Privacy-Preserving Contact Discovery with Applications to End-to-End Encrypted Messaging and Mobile-First Cryptocurrencies

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Abstract

Acknowledgements

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Chapter 1

Introduction

Privacy-oriented services such as end-to-end encrypted messaging are increasingly popular [10]. While they provide strong cryptographic guarantees for the confidentiality of message contents, many still leak or gather user-related data. This is particularly the case during a setup stage known as *contact discovery*. As a result, some of these applications gain access to their users' address books and therefore their mobile social graph [13, 14]. In this project, we are interested in performing *contact discovery* in a privacy-preserving manner while remaining practical for mobile applications with billions of users.

1.1 What is contact discovery?

Contact discovery (alternatively *contact matching*) simply refers to the process by which users of a service are able to find other users to interact with. The applied method is largely determined by the amount of information users choose to make public. In the case of networks such as Facebook or LinkedIn, users are encouraged to publish their legal names and can therefore be found through a simple search. In the cases we study, users are registered using pre-existing human-readable identifiers such as their phone numbers or email addresses. This information is kept private by the service such that only users with prior knowledge of each other's identifier can communicate.

As a user signs up to such a service, she will already hold an *address book* – a register that links people (often referred to as *contacts*) to their identifier. However, phone numbers and email addresses are identifiers generated by other services and there is no guarantee that all her *contacts* are using the new service. Thus in this context, *contact discovery*

is more precisely defined as the process by which a user can discover whether or not her *contacts* are using a specific service. Notice that such a process is not only a necessary initialisation step; it must also be regularly refreshed to ensure users keep an up-to-date view of the contacts they can address.

1.2 The privacy challenge

The simplest way to perform contact discovery is arguably to send one's address book to the service operator, allowing them to compute the intersection between the address book and the list of registered users. This is in fact how the popular messaging services WhatsApp and Telegram perform their contact matching [13, 14]. Although efficient, this approach reveals large amounts of private information about users and their contacts, including those that are not register for the service. The service operator is able to construct a social graph of its users and their first connections, allowing it to check for individual connections at will or under government pressure. Such information may discourage whistleblowers from ever speaking up, in fear that their identity may be revealed if they are linked to journalists.

Naive hashing – A naive approach using only cryptographic hash functions will also fail to meet our goal [5, 6]. A user could upload hashes of her contact's identifiers for the service operator to compare against hashes of the registered users' identifiers. While this approach is efficient and yields the desired result, it will still leak the user's address book.

Indeed, although the cryptographic hash function is pre-image resistant, the set of possible pre-images is small enough that hashes can be precomputed into a dictionary and used to find the identifiers that underly the uploaded hashes [6]. Salting these hashes to avoid offline computations renders the system unusable since the service operator would be required to hash the set of registered identifiers using a different salt for each attempt at contact discovery [5].

Advanced approaches and Efficiency – In light of the above, more advanced approaches have been developed to perform privacy-preserving contact discovery. We cover these in greater detail in chapter 2. The issue with such approaches is that they introduce additional complexity through computations, communication requirements, storage requirements or a combination thereof.

In the context of the services we study, contact discovery needs to be performed on mobile devices on a regular basis. These devices are less powerful than modern desktop computers and rely on rechargeable batteries. A computation-intensive process ran regularly on such a device could quickly drain its battery. Furthermore we must allow the process to scale elegantly with the number of registered users, and assume that it can grow to the order of billions.

Efficiency therefore constitutes a priority in the design of such contact discovery schemes. It will also provide a benchmark to evaluate systems against each other, provided that they guarantee a satisfactory level of privacy.

1.3 A peer-to-peer approach

In this report, we present a peer-to-peer approach that makes use of pairing-based cryptography. By doing so, we reduce the service operator's role to a minimum and provide clients with the tools to compute shared secret keys with their contacts. Computations on the client side are of linear order with respect to the size of their address book. Furthermore, clients are only expected to communicate with the service during setup and are only required to store short cryptographic material.

1.4 Structure

Chapter 2

Related Work

In this chapter we provide an overview of state-of-the-art methods for privacy-preserving contact discovery, as well as academic attempts at solving a similar problem. These methods can be divided according to their underlying approach: the first aims at computing the intersection between a list of registered users and an address book, the second aims at providing users with the necessary cryptographic material needed to authenticate and establish shared secrets between each other

In section 2.1, we cover Signal's approach which is to simply process each user's address book without storing her contacts [7]. To convince users that they are trustworthy, Signal publish their code and allow users to verify what code is being run by their servers. In section 2.2, we investigate cryptographic ways to perform a set intersection between two parties without either party learning the other's data. This is known as a private set intersection (PSI). The subsequent attempts fall under the second approach described above. Thus section 2.3 focuses on public key infrastructure and section 2.4 on identity-based key exchanges.

2.1 Public source code and remote attestation

2.1.1 Signal and Intel SGX

Signal's approach is arguably the simplest: request a user's address book, process it obliviously against the list of registered users and clear the servers from any knowledge linked to it [7]. While this process may seem trivial, it creates new challenges in terms

of security and user trust. First, Signal must guarantee that no knowledge of the address book remains on the server, be it obtained through regular or side channels. Secondly, Signal needs to earn the trust of its users. Not only do they need to convince users that their process is completely oblivious, they must also provide constant evidence that their servers are running that particular process rather than any other.

They meet both challenges by publishing their server-side code and performing all their processing within "secure enclaves" on their servers.

2.2 Private set intersection (PSI)

Kales // Basic Bloom Filter

2.3 Public key infrastructure (PKI)

CONIKs // DNS

2.4 Identity-based key exchange (IBKE)

Boneh Waters //

Chapter 3

Background

Before we introduce our system, we recall some definitions of lesser known cryptographic primitives and assumptions. Our aim is to provide the necessary technical background to then discuss our system's architecture. Alternatively, readers may proceed to chapter 4 and refer back to this section when needed.

3.1 Bilinear pairings

The following definition for a *pairing* is that provided by Boneh and Shoup in *A Graduate Course in Applied Cryptography* [3]. To remain consistent with the source text, group operations are represented multiplicatively.

Definition 3.1 (Pairing [3])**.** *Let $\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T$ be three cyclic groups of prime order q where $g_0 \in \mathbb{G}_0$ and $g_1 \in \mathbb{G}_1$ are generators. A* ***pairing*** *is an efficiently computable function $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$ satisfying the following properties:*

1. *bilinear: for all $u, u' \in \mathbb{G}_0$ and $v, v' \in \mathbb{G}_1$ we have*

$$e(u \cdot u', v) = e(u, v) \cdot e(u', v) \qquad \text{and} \qquad e(u, v \cdot v') = e(u, v) \cdot e(u', v) \tag{3.1}$$

2. *non-degenerate: $e(g_0, g_1)$ is a generator of $\mathbb{G}_T$*

When $\mathbb{G}_0 = \mathbb{G}_1$, we say that the pairing is a ***symmetric pairing****. When $\mathbb{G}_0 \neq \mathbb{G}_1$, we say that the pairing is an* ***asymmetric pairing****. We refer to $\mathbb{G}_0$ and $\mathbb{G}_1$ as the* ***pairing groups****, or source groups, and refer to $\mathbb{G}_T$ as the* ***target group****.*

From the bilinear property, we can derive the following equality which is central to our scheme:

$$\forall \alpha, \beta \in \mathbb{Z}_q,\ e({g_0}^{\alpha}, {g_1}^{\beta}) = e(g_0, g_1)^{\alpha\beta} = e({g_0}^{\beta}, {g_1}^{\alpha}) \tag{3.2}$$

Hard Problems in Pairing Groups – The existence of pairings has direct consequences on the discrete logarithm, the decisional Diffie-Hellman (DDH) and the computational Diffie-Hellman (CDH) assumptions. Most notably, the existence of a symmetric pairing on $\mathbb{G}_0$ provides a simple solution to the decisional Diffie Hellman problem. We summarise the effect of pairings on tradition cryptographic assumptions in Table 3.1 below.

