Exotic Options I

Asian Options

→ path-dependent option

→ Option on Average price

XYZ has monthly inflow of €100m

It's costs are fixed in \$.

Let X_i = dollar price of a euro in month i

$$12_{mo} \times €100_m \longrightarrow \sum_{i=1}^{12} \$\, 100 \cdot X_i e^{r(12-i)/12}$$

$$= 100 \sum_{i=1}^{12} X_i \, e^{r(12-i)/12}$$

$$\sum_{i=1}^{12} X_i = 12 \cdot \overline{X}$$

2 kinds of Average

Arithmetic : $A(T) = \frac{1}{N} \sum_{i=1}^{N} S_{ih}$ like $\overline{X}$,

Geometric : $G(T) = \left(S_h \cdot S_{2h} \cdot S_{3h} \cdots S_{Nh} \right)^{1/N}$

like $e^{\overline{\log(x)}}$

$$S_{ih} = 55, 72, 61, 85.$$

$$A(T) = \frac{55+72+61+85}{4} = 68,250$$

$$G(T) = (55 \cdot 72 \cdot 61 \cdot 85)^{1/4} = 67,315$$

Average as S_T or K

[illegible]

Average price option : Call : $\max[0,\ G(T) - K]$

$Av = S_T$ Put : $\max[0,\ K - G(T)]$

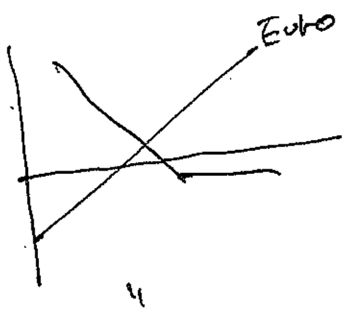
Average strike option : Call : $\max[0,\ S_T - G(T)]$

$Av = K$ Put : $\max[0,\ G(T) - S_T]$

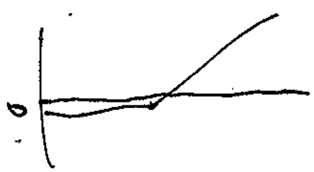
$2 \times 4 = 8$ types of Asian option.

XYZ could use ...

Arithmetic Average price ~~put~~ Asian put



$$\text{Payoff} = \max\left(0,\ K - \frac{1}{12}\sum_{i=1}^{12} X_i\right)$$



floor

TABLE 14.2 Comparison of costs for alternative hedging strategies for XYZ. The price in the second row is the sum of premiums for puts expiring after 1 month, 2 months, and so forth, out to 12 months. The first, third, and fourth row premiums are calculated assuming 1 year to maturity, and then multiplied by 12. Assumes the current exchange rate is \$0.9/€, option strikes are 0.9, $r_\$ = 6\%$, $r_€ = 3\%$, and dollar/euro volatility is 10%.

Hedge Instrument	Premium (\$)
Put option expiring in 1 year	0.2753
Strip of monthly put options	0.2178
Geometric average price put	0.1796
Arithmetic average price put	0.1764

Barrier Options

→ Set level = 'barrier'

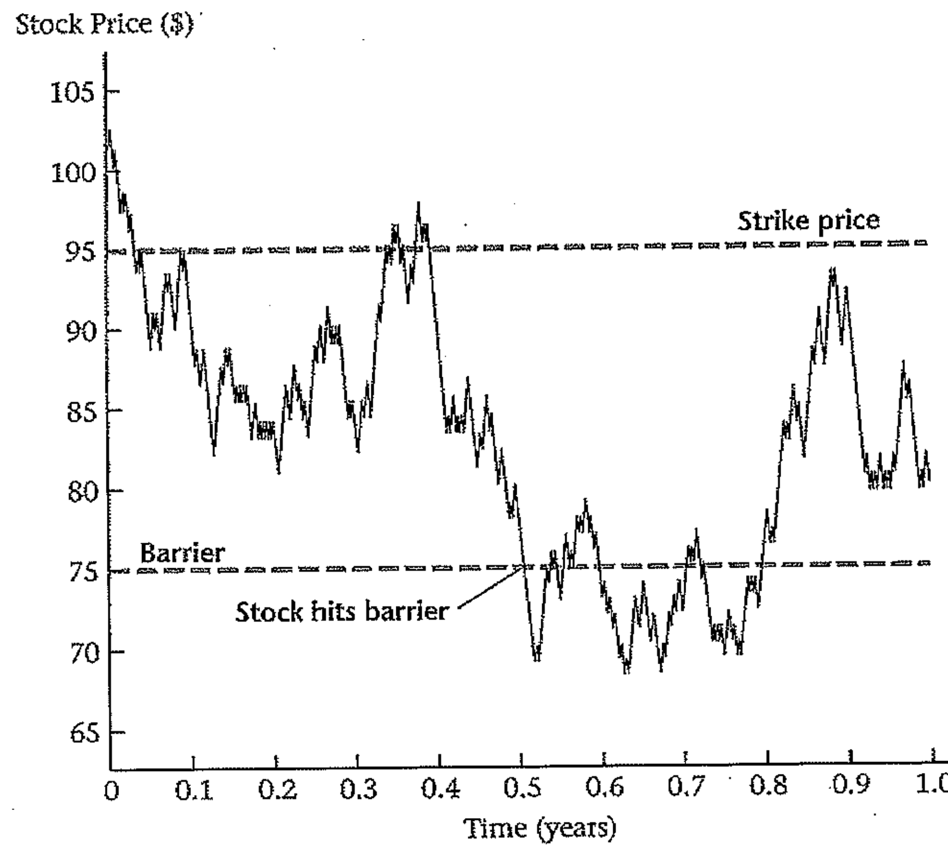
→ If stock price hit the barrier, then
the option may
- come into existance
- go out of existance.

