```
clc;
clear all;
```

Problem 2

```
syms q1 q2 q3 a b c;
syms dq1 dq2 dq3;
syms m1 mt g;

% calculate frame transforms for position and jacobian
T01 = [cos(q1)  0       sin(q1) 0; ...
       sin(q1)  0       cos(q1) 0; ...
       0        -1      0       a; ...
       0        0       0       1];


T12 = [1        0       0       0; ...
       0        0       1       0; ...
       0        -1      0       b+q2; ...
       0        0       0       1];

T23 = [0        1       0       0; ...
       -1       0       0       0; ...
       0        0       1       c+q3; ...
       0        0       0       1];

T02 = T01*T12;
T03 = T02*T23

% position vector
P03 = T03(1:3,4)

% jacobian
jacob = [diff(P03(1:3),q1), diff(P03(1:3),q2), diff(P03(1:3),q3)]

% K and P for mass 1
K1 = 0;
P1 = m1*g*a;

% K and P for mass 2
Vmt = jacob * [dq1; dq2; dq3];
K2 = 0.5*mt*(Vmt.' * Vmt);
P2 = mt*g*P03(3);

K = K1+K2;
P = P1+P2;

% Lagrange
L = K-P

syms Q1 Q2 Q3 Q1(t) Q2(t) Q3(t) ddq1 ddq2 ddq3
% Lagrange equation to calculate Tau 1
```

```
diffq1dot = diff(L,dq1);

diffq1dot = subs(diffq1dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
 diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq1dot = diff(diffq1dot, t);

diffq1dot = subs(diffq1dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
 diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
 q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq1 = diff(L,q1);

Tau1 = simplify(diffq1dot - diffq1)
% Lagrange equation to calculate F2
diffq2dot = diff(L,dq2);

diffq2dot = subs(diffq2dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
 diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq2dot = diff(diffq2dot, t);

diffq2dot = subs(diffq2dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
 diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
 q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq2 = diff(L,q2);

F2 = simplify(diffq2dot - diffq2)
% Lagrange equation to calculate F3
diffq3dot = diff(L,dq3);

diffq3dot = subs(diffq3dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
 diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq3dot = diff(diffq3dot, t);

diffq3dot = subs(diffq3dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
 diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
 q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq3 = diff(L,q3);

F3= simplify(diffq3dot - diffq3)


T03 =

[ sin(q1), cos(q1),  0, sin(q1)*(b + q2)]
[ cos(q1), sin(q1),  0, cos(q1)*(b + q2)]
[       0,       0, -1,       a - c - q3]
[       0,       0,  0,                1]
```

```
P03 =

 sin(q1)*(b + q2)
 cos(q1)*(b + q2)
       a - c - q3


jacob =

[  cos(q1)*(b + q2), sin(q1),  0]
[ -sin(q1)*(b + q2), cos(q1),  0]
[                 0,       0, -1]


L =

(mt*((dq2*sin(q1) + dq1*cos(q1)*(b + q2))^2 + (dq2*cos(q1) -
 dq1*sin(q1)*(b + q2))^2 + dq3^2))/2 + g*mt*(c - a + q3) - a*g*m1


Tau1 =

mt*(b + q2)*(b*ddq1 + 2*dq1*dq2 + ddq1*q2)


F2 =

-mt*(b*dq1^2 - ddq2 + dq1^2*q2)


F3 =

mt*(ddq3 - g)
```

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```
clc;
clear all;
```

Problem 3

```
syms q1 q2 q3 a1 a2 a3;
syms dq1 dq2 dq3;
syms m1 m2 m3 g;

% calculate frame transforms for position and jacobian
T01 = [cos(q1)  -sin(q1) 0       a1*cos(q1); ...
       sin(q1)  cos(q1) 0       a1*sin(q1); ...
       0        0       1       0; ...
       0        0       0       1];


T12 = [cos(q2)  -sin(q2) 0       a1*cos(q2); ...
       sin(q2)  cos(q2) 0       a1*sin(q2); ...
       0        0       1       0; ...
       0        0       0       1];

