1 - F(10))$

$F(20)$

$= (20)(1 - F(20))$

$- (10)(1 - F(20))$

Redo

$$F(20) = .96 = F(10)$$

B6. Klugman et al. define two functions:

i) The limited expected value function ($E[X \wedge d]$)
ii) The mean excess loss function [$e_X(d)$]

If $F(d) = \Pr\{X \leq d\}$ and the expected value of X is denoted by E[X], then which of the following equations expresses the relationship between $E[X \wedge d]$ and e(x)?

A. $E[X \wedge d] = E[X] - e_X(d)/[1 - F(d)]$ B. $E[X \wedge d] = E[X] - e_X(d)$
C. $E[X \wedge d] = E[X] - e_X(d)[1 - F(d)]$ D. $E[X \wedge d] = E[X][1 - F(d)] - e_X(d)$
E. None of these equations express that relationship. (90–4–53–2)

$$(X - d)_+ + (X \wedge d) = X$$

$$e_X(d)(1 - F(d)) = E[X - d \mid X > d] \cdot P(X > d)$$

$$= \int_d^\infty (x - d) f(x)\, dx$$

Ans. C

B7. The probability that an individual admitted to the hospital will stay k days or less is $1 - .8^k$ for $k = 0, 1, 2, \ldots$. A hospital indemnity policy provides a fixed amount per day for the fourth day through the tenth day (i.e., for a maximum of 7 days). Determine the percentage increase in the expected cost per admission if the maximum number of days paid is increased from 7 to 14.

A. 13 B. 15 C. 17 D. 19 E. 21 (90F–151–18)

Say, Cost : \$1 per day

$$\begin{array}{cccccccccc} & & & c & c & c & c & c & c & c \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}$$

$$P(X \le k) = 1 - .8^k$$

$$\text{Cost} = \begin{cases} 0 & \text{if } X \le 3 \\ c(X-3) & \text{if } X > 3 \end{cases}$$

(1)

$$P(\text{Cost} = 0) = P(X \le 3) = 1 - .8^3$$

$$P(\text{Cost} \le 1) = 1 - .8^4$$

$$P(\text{Cost} \le 2) = P(X \le 5) = 1 - .8^5$$

$$\vdots$$

$$P(\text{Cost} \le 6) = P(X \le 9) = 1 - .8^9$$

$$P(\text{Cost} \le 7) = 1$$

If max stay was changed to 17 (14 days paid)

② $P(\text{Cost} = 0) = 1 - .8^{3}$

$P(\text{Cost} \le 1) = 1 - .8^{4}$

$\vdots$

$P(\text{Cost} \le \cancel{14}\,13) = 1 - .8^{16}$

$P(\text{Cost} \le 14) = 1$

$$E(\text{Cost}) = \sum_{t=0}^{\infty} \left(1 - F_{\text{cost}}(t)\right)$$

under ①

$$= \sum_{t=0}^{6} \left(1 - F_{\text{cost}}(t)\right)$$

$$= \sum_{t=0}^{6} .8^{t+3}$$

under ②

$$= \sum_{t=0}^{13} \left(1 - F_{\text{cost}}(t)\right)$$

$$= \sum_{t=0}^{13} .8^{t+3}$$

$$= .8^3 \sum_{t=0}^{6} .8^t$$

$$= .8^3 \frac{1-.8^7}{1-.8}$$

$$= A$$

$$= .8^3 \sum_{t=0}^{13} .8^t$$

$$= .8^3 \frac{1-.8^{14}}{1-.8}$$

$$= B$$

$$\frac{B}{A} = \frac{1-.8^{14}}{1-.8^7} = \boxed{1.21}$$

21% increase in cost.

B8. Given the following, determine the probability that a claim exceeds \$3,000:

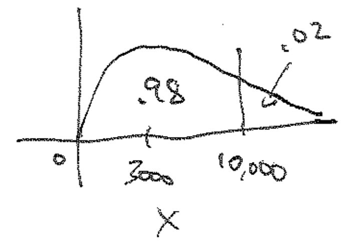
i) Based on observed data truncated from above at \$10,000, the probability of a claim exceeding \$3,000 is .30.

ii) Based on the underlying distribution of losses, the probability of a claim exceeding \$10,000 is .02.

