HW#3 Temporal Models

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In order to understand the underlying process of modeling a temporally autoregressive phenomena we will simulate data using the following structure.

$$S_t \sim \mathcal{N}(\beta S_{t-1}, \sigma_s^2)$$
$$Y_t \sim \mathcal{N}(S_t, \sigma_y^2)$$

Three parameterizations of β, $\beta = \{-.5, 0, .5\}$, and σ_y, $\sigma_y = \{.2, .4, .8\}$, will be used resulting in nine unique simulation sets where each set will consist of 100 simulation runs. From these 100 simulation runs a mean, sd, 2.5, and 97.5 quantiles can be calculated and histograms of the parameter estimates can be observed. Note that the temporal component of this model comes from the normal distribution around the prior state S_{t-1}. The observation that we see is a function of this past state, a growth or decay factor β and some measurement error captured by σ_y.

Parameter estimates from 9 unique simulations

b	sigma_proc	sigma.obs	b_hat	sigma_proc_hat	sigma_obs_hat
-0.5	0.4	0.2	-0.501	0.388	0.170
0.0	0.4	0.2	-0.006	0.270	0.200
0.5	0.4	0.2	0.503	0.379	0.198
-0.5	0.4	0.4	-0.502	0.377	0.391
0.0	0.4	0.4	0.002	0.310	0.285
0.5	0.4	0.4	0.500	0.386	0.382
-0.5	0.4	0.8	-0.493	0.386	0.774
0.0	0.4	0.8	0.006	0.511	0.428
0.5	0.4	0.8	0.500	0.301	0.808

b	sigma_proc	sigma.obs	b_sd	sigma_proc_sd	sigma_obs_sd
-0.5	0.4	0.2	0.036	0.062	0.111
0.0	0.4	0.2	0.051	0.204	0.209
0.5	0.4	0.2	0.037	0.069	0.100
-0.5	0.4	0.4	0.044	0.113	0.119
0.0	0.4	0.4	0.055	0.273	0.268
0.5	0.4	0.4	0.043	0.100	0.111
-0.5	0.4	0.8	0.071	0.195	0.123
0.0	0.4	0.8	0.085	0.410	0.424

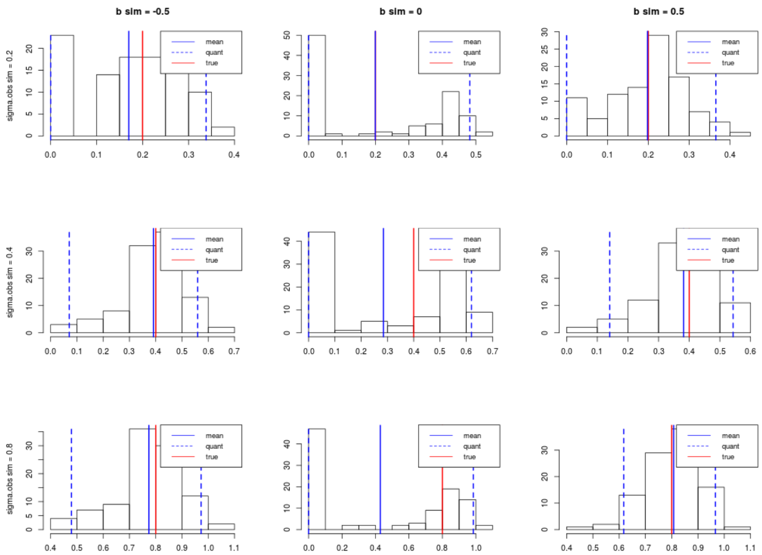
b	sigma_proc	sigma.obs	b_sd	sigma_proc_sd	sigma_obs_sd
0.5	0.4	0.8	0.060	0.201	0.104

b	sigma_proc	sigma.obs	b_q25	sigma_proc_q25	sigma_obs_q25
-0.5	0.4	0.2	-0.555	0.272	0.000
0.0	0.4	0.2	-0.098	0.000	0.000
0.5	0.4	0.2	0.442	0.226	0.000
-0.5	0.4	0.4	-0.586	0.156	0.071
0.0	0.4	0.4	-0.098	0.000	0.000
0.5	0.4	0.4	0.409	0.174	0.141
-0.5	0.4	0.8	-0.621	0.000	0.479
0.0	0.4	0.8	-0.140	0.000	0.000
0.5	0.4	0.8	0.397	0.000	0.617