T23 = [cos(q3)  -sin(q3) 0       a1 * cos(q3); ...
       sin(q3)  cos(q3) 0       a1 * sin(q3); ...
       0        0       1       0; ...
       0        0       0       1];

T02 = T01*T12;
T03 = T02*T23

% position vector
P01 = T01(1:3,4)
P02 = T02(1:3,4)
P03 = T03(1:3,4)

% Potential ENergies
P1 = m1 * g * subs(P01(3), a1, a1/2);
P2 = m2 * g * subs(P02(3), a2, a2/2);
P3 = m3 * g * subs(P03(3), a3, a3/2);


% Kinetic Energies
jacob1 = [diff(P01(1:3),q1)];
jacob2 = [diff(P02(1:3),q1), diff(P02(1:3),q2)];
jacob3 = [diff(P03(1:3),q1), diff(P03(1:3),q2), diff(P03(1:3),q3)];

Vm1 = subs(jacob1, a1, a1/2) * dq1;
Vm2 = subs(jacob2, a2, a2/2) * [dq1; dq2];
Vm3 = subs(jacob3, a3, a3/2) * [dq1; dq2; dq3];

K1 = 0.5*m1*(Vm1.' * Vm1);
K2 = 0.5*m2*(Vm2.' * Vm2);
K3 = 0.5*m3*(Vm3.' * Vm3);
```

```
% Lagrange
P = P1+P2+P3;
K = K1+K2+K3;
L = K-P

syms Q1 Q2 Q3 Q1(t) Q2(t) Q3(t) ddq1 ddq2 ddq3
% Lagrange equation to calculate Tau 1
diffq1dot = diff(L,dq1);

diffq1dot = subs(diffq1dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
 diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq1dot = diff(diffq1dot, t);

diffq1dot = subs(diffq1dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
 diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
 q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq1 = diff(L,q1);

Tau1 = simplify(diffq1dot - diffq1)
% Lagrange equation to calculate Tau2
diffq2dot = diff(L,dq2);

diffq2dot = subs(diffq2dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
 diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq2dot = diff(diffq2dot, t);

diffq2dot = subs(diffq2dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
 diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
 q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq2 = diff(L,q2);

Tau2 = simplify(diffq2dot - diffq2)
% Lagrange equation to calculate Tau3
diffq3dot = diff(L,dq3);

diffq3dot = subs(diffq3dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
 diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq3dot = diff(diffq3dot, t);

diffq3dot = subs(diffq3dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
 diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
 q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq3 = diff(L,q3);

Tau3= simplify(diffq3dot - diffq3)


T03 =
```

```
[ cos(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) -
 sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)), -
 cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) -
 sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)), 0, a1*cos(q1)
 + a1*cos(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) -
 a1*sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) + a1*cos(q1)*cos(q2) -
 a1*sin(q1)*sin(q2)]
[ cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) +
 sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)),
  cos(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) -
 sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)), 0, a1*sin(q1)
 + a1*cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) +
 a1*sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) + a1*cos(q1)*sin(q2) +
 a1*cos(q2)*sin(q1)]
[
                    0,
                                        0, 1,

                                                            0]
[
                    0,
                                        0, 0,

                                                            1]


P01 =

 a1*cos(q1)
 a1*sin(q1)
          0


P02 =

 a1*cos(q1) + a1*cos(q1)*cos(q2) - a1*sin(q1)*sin(q2)
 a1*sin(q1) + a1*cos(q1)*sin(q2) + a1*cos(q2)*sin(q1)
                                                    0


P03 =

 a1*cos(q1) + a1*cos(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) -
 a1*sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) + a1*cos(q1)*cos(q2) -
 a1*sin(q1)*sin(q2)
 a1*sin(q1) + a1*cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) +
 a1*sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) + a1*cos(q1)*sin(q2) +
 a1*cos(q2)*sin(q1)