	Symmetric Pairing $e : \mathbb{G}_0 \times \mathbb{G}_0 \to \mathbb{G}_T$	**Asymmetric Pairing** $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$
Discrete Logarithm	No harder in $\mathbb{G}_0$ than in $\mathbb{G}_T$	No harder in $\mathbb{G}_0$ or $\mathbb{G}_1$ than in $\mathbb{G}_T$
Decisional DH	Easy to solve in $\mathbb{G}_0$, assumed to hold in $\mathbb{G}_T$	Assumed to be hard in $\mathbb{G}_0$, $\mathbb{G}_1$ and $\mathbb{G}_T$
Computational DH	Assumed to be hard in $\mathbb{G}_0$ and $\mathbb{G}_T$	Assumed to be hard in $\mathbb{G}_0$, $\mathbb{G}_1$ and $\mathbb{G}_T$

Table 3.1: Summary table of classic cryptographic problems under pairings

There exist variants of the DDH and CDH assumptions that take into account the pairing operation: the decisional variant is known as the *decision Bilinear Diffie-Hellman* (DBDH) assumption and the computational variant is known as the *co-Computational Diffie-Hellman* (co-CDH) assumption. We provide formal definitions for both of the assumptions in Appendix A.

Implementation – Pairings have been implemented in practice on certain pairing-friendly elliptic curves. While the underlying constructions are outside of the scope of this project, we wish to emphasise a few of their features. In asymmetric pairings, the group $\mathbb{G}_0$ is usually built upon a finite field, while groups $\mathbb{G}_1$ and $\mathbb{G}_T$ are built on extensions of that field [3]. This implies that elements in $\mathbb{G}_0$ have a shorter representation than those in $\mathbb{G}_1$ or $\mathbb{G}_T$. Furthermore, operations in $\mathbb{G}_0$ are less computationally intensive.

Finally, a pairing operation is much more computationally intensive than exponentiation in any of the three groups [3].

3.2 BLS signatures

One application for pairings is to create deterministic and homomorphic signature schemes such as the one introduced by Boneh, Lynn and Shacham [2] – named BLS after all three of the authors. In this scheme, signatures are elements of one source group and public keys are elements of the other. Although we will make use of both variants, we only present the variant in which signatures are elements of $\mathbb{G}_0$ and public keys are elements of $\mathbb{G}_1$. Once again we write group operations multiplicatively to remain consistent with the source material.

Definition 3.2 (BLS Signatures [2])**.** *A BLS signature scheme $\mathcal{S}_{BLS}$ is composed of three efficient algorithms* $\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify}$*. Let $\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T$ be three cyclic groups of prime order q, with security parameter λ, such that there exists a pairing $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$. $g_0 \in \mathbb{G}_0$ and $g_1 \in \mathbb{G}_1$ are generators. Let H_0 a cryptographic hash function defined as $H_0 : \{0,1\}^* \to \mathbb{G}_0$, and "$\leftarrow_\$$" denote the "choose uniformly at random" operator, we define the three algorithms as:*

> KeyGen : *Choose uniformly at random $x \leftarrow_\$ \mathbb{Z}_q^*$ and set the secret key $\mathsf{sk} \leftarrow x$ and the public key $\mathsf{pk} \leftarrow {g_1}^x$. Output sk to the message signer and pk to the receiver.*
> $\mathsf{Sign}(\mathsf{sk}, m)$*: Output the signature $\sigma = H_0(m)^{\mathsf{sk}}$.*
> $\mathsf{Verify}(\sigma, m, \mathsf{pk})$*: If $e(\sigma, g_1) = e(H_0(m), \mathsf{pk})$ accept the signature. Otherwise reject.*

Theorem 3.1 (EUF-CMA Security of BLS Signatures [3])**.** *Let $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$ be a pairing, let $\mathcal{M}$ be the message space and let $H : \mathcal{M} \to \mathbb{G}_0$ be a hash function. Then the derived BLS signature scheme is* ***existentially unforgeable under chosen message attacks*** *(EUF-CMA) assuming the co-Computational Diffie-Hellman assumption[1] holds for e, and H is modelled as a random oracle.*

Blind and/or threshold variants of this scheme exist. The former allows to hide the original message from the signer, while the latter allows to hide the complete signature from

[1] see Appendix A

any individual (non-colluding) signer. We define a blind (t, n)-threshold BLS signature scheme below. Once again, we only present the variant in which signatures are elements of $\mathbb{G}_0$ and public keys are elements of $\mathbb{G}_1$.

Definition 3.3 (Blind (t, n)-threshold BLS Signature)**.** *A blind (t, n)-threshold BLS signature scheme $\mathcal{S}_{BTBLS}$ is composed of six algorithms* KeyGen*,* Blind*,* Sign*,* Combine*,* Unblind*,* Verify*. Let $\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T$ be three cyclic groups of prime order q such that there exists a pairing $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$. $g_0 \in \mathbb{G}_0$ and $g_1 \in \mathbb{G}_1$ are generators. Let H_0 a cryptographic hash function defined as $H_0 : \{0,1\}^* \to \mathbb{G}_0$, we define the six algorithms as:*

> $\mathsf{KeyGen}(\lambda)$ *: n participants $P_1, P_2, \ldots, P_n$ jointly execute a (t, n)-distributed key generation algorithm with security parameter λ to compute secret key shares $\mathsf{sk}_1, \mathsf{sk}_2, \ldots, \mathsf{sk}_n$ and public key pk. Output sk_i and pk to P_i and pk to the message receiver.*
> $\mathsf{Blind}(m)$*: Choose uniformly at random $\alpha \leftarrow_{\$} \mathbb{Z}_q$. Output $\sigma_\alpha \leftarrow H_0(m)^\alpha$ and α.*
> $\mathsf{Sign}(\mathsf{sk}_i, \sigma_\alpha)$*: Output the signature $\widehat{\sigma_i} \leftarrow {\sigma_\alpha}^{\mathsf{sk}_i}$.*
> $\mathsf{Combine}(\widehat{\sigma_{j_1}}, \widehat{\sigma_{j_2}}, \ldots, \widehat{\sigma_{j_t}})$*: Use Lagrange interpolation on a subset of t blinded partialn signatures $\widehat{\sigma_{j_1}}, \widehat{\sigma_{j_2}}, \ldots, \widehat{\sigma_{j_t}}$ to recover the full blinded signature $\widehat{\sigma}$.*
> $\mathsf{Unblind}(\widehat{\sigma}, \alpha)$*: Output $\sigma \leftarrow \widehat{\sigma}^{(\alpha^{-1})}$ as the full unblinded signature.*
> $\mathsf{Verify}(\sigma, m, \mathsf{pk})$*: If $e(\sigma, g_1) = e(H_0(m), \mathsf{pk})$ accept the signature. Otherwise reject.*

3.3 Left/Right constrained pseudorandom functions

Left/right constrained pseudorandom functions were first introduced by Boneh and Waters [4]. These pseudorandom functions (PRFs) are evaluated over a pair of inputs x, y with a random key k – we denote the output value as $F(k, (x, y))$ or $F_k(x, y)$. These functions can then be "constrained" to their left or their right input using *constraining keys*: knowing the left constraining key for a specific value w allows to compute $F(k, (w, \cdot))$ at all points $(w, \cdot)$ with no knowledge of k. Similarly, the right constraining key for a value w allows to compute $F(k, (\cdot, w))$ at all points $(\cdot, w)$ with no knowledge of k. Left/right constrained PRFs are formally defined in [4] as:

Definition 3.4 (Left/right constrained PRF [4])**.** *Let $F : \mathcal{K} \times \mathcal{X}^2 \to \mathcal{Y}$ be a PRF with security parameter λ. For all $w \in \mathcal{X}$ we wish to support constrained keys $k_{w,\mathrm{LEFT}}$ that enable the evaluation of $F(k, (x, y))$ at all points $(w, y) \in \mathcal{X}^2$, that is, at all points in which*

the left side is fixed to w. In addition, we want constrained keys $k_{w,\text{RIGHT}}$ that fix the right hand side of (x, y) to w. More precisely, for an element $w \in \mathcal{X}$ define the two predicates $p_w^{(L)}, p_w^{(R)} : \mathcal{X}^2 \to \{0, 1\}$ as

$$p_w^{(L)}(x, y) = 1 \iff x = w \quad \text{and} \quad p_w^{(R)}(x, y) = 1 \iff y = w$$

We say that F supports left/right fixing if it is constrained with respect to the set of predicates

$$P_{LR} = \left\{ p_w^{(L)}, p_w^{(R)} : w \in \mathcal{X} \right\}$$

Security – We now provide the definition of a secure left/right constrained PRF by adapting a more general definition provided in [4].

