`salt2`: Mutable References

`CS392-M1`: *Rust, In Practice and in Theory*

Syntax

$$
\begin{aligned}
&x && \text{(variables, } \mathcal{V}\text{)}\\
&n && \text{(integers, } \mathbb{Z}\text{)}\\
w &::= x \mid \texttt{*}\, w && \text{(place expression, } \mathcal{W}\text{)}\\
e &::= \texttt{()} \mid n \mid w \mid \texttt{\&}\, w \mid \texttt{\&mut}\ w \mid \texttt{copy}\ w \mid w \,\texttt{=}\, e && \text{(expressions, } \mathcal{E}\text{)}\\
s &::= e \mid \texttt{let}\ x \,\texttt{=}\, e \mid \texttt{let mut}\ x \,\texttt{=}\, e && \text{(statements, } \mathcal{S}\text{)}\\
p &::= e \mid s \,\texttt{;}\, p && \text{(programs, } \mathcal{P}\text{)}
\end{aligned}
$$

Typing

$$
\begin{aligned}
t &::= \texttt{()} \mid \texttt{i32} \mid \texttt{\&}\ w \mid \texttt{\&mut}\ w && \text{(types, } \mathcal{T}\text{)} \\
\tilde{t} &::= \lfloor t \rfloor \mid t && \text{(partial types, } \tilde{\mathcal{T}}\text{)} \\
m &::= \mathsf{imm} \mid \mathsf{mut} && \text{(mutability)} \\
u &::= \langle \tilde{t} \rangle^m && \text{(slot types, } \mathbb{S}_{\mathcal{T}}\text{)}
\end{aligned}
$$

$$
\Gamma \in \mathcal{V} \mapsto \mathbb{S}_{\mathcal{T}} \qquad \text{(contexts)}
$$

$$
\begin{aligned}
&\mathsf{copyable}(t) && \text{(copyability)} \\
&\Gamma \vdash w : u && \text{(place expressions)} \\
&\Gamma \vdash \mathsf{readable}(w) && \text{(readability)} \\
&\Gamma \vdash \mathsf{writable}(w) && \text{(writability)} \\
&\Gamma \vdash \tilde{t} \approx \tilde{t} && \text{(type compatibility)} \\
&\Gamma \vdash e : t \dashv \Gamma && \text{(expressions)} \\
&\Gamma \vdash s \dashv \Gamma && \text{(statments)} \\
&\Gamma \vdash p : t \dashv \Gamma && \text{(programs)}
\end{aligned}
$$

$$
\frac{(x \mapsto \langle \tilde{t} \rangle^m) \in \Gamma}{\Gamma \vdash x : \langle \tilde{t} \rangle^m}\ \text{(var)}
$$

$$
\frac{\Gamma \vdash w_1 : \langle \texttt{\&}\ w_2 \rangle^{m_1} \qquad \Gamma \vdash w_2 : \langle t \rangle^{m_2}}{\Gamma \vdash \texttt{*}\ w_1 : \langle t \rangle^{m_2}}\ \text{(deref)}
$$

$$
\frac{}{\Gamma \vdash \texttt{()} : \texttt{()} \dashv \Gamma}\ \text{(unit)}
$$

$$
\frac{n \in \mathbb{Z}}{\Gamma \vdash n : \texttt{i32} \dashv \Gamma}\ \text{(int)}
$$

$$
\frac{\nexists y, l.(y \mapsto \texttt{\&mut}\ \texttt{*}^l x) \in \Gamma}{\Gamma \vdash \mathsf{readable}(\texttt{*}^k x)}\ \text{(readable)}
$$

$$
\frac{}{\mathsf{copyable}(\texttt{()})}\ \text{(copy-unit)}
$$

$$\frac{}{\mathsf{copyable}(\texttt{i32})}\ (\text{copy-int})$$

$$\frac{}{\mathsf{copyable}(\texttt{\&}\ w)}\ (\text{copy-brw})$$

$$\frac{\Gamma \vdash w : \langle t \rangle^{m} \qquad \Gamma \vdash \mathsf{readable}(w) \qquad \mathsf{copyable}(t)}{\Gamma \vdash \mathsf{copy}\ w : t \dashv \Gamma}\ (\text{place-copy})$$

$$\frac{\Gamma \vdash \mathsf{readable}(\texttt{*}^{k} x) \qquad \nexists y, l.(y \mapsto \texttt{\&}\ \texttt{*}^{l} x) \in \Gamma}{\Gamma \vdash \mathsf{writable}(\texttt{*}^{k} x)}\ (\text{writable})$$

$$\frac{\Gamma \vdash x : \langle \texttt{\&mut}\ w \rangle^{m} \qquad \Gamma \vdash \mathsf{writable}(x)}{\Gamma \vdash x : \texttt{\&mut}\ w \dashv \Gamma[x \mapsto \lfloor \texttt{\&mut}\ w \rfloor]}\ (\text{place-move})$$

$$\frac{\Gamma \vdash w : \langle t \rangle^{m} \qquad \Gamma \vdash \mathsf{readable}(w)}{\Gamma \vdash \texttt{\&}\ w : \texttt{\&}\ w \dashv \Gamma}\ (\text{brw})$$