→ If they exist, they are same as ordinary puts and calls.

→ no more expensive than regular puts/calls.

FIGURE 14.1

Illustration of a barrier option where the initial stock price is \$100 and the barrier is \$75. At $t = 0.5$ the stock hits the barrier.



When does it 'hit' the barrier?

→ price of stock can be manipulated by large order.

→ barrier is defined by average over certain period of time.

→ different firms may use somewhat different definition.

Types of Barrier Options

1. Knock-out : go out of existance if it hits barrier. { down-and-out / up-and-out }

2. Knock-in : Comes into existence if it his barrier { down-and-in / up-and-in.

3. Rebate : pays Rebate wit it hits barrier { up-rebate / down-rebate.

Parity for Barrier Option

(Knock-in) + (Knock-out) = Ordinary Option

Ex.

$$\boxed{\text{Down-and-in Call}} + \boxed{\text{Down-and-out Call}} = \text{Call}$$

Compound Option

→ Option on Options

→ Put/Call option that has Put/Call option as underlying asset.

Compound Option Parity

P-C parity on regular options

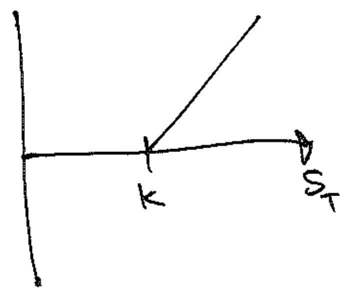
$$C - P = S - Ke^{-rT}$$

P-C parity on Compound Option

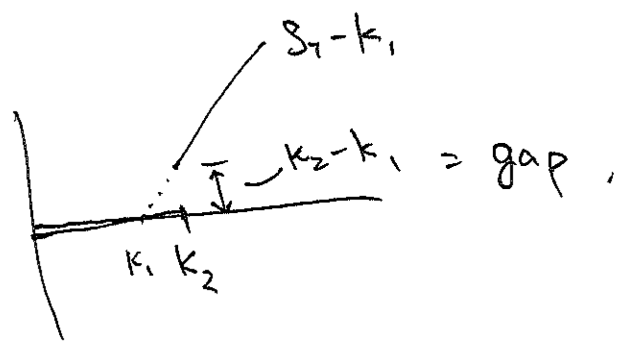
$$\begin{pmatrix} \text{Call-on-Call}_{\substack{K \\ T}} \\ {\scriptstyle x} \\ {\scriptstyle t_1} \end{pmatrix} - \begin{pmatrix} \text{Put-on-Call}_{\substack{K \\ T}} \\ {\scriptstyle x} \\ {\scriptstyle t_1} \end{pmatrix} = \text{Call}_K - x e^{-rt_1}$$

GAP Options

regular call : payoff $= S_T - K$ if $S_T > K$



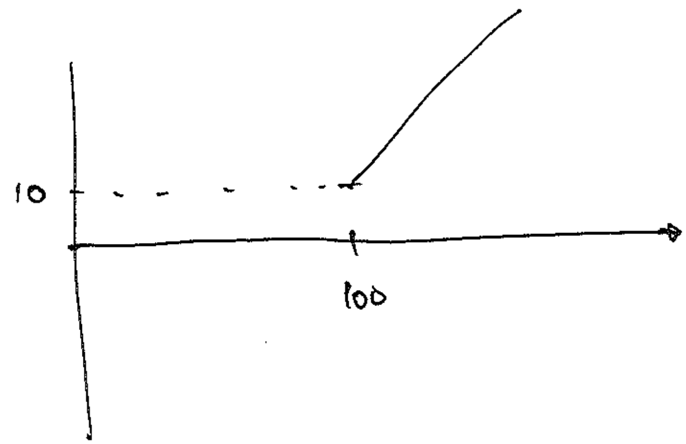
Gap Call : pay off $= S_T - K_1$ if $S_T > K_2$



Gap Call

$K_1 = 90$

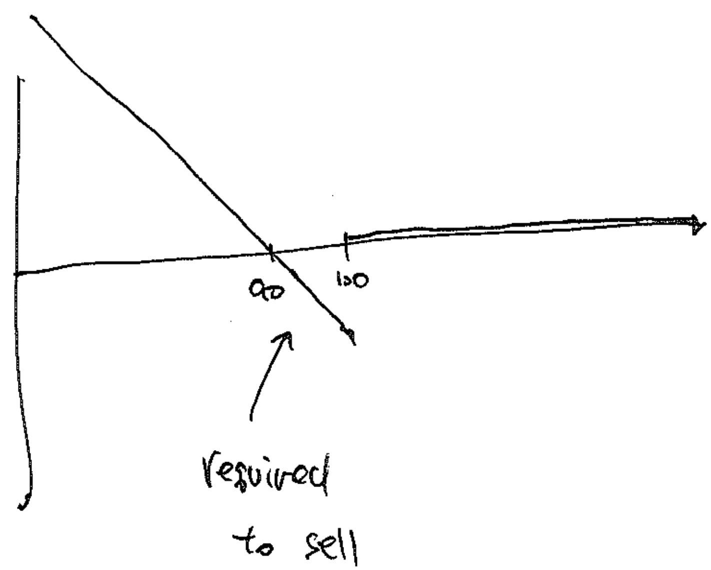
$K_2 = 100$



Gap Put

$K_1 = 90$

$K_2 = 100$



Exchange Options

(out performance option)

→ pays off only if underlying asset outperforms some other asset (benchmark)