An Adaptive Leapfrog Integrator to Model Self-Gravitating Systems

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Abstract

The field of computational particle modeling has proven imperative in initializing and controlling the motion of particles in plasma machines like tokamaks and stellarators. In order to better understand the nature of these particle interactions, we decided to design an adaptive leapfrog integrator to model the location and energies of self-gravitating systems. First, we modeled a simple harmonic oscillator in Python as a feasibility test under a second-order leapfrog scheme, and then plotted test cases of the "Two Body Problem" and "Three Body Problem" to confirm the strength of an integrator. To compute the locations and velocities of the particles in question, we applied the leapfrog scheme to each pair of particles in our test systems instead of making holistic approximations, improving the accuracy of the final plots. To circumvent the possibility of a time-step that is too large, we implemented an adaptive time-stepping algorithm in our C code. The algorithm halves the time step whenever at least one particle is moving too fast, which increases the accuracy of the data associated with close particle interactions, and doubles the time step whenever every particle is moving slowly, which speeds up the computation time. Next, we generalized the two/three-body code to support any number of bodies, and tested with a randomly distributed eight-body test. By using a more flexible time-step, the net energy of the system ended up being relatively stable and constant, even with the stress of computing 26 different pairs frequently undergoing high-energy close interactions. Additionally, the use of an adaptive time-stepping algorithm allows for the capture and transfer of binary systems, a gravitational mechanism not possible in many other orbital plotting software, due to the common inclusion of a pseudo repulsive force that prevents accuracy-wearing close-particle interactions. Finally, we worked to animate the plots, finally being able to demonstrate the motion of five of the particles in the eight-body test traversing about 330,000 data points in less than 10 seconds, giving users a reliable framework to observe the results of their simulations.