

# Assignment 1

CS320

Fall 2025

## Basic Problems

1. coeff. matrix:

$$\begin{bmatrix} 1 & -2 & -2 \\ 2 & -3 & -5 \\ -2 & 2 & 7 \end{bmatrix}$$

augmented matrix:

$$\begin{bmatrix} 1 & -2 & -2 & 2 \\ 2 & -3 & -5 & 2 \\ -2 & 2 & 7 & -1 \end{bmatrix}$$

2.  $x_1 + 2x_2 - x_3 + x_4 = 7$

$$x_1 + 3x_2 + 2x_4 = 15$$

$$-2x_1 - 6x_2 - 3x_4 = -27$$

3. Plug in values:

$$(1) - 2(3) + 2 - 2(3) = 1 - 6 + 2 - 6 = -9$$

$$1 - 3 - 2 - 2(3) = 1 - 3 - 2 - 6 = -10$$

$$-3(1) + 8(3) - 6(2) + 4(3) = -3 + 24 - 12 + 12 = 21$$

$$2(3) - 7(2) + 7(3) = 6 - 14 + 21 = 13$$

4. Augmented matrix:

$$\left[ \begin{array}{cccc} 1 & -2 & -2 & -7 \\ -1 & 3 & 2 & 10 \\ 2 & -6 & -3 & -18 \end{array} \right] \xrightarrow{R_2 \leftarrow R_2 + R_1}$$

$$\left[ \begin{array}{cccc} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 2 & -6 & -3 & -18 \end{array} \right] \xrightarrow{R_3 \leftarrow R_3 - 2R_1}$$

$$\left[ \begin{array}{cccc} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & 1 & -4 \end{array} \right] \xrightarrow{R_3 \leftarrow R_3 + 2R_2}$$

$$\left[ \begin{array}{cccc} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 \leftarrow R_1 + 2R_3 \\ R_1 \leftarrow R_1 + 2R_2 \end{array}}$$

$$\left[ \begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right] \quad \begin{array}{l} x_1 = 3 \\ x_2 = 3 \\ x_3 = 2 \end{array}$$

$(3, 3, 2)$  is the unique solution.

$$5. \begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 5 & -7 & 1 & -2 & -9 \\ 5 & 1 & -10 & 6 & -5 \\ 5 & 7 & -5 & 2 & 1 \end{bmatrix} \xrightarrow{R_4 \leftarrow -R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ -10 & -2 & 1 & -2 & -9 \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_3}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ -5 & -9 & 21 & -14 & 1 \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 5R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 20 & 26 & -4 & -4 & 6 \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 3R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 20 & 26 & -4 & -4 & 6 \\ -10 & -20 & 5 & 0 & -8 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ -10 & -20 & 5 & 0 & -8 \\ 20 & 26 & -4 & -4 & 6 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix}$$

6.

$$x_1 = -3 - x_2 + 4x_6 - 5x_7$$

$x_2$  is free

$$x_3 = -4 - x_6 - 3x_7$$

$x_4$  is free

$$x_5 = 2 - 5x_6 + 3x_7$$

$x_6$  is free

$x_7$  is free

7.

$$\begin{bmatrix} 1 & -1 & -2 & 1 \\ -1 & 2 & 4 & 0 \\ 2 & -3 & -6 & 2 \\ -2 & 1 & 2 & -1 \end{bmatrix}$$

Row operations indicated by arrows and numbers:

- $R_2 \leftarrow R_2 + R_1$  (red)
- $R_3 \leftarrow R_3 - 2R_1$  (green)
- $R_4 \leftarrow R_4 + 2R_1$  (purple)

$$\begin{aligned} R_2 &\leftarrow R_2 + R_1 \\ R_3 &\leftarrow R_3 - 2R_1 \\ R_4 &\leftarrow R_4 + 2R_1 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -2 & 1 \end{bmatrix}$$

The last two rows are circled in red, indicating a contradiction.

INCONSISTENT

True / False

1. True
2. False
3. True
4. True
5. False

Handwritten diagram showing the sequence of row operations for solving a system of linear equations. The matrices are arranged in a cycle:

- Top left:  $\begin{bmatrix} 1 & 1 \end{bmatrix}$
- Top right:  $\begin{bmatrix} 1 & 0 \end{bmatrix}$
- Middle left:  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$
- Middle right:  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
- Bottom left:  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$
- Bottom right:  $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

Arrows indicate the sequence of operations:

- A red arrow points from  $\begin{bmatrix} 1 & 1 \end{bmatrix}$  to  $\begin{bmatrix} 1 & 0 \end{bmatrix}$ .
- A black arrow points from  $\begin{bmatrix} 1 & 1 \end{bmatrix}$  down to  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ .
- A black arrow points from  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$  down to  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ .
- A black arrow points from  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$  to  $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ .
- A black arrow points from  $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$  up to  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ .
- A black arrow points from  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$  up to  $\begin{bmatrix} 1 & 0 \end{bmatrix}$ .
- A green curved arrow points from  $\begin{bmatrix} 1 & 0 \end{bmatrix}$  back to  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ .

6. True

7. False

8. False

$$x + y + z = 1$$

$$x + y + z = 2$$

9. False

$$x + y = 1$$

$$x + y = 2$$

$$x + y = 3$$

## More Difficult Problems

1.

$$\begin{bmatrix} 1 & 4 & -1 \\ 3 & -h & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -1 \\ 0 & -(h+12) & 10 \end{bmatrix}$$

The system is inconsistent exactly when  $h = 12$ .

There are no values of  $h$  so that the system has infinitely many solutions. This would require the second equation to be a multiple of the first, which is not possible.



$$2. \begin{bmatrix} h & 2 & 1 \\ 3 & 9 & k \end{bmatrix}$$

$$a) \quad h = \frac{2}{3}$$

$$k = 0$$

$$\begin{bmatrix} \frac{2}{3} & 2 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & 6 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 6 & 18 & 3 \\ 6 & 18 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 6 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$b) \quad h = 0$$

$$k = 0$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -\frac{3}{2} \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$c) \quad k = \frac{2}{3}$$

$$h = \frac{9}{2}$$

$$\begin{bmatrix} \frac{2}{3} & 2 & 1 \\ 3 & 9 & \frac{9}{2} \end{bmatrix} \sim \begin{bmatrix} 6 & 18 & 9 \\ 6 & 18 & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 6 & 18 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$