Question 4

Table of Contents

Part A Obtain State space

I took the solved dyanmics from 1.7.4 of Zaks textbook, but with a point mass and no friction adjustments

```
syms m M I l theta dtheta ddtheta ddx g fc u a

%a = 1/(m+M)

diffdiffx = -m*a*l*ddtheta*cos(theta)+m*a*l*dtheta^2*sin(theta)-a*fc
+a*u

eq = I*ddtheta == m*g*l*sin(theta)-m*l^2*ddtheta-
m*diffdiffx*l*cos(theta)

diffdifftheta = solve(eq,ddtheta)
diffdifftheta = simplify(subs(diffdifftheta, I, m*l^2))
diffdifftheta = simplify(subs(diffdifftheta, fc, 0))

diffdiffx = subs(diffdiffx, ddtheta, diffdifftheta)
diffdiffx = simplify(subs(diffdiffx, I, m*l^2))
diffdiffx = simplify(subs(diffdiffx, fc, 0))

syms x1 x2 x3 x4

output = x1

dx1 = x2
dx2 = diffdiffx
dx2 = subs(dx2, [theta dtheta], [x1 x2])
pretty(dx2)
dx3 = x4
dx4 = diffdifftheta
dx4 = subs(dx4, [theta dtheta], [x1 x2])

g1 = 0
f1 = dx1

g2 = coeffs(dx2, u);
g2 = g2(2)
f2 = simplify(dx2-g2*u)

g3 = 0
f3 = dx3
```

```
g4 = coeffs(dx4, u);
g4 = g4(2)
f4 = simplify(dx4-g4*u)
```

```
diffdiffx =

a*l*m*sin(theta)*dtheta^2 - a*fc + a*u - a*ddtheta*l*m*cos(theta)


eq =

I*ddtheta == g*l*m*sin(theta) - ddtheta*l^2*m + l*m*cos(theta)*(-
 a*l*m*sin(theta)*dtheta^2 + a*fc - a*u + a*ddtheta*l*m*cos(theta))


diffdifftheta =

-(l*m*cos(theta)*(a*l*m*sin(theta)*dtheta^2 - a*fc + a*u) -
 g*l*m*sin(theta))/(I + l^2*m - a*l^2*m^2*cos(theta)^2)


diffdifftheta =

-(l*m*cos(theta)*(a*l*m*sin(theta)*dtheta^2 - a*fc + a*u) -
 g*l*m*sin(theta))/(2*l^2*m - a*l^2*m^2*cos(theta)^2)


diffdifftheta =

(a*l*m*cos(theta)*sin(theta)*dtheta^2 - g*sin(theta) +
 a*u*cos(theta))/(l*(a*m*cos(theta)^2 - 2))


diffdiffx =

a*u - a*fc + a*dtheta^2*l*m*sin(theta) -
 (a*m*cos(theta)*(a*l*m*cos(theta)*sin(theta)*dtheta^2 - g*sin(theta)
 + a*u*cos(theta)))/(a*m*cos(theta)^2 - 2)


diffdiffx =

-(a*(2*u - 2*fc + a*fc*m*cos(theta)^2 + 2*dtheta^2*l*m*sin(theta) -
 g*m*cos(theta)*sin(theta)))/(a*m*cos(theta)^2 - 2)


diffdiffx =

-(a*(2*l*m*sin(theta)*dtheta^2 + 2*u - g*m*cos(theta)*sin(theta)))/
(a*m*cos(theta)^2 - 2)
```

```
output =

x1


dx1 =

x2


dx2 =

-(a*(2*l*m*sin(theta)*dtheta^2 + 2*u - g*m*cos(theta)*sin(theta)))/
(a*m*cos(theta)^2 - 2)


dx2 =

-(a*(2*l*m*sin(x1)*x2^2 + 2*u - g*m*cos(x1)*sin(x1)))/(a*m*cos(x1)^2 -
 2)

                    2
  a (l m sin(x1) x2  2 + 2 u - g m cos(x1) sin(x1))
- ------------------------------------------------
                               2
                    a m cos(x1)  - 2


dx3 =

x4


dx4 =

(a*l*m*cos(theta)*sin(theta)*dtheta^2 - g*sin(theta) +
 a*u*cos(theta))/(l*(a*m*cos(theta)^2 - 2))


dx4 =

(a*l*m*cos(x1)*sin(x1)*x2^2 - g*sin(x1) + a*u*cos(x1))/
(l*(a*m*cos(x1)^2 - 2))


g1 =

     0


f1 =

x2
```

```
g2 =

-(2*a)/(a*m*cos(x1)^2 - 2)


f2 =

(a*m*sin(x1)*(- 2*l*x2^2 + g*cos(x1)))/(a*m*cos(x1)^2 - 2)


g3 =

     0


f3 =

x4


g4 =

(a*cos(x1))/(l*(a*m*cos(x1)^2 - 2))


f4 =

-(sin(x1)*(- a*l*m*cos(x1)*x2^2 + g))/(l*(a*m*cos(x1)^2 - 2))
```

