An Adaptive Symplectic Integrator for Modeling the Mechanics of Self-Gravitating Systems

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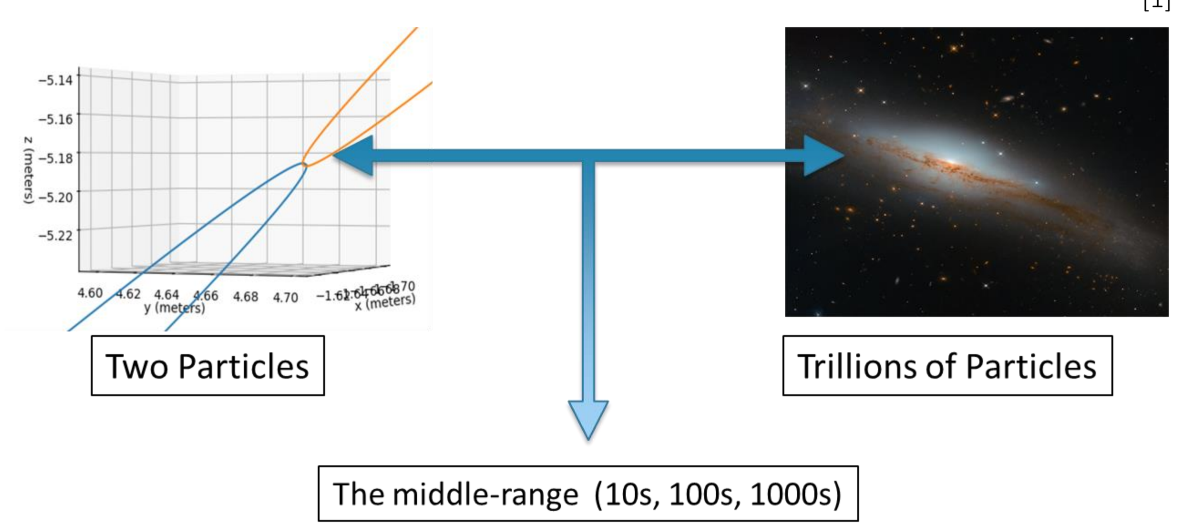
Introduction

The field of computational particle modeling has proven imperative in initializing and controlling the motion of particles in plasma machines like tokamaks and stellarators. Physicists have obtained a firm understanding of smaller particle systems, like the "Two-Body Problem", as well as massive ones, from the empirical data from galaxies. The gravitating systems of the tens, hundreds, and thousands range, however, still introduce an element of uncertainty. In order to better understand the nature of these particle interactions, we decided to design an adaptive leapfrog integrator to model the location and energies of self-gravitating systems.

