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clc;
clear all;
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## Problem 3

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syms q1 q2 q3 a1 a2 a3;
syms dq1 dq2 dq3;
syms m1 m2 m3 g;

% calculate frame transforms for position and jacobian
T01 = [cos(q1)  -sin(q1)  0      a1*cos(q1); ...
       sin(q1)   cos(q1)  0      a1*sin(q1); ...
       0         0       1       0; ...
       0         0       0       1];

T12 = [cos(q2)  -sin(q2)  0      a1*cos(q2); ...
       sin(q2)   cos(q2)  0      a1*sin(q2); ...
       0         0       1       0; ...
       0         0       0       1];

T23 = [cos(q3)  -sin(q3)  0      a1 * cos(q3); ...
       sin(q3)   cos(q3)  0      a1 * sin(q3); ...
       0         0       1       0; ...
       0         0       0       1];

T02 = T01*T12;
T03 = T02*T23;

% position vector
P01 = T01(1:3,4)
P02 = T02(1:3,4)
P03 = T03(1:3,4)

% Potential ENergies
P1 = m1 * g * subs(P01(3), a1, a1/2);
P2 = m2 * g * subs(P02(3), a2, a2/2);
P3 = m3 * g * subs(P03(3), a3, a3/2);

% Kinetic ENergies
jacob1 = [diff(P01(1:3),q1)];
jacob2 = [diff(P02(1:3),q1), diff(P02(1:3),q2)];
jacob3 = [diff(P03(1:3),q1), diff(P03(1:3),q2), diff(P03(1:3),q3)];

Vm1 = subs(jacob1, a1, a1/2) * dq1;
Vm2 = subs(jacob2, a2, a2/2) * [dq1; dq2];
Vm3 = subs(jacob3, a3, a3/2) * [dq1; dq2; dq3];

K1 = 0.5*m1*(Vm1.' * Vm1);
K2 = 0.5*m2*(Vm2.' * Vm2);
K3 = 0.5*m3*(Vm3.' * Vm3);
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% Lagrange
P = P1+P2+P3;
K = K1+K2+K3;
L = K-P

syms Q1 Q2 Q3 Q1(t) Q2(t) Q3(t) ddq1 ddq2 ddq3
% Lagrange equation to calculate Tau 1
diffq1dot = diff(L,dq1);

diffq1dot = subs(diffq1dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq1dot = diff(diffq1dot, t);

diffq1dot = subs(diffq1dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq1 = diff(L,q1);

Tau1 = simplify(diffq1dot - diffq1)
% Lagrange equation to calculate Tau2
diffq2dot = diff(L,dq2);

diffq2dot = subs(diffq2dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq2dot = diff(diffq2dot, t);

diffq2dot = subs(diffq2dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq2 = diff(L,q2);

Tau2 = simplify(diffq2dot - diffq2)
% Lagrange equation to calculate Tau3
diffq3dot = diff(L,dq3);

diffq3dot = subs(diffq3dot, [q1 q2 q3 dq1 dq2 dq3], [Q1 Q2 Q3
diff(Q1(t),t) diff(Q2(t),t) diff(Q3(t),t)]);

diffq3dot = diff(diffq3dot, t);

diffq3dot = subs(diffq3dot, [Q1 Q2 Q3 diff(Q1(t),t) diff(Q2(t),t)
diff(Q3(t),t) diff(Q1(t),t,t) diff(Q2(t),t,t) diff(Q3(t),t,t)], [q1
q2 q3 dq1 dq2 dq3 ddq1 ddq2 ddq3]);

diffq3 = diff(L,q3);

Tau3= simplify(diffq3dot - diffq3)

T03 =

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$$\begin{aligned}
& [ \cos(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)) - \\
& \sin(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)), - \\
& \cos(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)) - \\
& \sin(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)), 0, a1 * \cos(q_1) \\
& + a1 * \cos(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)) - \\
& a1 * \sin(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)) + a1 * \cos(q_1) * \cos(q_2) - \\
& a1 * \sin(q_1) * \sin(q_2)] \\
& [ \cos(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)) + \\
& \sin(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)), \\
& \cos(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)) - \\
& \sin(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)), 0, a1 * \sin(q_1) \\
& + a1 * \cos(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)) + \\
& a1 * \sin(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)) + a1 * \cos(q_1) * \sin(q_2) + \\
& a1 * \cos(q_2) * \sin(q_1)] \\
& [ \\
& \qquad \qquad \qquad 0, \\
& \qquad \qquad \qquad \qquad \qquad \qquad 0, 1, \\
& \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad 0] \\
& [ \\
& \qquad \qquad \qquad 0, \\
& \qquad \qquad \qquad \qquad \qquad \qquad 0, 0, \\
& \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad 1]
\end{aligned}$$

P01 =

$$\begin{aligned}
& a1 * \cos(q_1) \\
& a1 * \sin(q_1) \\
& 0
\end{aligned}$$

P02 =

$$\begin{aligned}
& a1 * \cos(q_1) + a1 * \cos(q_1) * \cos(q_2) - a1 * \sin(q_1) * \sin(q_2) \\
& a1 * \sin(q_1) + a1 * \cos(q_1) * \sin(q_2) + a1 * \cos(q_2) * \sin(q_1) \\
& 0
\end{aligned}$$

