`salt2`: Mutable References

`CS392-M1`: *Rust, In Practice and in Theory*

Syntax

$$
\begin{aligned}
&x && \text{(variables, } \mathcal{V}\text{)}\\
&n && \text{(integers, } \mathbb{Z}\text{)}\\
w &::= x \mid \texttt{*}\, w && \text{(place expression, } \mathcal{W}\text{)}\\
e &::= \texttt{()} \mid n \mid w \mid \texttt{\&}\, w \mid \texttt{\&mut}\ w \mid w \ \texttt{=}\ e && \text{(expressions, } \mathcal{E}\text{)}\\
s &::= e \mid \texttt{let}\ x \ \texttt{=}\ e \mid \texttt{let}\ \texttt{mut}\ x \ \texttt{=}\ e && \text{(statements, } \mathcal{S}\text{)}\\
p &::= e \mid s\ \texttt{;}\ p && \text{(programs, } \mathcal{P}\text{)}
\end{aligned}
$$

Typing

$$
\begin{aligned}
t &::= \texttt{()} \mid \texttt{i32} \mid \texttt{\&}\, w \mid \texttt{\&mut}\ w && \text{(types, } \mathcal{T}\text{)} \\
\tilde{t} &::= \lfloor t \rfloor \mid t && \text{(partial types, } \tilde{\mathcal{T}}\text{)} \\
m &::= \mathsf{imm} \mid \mathsf{mut} && \text{(mutability)}
\end{aligned}
$$

$$\Gamma \in \mathcal{V} \mapsto \mathcal{T} \times \{\mathsf{imm}, \mathsf{mut}\} \qquad \text{(contexts)}$$

$$
\begin{aligned}
&\Gamma \vdash w : \tilde{t}^m && \text{(place expressions)} \\
&\Gamma \vdash \mathsf{writable}(w) && \text{(writability)} \\
&\Gamma \vdash \tilde{t} \approx \tilde{t} && \text{(type compatibility)} \\
&\Gamma \vdash e : t \dashv \Gamma && \text{(expressions)} \\
&\Gamma \vdash s \dashv \Gamma && \text{(statments)} \\
&\Gamma \vdash p : t \dashv \Gamma && \text{(programs)}
\end{aligned}
$$

$$\mathsf{copy}(t) \equiv \nexists w. t = \texttt{\&mut}\ w$$

$$
\begin{aligned}
\mathsf{write}(\Gamma, x, t) &= \Gamma[x \mapsto t] \\
\mathsf{write}(\Gamma, \texttt{*}^{k+1} x, t) &= \mathsf{write}(\Gamma, \texttt{*}^{k} w, t) \quad \text{where } (x \mapsto \texttt{\&mut}\ w^m) \in \Gamma
\end{aligned}
$$

$$\frac{(x \mapsto t^m) \in \Gamma}{\Gamma \vdash x : t^m} \text{(var)}$$

$$\frac{\Gamma \vdash w_1 : \texttt{\&}\, w_2^{m_1} \qquad \Gamma \vdash w_2 : t^{m_2}}{\Gamma \vdash \texttt{*}\, w_1 : t^{m_2}} \text{(deref)}$$

$$\frac{}{\Gamma \vdash \texttt{()} : \texttt{()} \dashv \Gamma} \text{(unit)}$$

$$\frac{n \in \mathbb{Z}}{\Gamma \vdash n : \texttt{i32} \dashv \Gamma}\ (\text{int})$$

$$\frac{\nexists y.(y \mapsto \texttt{\&mut}\ *^{l} x) \in \Gamma}{\Gamma \vdash \mathsf{readable}(*^{k} x)}\ (\text{readable})$$

$$\frac{\nexists y.(y \mapsto \texttt{\&}\ *^{l} x) \in \Gamma \wedge \nexists y.(y \mapsto \texttt{\&mut}\ *^{l} x) \in \Gamma}{\Gamma \vdash \mathsf{writable}(*^{k} x)}\ (\text{writable})$$

$$\frac{\Gamma \vdash w : t^{m} \qquad \mathsf{copy}(t) \qquad \Gamma \vdash \mathsf{readable}(w)}{\Gamma \vdash w : t \dashv \Gamma}\ (\text{place-copy})$$

$$\frac{\Gamma \vdash x : \texttt{\&mut}\ w^{m} \qquad \Gamma \vdash \mathsf{writable}(x)}{\Gamma \vdash x : \texttt{\&mut}\ w \dashv \Gamma[x \mapsto \lfloor \texttt{\&mut}\ w \rfloor]}\ (\text{place-move})$$