Attack Game 3.1 ([4])**.** *Let $F : \mathcal{K} \times \mathcal{X}^2 \to \mathcal{Y}$ be a left-right constrained PRF with respect to a set system $\mathcal{S} \subseteq 2^{\mathcal{X}^2}$ and security parameter λ. We define constrained security using the following two experiments denoted $\text{EXP}(0)$ and $\text{EXP}(1)$ with an adversary $\mathcal{A}$. For $b = 0, 1$ experiment $\text{EXP}(b)$ proceeds as follows:*

A random key $k \in \mathcal{K}$ is selected and two helper sets $C, V \subseteq \mathcal{X}^2$ are initialised to $\emptyset$. The set $V \subseteq \mathcal{X}^2$ will keep track of all the points at which the adversary can evaluate $F(k, (\cdot, \cdot))$. The set $C \subseteq \mathcal{X}^2$ will keep track of all the points where the adversary has challenged. The sets C and V will ensure that the adversary cannot trivially decide whether challenge values are random or pseudorandom. In particular, the experiments maintain the invariant that $C \cap V = \emptyset$.

The adversary is then presented with three oracles as follows:

- *$F.\mathsf{eval}(x, y)$: given $(x, y) \in \mathcal{X}^2$ from $\mathcal{A}$ if $(x, y) \notin C$ the oracle returns $F(k, (x, y))$ and otherwise returns $\perp$. The set V is updated as $V \leftarrow V \cup \{(x, y)\}$.*
- *$F.\mathsf{constrain}(w, d)$: given a coordinate $w \in \mathcal{X}$ and a direction $d \in \{LEFT, RIGHT\}$ from $\mathcal{A}$ we define S as the set of all points p such that $p = (w, \cdot)$ if $d = LEFT$ or $p = (\cdot, w)$ if $d = RIGHT$. If $S \cap C = \emptyset$ the oracle returns the constraining key $k_{w,d}$ and the set V is updated $V \leftarrow V \cup S$. Otherwise, the oracle returns $\perp$.*
- *$\mathsf{Challenge}(x, y)$: given $(x, y) \in \mathcal{X}^2$ where $(x, y) \notin V$, if $b = 0$ the adversary is given $F(k, (x, y))$; otherwise the adversary is given a random (consistent) $z \in \mathcal{Y}$. The set C is updated $C \leftarrow C \cup \{(x, y)\}$.*

Once the adversary is done interrogating the oracles, it outputs $b' \in \{0,1\}$.

For $b = 0, 1$ *let* W_b *be the event that* $b' = 1$ *in* $\mathrm{EXP}(b)$. *We define the adversary's advantage as:*

$$\mathrm{AdvPRF}_{\mathcal{A},\mathrm{F}}(\lambda) = |\Pr[W_0] - \Pr[W_1]| \tag{3.3}$$

When experiences EXP(0) and EXP(1) are performed equally many times, an equivalent definition for the adversary's advantage is $\mathrm{AdvPRF}_{\mathcal{A},\mathrm{F}}(\lambda) = \left|\frac{1}{2} - \Pr[b' = b]\right|$. Indeed, using the law of total probability:

$$\begin{aligned}
\mathrm{AdvPRF}_{\mathcal{A},\mathrm{F}}(\lambda) &= |\Pr[W_0] - \Pr[W_1]| && (3.4)\\
&= |\Pr[b' = 1 \wedge b = 0] - \Pr[b' = 1 \wedge b = 1]| && (3.5)\\
&= |\Pr[b' = 1 \wedge b = 0] - (\Pr[b' = b] - \Pr[b' = 0 \wedge b = 0])| && (3.6)\\
&= |\Pr[b' = 1 \wedge b = 0] + \Pr[b' = 0 \wedge b = 0] - \Pr[b' = b]| && (3.7)\\
&= |\Pr[b = 0] - \Pr[b' = b]| && (3.8)\\
&= \left|\frac{1}{2} - \Pr[b' = b]\right| && (3.9)
\end{aligned}$$

Definition 3.5 (Secure left/right constrained PRF [4])**.** *The PRF* F *is a secure constrained PRF with respect to* $\mathcal{S}$ *if for all probabilistic polynomial time adversaries* $\mathcal{A}$ *the function* $\mathrm{AdvPRF}_{\mathcal{A},F}(\lambda)$ *is negligible in* λ.

Implementation – Boneh and Waters [4] present a secure left/right constrained PRF construction under the random oracle model by making use of a symmetric pairing. Here we present a variant that makes use of an asymmetric pairing. Let $\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T$ be three cyclic groups of prime order q such that there exists a pairing $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$. Let $H_0 : \{0,1\}^* \to \mathbb{G}_0$ and $H_1 : \{0,1\}^* \to \mathbb{G}_1$ be two hash functions modelled as random oracles. For a random key k, we define the left/right constrained PRF F as:

$$F(k, (x, y)) = e(H_0(x), H_1(y))^k \tag{3.10}$$

For $w \in \{0,1\}^*$, the constraining keys for the predicates $p_w^{(L)}$ and $p_w^{(R)}$ are:

$$k_{w,\mathrm{LEFT}} = H_0(w)^k \quad \text{and} \quad k_{w,\mathrm{RIGHT}} = H_1(w)^k \tag{3.11}$$

Using the bilinear property of the pairing, we can check that knowing $k_{w,\mathrm{LEFT}}$ allows

to evaluate $F(k,(w,y))$ for all $y \in \{0,1\}^*$:

$$e(k_{w,\text{LEFT}}, H_1(y)) = e(H_0(w)^k, H_1(y)) = e(H_0(w), H_1(y))^w = F(k,(w,y)) \tag{3.12}$$

A similar equality can be written to check that $k_{w,\text{RIGHT}}$ allows to evaluate $F(k,(x,w))$ for all $x \in \{0,1\}^*$ by computing $e\left(H_0(x), k_{w,\text{RIGHT}}\right)$.

Notice that left/right constrained PRFs and BLS signatures are closely related. Indeed they both make use of the same underlying pairing construction. Furthermore, BLS signatures take the same form as a constraining key, namely a group element raised to an unknown power.

Chapter 4

Pairing-Based Contact Discovery

In this chapter we present the architecture for our contact discovery scheme (section 4.2). We then provide outlines of security proofs (section 4.3), theoretical performance evaluations (section 4.4) and show how our system maps onto real-world applications such as end-to-end encrypted messaging and mobile-first cryptocurrencies (section 4.5).

4.1 Formal problem statement

First, we provide a formal definition for the problem of contact discovery. User A is registered to a third-party application from which she receives an opaque account identifier $\mathbf{acc}_A$, an address $\mathbf{addr}_A$ and a secret/public key pair $(\mathsf{sk}_A, \mathsf{pk}_A)$. User A also holds a human-readable discovery identifier $\mathtt{id}_A$ (mobile phone number or an email-address) and a list of contacts. We represent A's address book as a set of discovery identifiers $\mathcal{C}_A$. We assume that users exchanged discovery identifiers through out-of-bound communication but are unable to exchange cryptographic material, including their public keys and addresses. Thus for all users B such that $\mathtt{id}_B \in \mathcal{C}_A$ and $\mathtt{id}_A \in \mathcal{C}_B$, A wishes to learn the tuple $(\mathbf{addr}_B, \mathsf{pk}_B)$.