Part B

```
syms x1e x3e u1e u3e

dx2 = subs(dx2, a, (1/(m+M)))
dx4 = subs(dx4, a, (1/(m+M)))

dx2 = subs(dx2, [l m M g], [1 0.1 1 10])
dx4 = subs(dx4, [l m M g], [1 0.1 1 10])

dxAll = [dx1;dx2;dx3;dx4]

A = simplify([diff(dxAll,x1),
 diff(dxAll,x2),diff(dxAll,x3),diff(dxAll,x4)])

B = simplify(diff(dxAll,u))


dx2 =

-(2*l*m*sin(x1)*x2^2 + 2*u - g*m*cos(x1)*sin(x1))/((M +
 m)*((m*cos(x1)^2)/(M + m) - 2))
```

```
dx4 =

((l*m*cos(x1)*sin(x1)*x2^2)/(M + m) - g*sin(x1) + (u*cos(x1))/(M +
 m))/(l*((m*cos(x1)^2)/(M + m) - 2))


dx2 =

-(10*((sin(x1)*x2^2)/5 + 2*u - cos(x1)*sin(x1)))/(11*(cos(x1)^2/11 -
 2))


dx4 =

((cos(x1)*sin(x1)*x2^2)/11 - 10*sin(x1) + (10*u*cos(x1))/11)/
(cos(x1)^2/11 - 2)


dxAll =


        x2
        -(10*((sin(x1)*x2^2)/5 + 2*u - cos(x1)*sin(x1)))/
(11*(cos(x1)^2/11 - 2))

        x4
 ((cos(x1)*sin(x1)*x2^2)/11 - 10*sin(x1) + (10*u*cos(x1))/11)/
(cos(x1)^2/11 - 2)


A =

[

                                        0,
 1, 0, 0]
[                                       - (2*x2^2*cos(x1) - 20*cos(x1)^2 +
 10)/(cos(x1)^2 - 22) - (20*cos(x1)*sin(x1)*((sin(x1)*x2^2)/5 + 2*u -
 cos(x1)*sin(x1)))/(cos(x1)^2 - 22)^2,   (4*x2*sin(x1))/(sin(x1)^2 +
 21), 0, 0]
[

                                        0,
 0, 0, 1]
[ (22*cos(x1)*sin(x1)*((cos(x1)*sin(x1)*x2^2)/11 - 10*sin(x1)
 + (10*u*cos(x1))/11))/(cos(x1)^2 - 22)^2 - (110*cos(x1) -
 2*x2^2*cos(x1)^2 + 10*u*sin(x1) + x2^2)/(cos(x1)^2 - 22),
 (2*x2*sin(2*x1))/(cos(2*x1) - 43), 0, 0]


B =
```

```
                         0
          -20/(cos(x1)^2 - 22)
                         0
  (10*cos(x1))/(cos(x1)^2 - 22)
```

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