Homework 12 Solutions CAS CS 132

Fall 2024

Problem 1 1

$$A^{T}A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & G \end{bmatrix}$$

 $A^{T}b = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 6 & 1 \end{bmatrix}$ 

$$\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

Problem 1.2

$$\begin{array}{c} x \\ x \\ z \end{array} = \begin{array}{c} z \\ z \end{array}$$

$$\begin{array}{c} x \\ z \end{array} = \begin{array}{c} z \\ z \end{array}$$

Problem 1.3

Yes. The columns of A are

1 0 0 1 0.54 0.84 1 1 -0.42 0.91 2 1 -0.99 0.14 3 1 0.54 -0.84 -1 1 -0.46 -0.91 -2 1 -0.46 -0.91 -2 1

Problem 2.2

B,=6.32 B2=1.89 B3=0.36 By = 4.89

Problem 2.1

(determined ving numpy. linale. 1st sq)

7, this is the largest eigenvelve of  $A - 7I = \begin{bmatrix} -4 & 3 & 1 \\ 3 & -4 & 1 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 3^3 & 4^3 & 1^8 \end{bmatrix}$  $\begin{bmatrix}
1 & 1 & -2 \\
0 & -7 & 7
\end{bmatrix}
\begin{bmatrix}
1 & 1 & -2 \\
0 & 1 & -1
\end{bmatrix}$ X, = X3 X2 = X3 X3 is free

Problem 4.1
$$A^{T}A = \begin{bmatrix} 3 \end{bmatrix}$$

$$A^{T}A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & -2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$$

$$\det (A^{T}A - X) = (X - 17)^{2} + 64 = X^{2} - 34X + 225$$

$$= (X - 9)(X - 25)$$

$$\sum = \begin{pmatrix} 5 & 0 \\ 0 & 3 \\ 0 & 0 \end{pmatrix}$$

$$\sum_{n=0}^{\infty} \begin{bmatrix} 5 & 0 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

$$A^{T}A - 25 I = 0$$

$$A^{T}A - 9 I = \begin{bmatrix} 9 & 8 \\ 8 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 1/62 & 1/62 \\ 1/63 & 1/62 \end{bmatrix}$$

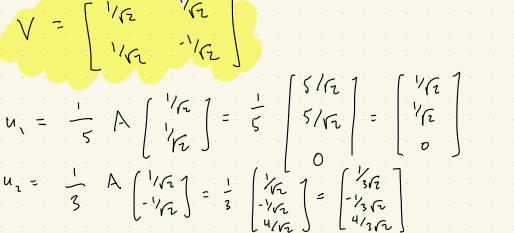
$$V = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

$$u_1 = \frac{1}{5} A \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 5/2 \\ 5/2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/6 \\ 1/6 \end{bmatrix}$$

$$\begin{bmatrix} 1/6 \\ 1/6 \end{bmatrix}$$

$$\begin{bmatrix} 1/6 \\ 1/6 \end{bmatrix}$$



$$u_3 = \begin{bmatrix} -2/3 \\ 2/3 \end{bmatrix}$$
 (by solving  $x + y = 0$   
 $x - y + 4z = 0$ )
$$\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

Problem 2.2  

$$A^{T} = (U \geq V^{T})^{T} = V \geq U^{T}$$