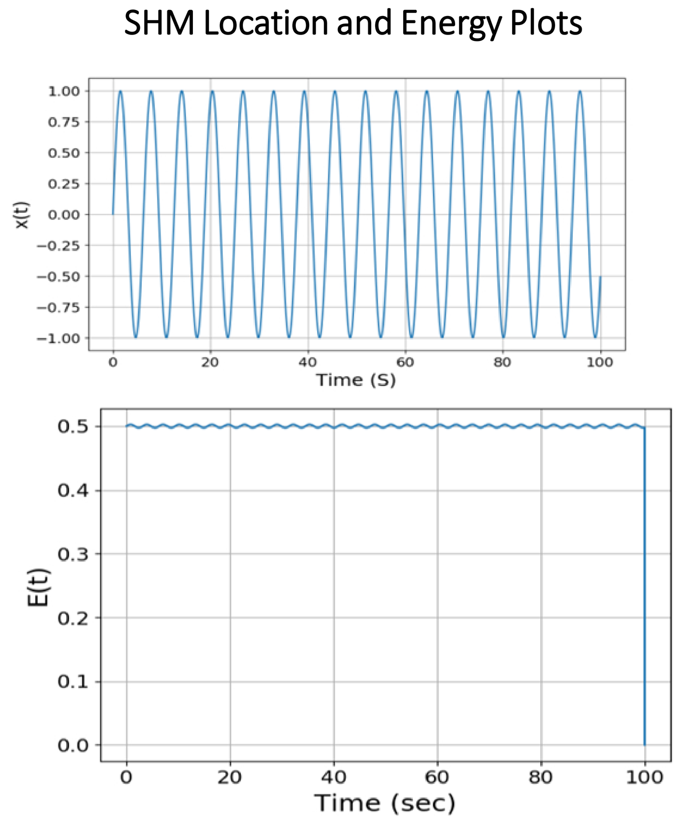


To improve the accuracy of our integrator, we also implemented an adaptive time-stepping algorithm in our C code that reduces the time step whenever at least one particle is moving too fast and increases the time step whenever every particle is moving slowly. Finally, we worked to animate the plots, finally being able to demonstrate the motion of five of the particles in the eight-body test traversing about 330,000 data points in less than 10 seconds, giving users a reliable framework to observe the results of their simulations.

Simple Harmonic Oscillator

To test the feasibility of using the leapfrog scheme, we started by modeling the motion and energy of a simple harmonic oscillator (SHM), modeled under a second-order leapfrog scheme. The position and energy of the plots were computed and plotted in Python. The location plot ended up being very accurate, but the energy plot was sinusoidal, instead of constant, marking a key characteristic in most adaptations of the leapfrog integrator[2].