$$\frac{\Gamma \vdash w : \langle t \rangle^{\mathsf{mut}} \qquad \Gamma \vdash \mathsf{writable}(w) \qquad \Gamma \vdash \mathsf{mutable}(w)}{\Gamma \vdash \texttt{\&mut}\ w : \texttt{\&mut}\ w \dashv \Gamma}\ (\text{mut-brw})$$

$$\frac{}{\Gamma \vdash \texttt{i32} \approx \texttt{i32}}\ (\approx\text{-int})$$

$$\frac{}{\Gamma \vdash \texttt{()} \approx \texttt{()}}\ (\approx\text{-unit})$$

$$\frac{\Gamma \vdash w_1 : \langle \tilde{t}_1 \rangle^{m_1} \qquad \Gamma \vdash w_2 : \langle \tilde{t}_2 \rangle^{m_2} \qquad \Gamma \vdash \tilde{t}_1 \approx \tilde{t}_2}{\Gamma \vdash \texttt{\&}\ w_1 \approx \texttt{\&}\ w_2}\ (\approx\text{-brw})$$

$$\frac{\Gamma \vdash w_1 : \langle \tilde{t}_1 \rangle^{m_1} \qquad \Gamma \vdash w_2 : \langle \tilde{t}_2 \rangle^{m_2} \qquad \Gamma \vdash \tilde{t}_1 \approx \tilde{t}_2}{\Gamma \vdash \texttt{\&mut}\ w_1 \approx \texttt{\&mut}\ w_2}\ (\approx\text{-mbrw})$$

$$\frac{\Gamma \vdash t_1 \approx \tilde{t}_2}{\Gamma \vdash \lfloor t_1 \rfloor \approx \tilde{t}_2}\ (\approx\text{-partial}_1)$$

$$\frac{\Gamma \vdash \tilde{t}_1 \approx t_2}{\Gamma \vdash \tilde{t}_1 \approx \lfloor t_2 \rfloor}\ (\approx\text{-partial}_2)$$

$$\begin{aligned}
\mathsf{write}(\Gamma, x, t) &= \Gamma[x \mapsto t] \\
\mathsf{write}(\Gamma, \texttt{*}^{k+1} x, t) &= \mathsf{write}(\Gamma, \texttt{*}^{k} w, t) \quad \text{where} \quad (x \mapsto \langle \texttt{\&mut}\ w \rangle^{m}) \in \Gamma
\end{aligned}$$

$$\begin{aligned}
\mathsf{replace}(\texttt{\&}\texttt{*}^{k+1} w_1, w_1, \texttt{\&}[\texttt{mut}] w_2) &= \texttt{\&}\texttt{*}^{k} w_2 \\
\mathsf{replace}(\texttt{\&mut}\ \texttt{*}^{k+1} w_1, w_1, \texttt{\&}[\texttt{mut}] w_2) &= \texttt{\&mut}\ \texttt{*}^{k} w_2 \\
\mathsf{replace}(t_1, w, t_2) &= t_1 \\
\mathsf{replace}(\lfloor t_1 \rfloor, w, t_2) &= \lfloor \mathsf{replace}(t_1, w, t_2) \rfloor \\
\mathsf{replace}(\Gamma, w, t_2) &= \{x \mapsto \mathsf{replace}(\tilde{t}_1, w, t_2) : (x \mapsto \tilde{t}_1) \in \Gamma\}
\end{aligned}$$

$$\mathsf{update}(\Gamma, w, t_2) = \mathsf{replace}(\mathsf{write}(\Gamma, w, t_2), w, t_2)$$

$$\frac{\Gamma_1 \vdash w : \langle \tilde{t}_1 \rangle^{\mathsf{mut}} \qquad \Gamma_1 \vdash e : t_2 \dashv \Gamma_2 \qquad \Gamma_2 \vdash \tilde{t}_1 \approx t_2 \qquad \Gamma_3 = \mathsf{update}(\Gamma, w, t_2) \qquad \Gamma_3 \vdash \mathsf{writable}(w)}{\Gamma_1 \vdash w \; \texttt{=} \; e : \texttt{()} \dashv \Gamma_3} \text{ (assign)}$$

$$\frac{\Gamma_1 \vdash e : t \dashv \Gamma_2}{\Gamma_1 \vdash e \dashv \Gamma_2} \text{ (expr-stmt)}$$

$$\frac{\Gamma_1 \vdash e : t \dashv \Gamma_2 \qquad x \notin \mathsf{dom}(\Gamma_2)}{\Gamma_1 \vdash \texttt{let} \; x \; \texttt{=} \; e \dashv \Gamma_2[x \mapsto t^{\mathsf{imm}}]} \text{ (let)}$$

$$\frac{\Gamma_1 \vdash e : t \dashv \Gamma_2 \qquad x \notin \mathsf{dom}(\Gamma_2)}{\Gamma_1 \vdash \texttt{let mut} \; x \; \texttt{=} \; e \dashv \Gamma_2[x \mapsto t^{\mathsf{mut}}]} \text{ (let-mut)}$$

$$\frac{\Gamma_1 \vdash s \dashv \Gamma_2 \qquad \Gamma_2 \vdash p : t \dashv \Gamma_3}{\Gamma_1 \vdash s \; \texttt{;} \; p : t \dashv \Gamma_3} \text{ (prog)}$$

