Reliability Theory

Indicator

$$X_i = \begin{cases} 1 & \text{component } i \text{ is working} \\ 0 & \text{Not} \end{cases}$$

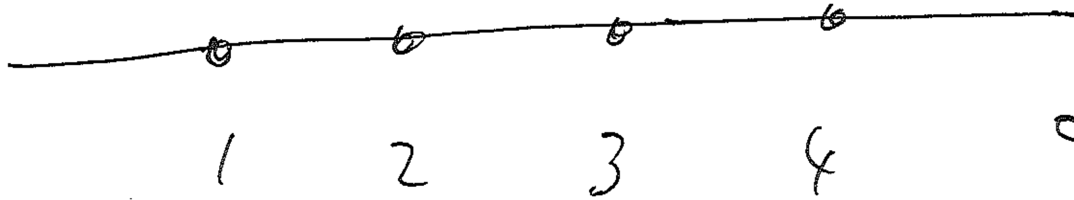
Structure Function

$$\phi(\underline{x}) = \begin{cases} 1 & \text{if system works} \\ 0 & \text{Not.} \end{cases} \quad \text{for given } \underline{x}.$$

$\uparrow$

State vector of Components

Series

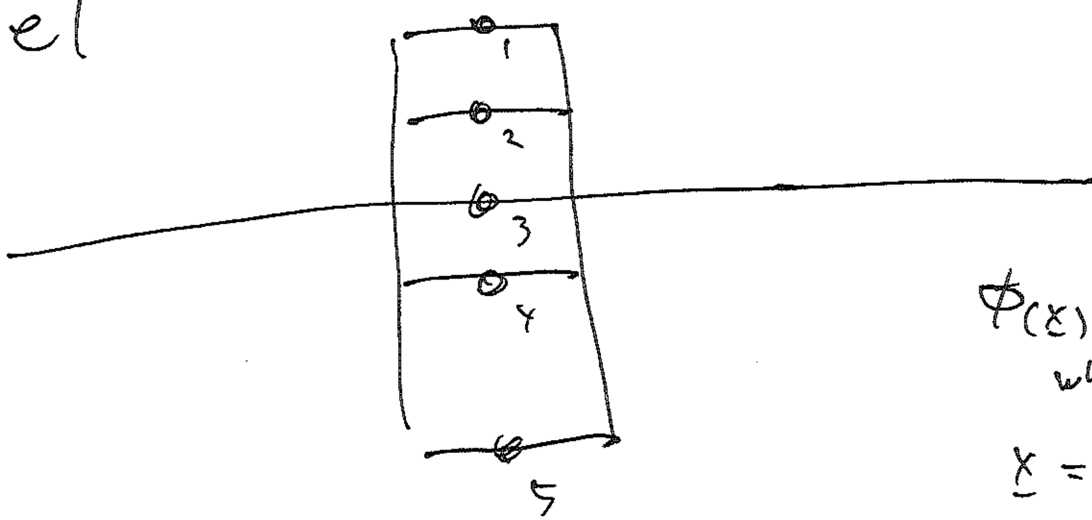


$\Phi(\underline{x}) = 1$

only when

$\underline{x} = [1 \; 1 \; 1 \; 1]$

Parallel



$\Phi(\underline{x}) = 1$

when

$\underline{x} = [1 \; 0 \; 0 \; 0 \; 0]$

or

$[0 \; 1 \; 0 \; 0 \; 0]$

$\vdots$

$[\qquad\qquad]$

If $\quad T_i \approx$ ~~component~~ component Lifetime.

Series $\quad S \sim \min(T_i)$

Sys. Lifetime

Parallel $\quad S = \max(T_i)$

Similarly,

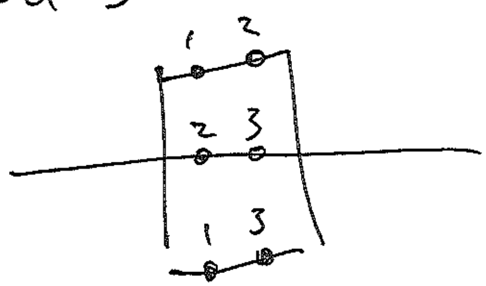
Series $\quad \phi(\underline{x}) = \overset{\min}{\cancel{\max}}(x_1, \ldots, x_n)$ $\quad$ one 0 makes sys = 0.

Parallel $\quad \phi(\underline{x}) = \max(x_1, \ldots, x_n)$. $\quad$ one 1 makes sys = 1.

