4-5

$$X_3 = \underbrace{\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}}_{X_0} \mathbb{P} \cdot \mathbb{P} \cdot \mathbb{P}$$

$$= \begin{bmatrix} .41 & .20 & .39 \end{bmatrix}$$

$$E(X_3) = 0 \cdot (.41) + 1 \cdot (.20) + 2 \cdot (.39) = \boxed{.98}$$

4-7

RR	0
SR	1
RS	2
SS	3

Day before	Yesterday	Today	Tomorrow
R	R	?	R
		R	
		S	

0 $\xrightarrow{.7}$ State 0 $\xrightarrow{.7}$ 0 (.49)

0 $\xrightarrow{.3}$ 2 $\xrightarrow{.5}$ 1 (.15)

$\boxed{.64}$ p(Rain tomorrow)

4-14

P_1 1 class. recurrent.

P_2 1 class recurrent.

P_3 $\{0,2\}$ recurrent

$\{1\}$ transient 3 classes.

$\{3,4\}$ recurrent

P_4 $\{0,1\}$ recurrent $\{4\}$ transient

$\{2\}$ recurrent 4 classes,

$\{3\}$ transient.

4-19

$$\begin{array}{cccc} 1 & 2 & 3 & \\ RR & SR & RS & SS \end{array}$$

$$\underline{\pi} = \begin{pmatrix} .25 & .15 & .15 & .45 \end{pmatrix}$$

$$\mathbb{P} = \begin{bmatrix} .7 & 0 & .3 & 0 \\ .5 & 0 & .5 & 0 \\ 0 & .4 & 0 & .6 \\ 0 & .2 & 0 & .8 \end{bmatrix}$$

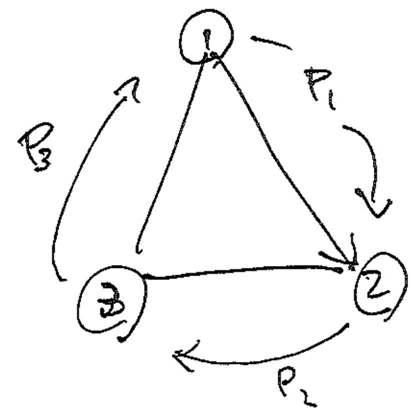
States 1 and 2

has today = Rain

$$\% \text{ of } (\text{today} = \text{Rain}) = .25 + .15$$

$$= \boxed{.4}$$

4-34



$$\mathbb{P} = \begin{bmatrix} 0 & P_1 & q_1 \\ q_2 & 0 & P_2 \\ P_3 & q_3 & 0 \end{bmatrix}$$

a) limit distribution (a, b, c) — see next page

From state 1, $P(1\ cc + 5\ cw) = q_1 P_3 P_1 P_2 P_3 P_1$

2 $= q_2 P_1 P_2 P_3 P_1 P_2$

3 $= q_3 P_2 P_3 P_1 P_2 P_3$

b) Ans

$$\boxed{\begin{aligned} & a(q_1 P_1^2 P_2 P_3^2) \\ & + b(q_2 P_1^2 P_2^2 P_3) \\ & + C(q_3 P_1 P_2^2 P_3) \end{aligned}}$$

a)

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 0 & p_1 & q_1 \\ q_2 & 0 & p_2 \\ p_3 & q_3 & 0 \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix}$$

$$\left.\begin{aligned} bq_2 + cp_3 &= a \quad &- \ (1) \\ ap_1 + cq_3 &= b \quad &- \ (2) \\ aq_1 + bp_2 &= c \quad &- \ (3) \\ a + b + c &= 1 \quad &- \ (4) \end{aligned}\right\} \text{ solve for } a, b, c \text{ using } p_1, p_2, p_3$$