4.2 Service architecture

The foundational design principle for our contact discovery scheme is to provide users with the means to perform contact discovery locally. As we have seen in chapter 2, sending a client the full list of registered users in a probabilistic data structures such as Bloom and Cuckoo filters requires the client to download and store large amounts of data. Instead,

we follow an approach similar to the IBKE protocols and is closely related to the NI-IBKE described in [4]. Our scheme runs in three phases which we will investigate individually:

1. **Setup:** a one-time step for each user. During the setup phase, a user interacts with the contact discovery service to obtain her unique cryptographic material.
2. **Key derivation:** using this cryptographic material, the user is able to compute shared secret keys with any of her contacts knowing only their discovery identifier.
3. **Discovery:** using their shared secret key, a pair of users can establish a secure meeting point on an untrusted online cache, thus allowing for asynchronous contact discovery.

Figure 4.1 shows a diagram of the process described above.

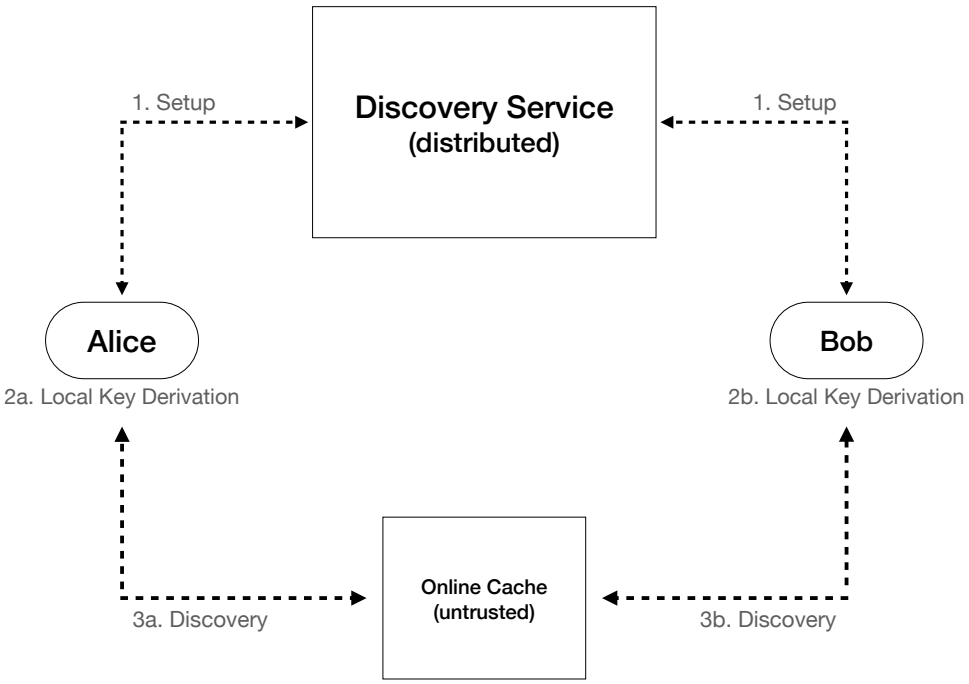

Figure 4.1: Contact discovery between a pair of users Alice and Bob, including setup. Numbers indicate the order of execution

4.2.1 Actors, assets and notation

We make a brief aside to clarify the actors and assets present in our scheme:

- **Users:** each user A holds an opaque account identifier $\mathbf{acc}_A$, an address $\mathbf{addr}_A$, a key pair $(\mathsf{sk}_A, \mathsf{pk}_A)$, a discovery identifier $\mathtt{id}_A$ and an address book $\mathcal{C}_A$ (see section 4.1). We denote $\mathcal{ID}$ the set of all existing discovery identifiers.
- **Discovery Service:** the discovery service is a distributed entity. We denote the set of all servers as $\mathcal{S}$ and the i-th server as S_i. All n servers have jointly executed a (t, n)-distributed key generation algorithm such as to hold shares s_i of an unknown master secret key, which we denote s. Furthermore, each server holds a list of tuples $(\mathbf{acc}, \mathsf{pk})$ for all registered users.
- **Online Cache:** the online cache may be operated by the discovery scheme or by a third party and is assumed to be untrusted. Its role is to manage key-value pairs.

Next we define the cryptographic setting for our scheme. For a security parameter λ:

- $\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T$ are three cyclic groups of prime order $q > 2^\lambda$ such that there exists a pairing $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$.
- $H_0 : \mathcal{ID} \to \mathbb{G}_0$ and $H_1 : \mathcal{ID} \to \mathbb{G}_1$ are two public hash functions modelled as random oracles.
- $F : \mathbb{Z}_q \times \mathcal{ID}^2 \to \mathbb{G}_T$ is a left/right constrained PRF defined as:

$$F(k, (\mathtt{id}_A, \mathtt{id}_B)) = F_k(\mathtt{id}_A, \mathtt{id}_B) = e\left(H_0(\mathtt{id}_A), H_1(\mathtt{id}_B)\right)^k \tag{4.1}$$

- **KDF** is a public, deterministic key derivation function.
- BTBLS is a blind (t, n)-threshold BLS signature scheme (see definition 3.3). We denote this scheme's algorithms as BTBLS.KeyGen, BTBLS.Sign, *etc...*
- DSA is a strong existentially unforgeable signature scheme which makes use of the third-party provided user keys $(\mathsf{sk}_A, \mathsf{pk}_A)$ and is composed of algorithms DSA.Sign and DSA.Verify.
- The master secret key is set to an integer $s \in \mathbb{Z}_q$ chosen uniformly at random. We define two corresponding master public keys ${g_0}^s$ and ${g_1}^s$, for which there exists i public shares denoted as ${g_0}^{s_i}$ and ${g_1}^{s_i}$ respectively.

- Let n the number of servers ($n = |\mathcal{S}|$)) and t a fixed threshold such that $1 \leq t \leq n$, we assume that the master secret key is shared according to a secure t-out-of-n secret sharing scheme and that no single entity holds the master secret key.

4.2.2 Key derivation

We first introduce the essential key derivation step. In doing so, we provide the reader with the necessary material to understand the security constraints under which the initial setup phase operates.

For all users B such that $\texttt{id}_B \in \mathcal{C}_A$, user A can compute shared key material with B by evaluating $F_s(\texttt{id}_A, \texttt{id}_B)$ and $F_s(\texttt{id}_B, \texttt{id}_A)$. From the definition of left/right constrained PRFs, A can do so with the constraining keys $k_{\texttt{id}_A,\text{LEFT}}$ and $k_{\texttt{id}_A,\text{RIGHT}}$:

$$f_{AB} = F_s(\texttt{id}_A, \texttt{id}_B) = e\left(k_{\texttt{id}_A,\text{LEFT}}, H_1(\texttt{id}_B)\right) \tag{4.2}$$

$$f_{BA} = F_s(\texttt{id}_B, \texttt{id}_A) = e\left(H_0(\texttt{id}_B), k_{\texttt{id}_A,\text{RIGHT}}\right) \tag{4.3}$$

Similarly, B can evaluate F at the same points using the constraining keys $k_{\texttt{id}_B,\text{LEFT}}$ and $k_{\texttt{id}_B,\text{RIGHT}}$:

$$f_{AB} = F_s(\texttt{id}_A, \texttt{id}_B) = e\left(H_0(\texttt{id}_A), k_{\texttt{id}_B,\text{RIGHT}}\right) \tag{4.4}$$

$$f_{BA} = F_s(\texttt{id}_B, \texttt{id}_A) = e\left(k_{\texttt{id}_B,\text{LEFT}}, H_1(\texttt{id}_A)\right) \tag{4.5}$$

Using this key material, A and B can establish a symmetric secret key using a standardised key derivation function:

$$k_{AB} = k_{BA} = \mathbf{KDF}\left(f_{AB} \oplus f_{BA}\right) = \mathbf{KDF}(f_{BA} \oplus f_{AB}) \tag{4.6}$$

A note on security – The constraining keys $k_{\texttt{id}_A,\text{LEFT}}$ and $k_{\texttt{id}_A,\text{RIGHT}}$ allow to compute every symmetric key that A may establish with her contacts. As such, those **constraining keys must remain private** to A. The consequences of a leak range from impersonation to a total leak of A's address book and are further detailed in section 4.3.

