Assignment 2 Solutions

CS132

Fall 2025

Basic Problems

1.

$$A = \begin{bmatrix} -4 & -7 & 4 \\ 3 & 8 & 2 \\ -10 & 1 & -9 \end{bmatrix}$$

① $R_1 \leftrightarrow R_2$

$\longrightarrow$

$$\begin{bmatrix} 3 & 8 & 2 \\ -4_{-10} & -7_{+1} & 4_{-9} \\ -10 & 1 & -9 \end{bmatrix}$$

② $R_2 \leftarrow R_2 + R_3$

$\longrightarrow$

$$\begin{bmatrix} 3 & 8 & 2 \\ -14 & -6 & -5 \\ -10 & 1 & -9 \end{bmatrix} = B$$

2.

$$\begin{bmatrix} 1 & -6 & 0 & 1 & 2 \\ 2 & -12 & 1 & -4 & 7 \\ 1 & -6 & -2 & 13 & -4 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - R_1$

(annotations above matrix: row 2: -2, $+12$, -0, -2, -4; row 3: -1, $+6$, -0, -1, -2)

$$\begin{bmatrix} 1 & -6 & 0 & 1 & 2 \\ 0 & 0 & 1 & -6 & 3 \\ 0 & 0 & -2 & 12 & -6 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 2R_2$

(annotations: $+2$, -12, $+6$)

$$\begin{bmatrix} 1 & -6 & 0 & 1 & 2 \\ 0 & 0 & 1 & -6 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 2 + 6x_2 - x_4$

x_2 is free

$x_3 = 3 + 6x_4$

x_4 is free

3.

$$x_1 = 1 - x_2 + 3x_4 + 3x_7$$
x_2 is free
$$x_3 = 3 + x_4 - 3x_6 - 2x_7$$
x_4 is free
$$x_5 = -3 - 3x_6 + 4x_7$$
x_6 is free
x_7 is free

$x_2 = 0$ $x_1 = 1$
$x_4 = 0$ $\Rightarrow$ $x_3 = 3$
$x_6 = 0$
$x_7 = 0$ $x_5 = -3$

$(1, 0, 3, 0, -3, 0, 0)$
$(0, 1, 3, 0, -3, 0, 0)$
$(4, 0, 4, 1, -3, 0, 0)$

$x_2 = 1$ $x_1 = 0$
$x_4 = 0$ $\Rightarrow$
$x_6 = 0$ $x_3 = 3$
$x_7 = 0$ $x_5 = -3$

$x_2 = 0$ $x_1 = 4$
$x_4 = 1$ $x_3 = 4$
$x_6 = 0$ $\Rightarrow$
$x_7 = 0$ $x_5 = -3$

4.

$$7 \begin{bmatrix} 2 \\ -3 \\ -8 \\ 9 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ -3 \\ -4 \\ -3 \end{bmatrix} - 4 \begin{bmatrix} 5 \\ -1 \\ 5 \\ -7 \end{bmatrix} - 2 \begin{bmatrix} -2 \\ -9 \\ -2 \\ -10 \end{bmatrix} =$$

$$\begin{bmatrix} 14 + 6 - 20 + 4 \\ -21 - 9 + 4 + 18 \\ -56 - 12 - 20 + 4 \\ 63 - 9 + 28 + 20 \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -84 \\ 102 \end{bmatrix}$$

5.

$$x_1 \begin{bmatrix} 8 \\ 4 \\ 4 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 6 \\ 3 \\ 5 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -9 \\ 9 \\ 0 \\ 8 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} -5 \\ -1 \\ 9 \\ 2 \end{bmatrix}$$

6.

$$\begin{bmatrix} -7 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -8 \\ 9 \\ -7 \end{bmatrix} = \begin{bmatrix} -15 \\ 8 \\ -7 \end{bmatrix}$$

This vector is not a scaler multiple of either $\vec{v}_1$ or $\vec{v}_2$ but is a linear combination of $\vec{v}_1$ and $\vec{v}_2$.

7.
$$
\begin{bmatrix} 1 & -2 & -8 & 16 \\ -2^{+2} & 5^{-4} & 21^{-16} & -38^{+32} \\ 0 & 2 & 10 & -12 \end{bmatrix}
\qquad R_2 \leftarrow R_2 + 2R_1 \longrightarrow
$$

$$
\begin{bmatrix} 1 & -2 & -8 & 16 \\ 0 & 1 & 5 & -6 \\ 0 & 2^{-2} & 10^{-10} & -12^{+12} \end{bmatrix}
\qquad R_3 \leftarrow R_3 - 2R_2 \longrightarrow
$$

$$
\begin{bmatrix} 1 & -2^{+2} & -8^{+10} & 16^{-12} \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\qquad R_1 \leftarrow R_1 + 2R_2 \longrightarrow
$$

$$
\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
x_1 = 4 - 2x_3
$$
$$
x_2 = -6 - 5x_3
$$
$$
x_3 \text{ is free}
$$

8.

$$\begin{bmatrix} 0 & 5 & 1 & 1 \\ 1 & 7 & 2 & 4 \\ -3 & -1 & -2 & -7 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\begin{bmatrix} 1 & 7 & 2 & 4 \\ 0 & 5 & 1 & 1 \\ -3 & -1 & -2 & -7 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 3R_1}$$

with red annotations: $+3$ $+21$ $+6$ $+12$

$$\begin{bmatrix} 1 & 7 & 2 & 4 \\ 0 & 5 & 1 & 1 \\ 0 & 20 & 4 & 5 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - 4R_2}$$

with red annotations: -20 -4 -4

$$\begin{bmatrix} 1 & 7 & 2 & 4 \\ 0 & 5 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NO SOLUTION

True / False

1. true

2. false
$$x + y = 5$$
$$2x + y = 7$$

3. true

4. true

5. false
$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

6. false
$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

7. false
$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

More Difficult Problems

1.

$$\begin{bmatrix} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7   total

2.

$$a(-1)^3 + b(-1)^2 + c = 4$$
$$a(1)^3 + b(1)^2 + c = 5$$
$$a(2)^3 + b(2)^2 + c = 10$$

$$-x_1 + x_2 + x_3 = 4$$
$$x_1 + x_2 + x_3 = 5$$
$$8x_1 + 4x_2 + x_3 = 10$$

3.

$$\begin{bmatrix} -2 & -1 & b_1 \\ 1 & 0 & b_2 \\ 2 & 2 & b_3 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & b_2 \\ -2^{+2} & -1^{+0} & b_1^{+2b_2} \\ 2^{-2} & 2^{-0} & b_3^{-2b_2} \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & b_2 \\ 0 & -1 & b_1 + 2b_2 \\ 0 & 2^{-2} & b_3 - 2b_2 \, ^{+\,2(b_1+2b_2)} \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & b_2 \\ 0 & -1 & b_1 + 2b_2 \\ 0 & 0 & (b_3 - 2b_2) + 2(b_1 + 2b_2) \end{bmatrix}$$

$$b_3 - 2b_2 + 2b_1 + 4b_2 =$$

$$2b_1 + 2b_2 + b_3 = 0$$

$$2x_1 + 2x_2 + x_3 = 0$$