id of id ?

(annotations in blue)

$\{(x, x) : x \in ?\}$

the identity function as a map.

$\mathbb{N}, \mathbb{Z}$, String

what is the domain / range

$id(id)$

$\{(id, id) : id \in ?\}$

$(\{(x, x) : x \in \underline{?}\}, \{(x, x) : x \in \underline{?}\}) \in \underline{?}$

"?" needs to be the same if we want id to apply to itself

λ - Terms

$V = \{ v_1, v_2, \ldots$

<u>**Def'n**</u> $\underline{\Lambda}$ of λ-terms

(1) $V \subset \underline{\Lambda}$

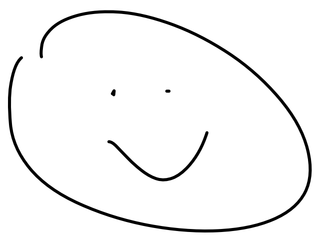
(2) $M \in \underline{\Lambda}$ $v_i \in V$, $\lambda x . M \in \underline{\Lambda}$

(3) $M, N \in \underline{\Lambda} \Rightarrow (M N) \in \underline{\Lambda}$

Ex. v_{10}, v_{20}, $(v_{20} v_{10})$

$(\lambda v_{30}. (v_{20} v_{10})) v_1$

not particularly meaningful examples of Λ-terms

 v_1

a non-example, emphasizing the definition of Λ is <u>injective</u>

Reduction of $(\lambda f g x.\ f(g x))\ M N P$

$$(\lambda f g x.\ f(g x))\ M\ N\ P \rightarrow$$

$$(\lambda g x.\ M(g x))\ N\ P \rightarrow$$

$$(\lambda x.\ M(N x))\ P \rightarrow$$

$$M(N P)$$

$$\boxed{(\lambda y . y)[\lambda q . q / y] = ?}$$

$$\underbrace{\text{'given } y,}_{\lambda y .} \quad \underbrace{\text{return the id func'}}_{\lambda y . y}$$

$$(\lambda y . \lambda y . y)(\lambda q . q) \to \underbrace{\lambda y . \lambda y . \lambda q . q}$$

this would be the result of β-reducing if we used the naive definition

$\lambda y . z [y / z] = ?$

Calculating $FV(x(\lambda x. xy))$

$$FV(x(\lambda x. xy)) =$$

$$FV(x) \cup FV(\lambda x. xy) =$$

$$\{x\} \cup (FV(xy) - \{x\}) =$$

$$\{x\} \cup (\{x, y\} - \{x\}) =$$

$$\{x\} \cup \{y\} = \{x, y\}$$

Capture Avoiding Substitution

Practice Problems

$$(\lambda y.\, y y x)[\{yx / x] =$$

$$\lambda y.\, y y (y x)$$

the ~~wrong~~ solution based on the naive definition of substitution

$$(\lambda z.\, z z x)[yx / x] =$$

$$\lambda z.\, z z (y x)$$

Reducing $(\lambda x.(\lambda y.yx)z)r$

Reducing $(\lambda x.\, xx)(\lambda x.\, xx)$

β - Equivalence

Practice Problem

Challenge Problem