4.2.3 Discovery

Using their shared key material (k_{AB}, f_{AB}, f_{BA}), users A and B can determine secret memory locations on the online cache to leave an encrypted message for each other. Let Enc, Dec be a secure symmetric encryption scheme and H a hash function modelled as a random oracle, we define two cache operations **Write** and **Read**:

- **Write**(f_{AB}): store the key-value pair $(H(f_{AB}), \mathsf{Enc}_{k_{AB}}(\mathsf{pk}_A || \mathbf{addr}_A))$ on the online cache.
- **Read**(f_{BA}): retrieve the key-value pair $(H(f_{BA}), c_{BA})$. If B has already run the discovery phase of our scheme then $c_{BA} = \mathsf{Enc}_{k_{BA}}(\mathsf{pk}_B || \mathbf{addr}_B)$. Decrypt c_{BA} using the key $k_{AB} = k_{BA}$.

Using these two operations, A is able to leave a message for B to find (**Write**) and check whether B has previously completed the contact matching process (**Read** at the address $H(f_{BA})$). Both users regularly check the relevant memory locations for a message. Once both users have completed the contact discovery process, they will hold each other's public keys and address, allowing them to communicate securely.

4.2.4 Setup

The setup stage serves to provide user A with the constraining keys $k_{\mathtt{id}_A,\mathrm{LEFT}}$ and $k_{\mathtt{id}_A,\mathrm{RIGHT}}$. Consequently, the setup is a security-critical task. As we have shown in Equation 3.11, under our construction of F the constraining keys can be expressed as:

$$k_{\mathtt{id}_A,\mathrm{LEFT}} = H_0(\mathtt{id}_A)^s \quad \text{and} \quad k_{\mathtt{id}_A,\mathrm{RIGHT}} = H_1(\mathtt{id}_A)^s \tag{4.7}$$

These constraining keys are equivalent to BLS signatures on $\mathtt{id}_A$ by at least t out of n servers of the discovery service. Notice that the service needs to produce signatures under both variants of the BLS scheme: one with signatures in $\mathbb{G}_0$ and one with signatures in $\mathbb{G}_1$.

The setup protocol between user A and a server S_i is described as follows:

1. S_i issues a challenge c

2. A chooses a random blinding factor $\alpha \leftarrow_{\$} \mathbb{Z}_q$ and sends $\mathbf{acc}_A$, $\mathbf{sig}_A \leftarrow \mathsf{DSA.Sign}(\mathsf{sk}_A, \mathbf{acc}_A||c)$, $\sigma_{\alpha,0} \leftarrow H_0(\mathtt{id}_A)^\alpha$, $\sigma_{\alpha,1} \leftarrow H_1(\mathtt{id}_A)^\alpha$ to S_i.

3. Upon reception of A's request, S_i retrieves the associated public key and checks that the signature $\mathbf{sig_A}$ is valid:

$$\mathsf{DSA.Verify}(\mathsf{pk}_A, \mathbf{acc}_A||c, \mathbf{sig}_A) = 1 \tag{4.8}$$

4. If the check succeeds, S_i sends $\widehat{\sigma}_{i,0} \leftarrow {\sigma_{\alpha,0}}^{s_i}$ and $\widehat{\sigma}_{i,1} \leftarrow {\sigma_{\alpha,1}}^{s_i}$ to A.

5. Using S_i's public keys $({g_0}^{s_i}, {g_1}^{s_i})$, A checks the following equalities:

$$e\left(\widehat{\sigma}_{i,0}, g_0\right) = e\left(H_0(\mathtt{id}_A)^\alpha, {g_0}^{s_i}\right) \tag{4.9}$$
$$e\left(g_1, \widehat{\sigma}_{i,1}\right) = e\left({g_1}^{s_i}, H_1(\mathtt{id}_A)^\alpha\right) \tag{4.10}$$

6. If the checks succeed (in other words, if A receives valid signatures from the service), A removes the blinding factor α to obtain $H_0(\mathtt{id}_A)^{s_i}$ and $H_1(\mathtt{id}_A)^{s_i}$.

A repeats the above procedure with at least t servers. Using the obtained signature shares, A can recover the full signatures $H_0(\mathtt{id}_A)^s$ and $H_1(\mathtt{id}_A)^s$ using $\mathsf{BTBLS.Combine}$.

This completes our description of the contact discovery scheme. We have seen how the setup process allows users to obtain their private constraining keys. Using those keys, users can locally and asynchronously derive shared key material with their contacts by evaluating a left/right constrained PRF at specific points. Finally, using the shared key material, users can leave and read messages from an untrusted online cache, thus completing the contact discovery process.

4.3 Privacy

We will now evaluate the privacy guarantees of our scheme when there are strictly less than t malicious servers. Our scheme hides the links between users as long as the decisional bilinear Diffie-Hellman assumption holds for the pairing e, the master secret key s does not leak and both constraining keys $k_{X,\text{LEFT}}, k_{X,\text{RIGHT}}$ are known only to user X. We present the threat model, potential attacks, outlines of security proofs as well as the consequences of a security breach.

4.3.1 Threat model

An adversary $\mathcal{T}$ wishing to break our scheme's privacy property aims to gain information about the contents of any user's address book. This goal is equivalent to determining whether $\texttt{id}_B \in \mathcal{C}_A$, for any user A and any identifier $\texttt{id}_B$ that is not owned by $\mathcal{T}$. $\mathcal{T}$ is characterised as:

- having access to all public information.
- having access to the present and past states of the online cache.
- may eavesdrop on any communication between the users, servers and online cache.
- may spawn any number of users for which $\mathcal{T}$ owns the discovery identifier.
- may control up to $t - 1$ servers in the discovery service.

Notice that we are working under the assumption that discovery identifiers are correctly linked to the users who own them. We discuss ways in which this assumption can be upheld in practice in section 4.3.2, under "**Impersonating a user**"

4.3.2 Proof outline

To guide our analysis, we provide an attack tree[1] against the privacy property of our scheme in Figure 4.2. The root node represents the attacker's goal and each child node represents an option to solve the problem indicated in the parent node. Consequently, leaf nodes represent the attacker's entry points: breaking the security of F, obtaining the master secret key s, forging BLS signatures on another user's discovery identifier, impersonating a user or computing the shared key material (k_{AB}, f_{AB}, f_{BA}). We will therefore consider each leaf node and show that our scheme is resistant against these attacks.

[1] as defined by Schneier [11]

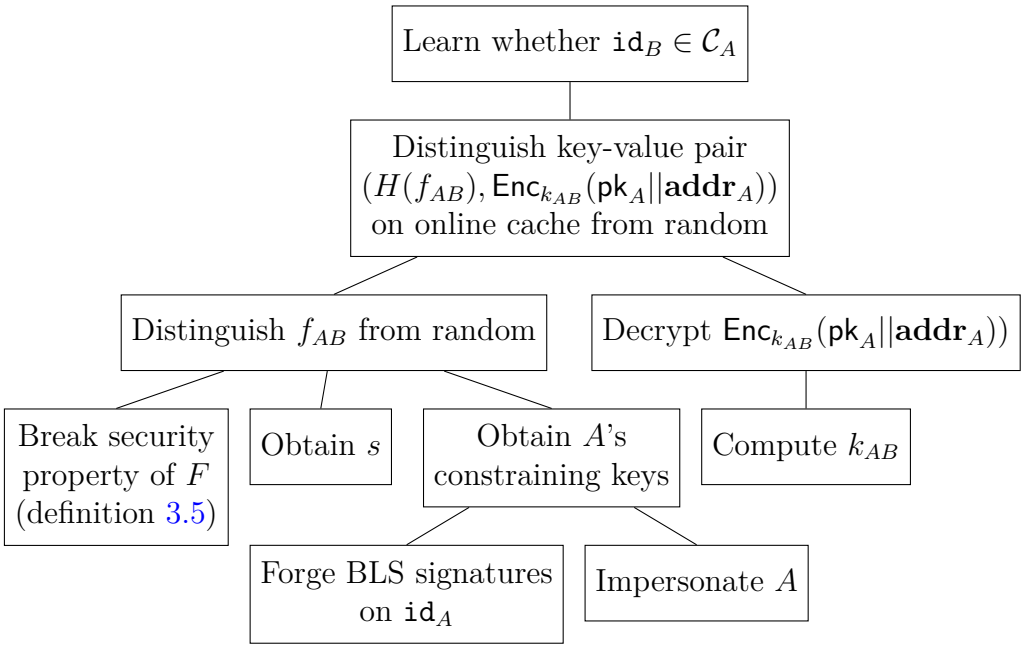

Figure 4.2: Attack tree against our discovery scheme. Branches represent "OR" statements

