B1 Numerical Algorithms: Computational Class

N.B. You may use MATLAB to help answer any of these questions.

Question 1 – Difference formulas

a. Derive the five point central difference formula:

$$f_n' \approx \frac{f_{n-2} - 8f_{n-1} + 8f_{n+1} - f_{n+2}}{12h}$$

b. Demonstrate that the error is $O(h^4)$

Question 2– Numerical integration, the left point rule

a. Using MATLAB, write a code to implement left point rectangular dissection and integrate the following function between $x = -4$ and $x = 4$

$$f(x) = \frac{1}{2\pi\sigma^2} exp(-(x^2)/2\sigma^2)$$

b. Estimate the error in your result. What do you notice?

c. Perform a) and b) above for the range $x = 0$ and $x = 4$.

Question 3 – Numerical integration, the midpoint rule

a. Using MATLAB, implement the midpoint rule to integrate the following function between $x = -4$ and $x = 4$

$$f(x) = \frac{1}{2\pi\sigma^2} exp(-(x^2)/2\sigma^2)$$

b. Derive the error of the midpoint rule.

Question 4 – Richardson extrapolation

a. Use Richardson's method and the forward difference to gain an accurate estimate for the derivative of

$$f(x) = x^x$$

At $x = 0.3679$

b. Derive the analytic derivative of the function, use this to find the minima of the function.

Question 5 – Solving ODEs and the shooting method

a. Write a predictor-corrector code to solve the following ODE

$$y'' = \frac{d^2y}{dx^2} = -y$$

b. Given the boundary values $y(0) = 1$ and $y(11) = 0$, find the value of $y'(0)$ using the shooting method. Verify your answer.

Question 6 – Solving PDEs

a. Solve the following PDE over the range $x = 0$ to $x = 20$, $y = 0$ to $y = 100$, with T=67.5 applied to one of the boundaries (for example T(1:xdim,1) = 67.5) and all other boundary points held at zero.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

b. Suggest how you might reach a quicker numerical solution.

c. Implement your idea in MATLAB.