$$\frac{\Gamma \vdash x : t^{m}}{\Gamma \vdash \texttt{\&}\ x : \texttt{\&}\ x \dashv \Gamma}\ (\text{brw-var})$$

$$\frac{\Gamma \vdash w_1 : \texttt{\&}\ w_2^{m} \qquad \Gamma \vdash \mathsf{readable}(w_1)}{\Gamma \vdash \texttt{\&}\ *\ w_1 : \texttt{\&}\ w_2 \dashv \Gamma}\ (\text{brw-drf})$$

$$\frac{\Gamma \vdash w_1 : \texttt{\&mut}\ w_2^{m} \qquad \Gamma \vdash \mathsf{readable}(w_1)}{\Gamma \vdash \texttt{\&}\ *\ w_1 : \texttt{\&}\ *\ w_1 \dashv \Gamma}\ (\text{brw-drf-mut})$$

$$\frac{(x \mapsto t^{\mathsf{mut}}) \in \Gamma}{\Gamma \vdash \mathsf{mutable}(x)}\ (\text{mut-var})$$

$$\frac{\Gamma \vdash w_1 : \texttt{\&mut}\ w_2^{m} \qquad \Gamma \vdash \mathsf{mutable}(*^{k} w_2)}{\Gamma \vdash \mathsf{mutable}(*^{k+1} w_1)}\ (\text{mut-deref})$$

$$\frac{\Gamma \vdash w : t^{\mathsf{mut}} \qquad \Gamma \vdash \mathsf{writable}(w) \qquad \Gamma \vdash \mathsf{mutable}(w)}{\Gamma \vdash \texttt{\&mut}\ w : \texttt{\&mut}\ w \dashv \Gamma}\ (\text{mut-brw})$$

$$\frac{}{\Gamma \vdash \texttt{i32} \approx \texttt{i32}}\ (\approx\text{-int})$$

$$\frac{}{\Gamma \vdash \texttt{()} \approx \texttt{()}}\ (\approx\text{-unit})$$

$$\frac{\Gamma \vdash w_1 : \tilde{t}_1^{m} \qquad \Gamma \vdash w_2 : \tilde{t}_2^{m} \qquad \Gamma \vdash \tilde{t}_1 \approx \tilde{t}_2}{\Gamma \vdash \texttt{\&}\ w_1 \approx \texttt{\&}\ w_2}\ (\approx\text{-brw})$$

$$\frac{\Gamma \vdash w_1 : \tilde{t}_1^{m} \qquad \Gamma \vdash w_2 : \tilde{t}_2^{m} \qquad \Gamma \vdash \tilde{t}_1 \approx \tilde{t}_2}{\Gamma \vdash \texttt{\&mut}\ w_1 \approx \texttt{\&mut}\ w_2}\ (\approx\text{-mbrw})$$

$$\frac{\Gamma \vdash t_1 \approx \tilde{t}_2}{\Gamma \vdash \lfloor t_1 \rfloor \approx \tilde{t}_2}\ (\approx\text{-partial}_1)$$

$$\frac{\Gamma \vdash \tilde{t}_1 \approx t_2}{\Gamma \vdash \tilde{t}_1 \approx \lfloor t_2 \rfloor}\ (\approx\text{-partial}_2)$$

$$\frac{\Gamma_1 \vdash w : \tilde{t}_1^{\mathsf{mut}} \qquad \Gamma_1 \vdash e : t_2 \dashv \Gamma_2 \qquad \Gamma_2 \vdash \tilde{t}_1 \approx t_2 \qquad \Gamma_3 = \mathsf{write}(\Gamma, w, t_2) \qquad \Gamma_3 \vdash \mathsf{writable}(x)}{\Gamma_1 \vdash w \mathrel{\texttt{=}} e : \texttt{()} \dashv \Gamma_3}\ \text{(assign)}$$

$$\frac{\Gamma_1 \vdash e : t \dashv \Gamma_2}{\Gamma_1 \vdash e \dashv \Gamma_2}\ \text{(expr-stmt)}$$

$$\frac{\Gamma_1 \vdash e : t \dashv \Gamma_2 \qquad x \notin \mathsf{dom}(\Gamma_2)}{\Gamma_1 \vdash \texttt{let}\ x \mathrel{\texttt{=}} e \dashv \Gamma_2[x \mapsto t^{\mathsf{imm}}]}\ \text{(let)}$$

