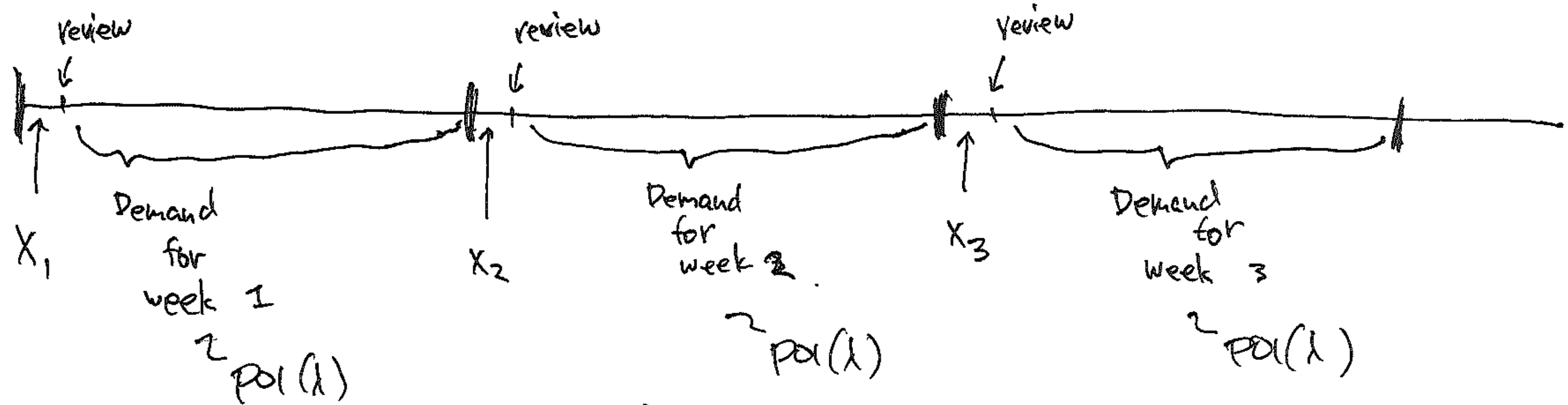
Stock - Control Problem

(Tijms Ex 3.1.2)



→ If stock is less than k, replenish to S.

→ replenishing occurs at time of review, and happens instantly.

→ Markov Property is satisfied.

→ All states communicates. → all state pos. recurrent.

Q: what is Av. ordering freq.

what is Av. # of demand lost ?

Let

X_n = Stock on hand at beginning of n th week just prior to review.

X_n : MC

k = replenish line

states $= \{0, 1, \ldots, S\}$

$$P_{ij} = P\{X_{n+1} = j \mid X_n = i\}.$$

If $\boxed{i \geq \cancel{s} \, k \text{ (no replenishing)} \quad j \neq 0}$

$$P_{ij} = P(\text{Demand for week } n \text{ was } i-j)$$

$$= P(Y_n = i-j) \qquad Y_n \sim Poi(\lambda)$$

$$= \frac{e^{-\lambda} \lambda^{(i-j)}}{(i-j)!} \qquad j = 1, \ldots, i.$$

not $j = 0$

In R : $=$ dpoi (i-j , λ)

If $\boxed{i \geq k \text{ (no replenishing)} \quad j = 0}$

$$P_{i0} = P(\text{Demand was } \geq i)$$

$$= P(Y_n \geq i)$$

$$= \sum_{k=i}^{\infty} \frac{e^{-\lambda} \lambda^{k}}{k!}$$

pmf of POI(λ) at k

$$= 1 - \underbrace{\sum_{k=0}^{i-1} \frac{e^{-\lambda} \lambda^{k}}{k!}}$$

CDF of POI(λ) at $i-1$

In R: 1- ppoi (i-1, λ)

$\boxed{i \geq k}$

R commands.