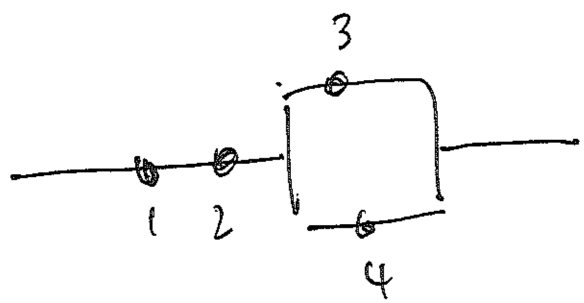
K-out-of-N structure

$$\phi(\underline{x}) = \begin{cases} 1 & \sum_i x_i \geq k \\ 0 & < k \end{cases}$$

2-out-of-3



Ex 9.4 Four component



$$\phi(\underline{x}) = x_1 x_2 \max(x_3, x_4)$$

$$= x_1 x_2 (x_3 + x_4 - x_3 x_4)$$

$$\boxed{\max(x_i) = 1 - \prod_{i=1}^{n} (1 - x_i)}$$

if $x_i = \begin{cases} 1 \\ 0 \end{cases}$.

For binary X_i

$$\min(X_1, \dots, X_n) = X_1 \cdots X_n.$$

$$\max(X_1, \dots, X_n) = 1 - \prod_{i=1}^{n} (1 - X_i)$$

$$\max(X_1, X_2) = 1 - (1 - X_1)(1 - X_2)$$

$$= X_1 + X_2 - X_1 X_2$$

Any system can be represented as

→ Parallel of series systems. (1)

or

→ Series of parallel systems. (2)

Minimal Path

$\phi(\underline{x})$ is a <u>monotone</u> function

i.e. if $x_i \le y_i$ for all $i = 1, \ldots, n$

then $\phi(\underline{x}) \le \phi(\underline{y})$.

~~$\underline{x}$ is a minimal path vector if $\phi(\underline{x}) = 1$~~

$\underline{x}$ is a <u>path vector</u> if $\phi(\underline{x}) = 1$.

$\underline{x}$ is a minimal path vector if $\phi(\underline{x}) = 1$ and

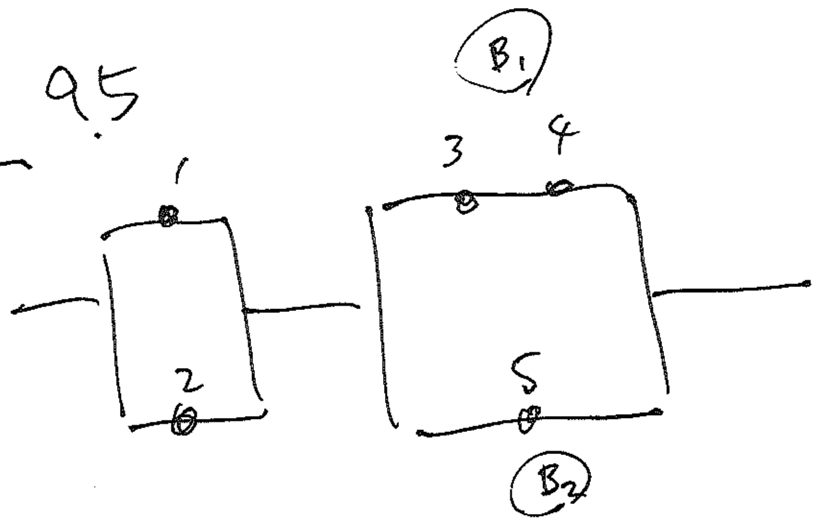
$\phi(\underline{y}) = 0$ for all $\underline{y} < \underline{x}$.

$\underline{y} \leq \underline{x}$ means $y_i \leq x_i \quad \forall i$, and

$y_i < x_i$ for some i

$\{i : x_i = 1\}$: minimal path set.

Ex 9.5



$$\phi(\underline{x}) = A_1 A_2$$

$$= \max(X_1, X_2) \cdot \max(B_1, B_2)$$

$$= \max(X_1, X_2) \cdot \max(X_3 X_4, X_5)$$

$$= (X_1 + X_2 - X_1 X_2)(X_3 X_4 + X_5 - X_3 X_4 X_5)$$

min. path set

1 3 4   2 3 4

1 5   2 5 .

1

$\exists$ A_j is met, Sys works.
but all in A_j must be 1

Let $A_1, \ldots, A_s$ be minimal path sets.