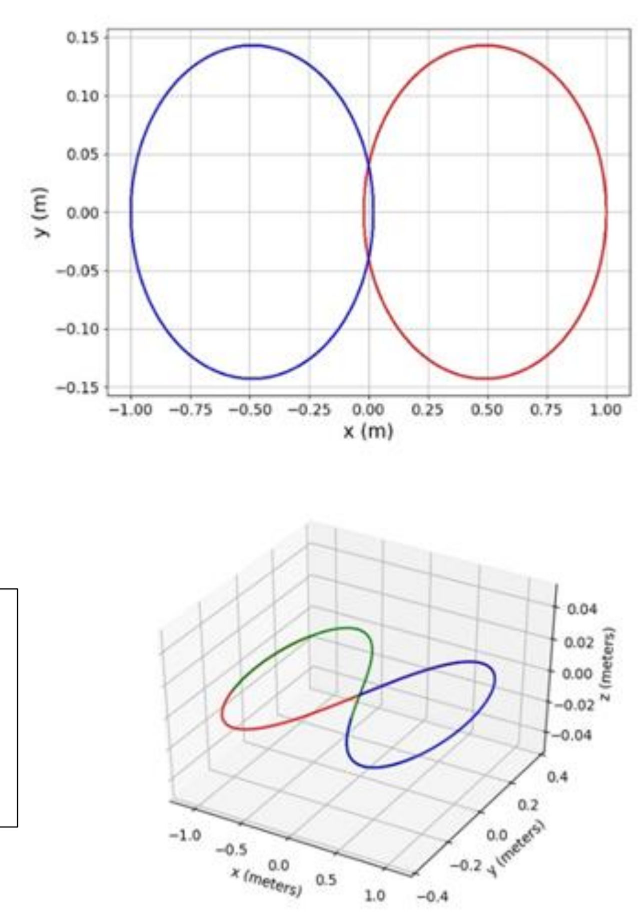
SHM Location and Energy Plots



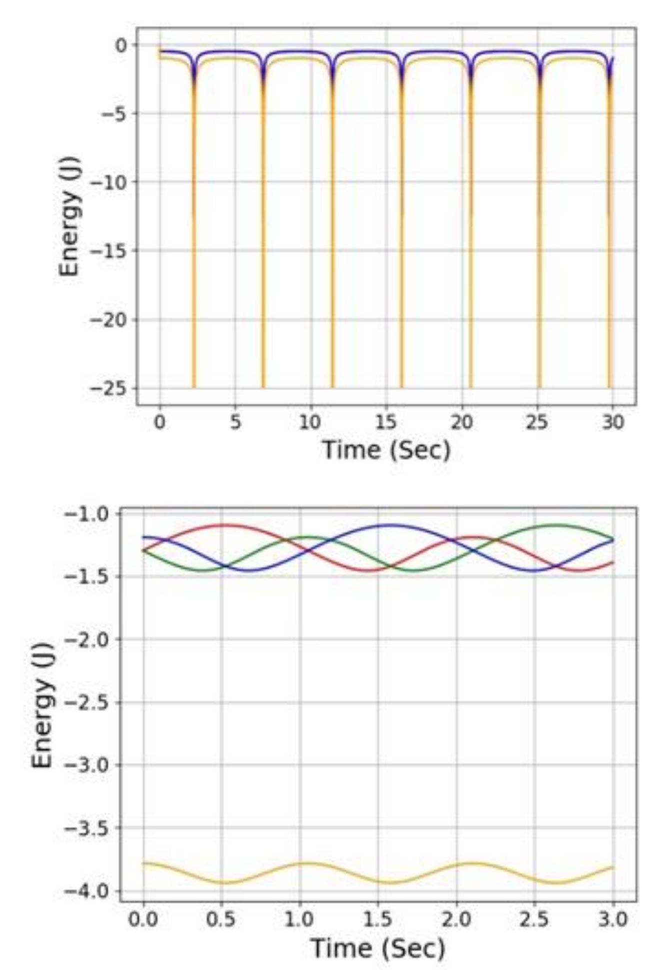
Two/Three-Body Problem Tests

Location Plot Key
Each curve (Red, Green, Blue) represents the path/behavior of a different particle.

2- and 3-Body Location Plots



2- and 3-Body Energy Plots



Total Energy Key
Orange/Green – Total Kinetic/Potential Energy
Blue - Combined Energy (Potential + Kinetic)

Next, it was decided to plots specific test cases of the two and three body problem under the leapfrog scheme (computed with C, plotted in Python). The computation was done between each pair of particles in the system, instead of relying on holistic system approximations, improving the accuracy of the program, while also increasing the computation time required to create the location and energy data files.

Integrator Weaknesses

- Fixed time step (dt) makes it possible to miss finer levels of physical interaction when particles are near each other
- "Total Energy" plot is sinusoidal instead of being constant
- Hard-coded nature of this version of the code limits tests to two/three bodies

Adaptive Time-Stepping Solution

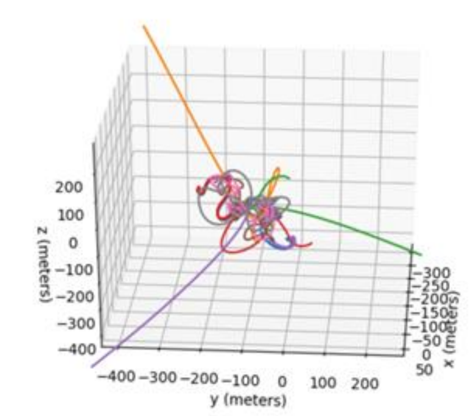
To circumvent the possibility of missing physical interaction with a time step that is too large, we decided to add an adaptive time-stepping algorithm to the original C code. The new algorithm computes three values each loop – the locations of the particle with the original time step, a doubled time step, and a halved time step.

- If the distance between the locations of the halved time step and the original is too large -> the **halved** time step becomes the new default
  - Improves the accuracy of the integrator during close particle interactions
- If the distance between the locations of the doubled time step and the original is too small -> the **doubled** time step becomes the new default
  - Helps speed up the computation time of the algorithm