A. < .28 B. ≥ .28 but < .3 C. ≥ .3 but < .32 D. ≥ .32 but < .34 E. ≥ .34 (92F–4B–3–1)

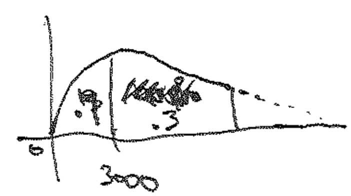
Call it X.

$$P(X > 10{,}000) = .02$$



$$Y = [X \mid X < 10{,}000]$$

$$P(Y > 3000) = .30$$



$$P(X > 3000) = .98(.3) + .02 = \boxed{.314}$$

B9. The following random sample has been observed:

2.0 10.3 4.8 16.4 21.6 3.7 21.4 34.4

d

Calculate the value of the empirical mean excess loss [$e_X(d)$] for ~~x~~ = 8.

A. < 7 B. ≥ 7 but < 9 C. ≥ 9 but < 11 D. ≥ 11 but < 13 E. ≥ 13 (93S–4B–25–2)

$$e_x(8) = E[X - 8 \mid X > 8]$$

empirical version

$$= \left.\begin{matrix} 16.4 - 8 \\ 21.6 - 8 \\ 10.3 - 8 \\ 21.4 - 8 \\ 34.4 - 8 \end{matrix}\right\} \text{take average} = \boxed{12.82}$$

B10. The limited expected value function evaluated at any point $d \geq 0$ equals

$$E[X \wedge d] = \int_0^d x f_X(x)\,dx + d[1 - F_X(d)]$$

where $f_X(x)$ and $F_X(x)$ are the probability density and distribution functions, respectively, of the loss random variable X. (93F–4B–16–MC)

true.

$$E[\min(X, d)] = \int_0^d x f(x)\,dx + \underbrace{\int_d^\infty d\, f(x)\,dx}_{d(1-F(d))}.$$

B11. A random sample of auto glass claims has yielded the following five observed claim amounts:

100 125 200 250 300

What is the value of the empirical mean excess loss at x = 150?

A. 75 B. 100 C. 200 D. 225 E. 250 (94F–4B–16–1)

$$e_X(150) = \frac{200 - 150 + (250 - 150) + (300 - 150)}{3} = \boxed{100}$$

C2 ~~The mean excess loss at d [$e_X(d)$] is linear in x for the Pareto distribution. (87–4–59–MC)~~ True

C3. If $e_X(d)$ increases as d increases, this suggests that a Pareto model may be appropriate. (92S–4B–14–MC) True

Y - heavy tail

Pareto

$$F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^{r} \qquad x > 0$$

$$S(x) = \left(\frac{\theta}{x+\theta}\right)^{r}$$

$$e_X(d) = \frac{\int_d^\infty S(x)\,dx}{S(d)} = \left(\frac{d+\theta}{\theta}\right)^{r} \cdot \int_d^\infty \left(\frac{\theta}{x+\theta}\right)^{r} dx$$

$$= \left(\frac{d+\theta}{\cancel{\theta}}\right)^{r} \cdot -\frac{(x+\theta)^{-r+1}}{r+1} \Bigg|_d^\infty$$

$$= \boxed{\frac{d+\theta}{r+1}}$$

need $r \geq 1$ otherwise ∞

C4. Losses follow a Pareto distribution. Determine the ratio of the mean excess loss at d = 2λ to the mean excess loss at d = λ.

A. 1/2 B. 1 C. 3/2 D. 2 E. It cannot be determined from the given information.
(95S–4B–21–3)

when $\theta = \lambda$

C5. If it exists, the mean excess loss function of a Pareto distribution is decreasing. (97S–4B–13–MC)

FALSE

$$\frac{d+\theta}{r+1}$$

$$\frac{\frac{2\lambda+\cancel{\theta}}{r+1}}{\frac{\lambda+\cancel{\theta}}{r+1}} = \frac{3}{2}$$

C6. You are given the following:

i) Claim sizes follow a Pareto distribution with parameters α (unknown) and $\lambda = 10{,}000$.
ii) The null hypothesis (H_0), $\alpha = .5$, is tested against the alternative hypothesis (H_1), $\alpha < .5$.
iii) One claim of 9,600,000 is observed.