Security of our PRF construction

We will first show that our construction for the left/right constrained PRF using an asymmetric pairing is secure as per definition 3.5. Our construction is closely related to the one presented by Boneh and Waters [4]. As such our proof sketch makes use of very similar ideas.

Theorem 4.1. *The PRF F defined as $F(k,(x,y)) = e\left(H_0(x), H_1(y)\right)^k$ is a secure constrained PRF with respect to its constraining keys assuming the decisional bilinear Diffie-Hellman assumption holds for e and the functions H_0 and H_1 are modelled as random oracles.*

Proof sketch. We assume for contradiction the existence of a probabilistic polynomial-time adversary $\mathcal{A}$ that distinguishes F from random as in definition 3.5, however $\mathcal{A}$ is limited to a single $\mathsf{Challenge}$ query. We can then construct an adversary $\mathcal{B}$ that breaks the decisional bilinear Diffie-Hellman (DBDH) assumption.

Given $(g_0, g_1, u_0 \leftarrow {g_0}^{\alpha}, u_1 \leftarrow {g_1}^{\alpha}, v_0 \leftarrow {g_0}^{\beta}, w_1 \leftarrow {g_1}^{\gamma}, z^{(b)})$, $\mathcal{B}$'s goal is to determine whether $z^{(b)} = z^{(0)} = {g_0}^{\alpha\beta\gamma}$ or $z^{(b)} = z^{(1)} = {g_0}^{\delta}$, where $\delta \leftarrow_{\$} \mathbb{Z}_q$ (see Attack Game A.2 in

Appendix A). Using the pairing operation, this game can be viewed as distinguishing the output of F from a random element of $\mathbb{G}_T$. Indeed let $b, c \in \mathcal{X}$ such that $H_0(b) = {g_0}^{\beta}$ and $H_1(c) = {g_1}^{\gamma}$, then:

$$e\left(z^{(b)}, g_1\right) = \begin{cases} e\left(g_0, g_1\right)^{\alpha\beta\gamma} = e\left({g_0}^{\beta}, {g_1}^{\gamma}\right)^{\alpha} = F(\alpha, (b, c)), & \text{if } b = 0 \\ e\left(g_0, g_1\right)^{\delta} = {g_T}^{\delta}, & \text{if } b = 1 \end{cases} \tag{4.11}$$

Thus, $\mathcal{B}$ will run $\mathcal{A}$ as a sub-routine and must therefore emulate its oracles, namely $F.\mathsf{eval}$, $F.\mathsf{constrain}$, $\mathsf{Challenge}$ and oracles for the hash functions H_0, H_1.

When $\mathcal{A}$ issues a query to $H_0(x)$, $\mathcal{B}$ chooses a consistent random $\widehat{x} \leftarrow_{\$} \mathbb{Z}_q$ and sets $H_0(x) \leftarrow {g_0}^{\widehat{x}}$. To one of $\mathcal{A}$'s queries to H_0 which we denote x^*, $\mathcal{B}$ responds with $H_0(x^*) \leftarrow v_0$. Queries to H_1 are answered in a similar fashion where one query is responded to with $H_1(y^*) \leftarrow w_1$. Using these values, queries to $F.\mathsf{constrain}(x, LEFT)$ where $x \neq x^*$ are answered with $k_{x,LEFT} \leftarrow {u_0}^{\widehat{x}}$. Notice that as required

$${u_0}^{\widehat{x}} = ({g_0}^{\alpha})^{\widehat{x}} = {g_0}^{\alpha\widehat{x}} = ({g_0}^{\widehat{x}})^{\alpha} = H_0(x)^{\alpha}$$

Similarly, queries to $F.\mathsf{constrain}(y, RIGHT)$ where $y \neq y^*$ are answered with $k_{y,RIGHT} \leftarrow {u_1}^{\widehat{y}}$. Queries to $F.\mathsf{eval}(x, y)$ are answered for $x \neq x^*$ or $y \neq y^*$ by building the constraining keys as it is done for the $F.\mathsf{constrain}$ oracle. Notice that $\mathcal{B}$ does not hold the values β, γ and is therefore unable to answer queries to the $F.\mathsf{constrain}$ oracle for $(x^*, LEFT)$ and $(y^*, RIGHT)$, nor can it answer the $F.\mathsf{eval}$ query for (x^*, y^*). This is in fact equivalent to starting Attack Game 3.1 with the set C initialised to $\{(x^*, y^*)\}$.

After n queries to the H_0 oracle and m queries to the H_1 oracle, $\mathcal{A}$ will hold at most $n \times m$ pairs on which it could challenge. Some of these pairs may have been added to the set V due to queries to $F.\mathsf{constrain}$ and $F.\mathsf{eval}$, and thus become ineligible for challenging. However, since the experiment started with $C = \{(x^*, y^*)\}$, we can be sure that (x^*, y^*) is an eligible pair (remember that the attack game maintains the invariant $C \cap V = \emptyset$). Therefore, $\mathcal{A}$ will challenge the pair (x^*, y^*) with probability $p \geq \frac{1}{n \times m}$, to which $\mathcal{B}$ answers with $z^{(b)}$.

If $b = 0$, then $z^{(b)} = F(\gamma, (x^*, y^*))$ and $\mathcal{A}$ will answer as in experiment EXP(0). On the other hand if $b = 1$, then $z^{(b)} = {g_T}^{\delta}$ and $\mathcal{A}$ will answer as in experiment EXP(1). Let $b'_{\mathcal{A}}$

be the output of $\mathcal{A}$, we define as $b'_{\mathcal{B}} \leftarrow b'_{\mathcal{A}}$ the return value of $\mathcal{B}$. Thus

$$\Pr[b'_{\mathcal{B}} = b] \geq \frac{1}{n \times m} \times \Pr[b'_{\mathcal{A}} = b] \tag{4.12}$$

Given that $\mathcal{A}$ is a probabilistic polynomial-time adversary, $n \times m$ must necessarily be polynomial in λ. Therefore, if $\mathcal{A}$'s advantage is non-negligible then so is $\mathcal{B}$'s, thus breaking the DBDH assumption and yielding a contradiction. □

Computing A and B's shared key material

To compute A and B's shared key material, an attacker needs to compute f_{AB} and f_{BA}. This is in fact a harder problem than the decisional problem investigated above. We have already shown that no probabilistic polynomial-time adversary can break the security of F under the DBDH assumption. Similarly, no probabilistic polynomial-time adversary will be able to compute either f_{AB} or f_{BA} under the DBDH assumption without access to the relevant constraining keys or the master secret key.

Obtaining the master secret key

Next, we consider the option for an attacker to obtain the master secret key s. It is part of our assumption that there are strictly less than t malicious servers. Therefore, they do not meet the threshold required to construct the master secret key. An attacker aiming to break our scheme through this attack will need to steal at least one of the key shares.

