471/571 - HW5

due Wed, Nov 16th

Name: _Solution_

1. Project P requires an investment of 4000 at time 0. The investment pays 2000 at time 1 and 4000 at time 2. Project Q requires an investment of X at time 2. The investment pays 2000 at time 0 and 4000 at time 1. The net present values of the two projects are equal at an interest rate of 10%. Calculate X.
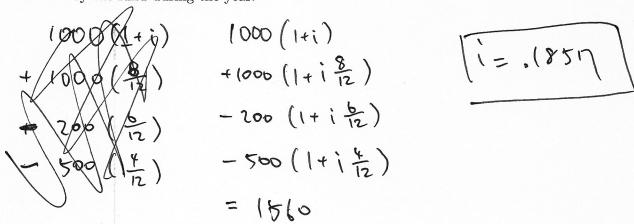
$$-4000 + \frac{2000}{1.1} + \frac{4000}{1.1^2} = 2000 + \frac{4000}{1.1} - \frac{X}{1.1^2}$$

$$\boxed{X = 5460}$$

2. An investor pays \$100,000 today for a 4-year investment that returns cash flows of \$60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective. If the rate of interest at which the investment is to be valued is 5.0%, what is the net present value of this investment today?

$$-100,000 + \frac{60000\,(1.04)}{1.05^4} + \frac{60000}{1.05^4} = \boxed{698.72}$$

3. At the beginning of the year, an investment fund was established with an initial deposit of 1000. A new deposit of 1000 was made at the end of 4 months. Withdrawals of 200 and 500 were made at the end of 6 months and 8 months, respectively. The amount in the fund at the end of the year is 1560. Calculate the dollar-weighted yield rate earned by the fund during the year.

$$1000(1+i)$$
$$+1000\left(\frac{8}{12}\right)$$
$$200\left(\frac{6}{12}\right)$$
$$500\left(\frac{4}{12}\right)$$

$$1000(1+i)$$
$$+1000\left(1+i\frac{8}{12}\right)$$
$$-200\left(1+i\frac{6}{12}\right)$$
$$-500\left(1+i\frac{4}{12}\right)$$
$$= 1560$$

$$\boxed{i = .1851}$$

4. Yield rates to maturity for zero coupon bonds are currently quoted at 8.5% for one-year maturity, 9.5% for two-year maturity, and 10.5% for three-year maturity. Let i be the one-year forward rate for year two implied by current yields of these bonds. Calculate i.

$$\frac{1.095^2}{1.085} = 1+i = 1.1051$$

$$\boxed{i = .1051}$$

5. You are given the following term structure of spot interest rates:

Term (in years)	Spot interest rate
1	5.00%
2	5.75%
3	6.25%
4	6.50%

A three-year annuity-immediate will be issued a year from now with annual payments of 5000. (so 5000 is coming at t=2,3,and 4). Using the forward rates, calculate the present value of this annuity a year from now.

$$\frac{\frac{5000}{1.0575^2}}{1.05} + \frac{\frac{5000}{1.0625^3}}{1.05} + \frac{\frac{5000}{1.0650^4}}{1.05} = \boxed{13152.50}$$

6. The current price of an annual coupon bond is 100. The derivative of the price of the bond with respect to the yield to maturity is -700. The yield to maturity is an annual effective rate of 8%. Calculate the duration of the bond.

$$DM = \frac{-P'}{P} = \frac{D}{1+i} = \frac{700}{100} = 7$$

$$D = 7\left(\frac{1}{1.08}\right) = \boxed{7.56}$$

7. Calculate the duration of a common stock that pays dividends at the end of each year as perpetuity. Assume that the dividend is constant, and that the effective rate of interest is 10%.

$$\text{dividend} = d.$$

$$PV = \frac{d}{i}$$

$$PV' = -\frac{d}{i^2}$$

$$D = \frac{-PV'}{PV}(1.1) = \frac{d/i^2}{d/i}(1+i) = \frac{1+i}{i}$$

$$\boxed{D = \frac{1.1}{.1} = 11}$$