Determine the mean excess loss at 10,000 under the assumption that H_0 is true.

A. 5,000 B. 10,000 C. 20,000 D. 40,000 E. ∞ (98F–4B–6–2)

USE the following information for the next three questions. You are given:

i) The random variable X follows a Pareto distribution with parameters $\theta = 100$ and $\alpha = 2$.
ii) The mean residual life function, $e_X(k)$, is defined to be $E[X - k \mid X \geq k]$.

$\alpha = .5$
$\lambda = 10{,}000$

$$e_X(d) = \frac{d+\lambda}{\alpha+1} = \infty \quad b/c \ \alpha < 1$$

C7. Determine the range of $e_X(k)$ over its domain of $[0, \infty)$.

A. $[0, 100]$ B. $[0, \infty)$ C. 100 D. $[100, \infty)$ E. ∞ (99F–4B–25–2)

C8. $Y = 1.10X$. Determine the range of the function $e_Y(k)/e_X(k)$ over its domain of $[0, \infty)$.

A. $(1, 1.10]$ B. $(1, \infty)$ C. 1.10 D. $[1.10, \infty)$ E. ∞ (99F–4B–26–1)

C9. $Z = \min(X, 500)$. Determine the range of $e_Z(k)$ over its domain of $[0, 500]$.

A. $[0, 150]$ B. $[0, \infty)$ C. $[100, 150]$ D. $[100, \infty)$ E. $[150, \infty)$ (99F–4B–27–2)

C7. $e_X(d) = \dfrac{d+\theta}{\alpha - 1}$ $\quad \theta = 100$, $\alpha = 2$

$d \in [0, \infty)$ then $e_X(d) \in [100, \infty)$

C8. $Y = 1.1X \Rightarrow Y \sim \text{Pareto}\ (\theta = 110)$

$$\frac{e_Y(d)}{e_X(d)} = \frac{d+110}{d+100} \Rightarrow \left[\overset{d=0}{\frac{110}{100}}, \overset{d=\infty}{\frac{\infty+110}{\infty+100}}\right]$$

$= (1, 1.1]$

C4. $e_Z(k)$ $\quad k \in [0, 500]$

$Z = \min(X, 500)$

$e_Z(\overset{500}{\cancel{k}}) = 0$. $\to$ choice C, D, E impossible.

$e_Z(0) = ?$ but can't be bigger than 500.

$\to$ Answer $A = [0, 150]$

D1. Let X be a random variable with the following density function:

$$f(x) = \begin{cases} ae^{-ax} \text{ for } x > 0 \\ 0 \text{ otherwise} \end{cases}$$

where a > 0. If M(t) denotes the moment-generating function of X, what is M(−3a)?

A. e^{-3a} B. 1/30 C. 1/3 D. 1/4 E. $+\infty$ (81F–2–29)

$$\cancel{E\left[e^{tX}\right] = \int_0^\infty e^{tx} \cdot ae^{-ax}\, dx}$$

$$X \sim Exp(a) \quad \text{with} \quad E(X) = \frac{1}{a}$$

$$M(t) = \frac{1}{1 - \frac{1}{a}t}$$

$$M(-3a) = \frac{1}{1 - \left(\frac{-3a}{a}\right)} = \boxed{\frac{1}{4}}$$

D3. $S = X_1 + X_2 + \cdots + X_6$. The X_i's, $i = 1, 2, \ldots, 6$, are independent random variables each with a gamma distribution. $E[X_i] = Var(X_i) = i$ for $i = 1, 2, \ldots, 6$. Determine $E[S^3]$

A. 9,261 B. 9,606 C. 9,896 D. 9,996 E. 10,626 (86F–151–5)

$X_i \sim GAM(\alpha, \beta)$

$\alpha\beta = i$

$\alpha\beta^2 = i$

$\beta = 1$

$\alpha = i$

$M_X(t) = (1-\beta t)^{-\alpha}$

$M_S(t) = (1-t)^{-1} \cdot (1-t)^{-2} \cdot \cdots \cdot (1-t)^{-6} = (1-t)^{-21}$

$M_S'(t) = -21(1-t)^{-22}(-1)$

$M_S''(t) = 21(-22)(1-t)^{-23}(-1)$

$M_S'''(t) = 21(22)(-23)(1-t)^{-18}(-1)$

$M_S'''(0) = 21(22)(23) = \boxed{\cancel{}\ 10626}$

D6. X_1, X_2, X_3, and X_4 are independent random variables for a gamma distribution $G(\alpha, \theta)$ with the parameter $\alpha = 2.2$ and the parameter $\theta = 1/5$. If $S = X_1 + X_2 + X_3 + X_4$, then what is the distribution function for S?

