Flomework 10 Solutions CAS CS 132 Fall 2024

Problem 1. (
$$A - \lambda T = \begin{bmatrix} a - \lambda & 4 \\ -14 & -6 - \lambda \end{bmatrix}$$

$$det(A - \lambda T) = (a - \lambda)(-6 - \lambda) + 56 = (a - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-6 - \lambda)(-6 - \lambda) + 6\lambda + \lambda^2 + 56 = (a - \lambda)(-6 - \lambda)(-$$

4 ] ~ [1 4/2] -8] ~ [0 0] A-2I = 17 x,=-4/4 x2  $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ ,  $\lambda = 2$ 

x2 is free

$$D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} -4 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{8+1} \begin{bmatrix} 2 & 1 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 7 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -1 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ 7 & 4 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -6 - \lambda & -7 \\ 7 & 8 - \lambda \end{bmatrix}$$

$$det(A-XI) = (-6-X)(8-X) + 49 =$$

$$118+6X-8X+X^2+49 =$$

$$-48 + 6\lambda - 8\lambda + \lambda^{2} + 49 = (\lambda - 1)^{2}$$

$$\lambda^{2} - 2\lambda + 1 = (\lambda - 1)^{2}$$

not diagonalizable

$$A - I = \begin{bmatrix} -7 & -7 \\ 7 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Problem 1.3
$$(2-x)^{2}(3-x)(-1-x) =$$

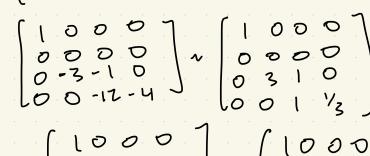
$$(2-\lambda)(3-\lambda)(-1-\lambda)-$$

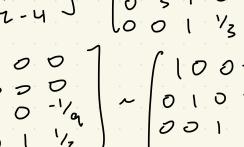
$$(\lambda-2)^{2}(\lambda-3)(\lambda+1)$$

$$(\lambda - 2)(\lambda - 3)(\lambda - 2)$$

$$(\lambda - 3I) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -3 & -3 & -1 & 0 \\ -3 & 18 & -6 & -4 \end{bmatrix}$$

$$A - 3I$$
) =  $\begin{vmatrix} -1 & 0 \\ -1 & 0 \\ 3 & -3 \\ -3 & 18 \end{vmatrix}$ 





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X3=0

X4 is free

0 0

Problem 1.4

$$\det(A - \lambda T) = \det \begin{bmatrix} -1 - \lambda & -3 & 0 \\ -1 - \lambda & 0 & 0 \end{bmatrix}$$

$$= (-1 - \lambda)(4 - \lambda)(1 - \lambda) + -3(0)(0) + 0(2)(0)$$

$$= (-1 - \lambda)(4 - \lambda)(1 - \lambda) + (-1 - \lambda)(0) - (-1 - \lambda)(0)$$

$$= (-1 - \lambda)(4 - \lambda)(1 - \lambda) + 6(1 - \lambda)$$

$$= (-1 - \lambda)(4 - \lambda)(1 - \lambda) + 6(1 - \lambda)$$

$$= (1-\lambda)(-4+\lambda-4\lambda+\lambda^{2}+6)$$

$$= (1-\lambda)(\lambda^{2}-\lambda^{3}+2)$$

$$= (1-\lambda)(\lambda^{-2})(\lambda-1)$$

= (1-1) ((-1-1)(4-1)+6)

$$= -(\lambda - 1)^{2}(\lambda - 2) \quad \lambda = 2, 1$$

$$A - 2I = \begin{bmatrix} -3 & -3 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
 $X, = -X^2$ 
 $X, = -X^2$ 

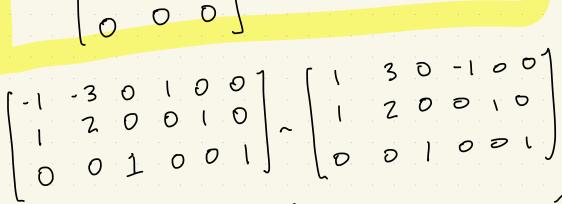
$$x_1 = -x_2$$
 $x_2 = 0$ 
 $x_3 = 0$ 
 $x_4 = 0$ 
 $x_5 = 0$ 
 $x_6 = 0$ 
 $x_7 = 0$ 

$$A-I = \begin{bmatrix} -2 & -3 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3/2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \lambda = 1$$

$$P = \begin{bmatrix} -1 & -3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad P = \begin{bmatrix} 2 & 3 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$



Problem 2.1  $\det \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ det [ 1 0] = 1 det [ -1 0 ] = 1 Proben 2.2 (-6-7) 78] by problem 1.2 Problem 2.2  $\begin{bmatrix} -6 & -1 \\ 7 & 8 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$ (01) is diagonalizable, [-6-7] is not

Problem 3
$$\lambda = 9, 1, 4$$

$$A - 9I \sim \begin{bmatrix} 1 & 6 & 6 \\ 0 & 6 & 6 \end{bmatrix}$$

$$A - I \sim \begin{bmatrix} 1 & 6 & 6 \\ 0 & 6 & 6 \end{bmatrix}$$

$$A - 4I \sim \begin{bmatrix} 1 & 6 & 6 \\ 0 & 6 & 6 \end{bmatrix}$$

$$A - I \wedge \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -\frac{1}{2} \\ 1 \end{bmatrix}, \lambda = 1$$

$$A - 4I \wedge \begin{bmatrix} 1 & 0 & -\frac{3}{2} \\ 0 & 1 & 4/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -\frac{4}{7} \\ 7 \end{bmatrix}, \lambda = 4$$

$$P = \begin{bmatrix} 1 & 3 \\ 0 & 1 & -4 \\ 2 & 7 \end{bmatrix} D = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 4 \end{bmatrix}$$

$$B = P \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 7 \end{bmatrix} P = \begin{bmatrix} 25 & -7 & -11 \\ 8 & 1 & -4 \\ 46 & -4 & -70 \end{bmatrix}$$

$$A - 9I \sim \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad \lambda = 9$$

$$A - I \sim \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \lambda = 2$$