$$\frac{\Gamma_1 \vdash e : t \dashv \Gamma_2 \qquad x \notin \mathsf{dom}(\Gamma_2)}{\Gamma_1 \vdash \texttt{let mut}\ x \mathrel{\texttt{=}} e \dashv \Gamma_2[x \mapsto t^{\mathsf{mut}}]}\ \text{(let-mut)}$$

$$\frac{\Gamma_1 \vdash s \dashv \Gamma_2 \qquad \Gamma_2 \vdash p : t \dashv \Gamma_3}{\Gamma_1 \vdash s\ \texttt{;}\ p : t \dashv \Gamma_3}\ \text{(prog)}$$

Evaluation

$$
\begin{aligned}
\ell &::= \ell_x && \text{(locations, } \mathcal{L}\text{)} \\
v &::= \texttt{()} \mid n \mid \ell^m && \text{(values, } \mathbb{V}\text{)} \\
\tilde{v} &::= \bot \mid v && \text{(partial values, } \tilde{\mathbb{V}}\text{)}
\end{aligned}
$$

$$
S \in \mathcal{L} \mapsto \tilde{\mathbb{V}} \times \{\mathsf{imm}, \mathsf{mut}\} \qquad \text{(store)}
$$

$$
\begin{aligned}
&\langle\, S \,,\, e \,\rangle \Downarrow \langle\, S \,,\, v \,\rangle && \text{(expressions)} \\
&\langle\, S \,,\, s \,\rangle \Downarrow S && \text{(statements)} \\
&\langle\, S \,,\, p \,\rangle \Downarrow \langle\, S \,,\, v \,\rangle && \text{(programs)}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{loc}(S, x) &= \ell_x \\
\mathsf{loc}(S, *\, w) &= \ell_x \quad \text{where } S(\mathsf{loc}(S, w)) = \ell_x^m
\end{aligned}
$$

$$
\frac{}{\langle\, S \,,\, \texttt{()} \,\rangle \Downarrow \langle\, S \,,\, \texttt{()} \,\rangle}\ \text{(unit)}
$$

$$
\frac{n \in \mathbb{Z}}{\langle\, S \,,\, n \,\rangle \Downarrow \langle\, S \,,\, n \,\rangle}\ \text{(int)}
$$

$$
\frac{S(\mathsf{loc}(S, w)) \neq \ell_x^{\mathsf{mut}}}{\langle\, S \,,\, w \,\rangle \Downarrow \langle\, S \,,\, S(\mathsf{loc}(S, w)) \,\rangle}\ \text{(place-copy)}
$$

$$
\frac{S(\mathsf{loc}(S, w)) = \ell_x^{\mathsf{mut}}}{\langle\, S \,,\, w \,\rangle \Downarrow \langle\, S[\mathsf{loc}(S, w) \mapsto \bot] \,,\, S(\mathsf{loc}(S, w)) \,\rangle}\ \text{(place-move)}
$$

$$
\frac{}{\langle\, S \,,\, \texttt{\&}\, w \,\rangle \Downarrow \langle\, S \,,\, \mathsf{loc}(S, w)^{\mathsf{imm}} \,\rangle}\ \text{(brw)}
$$

$$
\frac{}{\langle\, S \,,\, \texttt{\&mut}\, w \,\rangle \Downarrow \langle\, S \,,\, \mathsf{loc}(S, w)^{\mathsf{mut}} \,\rangle}\ \text{(mbrw)}
$$

$$\frac{\langle\, S_1 \,,\, e \,\rangle \Downarrow \langle\, S_2 \,,\, v \,\rangle}{\langle\, S_1 \,,\, e \,\rangle \Downarrow S_2}\ \text{(expr-stmt)}$$

$$\frac{\langle\, S_1 \,,\, e \,\rangle \Downarrow \langle\, S_2 \,,\, v \,\rangle}{\langle\, S_1 \,,\, \texttt{let}\ x\ \texttt{=}\ e \,\rangle \Downarrow S_2[x \mapsto v]}\ \text{(let)}$$

$$\frac{\langle\, S_1 \,,\, s \,\rangle \Downarrow S_2 \qquad \langle\, S_2 \,,\, p \,\rangle \Downarrow \langle\, S_3 \,,\, v \,\rangle}{\langle\, S_1 \,,\, s\ \texttt{;}\ p \,\rangle \Downarrow \langle\, S_3 \,,\, v \,\rangle}\ \text{(prog)}$$