Forging a BLS signature

As we have seen in subsection 4.2.4, the service generates constraining keys by signing a user's discovery identifier. The signing algorithm is a blind (t, n)-threshold BLS algorithm. As per Theorem 3.1, the BLS signature is existentially unforgeable against chosen message attacks under the co-Computational Diffie Hellman assumption. This assumption is in fact a weaker than the DBDH assumption which is required for the left/right constrained PRF security.

Impersonating a user

Impersonation attacks are the most threatening to our scheme and lead to open questions. We first describe the issue and offer two solutions, neither of which are fully sat-

isfying. The attack is performed by running the setup process maliciously from the user side: upon receiving a challenge c, $\mathcal{T}$ can send the tuple ($\mathbf{acc}_{\mathcal{T}}$, $\mathsf{DSA.Sign}(\mathsf{sk}_{\mathcal{T}}, \mathbf{acc}_{\mathcal{T}}||c)$, $H_0(\mathtt{id}_A)^\alpha$, $H_1(\mathtt{id}_A)^\alpha$). The server S_i receiving this tuple will find that the signature $\mathsf{DSA.Sign}(\mathsf{sk}_{\mathcal{T}}, \mathbf{acc}_{\mathcal{T}}||c)$ does verify for the specified account and challenge. Furthermore, S_i will be unable to distinguish the blinded values $H_0(\mathtt{id}_A)^\alpha, H_1(\mathtt{id}_A)^\alpha$ from random elements in $\mathbb{G}_0$ and $\mathbb{G}_1$ respectively. As such S_i will issue partial constraining keys for $\mathtt{id}_A$ to $\mathcal{T}$. Repeating this process with t servers, $\mathcal{T}$ will obtain the full constraining keys for user A.

The first solution is for A to transmit her discovery identifier in clear to S_i. The server can then use out-of-bound communication to verify that A indeed owns $\mathtt{id}_A$ (possible techniques include sending a one-time code via text message or email). If A proves that she owns $\mathtt{id}_A$, S_i provides the partial signatures for that discovery identifier. Notice that under this approach, A cannot blind the hash of her discovery identifier. Consequently, the communication between A and S_i must be encrypted to prevent an eavesdropping adversary from learning the signature share on $\mathtt{id}_A$. Furthermore, this identification method allows the servers to build a list of identifiers for the users registered to the mobile application. In some cases, this may be a breach of privacy.

The second solution makes use of identification tokens to delegate the task of linking a user to their discovery identifier. Suppose an entity V (centralised or distributed) is trusted to verify whether a user owns a discovery identifier. Using a secret key $v \leftarrow_\$ \mathbb{Z}_q$ and the corresponding public keys ${g_0}^v$ and ${g_1}^v$, V could issue ownership tokens in the form of BLS signatures $t_{0,A} = H_0(\mathtt{id}_A)^v, t_{1,A} = H_1(\mathtt{id}_A)^v$. These tokens can then be blinded and verified against a blinded discovery identifier $H_0(\mathtt{id}_A)^\alpha, H_1(\mathtt{id}_A)^\alpha$:

$$e\left({t_{0,A}}^\alpha, g_1\right) = e\left(H_0(\mathtt{id}_A)^\alpha, {g_1}^v\right) \iff t_{0,A} = H_0(\mathtt{id}_A)^v \tag{4.13}$$

$$e\left(g_0, {t_{1,A}}^\alpha\right) = e\left({g_0}^v, H_1(\mathtt{id}_A)^\alpha\right) \iff t_{1,A} = H_1(\mathtt{id}_A)^v \tag{4.14}$$

Users can therefore send ${t_{0,A}}^\alpha$, ${t_{1,A}}^\alpha$ along with the setup tuple ($\mathbf{acc}_A$, $\mathbf{sig}_A$, $H_0(\mathtt{id}_A)^\alpha$, $H_1(\mathtt{id}_A)^\alpha$), to allow each server to perform the checks in Equation 4.13 and Equation 4.14.

While this method allows identification without revealing the discovery identifier to the contact discovery servers, it relies on a trusted verification entity V. In fact, this entity faces the same problem we were trying to avoid: it must output a BLS signature on an identifier only if the request was made by the identifier's owner. This raises the question of

trust within our system. The contact discovery scheme can be made secure and oblivious to which user owns which discovery identifier. However, for that to happen, we need another entity to perform that check and gather private information about the users.

4.3.3 Consequences of a breach

To conclude our investigation of the scheme's privacy properties, we evaluate the consequences of various breaches of the protocol. Let us first consider the scenario in which a pair of constraining keys $k_{\mathtt{id}_A,\text{LEFT}}, k_{\mathtt{id}_A,\text{RIGHT}}$ is leaked. Using these keys, an attacker will be able to compute the shared key material (k_{AX}, f_{AX}, f_{BX}) between A and any other user X. The attacker is then able to:

(a) check whether X has written to the cache in location $H(f_{XA})$, thus uncovering whether $\mathtt{id}_A \in \mathcal{C}_X$. Iterating over all $\mathtt{id}_X \in \mathcal{ID}$ allows to determine which users hold $\mathtt{id}_A$ in their contacts.

(b) decrypt the message (if any) left by X at location $H(f_{XA})$ using the key k_{AX}, thus linking $\mathtt{id}_X$ to an address and public key.

(c) check whether A has written to the cache in location $H(f_{AX})$, thus uncovering whether $\mathtt{id}_X \in \mathcal{C}_A$. Iterating over all $\mathtt{id}_X \in \mathcal{ID}$ allows to recover all of A's contacts.

(d) overwrite the value that A wrote in location $H(f_{AX})$ using the key k_{AX}. This allows the attacker to send any address and public key to X, thus hijacking any channel that A and X were wishing to establish.

Should only one of the constraining keys leak, the attacker will only be able to perform a subset of the actions above. Indeed, obtaining a $LEFT$ key only allows to compute f_{AX}. The attacker will therefore only be able to perform action (c). Similarly, obtaining only a $RIGHT$ key limits the attacker's possibilities to (a). In either case, these breaches represent a complete loss of privacy. Finally, obtaining the master secret key allows to compute any constraining key, thus allowing to perform the above operations for any pair of users.

4.4 Theoretical performance evaluation

In this section we present a brief evaluation of the scheme's efficiency. Importantly, we show that all computational costs grow linearly with respect to the input size. We estimate the computation time associated with each phase of our discovery scheme using benchmark timings from the mobile-friendly elliptic curve pairing library MCL [9]. These benchmark tests were performed on an iPhone 7 running iOS 11.2.1, executing operations over three Barreto-Naehrig (BN) elliptic curves with varying security parameters [8]. The results are summarised in Table 4.1. Notice that we are now working with elliptic curves and therefore adopt an additive notation for the group operation.

Operation	Notation	BN254	BN381_1	BN462
Pairing	$\mathbf{p}$	3.9	11.752	22.578
Addition in $\mathbb{G}_0$	$\mathbf{add}_{\mathbb{G}_0}$	0.006	0.015	0.018
Point doubling in $\mathbb{G}_0$	$\mathbf{dbl}_{\mathbb{G}_0}$	0.005	0.01	0.019
Multiplication in $\mathbb{G}_0$	$\mathbf{mul}_{\mathbb{G}_0}$	0.843	2.615	5.339
Addition in $\mathbb{G}_1$	$\mathbf{add}_{\mathbb{G}_1}$	0.015	0.03	0.048
Point doubling in $\mathbb{G}_1$	$\mathbf{dbl}_{\mathbb{G}_1}$	0.011	0.022	0.034
Multiplication in $\mathbb{G}_1$	$\mathbf{mul}_{\mathbb{G}_1}$	1.596	4.581	9.077
Hash to $\mathbb{G}_0$	$\mathbf{hash}_{\mathbb{G}_0}$	0.212	0.507	1.201
Hash to $\mathbb{G}_1$	$\mathbf{hash}_{\mathbb{G}_1}$	3.486	9.93	21.817