Define

$$\alpha_j(\underline{x}) = \begin{cases} 1 & \text{if all comp. in } A_j \text{ are working.} \\ 0 & \text{not} \end{cases} = \prod_{l \in A_j} x_l$$

all $\rightarrow$ series

System works if there's at least one $\rightarrow$ Parallel

$$\alpha_j(\underline{x}) = 1.$$

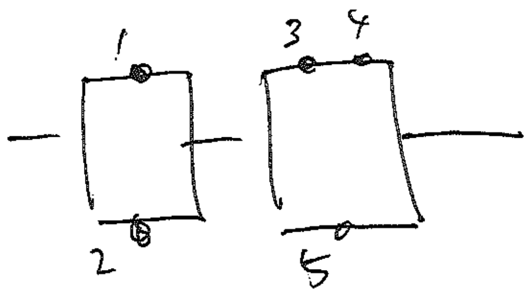
Hence

$$\phi(\underline{x}) = \max[\alpha_1(\underline{x}), \ldots, \alpha_s(\underline{x})]$$

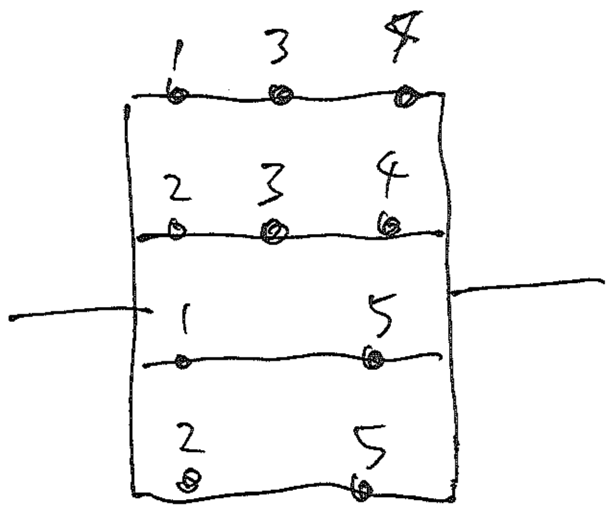
$$= \max\left[\prod_{l \in A_1} X_l, \prod_{l \in A_2} X_l, \ldots, \prod_{l \in A_s} X_l\right].$$

→ this is Parallel arrangement of series sys.

we can write

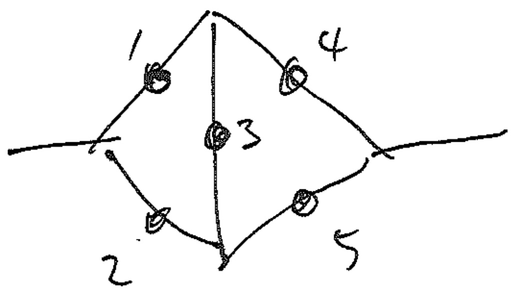


as



$$\phi(\underline{x}) = \max[x_1x_3x_4,\ x_2x_3x_4,\ x_1x_5,\ x_2x_5]$$

Ex 9.8 Bridge System



min path sets

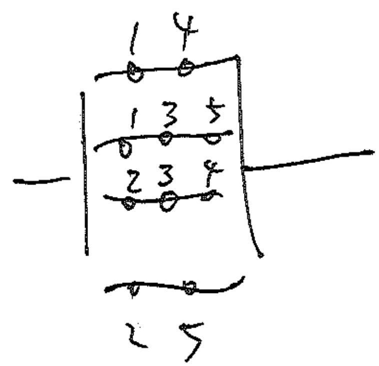
(1 4)

(1 3 5)

(2 3 4)

(2 5)

$$\phi(\underline{x}) = \max\left[x_1x_4,\ x_1x_3x_5,\ x_2x_3x_4,\ x_2x_5\right]$$



cut vector

If $\phi(\underline{x}) = 0$, then $\underline{x}$ is a cut vector.

If $\phi(\underline{y}) = 1$ for all $\underline{y} > \underline{x}$, then

$\underline{x}$ is minimal cut vector.

$C = \{i : x_i = 0\}$ is a minimal cut set.

(2)

$$\beta_j(\underline{x}) = \begin{cases} 1 & \text{if at least one in } C_j \text{ is working} \\ 0 & \text{if all comp in } C_j \text{ is } 0. \end{cases}$$

Parallel

$$= \max_{i \in C_j} X_i$$

One $\beta_j = 0$, then Sys $= 0$.
All in β_j must be 0.

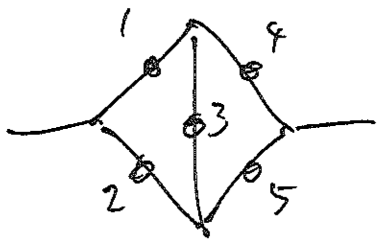
$\phi(\underline{x}) = 0$ if At least one β_j are 0.