Evaluation

$$\begin{aligned}
\ell &::= \ell_x && \text{(locations, } \mathcal{L}\text{)}\\
v &::= \texttt{()} \mid n \mid \ell && \text{(values, } \mathbb{V}\text{)}\\
\tilde{v} &::= \bot \mid v && \text{(partial values, } \tilde{\mathbb{V}}\text{)}
\end{aligned}$$

$$S \in \mathcal{L} \mapsto \tilde{\mathbb{V}} \qquad \text{(store)}$$

$$\begin{aligned}
&\langle\, S \,,\, e \,\rangle \Downarrow \langle\, S \,,\, v \,\rangle && \text{(expressions)}\\
&\langle\, S \,,\, s \,\rangle \Downarrow S && \text{(statements)}\\
&\langle\, S \,,\, p \,\rangle \Downarrow \langle\, S \,,\, v \,\rangle && \text{(programs)}
\end{aligned}$$

$$\frac{}{x \rightsquigarrow \ell_x}\ \text{(loc-var)}$$

$$\frac{w \rightsquigarrow \ell_x \qquad (\ell_x \mapsto \ell_y) \in S}{*\, w \rightsquigarrow \ell_y}\ \text{(loc-drf)}$$

$$\frac{}{\langle\, S \,,\, \texttt{()} \,\rangle \Downarrow \langle\, S \,,\, \texttt{()} \,\rangle}\ \text{(unit)}$$

$$\frac{n \in \mathbb{Z}}{\langle\, S \,,\, n \,\rangle \Downarrow \langle\, S \,,\, n \,\rangle}\ \text{(int)}$$

$$\frac{w \rightsquigarrow \ell_x \qquad (\ell_x \mapsto v) \in S}{\langle\, S \,,\, \texttt{copy}\ w \,\rangle \Downarrow \langle\, S \,,\, v \,\rangle}\ \text{(place-copy)}$$

$$\frac{w \rightsquigarrow \ell_x \qquad (\ell_x \mapsto v) \in S}{\langle\, S \,,\, w \,\rangle \Downarrow \langle\, S[\ell_x \mapsto \bot] \,,\, v \,\rangle}\ \text{(move)}$$

$$\frac{w \rightsquigarrow \ell}{\langle\, S \,,\, \texttt{\&}\, w \,\rangle \Downarrow \langle\, S \,,\, \ell \,\rangle}\ \text{(brw)}$$

$$\frac{w \rightsquigarrow \ell}{\langle\, S \,,\, \texttt{\&mut}\ w \,\rangle \Downarrow \langle\, S \,,\, \ell \,\rangle}\ \text{(mut-brw)}$$

$$\frac{w \rightsquigarrow \ell_x \qquad (\ell_x \mapsto \tilde{v}) \in S_1 \qquad \langle\ S_1\ ,\ e\ \rangle \Downarrow \langle\ S_2\ ,\ v\ \rangle}{\langle\ S_1\ ,\ w\ \texttt{=}\ e\ \rangle \Downarrow \langle\ S_2[\ell_x \mapsto v]\ ,\ \texttt{()}\ \rangle}\ \text{(assign)}$$

$$\frac{\langle\ S_1\ ,\ e\ \rangle \Downarrow \langle\ S_2\ ,\ v\ \rangle}{\langle\ S_1\ ,\ e\ \rangle \Downarrow S_2}\ \text{(expr-stmt)}$$

$$\frac{\langle\ S_1\ ,\ e\ \rangle \Downarrow \langle\ S_2\ ,\ v\ \rangle}{\langle\ S_1\ ,\ \texttt{let}\ x\ \texttt{=}\ e\ \rangle \Downarrow S_2[\ell_x \mapsto v]}\ \text{(let)}$$

$$\frac{\langle\ S_1\ ,\ e\ \rangle \Downarrow \langle\ S_2\ ,\ v\ \rangle}{\langle\ S_1\ ,\ \texttt{let mut}\ x\ \texttt{=}\ e\ \rangle \Downarrow S_2[\ell_x \mapsto v]}\ \text{(let-mut)}$$

$$\frac{\langle\ S_1\ ,\ e\ \rangle \Downarrow \langle\ S_2\ ,\ v\ \rangle}{\langle\ S_1\ ,\ e\ \rangle \Downarrow S_2}\ \text{(expr-stmt)}$$

$$\frac{\langle\ S_1\ ,\ s\ \rangle \Downarrow S_2 \qquad \langle\ S_2\ ,\ p\ \rangle \Downarrow \langle\ S_3\ ,\ v\ \rangle}{\langle\ S_1\ ,\ s\ \texttt{;}\ p\ \rangle \Downarrow \langle\ S_3\ ,\ v\ \rangle}\ \text{(prog)}$$