451/551 - HW on Ch. 3

Due Fri Sep. 30th

The final answer must be clearly indicated.

Name: Solution

1. An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. What is the expected benefit under this policy?

$4000\,(.4)$

$3000\,(.6)(.4)$

$2000\,(.6)^2(.4)$

$1000\,(.6)^3(.4)$

$\doteq \boxed{2694.4}$

2 ~~b~~ A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist. What is the expected revenue of the tour operator?

X = # of tourist who does not shows up $\quad X \sim BIN(21, .02)$

If $X \geq 1$ revenue $= 21(50)$

If $X = 0$, revenue $= 21(50) - 100$

$$P(X=0) = \binom{21}{0}(.02)^0(.98)^{21} = .654$$
$$P(X\geq 1) = 1 - .654 = .346$$

$$E[\text{Revenue}] = 21(50)\cdot P(X\geq 1) + [21(50) - 100]\cdot P(X=0)$$

$$= \boxed{984.57}$$

- he should have just sold 20 tickets, get \$1000 with 100% probability.

3. Suppose you roll a die until the number 3 comes up, and record X = number of rolls it took. What is the smallest value of a, so that $P(X \leq a)$ is more than 60%?

X = # of rolls until 1st 3

$X \sim NB(1, \frac{1}{6})$

b/c # of tails

In R:

$P(X \leq a) = \text{pnbinom}(a-1, 1, \frac{1}{6})$

CDF

$P(X \leq 5) = .598$

$\boxed{P(X \leq 6) = .665}$

4. In a pond, there are 40 catfish, where 20 of them are tagged. If 10 of them are caught without replacement. Let X be the number of tagged fish caught. What is $P(4 \le X \le 6)$?

$$X \sim HG(\overset{n}{10}, \overset{m}{20}, \overset{N}{40})$$

$$P(4 \le X \le 6) = P(X=4) + P(X=5) + P(X=6)$$

$$= P(X \le 6) - P(X \le 3)$$

$$= \text{phyper}(6, \overset{m}{20}, \overset{N-m}{20}, \overset{n}{10})$$

$$- \text{phyper}(3, 20, 20, 10) = \boxed{.7266}$$

— or —

$$= \frac{\binom{20}{4}\binom{20}{6}}{\binom{40}{10}} + \frac{\binom{20}{5}\binom{20}{5}}{\binom{40}{10}} + \frac{\binom{20}{6}\binom{20}{4}}{\binom{40}{10}}$$

5. In a city, there are 30 factories, where 5 of them are violating the environmental protocol. If inspectors rondomly choose the factory for inspection, what is the probability all 5 violators are caught before the 9th inspection?

X = # of violators caught in 8 inspections.

$$X \sim HG(\overset{n}{8}, \overset{m}{5}, \overset{N}{30})$$

$$P(X=5)$$

$$= \frac{\binom{5}{5}\binom{25}{3}}{\binom{30}{8}}$$

6. An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2. Assume the numbers of claims filed by distinct policyholders are independent of one another. What is the mean and variance of a number of total claims during a one-year period?

\# of claim $\sim$ Poi (2) per customer per year

X_i = # of claim from customer i $\quad E[X_1] = 2$

$V[X_1] = 2$

$E[\text{Total \# of claim}]$

$= E[X_1 + X_2 + \cdots + X_{1250}] = 1250\, E[X_1]$

$= 1250 \cdot 2 = \boxed{2500}$

$V[X_1 + \cdots + X_{1250}] = V[X_1] + \cdots + V[X_{1250}] = 1250\, V[X_1]$

by independence

$= 1250 \cdot 2$

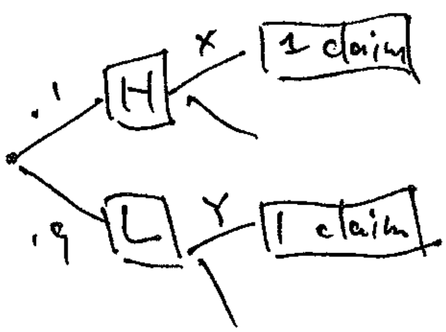
$= \boxed{2500}$

7 ~~14~~. An insurance company designates 10% of its customers as high risk and 90% as low risk. The number of claims made by a customer in a calendar year is Poisson distributed with mean θ and is independent of the number of claims made by a customer in the previous calendar year. For high risk customers $\theta = 0.6$, while low risk customers $\theta = 0.1$. Calculate the expected number of claims made in calendar year 1998 by a customer who made one claim in calendar year 1997.

If there was no information, then

$$E[\#\text{ of claims}] = .6\,\underbrace{P(\text{High Risk})}_{.1} + .1\,\underbrace{P(\text{Low Risk})}_{.9}$$

With information given, we need to update H/L prob. distribution.



$$X = P(1\text{cl} \mid H) = \frac{.6e^{-.6}}{1} = .329$$

$$Y = P(1\text{cl} \mid L) = \frac{.1e^{-.1}}{1} = .0905$$

$$P(H \mid 1\text{cl in }97) = \frac{.329}{.329 + .0905} = .2879$$

$$P(L \mid 1\text{cl in }97) = \frac{.0905}{.329 + .0905} = .7121$$

$$E[\#\text{ of claims} \mid 1\text{cl in }97] = .6(.2879) + .1(.7121)$$

$$= \boxed{.2440}$$

8. An actuary determines that the number of claims for a certain class of accidents is a random variable, X, with moment generating function $M(t) = exp(10e^t - 10)$. Determine the standard deviation of the number of claims for this class of accidents.

$$M(t) = \exp(10(e^t - 1))$$

this is MGF of $Poi(\underset{10}{\underset{\shortparallel}{\lambda}})$.

$\Rightarrow$ $E(X) = 10$

$V(X) = 10$ $\quad SD(X) = \sqrt{10}$.