A. Gamma (8.8, 4/5) B. Gamma (8.8, 1/5) C. Gamma (2.2, 4/5) D. Gamma (2.2, 1/5)
E. None of these answers are correct. (94F–5A–23–1)

$X_1 \sim GAM(2.2, \frac{1}{5})$

$S \sim GAM(4(2.2), \frac{1}{5})$

$\boxed{B}$

D10. The following information is available for a collective risk model:

i) X is a random variable representing the size of each loss.
ii) X follows a gamma distribution with $\alpha = 2$ and $\theta = 100$.
iii) N is a random variable representing the number of claims.
iv) S is a random variable representing aggregate losses.
v) $S = X_1 + \cdots + X_N$

Calculate the mode of S when N = 5.

A. < 950 B. ≥ 950 but < 1,050 C. ≥ 1,050 but < 1,150 D. ≥ 1,150 but < 1,250 E. ≥ 1,250
(06S-3-36-2)

$X \sim GAM(2, 100)$

$S \sim GAM(2(5), 100)$

mode = peak of $f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \cdot x^{\alpha-1} e^{-x/\beta}$

$f'(x) = (\alpha-1) f(x)(x)^{-1} + \left(-\frac{1}{\beta}\right) f(x) \overset{\text{set}}{=} 0$

$(\alpha-1)(x)^{-1} = \frac{1}{\beta}$

$\beta(\alpha-1) = x$ mode

$(100)(10-1) = \boxed{900}$

D13. The random variables, $X_1, X_2, \ldots, X_n$ are independent and identically distributed with probability density function

$$f(x) = e^{-x/\theta} \quad x \geq 0 \qquad = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

$\alpha\beta^2 + \alpha^2\beta^2 = \alpha$

$= \beta^2(\alpha+1)\alpha$

Determine $E[X^2]$.

A. $\frac{(n+1)\theta^2}{n}$ B. $\frac{(n+1)\theta^2}{n^2}$ C. $\frac{\theta^2}{n}$ D. $\frac{\theta^2}{\sqrt{n}}$ E. θ^2 (06F–C–26)

$\frac{1}{\theta}$ missing, or $\theta = 1$.

$X \sim GAM(1, \theta) = Exp(\theta)$

$E(X) = \theta$

$V(X) = \theta^2$

$E[X^2] = \theta^2 + \theta^2 = 2\theta^2$

this is $\bar{X}$.

$$V(\bar{X}) = \frac{V(X)}{n} = \cancel{\theta^2} \quad \frac{2\theta^2}{n}$$

$$E(\bar{X}^2) = V(\bar{X}) + [E(\bar{X})]^2 = \frac{\theta^2}{n} + (\theta)^2 = \boxed{\frac{(n+1)\theta^2}{n}}$$

$\boxed{A}$

E1. The tail of a Pareto distribution does not approach zero as fast as does the tail of a lognormal distribution. (86–4–49–MC) T

E2. The tail of a Pareto distribution fades to zero more slowly than does that of a lognormal, for large enough X. (88–4–47–MC) T

E3. According to Klugman et al., when the approximation to the distribution of aggregate claims needs to accommodate skewness, a gamma distribution may be used. (92–5–24–MC) T

E4. The lognormal and the Pareto distributions are positively skewed. (92F–4B–2–MC) T

E5. The lognormal distribution generally has greater probability in the tail than the Pareto distribution. (92F–4B–2–MC) F

B6. The lognormal distribution is often useful as a model for a claim size distribution because it is positively skewed. (93F–4B–21–MC) T

E7. The Pareto probability density function tapers away to zero much more slowly than the lognormal probability density function. (93F–4B–21–MC) T

E8. According to Klugman et al., if the claim amount random variable is only positive and its distribution is skewed to the right, we should choose an exponential distribution for the individual claim amount distribution. (97F–5A–25–MC) F