          0
```

```
L =

(m2*((dq2*(a1*cos(q1)*cos(q2) - a1*sin(q1)*sin(q2)) +
 dq1*(a1*cos(q1) + a1*cos(q1)*cos(q2) - a1*sin(q1)*sin(q2)))^2 +
 (dq2*(a1*cos(q1)*sin(q2) + a1*cos(q2)*sin(q1)) + dq1*(a1*sin(q1)
 + a1*cos(q1)*sin(q2) + a1*cos(q2)*sin(q1)))^2))/2 +
 (m1*((a1^2*dq1^2*cos(q1)^2)/4 + (a1^2*dq1^2*sin(q1)^2)/4))/2
 + (m3*((dq1*(a1*cos(q1) + a1*cos(q3)*(cos(q1)*cos(q2)
 - sin(q1)*sin(q2)) - a1*sin(q3)*(cos(q1)*sin(q2) +
 cos(q2)*sin(q1)) + a1*cos(q1)*cos(q2) - a1*sin(q1)*sin(q2))
 + dq2*(a1*cos(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) -
 a1*sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) + a1*cos(q1)*cos(q2)
 - a1*sin(q1)*sin(q2)) + dq3*(a1*cos(q3)*(cos(q1)*cos(q2) -
 sin(q1)*sin(q2)) - a1*sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1))))^2
 + (dq1*(a1*sin(q1) + a1*cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) +
 a1*sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) + a1*cos(q1)*sin(q2)
 + a1*cos(q2)*sin(q1)) + dq2*(a1*cos(q3)*(cos(q1)*sin(q2)
 + cos(q2)*sin(q1)) + a1*sin(q3)*(cos(q1)*cos(q2) -
 sin(q1)*sin(q2)) + a1*cos(q1)*sin(q2) + a1*cos(q2)*sin(q1))
 + dq3*(a1*cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) +
 a1*sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2))))^2))/2


Tau1 =

(a1^2*(ddq1*m1 + 8*ddq1*m2 + 12*ddq1*m3 + 4*ddq2*m2 + 8*ddq2*m3
 + 4*ddq3*m3 + 8*ddq1*m2*cos(q2) + 8*ddq1*m3*cos(q2) +
 4*ddq2*m2*cos(q2) + 8*ddq1*m3*cos(q3) + 4*ddq2*m3*cos(q2) +
 8*ddq2*m3*cos(q3) + 4*ddq3*m3*cos(q3) - 4*dq2^2*m3*sin(q2 + q3) -
 4*dq3^2*m3*sin(q2 + q3) - 4*dq2^2*m2*sin(q2) - 4*dq2^2*m3*sin(q2)
 - 4*dq3^2*m3*sin(q3) + 8*ddq1*m3*cos(q2 + q3) + 4*ddq2*m3*cos(q2
 + q3) + 4*ddq3*m3*cos(q2 + q3) - 8*dq1*dq2*m3*sin(q2 + q3)
 - 8*dq1*dq3*m3*sin(q2 + q3) - 8*dq2*dq3*m3*sin(q2 + q3) -
 8*dq1*dq2*m2*sin(q2) - 8*dq1*dq2*m3*sin(q2) - 8*dq1*dq3*m3*sin(q3) -
 8*dq2*dq3*m3*sin(q3)))/4


Tau2 =

a1^2*(ddq1*m2 + 2*ddq1*m3 + ddq2*m2 + 2*ddq2*m3 + ddq3*m3
 + ddq1*m2*cos(q2) + ddq1*m3*cos(q2) + 2*ddq1*m3*cos(q3) +
 2*ddq2*m3*cos(q3) + ddq3*m3*cos(q3) + dq1^2*m3*sin(q2 + q3)
 + dq1^2*m2*sin(q2) + dq1^2*m3*sin(q2) - dq3^2*m3*sin(q3) +
 ddq1*m3*cos(q2 + q3) - 2*dq1*dq3*m3*sin(q3) - 2*dq2*dq3*m3*sin(q3))


Tau3 =

a1^2*m3*(ddq1 + ddq2 + ddq3 + dq1^2*sin(q3) + dq2^2*sin(q3) +
 ddq1*cos(q2 + q3) + ddq1*cos(q3) + ddq2*cos(q3) + dq1^2*sin(q2 + q3)
 + 2*dq1*dq2*sin(q3))
```

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