$$P_{i0} = 1 - \text{ppoi}(i-1, \lambda)$$

$$P_{ij} = \begin{cases} \text{dpoi}(i-j, \lambda) & j = 1, \dots, i \\ 0 & j = i+1, \dots, s \end{cases}$$

If $\boxed{i < s}$ stock replenished to S instantly.

j≠0

$$p_{ij} = P(\overset{\text{Demand}}{Y_n} = S - j)$$

$$= \text{dpoi}(S-j, \lambda) \qquad j = 1, \dots, S$$

j=0

$$P_{i0} = P(\overset{\text{Demand}}{Y_n} \ge S)$$

$$= 1 - P(Y_n < S) = 1 - \underbrace{P(Y_n \le S-1)}_{\text{CDF}}$$

$$= 1 - \text{ppoi}(S-1, \lambda)$$

$$P(A) = 1 - P(A^c)$$

$$\mathbb{P} = \begin{array}{cc} & \begin{array}{cc} 0 \cdots\; k, & k+1, \cdots\cdots S \end{array} \\ \begin{array}{c} 0 \\ \vdots \\ k \\ i \quad k+1 \\ \vdots \\ S \end{array} & \left[\begin{array}{c|c} A & \text{dpois}(S-j, \lambda) \\ \hline B & \begin{array}{cc} \text{dpois}(i-j, \lambda) & 0 \\ j \le i & \end{array} \end{array}\right] \end{array}$$

$$A = 1 - \text{ppois}(S-1, \lambda)$$

$$B = 1 - \text{ppois}(i-1, \lambda)$$

Solution to

$$\underline{\Pi} = \underline{\Pi}\,\mathbb{P} \qquad \text{exists.}$$

$$\underline{\Pi} = [\pi_0, \ldots, \pi_S] \qquad \text{stationary distribution.}$$

long-run Av. of ordering frequency. $= \pi_0 + \cdots + \pi_{s-1}$

long-run Av. of stock level (just before review) $= \sum_{j=0}^{S} j\,\pi_j$

Modification.

ordering cost : K fixed.

stocking cost : h per stock.

penalty for lost demand : b per unit.

Let

$C(j) = E(\text{cost in a week when } X_n = j)$
prior to review.

$$C(j) = E(\text{order}) + E\left(\begin{matrix}\text{Stocking}\\ \text{cost}\end{matrix}\right) + E(\text{lost Demand})$$

$$= E(\text{order}) + h\,E\left(\begin{matrix}\#\text{ of Stock}\\ \text{left}\end{matrix}\right) + b\,E(\#\text{ of lost demand})$$

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get this from ordering frequency.

$$E(\text{order}) = \begin{cases} K \cdot P(\text{order}) \\ + 0 \cdot P(\text{not order}) \end{cases}$$

$$E(\#\text{ of Stock left}) = \begin{cases} \displaystyle\sum_{l=0}^{S} (S-l)\cdot P(\text{Demand} = l) & j < k \text{ (repl.)} \\[2ex] \displaystyle\sum_{l=0}^{j} (j-l)\cdot P(\text{Demand} = l) & j \geq k \text{ (No repl.)} \end{cases}$$

~~use~~

$$E(\#\text{ of lost demand}) = \begin{cases} \displaystyle\sum_{l=S+1}^{\infty} (l-S)\cdot P(\text{Demand} = l) & j < k \text{ (repl.)} \\[2ex] \displaystyle\sum_{l=j+1}^{\infty} (l-j)\cdot P(\text{Demand} = l) & j \geq k \text{ (No repl.)} \end{cases}$$

Then,

$$E(\text{Cost per week})$$

$$= E\left(E\left(\text{Cost} \,\middle|\, j \text{ stock left at the beggining}\right)\right)$$

$$= \sum_{j=0}^{S} E(\text{Cost} \mid X_n = j) \cdot P(X_n = j)$$

$$= \sum_{j=0}^{S} C(j) \cdot \pi_j$$