Quiz 1

CAS CS 132: *Geometric Algorithms*

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▷ You will have approximately 30 minutes to complete this exam.

▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.

1 Row Operations

Apply the row operations:

$$R_1 \leftarrow -5R_1$$
$$R_1 \leftarrow R_1 - 4R_2$$
$$R_2 \leftarrow R_2 + 3R_1$$

from top to bottom to the following matrix. You must write down the intermediate matrices.

$$\begin{bmatrix} -2 & 7 & 9 \\ 7 & -9 & -9 \\ 4 & -1 & 9 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 7 & 9 \\ 7 & -9 & -9 \\ 4 & -1 & 9 \end{bmatrix} \xrightarrow{R_1 \leftarrow -5R_1} \begin{bmatrix} 10 & -35 & -45 \\ 7 & -9 & -9 \\ 4 & -1 & 9 \end{bmatrix}$$

$$-28 \quad +36 \quad +36$$

$$\xrightarrow{R_1 \leftarrow R_1 - 4R_2} \begin{bmatrix} -18 & 1 & -9 \\ 7 & -9 & -9 \\ 4 & -1 & 9 \end{bmatrix}$$

$$-54 \quad +3 \quad -27$$

$$\xrightarrow{R_2 \leftarrow R_2 + 3R_1} \begin{bmatrix} -18 & 1 & -9 \\ -47 & -6 & -36 \\ 4 & -1 & 9 \end{bmatrix}$$

$$\begin{matrix} 2 \\ 18 \\ 3 \\ \hline 54 \end{matrix}$$

Solution. *(the final matrix only)*

$$\begin{bmatrix} -18 & 1 & -9 \\ -47 & -6 & -36 \\ 4 & -1 & 9 \end{bmatrix}$$

2 Reduced Echelon Forms

Determine the reduced echelon form of the following matrix. You must write down the intermediate matrices and row operations you used in your calculation.

$$\begin{bmatrix} 0 & 1 & -6 & -2 \\ 1 & 1 & -3 & -6 \\ -2 & -4 & 18 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & -6 & -2 \\ 1 & 1 & -3 & -6 \\ -2 & -4 & 18 & 16 \end{bmatrix} \xrightarrow{R_1 \Leftrightarrow R_2} \begin{bmatrix} 1 & 1 & -3 & -6 \\ 0 & 1 & -6 & -2 \\ -2 & -4 & 18 & 16 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + 2R_1} \begin{bmatrix} 1 & 1 & -3 & -6 \\ 0 & 1 & -6 & -2 \\ 0 & -2 & 12 & 4 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + 2R_2} \begin{bmatrix} 1 & 1 & -3 & -6 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution. (the final RREF only)

$$\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3 General Form Solutions

Determine a general form solution for a linear system whose augmented matrix is row equivalent to the following matrix.

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 & 5 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 \\ 0 & 1 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

Solution.

$$x_1 = 1 + 3x_3$$
$$x_2 = -4 - x_3$$
$$x_3 \text{ is free}$$
$$x_4 = 5$$