$\Rightarrow \phi(\underline{x}) = 1$ if all of β_j are 1

$$\phi(\underline{x}) = \prod_{j=1}^{k} \beta_j(\underline{x}) = \prod_{j=1}^{k} \max_{i \in C_j} X_i.$$

Series sys of β_j's. each β_j are parallel sys.

Ex Bridge Sys.



Min Cut sets

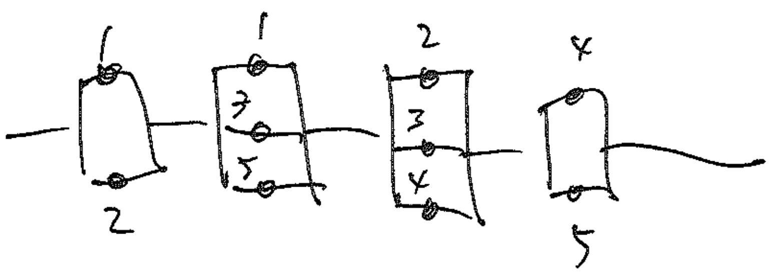
1 2

1 3 5

2 3 4

4 5

$$\phi(\underline{x}) = \max(x_1, x_2) \cdot \max(x_1, x_3, x_5) \cdot \max(x_2, x_3, x_4) \cdot \max(x_4, x_5)$$



①

A_j : min path sets.

If there's one A_j that's ~~works~~ satisfied, then

Sys. works. $\phi(\underline{x}) = 1$.

All comp in the A_j must be 1,

to ~~make~~ satisfy A_j.

Sys = Parallel of A_j. A_j is series sys.

② B_j : Cut path sets

If there's one B_j that's satisfied,
then Sys does not work. $\phi(\underline{x}) = 0$.

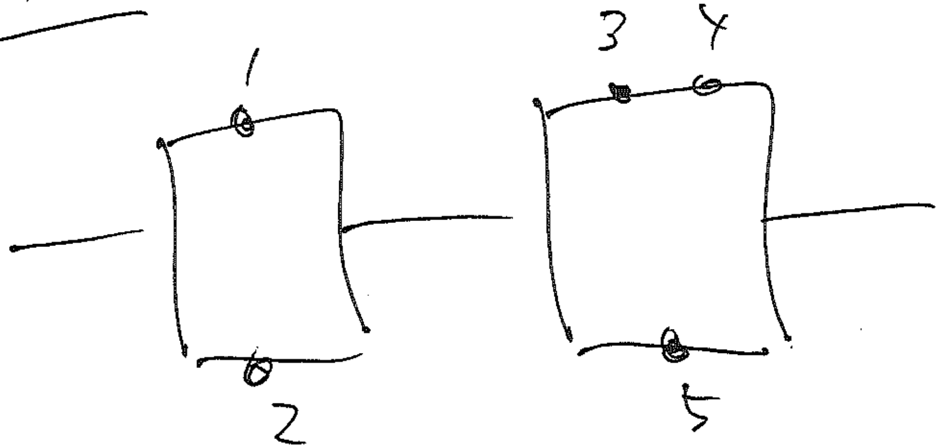
$\Rightarrow$ All B_j are not satisfied, then
Sys will work, (Series)

All comp in B_j must be 0 to satisfy B_j

$\Rightarrow$ at least one "1" will not satisfy B_j (Parallel)

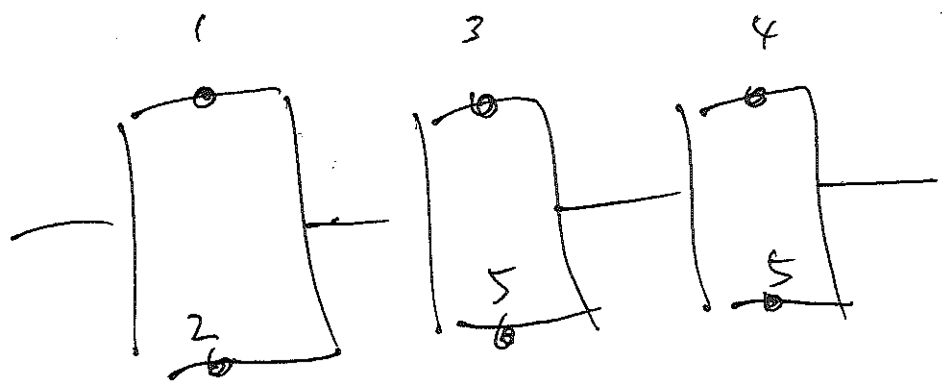
Sys = Series of B_j, B_j is a parallel sys.

Ex



Min Cut set

1	2
3	5
4	5



$$\phi(\underline{x}) = \max(X_1, X_2)\max(X_3, X_5)\max(X_4, X_5)$$