(1) + (2)

$$ap_1 + bq_2 + c = a + b$$

$$ap_1 + bq_2 = a + b - c \quad - \ (5)$$

use (4) + (5) $\quad 1 + ap_1 + bq_2 = 2a + 2b$

$$a(p_1 - 2) + b(q_2 - 2) = -1 \qquad - (6)$$

~~(5) gives~~ $\cancel{a(1-p_1) + b(1-q_2)}$

$\cancel{= aq_1 + bq_2}$

(2) & (3) $\quad ap_1 + q_3(aq_1 + bp_2) = b$

$$a(p_1 + q_1 q_3) + b(p_2 q_3 - 1) = 0$$

$$b = \frac{-a(p_1 + q_1 q_3)}{(p_2 q_3 - 1)} \qquad - (7)$$

6 ← 2

$$a(P_1 - 2) - \frac{a(P_1 + q_1 q_3)}{(P_2 q_3 - 1)}(q_2 - 2) = -1$$

$$\underbrace{a\left[(P_1 - 2) + \frac{(2 - q_2)(P_1 + q_1 q_3)}{(P_2 q_3 - 1)}\right]}_{K} = -1$$

$$\begin{cases} a = -\dfrac{1}{K} \\ b = \dfrac{1}{K}\dfrac{(P_1 + q_1 q_3)}{(P_2 q_3 - 1)} \\ c = 1 - (a + b) \end{cases}$$

limit distribution.

4-36

$$\mathbb{P} = \begin{bmatrix} .4 & .6 \\ .2 & .8 \end{bmatrix}$$

P_0

P_1

a) $\begin{array}{ccc} \text{Mon} & & \text{Tue} \\ [1 \ \ 0]\,\mathbb{P} & = & [.4 \ \ .6] \end{array}$

$$P(\text{Good}) = [.4 \ \ .6] \begin{bmatrix} P_0 \\ P_1 \end{bmatrix} = \boxed{.4P_0 + .6P_1}$$

b) $\begin{array}{ccc} & & \text{Fri} \\ [1 \ \ 0]\,\mathbb{P}^4 & = & [.251 \ \ .749] \end{array}$

$$P(\text{Good}) = \boxed{.251 P_0 + .749 P_1}$$

c) long-run dist.

$$\begin{array}{cc} 0 & 1 \\ \end{array}$$
$$\begin{bmatrix} .25 & .75 \end{bmatrix}$$

$$\left.\begin{aligned} P(\text{Good}) &= .25 P_0 + .75 P_1 \\ P(\text{Bad}) &= .25 q_0 + .75 q_1 \end{aligned}\right\} \text{long-run. \%.}$$

(d)

If Message was "Good"

$$P(\text{State} = 0 \mid \text{Good}) = \frac{P(G|0)\,P(0)}{P(G|0)\,P(0) + P(G|1)\,P(1)}$$

Baye's Rule

$$= \frac{P_0\, P(0)}{P_0\, P(0) + P_1\, P(1)} = a$$

$$P(\text{State} = 1 \mid \cancel{\text{Bad}}\ \text{Good})$$

$$= \frac{P_1\, P(1)}{P_0\, P(0) + P_1\, P(1)} = b$$

Then Tomorrow's state is

$$[a \;\; b]\, \mathbb{P}$$

Then Tomorrow's ~~stat~~ "Good" message prob. is

$$[a \quad b] \, \mathbb{P} \begin{bmatrix} P_0 \\ P_1 \end{bmatrix} = P(\text{"Good" today} \to \text{"Good" tom.})$$

All we need is $P(0) = P(\text{State } 0)$
$P(1) = P(\text{State } 0)$ for today.

In the short-run, we don't have this.

So we can't make this into MC.

In the long-run use

$$\left.\begin{aligned} P(0) &= .25 \\ P(1) &= .75 \end{aligned}\right\} \text{long-run prob.}$$

//.

HW 2

a) $\{1, 2, 3\}$ transient ← once you enter $\{4, 5\}$ you won't come back.

b) $\{4, 5\}$ recurrent

c) P^{100} => $\Pi = [0 \quad 0 \quad 0 \quad .3 \quad .7]$

d) For $\{4, 5\}$ Av time to come back is

$$\frac{1}{.3} = 3.33$$

$$\frac{1}{.7} = 1.43$$

e)

$$\underline{m} = (\mathbb{I} - \mathbb{Q})^{-1} \underline{e}$$

$$\mathbb{Q} = \mathbb{P} \text{ (remove 4th col. 4th row)}$$

$$\underline{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\underline{m} = \begin{bmatrix} 4.2 \\ 5.3 \\ 4.47 \\ 3.33 \end{bmatrix} = \begin{bmatrix} m_{14} \\ m_{24} \\ m_{34} \\ m_{54} \end{bmatrix}$$