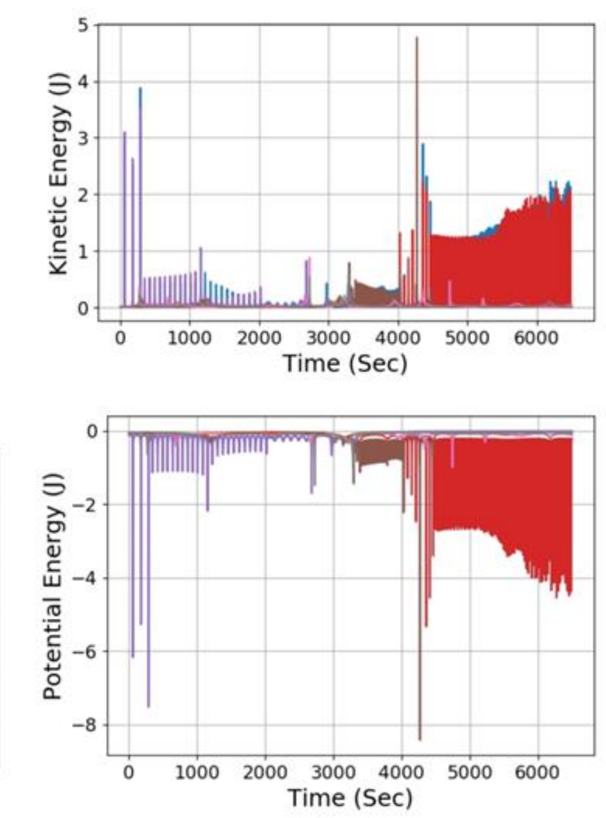
8-Body Distribution

Location, KE, PE Key
Each curve represents the path/behavior of a different particle, with the eight different colors corresponding across the three plots

8-Body Location Plot



8-Body KE/PE Plots



Total Energy Key
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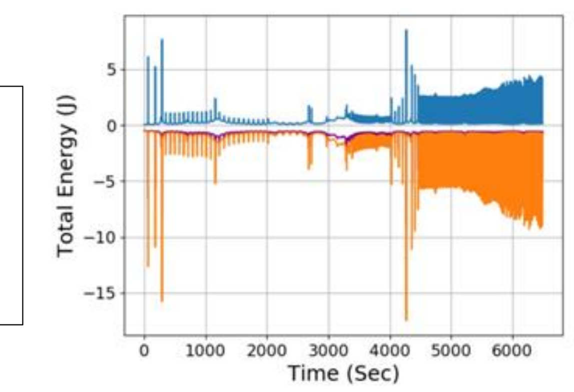
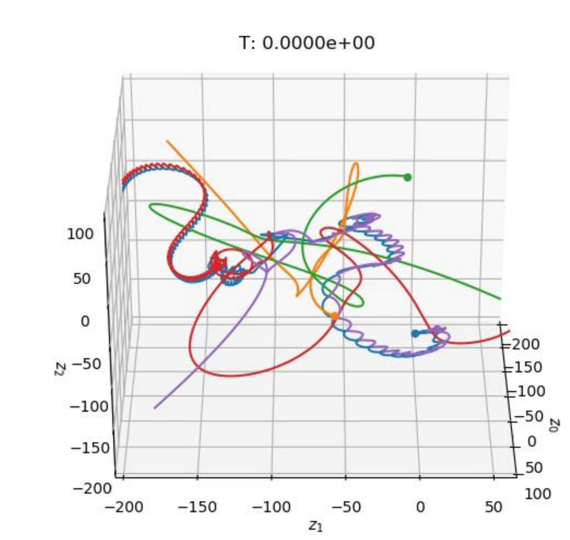


Next, the leapfrog integrator code was generalized to plot any number of bodies for a set time interval. As a test, eight particles of equal mass were distributed randomly within a 100-meter sphere and given random velocities within -.1 m/s to 0.1 m/s. The location plot reveals a couple of binary sub-systems and three particle ejections, and the "Combined Energy" curve (purple) in the "Total Energy" plot is fairly level, confirming the improved strength introduced by the adaptive time-stepping.

Animation Loop

- Animated to model five of the particles in the eight-body system (~6.5k seconds, ~330k data points)
- blue/purple particle binary gets separated by close interactions (~3.1k sec) and eventually yields a blue/red particle binary
- Absence of pseudo-force commonly found in other integrators allows for the modeling of these binary captures/transfers

Animated 8-Body Location Plot (5 Particles)



Conclusions and Future Directions

Conclusion

- A user-friendly, adaptive leapfrog integrator that can be used to simulate a wide range of self-gravitating systems consisting of large number of particles – useful for studying the statistical mechanics of gravitational/electromagnetic systems.

Future Directions

- Swap out the Leapfrog scheme with other schemes (ex. Runge-Kutta) and compare the difference in location and energy plot
- Determine the stability of stochasticity of systems in question – take each particle and add trace "ghost" particles that move under the influence of forces already existing in the system with slightly perturbed initial velocities.

References

[1]: ESA/Hubble & NASA, D. Rosario et al.
[2]: Time-Reversibility. (n.d.). Retrieved August 18, 2020, from http://www.physics.drexel.edu/~valliere/PHYS305/Diff_Eq_Integrators/time_reversal/

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