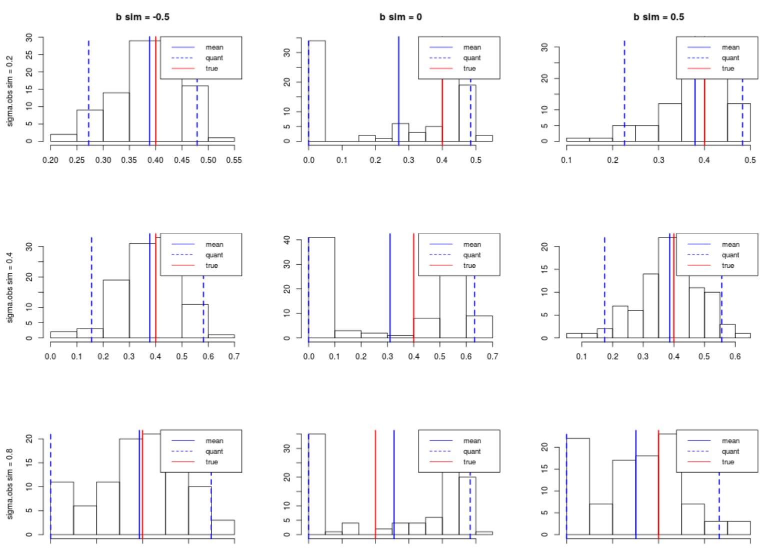
b	sigma_proc	sigma.obs	b_q975	sigma_proc_q975	sigma_obs_q975
-0.5	0.4	0.2	-0.422	0.479	0.338
0.0	0.4	0.2	0.096	0.485	0.482
0.5	0.4	0.2	0.572	0.483	0.365
-0.5	0.4	0.4	-0.420	0.582	0.560
0.0	0.4	0.4	0.091	0.632	0.620
0.5	0.4	0.4	0.570	0.556	0.543
-0.5	0.4	0.8	-0.341	0.699	0.973
0.0	0.4	0.8	0.147	0.966	0.985
0.5	0.4	0.8	0.613	0.664	0.966

The values of the 95% confidence intervals cover the true parameter in all simulation parameterizations. When β is 0 the estimates of either σ has a bimodal distribution. The histograms below show this effect

Estimates of σ_s



Estimates of σ_y



This is likely do to the inability of the model to distinguish where the variance is coming from when β is zero since there is no auto-regression.

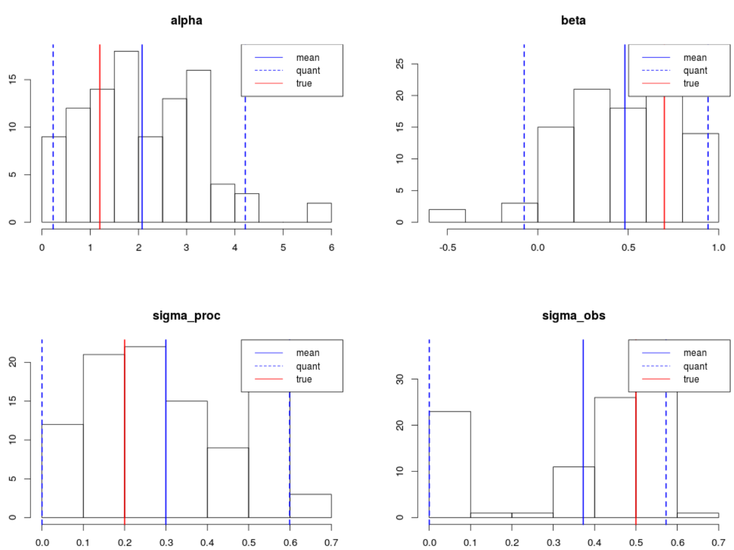
Gompertz model simulation

A Gompertz model can be defined as the following

$$S_t \sim \mathcal{N}(\alpha + \beta S_{t-1}, \sigma_s^2)$$
$$Y_t \sim \mathcal{N}(S_t, \sigma_y^2)$$

The primary difference is the inclusion of the intercept term α which affects the trend of the population. In this model a population will converge on a stable state. We will simulate data following this structure with $\alpha = 1.2$, $\beta = .7$, $\sigma_s = .2$, $\sigma_y = .5$. We will start with an initial value of 4 for the first observation y which leads to a difficult estimation of the variance terms σ_s & σ_y and even the means of α and β. This is shown in the very non-normal distributions of the parameter estimates and lack of convergence to the mean.

Estimates of Gompertz parameters when start values are at stable point



This effect is attenuated when we change the start value. So that the model may more readily identify the parameters within 100 time points.

Estimates of Gompertz parameters when start values are not at stable point