Assignment 1
CS320
Fall 2025

Basic Problems

1. coeff. matrix:

$$\begin{bmatrix} 1 & -2 & -2 \\ 2 & -3 & -5 \\ -2 & 2 & 7 \end{bmatrix}$$

augmented matrix:

$$\begin{bmatrix} 1 & -2 & -2 & 2 \\ 2 & -3 & -5 & 2 \\ -2 & 2 & 7 & -1 \end{bmatrix}$$

2.

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 7 \\ x_1 + 3x_2 + 2x_4 &= 15 \\ -2x_1 - 6x_2 - 3x_4 &= -27 \end{aligned}$$

3. Plug in values:

$$(1) - 2(3) + 2 - 2(3) = 1 - 6 + 2 - 6 = -9$$

$$1 - 3 - 2 - 2(3) = 1 - 3 - 2 - 6 = -10$$

$$-3(1) + 8(3) - 6(2) + 4(3) = -3 + 24 - 12 + 12 = 21$$

$$2(3) - 7(2) + 7(3) = 6 - 14 + 21 = 13$$

4. Augmented matrix:

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ -1^{+1} & 3^{-2} & 2^{-2} & 10^{-7} \\ 2 & -6 & -3 & -18 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 + R_1}$$

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 2^{-2} & -6^{+4} & -3^{+4} & -18^{+14} \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - 2R_1}$$

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & -2^{+2} & 1^{+0} & -4^{+6} \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 2R_2}$$

$$\begin{array}{cccc} +2 & & +2 & +4 \; +6 \end{array}$$

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow[R_1 \leftarrow R_1 + 2R_2]{R_1 \leftarrow R_1 + 2R_3}$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = 3$

$x_2 = 3$

$x_3 = 2$

$(3, 3, 2)$ is the unique solution.

5.

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 5 & -7 & 1 & -2 & -9 \\ 5 & 1 & -10 & 6 & -5 \\ 5 & 7 & -5 & 2 & 1 \end{bmatrix} \xrightarrow{R_4 \leftarrow -R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ \overset{-10}{5} & \overset{-2}{-7} & \overset{+20}{1} & \overset{-12}{-2} & \overset{+10}{-9} \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_3}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ \overset{+25}{-5} & \overset{+35}{-9} & \overset{-25}{21} & \overset{+10}{-14} & \overset{+5}{1} \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 5R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 20 & 26 & -4 & -4 & 6 \\ \overset{-15}{5} & \overset{-21}{1} & \overset{+15}{-10} & \overset{-6}{6} & \overset{-3}{-5} \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 3R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 20 & 26 & -4 & -4 & 6 \\ -10 & -20 & 5 & 0 & -8 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ -10 & -20 & 5 & 0 & -8 \\ 20 & 26 & -4 & -4 & 6 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix}$$

6.

$x_1 = -3 - x_2 + 4x_6 - 5x_7$

x_2 is free

$x_3 = -4 - x_6 - 3x_7$

x_4 is free

$x_5 = 2 - 5x_6 + 3x_7$

x_6 is free

x_7 is free

7.

$$
\begin{bmatrix} 1 & -1 & -2 & 1 \\ -1^{\,+1} & 2^{\,-1} & 4^{\,-2} & 0^{\,+1} \\ 2^{\,-2} & -3^{\,+2} & -6^{\,+4} & 2^{\,-2} \\ -2^{\,+2} & 1^{\,-2} & 2^{\,-4} & -1^{\,+2} \end{bmatrix}
\xrightarrow[R_4 \leftarrow R_4 + 2R_1]{\substack{R_2 \leftarrow R_2 + R_1 \\ R_3 \leftarrow R_3 - 2R_1}}
$$

$$
\begin{bmatrix} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -2 & 1 \end{bmatrix}
$$

INCONSISTENT

$\boxed{\text{True / False}}$

1. True
2. False
3. True
4. True
5. False

$$\begin{tikzcd}
\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \arrow[r, shift left] \arrow[r, shift right] \arrow[d] \arrow[r, bend right=80] & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \arrow[d] & \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \arrow[u] \\
\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \arrow[r] & \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \arrow[u]
\end{tikzcd}$$

6. True

7. False

8. False

$$x + y + z = 1$$
$$x + y + z = 2$$

9. False

$$x + y = 1$$
$$x + y = 2$$
$$x + y = 3$$

More Difficult Problems

1.
$$\begin{bmatrix} 1 & 4 & -1 \\ 3^{-3} & -h^{-12} & 7^{+3} \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -1 \\ 0 & -(h+12) & 10 \end{bmatrix}$$

The system is inconsistent exactly when $h = 12$.

There are no values of h so that the system has infinitely many solutions. This would require the second equation to be a multiple of the first, which is not possible.

2. $\begin{bmatrix} h & 2 & 1 \\ 3 & 9 & k \end{bmatrix}$

a) $h = 2/3$

$K = 0$

$$\begin{bmatrix} 2/3 & 2 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & 6 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 6 & 18 & 3 \\ 6 & 18 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 6 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $h = 0$

$K = 0$

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1/2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 1/2 \end{bmatrix}$$

c) $k = 2/3$

$h = 9/2$

$$\begin{bmatrix} 2/3 & 2 & 1 \\ 3 & 9 & 9/2 \end{bmatrix} \sim \begin{bmatrix} 6 & 18 & 9 \\ 6 & 18 & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 6 & 18 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$