P03 =

$$\begin{aligned}
& a1 * \cos(q_1) + a1 * \cos(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)) - \\
& a1 * \sin(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)) + a1 * \cos(q_1) * \cos(q_2) - \\
& a1 * \sin(q_1) * \sin(q_2) \\
& a1 * \sin(q_1) + a1 * \cos(q_3) * (\cos(q_1) * \sin(q_2) + \cos(q_2) * \sin(q_1)) + \\
& a1 * \sin(q_3) * (\cos(q_1) * \cos(q_2) - \sin(q_1) * \sin(q_2)) + a1 * \cos(q_1) * \sin(q_2) + \\
& a1 * \cos(q_2) * \sin(q_1)
\end{aligned}$$

$$0$$

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$$L =$$

$$\begin{aligned}
& (m2*((dq2*(a1*cos(q1)*cos(q2) - a1*sin(q1)*sin(q2)) + \\
& dq1*(a1*cos(q1) + a1*cos(q1)*cos(q2) - a1*sin(q1)*sin(q2)))^2 + \\
& (dq2*(a1*cos(q1)*sin(q2) + a1*cos(q2)*sin(q1)) + dq1*(a1*sin(q1) \\
& + a1*cos(q1)*sin(q2) + a1*cos(q2)*sin(q1)))^2))/2 + \\
& (m1*((a1^2*dq1^2*cos(q1)^2)/4 + (a1^2*dq1^2*sin(q1)^2)/4))/2 \\
& + (m3*((dq1*(a1*cos(q1) + a1*cos(q3)*(cos(q1)*cos(q2) \\
& - sin(q1)*sin(q2)) - a1*sin(q3)*(cos(q1)*sin(q2) + \\
& cos(q2)*sin(q1)) + a1*cos(q1)*cos(q2) - a1*sin(q1)*sin(q2)) \\
& + dq2*(a1*cos(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) - \\
& a1*sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) + a1*cos(q1)*cos(q2) \\
& - a1*sin(q1)*sin(q2)) + dq3*(a1*cos(q3)*(cos(q1)*cos(q2) - \\
& sin(q1)*sin(q2)) - a1*sin(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)))^2 \\
& + (dq1*(a1*sin(q1) + a1*cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) + \\
& a1*sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)) + a1*cos(q1)*sin(q2) \\
& + a1*cos(q2)*sin(q1)) + dq2*(a1*cos(q3)*(cos(q1)*sin(q2) \\
& + cos(q2)*sin(q1)) + a1*sin(q3)*(cos(q1)*cos(q2) - \\
& sin(q1)*sin(q2)) + a1*cos(q1)*sin(q2) + a1*cos(q2)*sin(q1)) \\
& + dq3*(a1*cos(q3)*(cos(q1)*sin(q2) + cos(q2)*sin(q1)) + \\
& a1*sin(q3)*(cos(q1)*cos(q2) - sin(q1)*sin(q2)))^2))/2
\end{aligned}$$

$$Tau1 =$$

$$\begin{aligned}
& (a1^2*(ddq1*m1 + 8*ddq1*m2 + 12*ddq1*m3 + 4*ddq2*m2 + 8*ddq2*m3 \\
& + 4*ddq3*m3 + 8*ddq1*m2*cos(q2) + 8*ddq1*m3*cos(q2) + \\
& 4*ddq2*m2*cos(q2) + 8*ddq1*m3*cos(q3) + 4*ddq2*m3*cos(q2) + \\
& 8*ddq2*m3*cos(q3) + 4*ddq3*m3*cos(q3) - 4*dq2^2*m3*sin(q2 + q3) - \\
& 4*dq3^2*m3*sin(q2 + q3) - 4*dq2^2*m2*sin(q2) - 4*dq2^2*m3*sin(q2) \\
& - 4*dq3^2*m3*sin(q3) + 8*ddq1*m3*cos(q2 + q3) + 4*ddq2*m3*cos(q2 \\
& + q3) + 4*ddq3*m3*cos(q2 + q3) - 8*dq1*dq2*m3*sin(q2 + q3) \\
& - 8*dq1*dq3*m3*sin(q2 + q3) - 8*dq2*dq3*m3*sin(q2 + q3) - \\
& 8*dq1*dq2*m2*sin(q2) - 8*dq1*dq2*m3*sin(q2) - 8*dq1*dq3*m3*sin(q3) - \\
& 8*dq2*dq3*m3*sin(q3)))/4
\end{aligned}$$

$$Tau2 =$$

$$\begin{aligned}
& a1^2*(ddq1*m2 + 2*ddq1*m3 + ddq2*m2 + 2*ddq2*m3 + ddq3*m3 \\
& + ddq1*m2*cos(q2) + ddq1*m3*cos(q2) + 2*ddq1*m3*cos(q3) + \\
& 2*ddq2*m3*cos(q3) + ddq3*m3*cos(q3) + dq1^2*m3*sin(q2 + q3) \\
& + dq1^2*m2*sin(q2) + dq1^2*m3*sin(q2) - dq3^2*m3*sin(q3) + \\
& ddq1*m3*cos(q2 + q3) - 2*dq1*dq3*m3*sin(q3) - 2*dq2*dq3*m3*sin(q3))
\end{aligned}$$

$$Tau3 =$$

$$\begin{aligned}
& a1^2*m3*(ddq1 + ddq2 + ddq3 + dq1^2*sin(q3) + dq2^2*sin(q3) + \\
& ddq1*cos(q2 + q3) + ddq1*cos(q3) + ddq2*cos(q3) + dq1^2*sin(q2 + q3) \\
& + 2*dq1*dq2*sin(q3))
\end{aligned}$$