Table 4.1: Timing benchmarks for elliptic curve operations over three pairing-friendly curves (BN254, BN381_1 and BN462) executed on an iPhone 7 running the MCL library [9] on iOS 11.2.1. All timings are given in milliseconds. [8]

Computational cost: Setup with identification tokens

First, let us inspect the setup phase with identification tokens. This phase only needs to be performed once per user and per server. As such we will evaluate the computational cost of a single user-server interaction, then consider the scaling with respect to the total number of users N and the threshold of servers t. We assume that users already hold a hash of their own discovery identifier. By inspection of the protocol, the setup phase requires:

> **User:** 2 multiplications in $\mathbb{G}_0$, 2 multiplications in $\mathbb{G}_1$ (blind and unblind identifier), 4 pairing operations (Equation 4.9, Equation 4.10), one standard digital signature. Identification tokens introduce one additional multiplication in $\mathbb{G}_0$,

and one multiplication in $\mathbb{G}_1$ (blind tokens). After repeating these operations with enough servers to meet the system's threshold, a user is required to recombine t partial BLS signatures in both $\mathbb{G}_0$ and $\mathbb{G}_1$. Using Lagrange interpolation, each of these recombinations require t multiplications and t additions in their respective source group. We can now express the time spent by each user on computations for the setup phase as a function of t:

$$\text{comp}_{\text{setup,user}}(t) = t(4(\mathbf{mul}_{\mathbb{G}_0} + \mathbf{mul}_{\mathbb{G}_1} + \mathbf{p}) + \mathbf{add}_{\mathbb{G}_0} + \mathbf{add}_{\mathbb{G}_0} + \mathbf{DSA}_S) \quad (4.15)$$

where $\mathbf{DSA}_S$ is the time to execute the digital signature algorithm.

Server: 1 multiplication in $\mathbb{G}_0$, 1 multiplication in $\mathbb{G}_1$ (BLS signature in each source group) and one standard digital signature verification. Identification tokens introduce 4 additional pairing operations (Equation 4.13, Equation 4.14). We express the time spent by each server on computations for the setup phase as a function of N:

$$\text{comp}_{\text{setup,server}}(N) = N(4\mathbf{p} + \mathbf{mul}_{\mathbb{G}_0} + \mathbf{mul}_{\mathbb{G}_1} + \mathbf{DSA}_V) \quad (4.16)$$

where $\mathbf{DSA}_V$ is the time to verify the digital signature.

Assuming we are using curve BN381_1 and using realistic values for $\mathbf{DSA}_S$ and $\mathbf{DSA}_V$ on a mobile device (respectively 0.82 ms and 3.02 ms [15]) yields:

$$\text{comp}_{\text{setup,user}}(t) = 76.66\text{ms} \quad \text{and} \quad \text{comp}_{\text{setup,server}}(N) = 57.22\text{ms} \quad (4.17)$$

For a threshold of servers set to 6, a single user can complete the setup phase while spending less than 0.5 seconds performing local computations. Similarly, servers can setup a new user within tens of milliseconds. Launching our discovery service for an application with 10 million users can be done with slightly more than one day of serial computations. However, with no optimisations, an application with one billion users would require a roll out period of almost two years.

In order to render the service practical for widely used applications, we must investigate methods to optimise the server-side setup process. The most costly operation performed by the server are the verifications of the identity tokens. These are in fact verification

of BLS signatures, which can be aggregated to reduce the number of pairing operations needed [1]. We can establish optimal batching strategies by placing assumptions on the rate of false signatures; however, such strategies are outside of the scope of this report. Promising approaches to establishing an optimal strategy may come from other fields of research [12].

Computational cost: Shared key derivation

We now consider the local key derivation phase on a per-contact basis, before investigating how computations scale with the number of contacts N_c. A user must perform two evaluations of a left/right constrained PRF. Under our construction, this amounts to performing one hash to each source group and two pairing operations. Thus:

$$\text{comp}_{\text{key,user}}(N_c) = N_c(2\mathbf{p} + \mathbf{hash}_{\mathbb{G}_0} + \mathbf{hash}_{\mathbb{G}_1}) \tag{4.18}$$

Using the values from curve BN381_1 in Table 4.1 yields a very fast 34ms per address book entry. Thus an initial computation over an address book of one thousand entries would only require 34 seconds of computations.

Concluding remarks

The discovery phase makes use of standard cryptographic operations and is of lesser interest when estimating the computational costs of our scheme. However, a similar analysis of the scheme can be performed for communication costs. These are largely dependent on the type of mobile network that is being used as well as the availability of the servers and the online cache. We do not perform this analysis in our report, however we wish to emphasise that our scheme requires very few communications rounds to complete all three phases.

Overall, we have seen that all computations grow linearly with respect to their inputs. Using realistic numbers of registered users, we have seen that our service may be immediately applicable to relatively popular applications (in the range of tens of millions of users). For more popular applications such as WhatsApp, our system will require optimisations of the server-side setup process. Under our construction of the left/right constrained PRF, the most promising approach is to batch token verifications to reduce the number of pairing operations required.

4.5 Applications

4.5.1 End-to-end encrypted messaging

4.5.2 Mobile-first cryptocurrencies

Chapter 5

Proof-of-Concept Implementation

5.1 Local server emulation

5.2 Local key derivation

5.3 Online meeting point via IPFS

Chapter 6

Conclusion

Appendix A

Bilinear variants of the CDH and DDH problems

A.1 The co-computational Diffie-Hellman (co-CDH) Problem and Assumption

The co-Computational Diffie-Hellman (co-CDH) assumption is a variant of the Computational Diffie-Hellman assumption that applies for asymmetric pairings. Let us recall the definition for the co-Computational Diffie-Hellman assumption given in [3], using a multiplicative notation for the group operation as in the source text..

Attack Game A.1 (co-CDH [3])**.** *Let $\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T$ be three cyclic groups of prime order q such that there exists a pairing $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$. For a given adversary $\mathcal{A}$, the attack game runs as follows:*

- *The challenger picks at random $\alpha, \beta \leftarrow_{\$} \mathbb{Z}_q$ and computes*

$$u_0 \leftarrow {g_0}^{\alpha}, \qquad u_1 \leftarrow {g_1}^{\alpha}, \quad v_0 \leftarrow {g_0}^{\beta}, \quad z_0 \leftarrow {g_0}^{\alpha\beta}$$

- *The adversary $\mathcal{A}$ receives the tuple (u_0, u_1, v_0) and outputs $\hat{z}_0 \in \mathbb{G}_0$*

We define the advantage of $\mathcal{A}$ in solving the co-CDH problem for e as:

$$\text{coCDHadv}[\mathcal{A}, e] := \Pr(\hat{z}_0 = z_0) \tag{A.1}$$

Notice that for symmetric pairings, $\mathbb{G}_0 = \mathbb{G}_1$ therefore $g_0 = g_1$, $u_0 = u_1$ and attack game A.1 is identical to the Computational Diffie-Hellman attack game.

Definition A.1 (co-CDH Assumption [3]). *We say that the co-CDH assumption holds for the pairing e if for all efficient adversaries $\mathcal{A}$ the quantity* $\mathrm{coCDHadv}[\mathcal{A}, e]$ *is negligible.*

A.2 The decision bilinear Diffie-Hellman (DBDH) Problem and Assumption

The decisional variant is relatively straight-forward having already defined the co-CDH assumption. The attack setting is closely related, however the adversary is expected to distinguish an element from random (rather than required to computed it). Once again, the definition is adapted from [3] and uses a multiplicative notation for group operations.

Attack Game A.2 (Decision bilinear Diffie-Hellman [3]). *Let $e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T$ be a pairing where $\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T$ are cyclic groups of prime order q with generators $g_0 \in \mathbb{G}_0$ and $g_1 \in \mathbb{G}_1$. For a given adversary $\mathcal{A}$, we define the following experiment:*

- *The challenger picks at random $\alpha, \beta, \gamma, \delta \xleftarrow{\$} \mathbb{Z}_q$, computes*

$$u_0 \leftarrow {g_0}^{\alpha}, \quad u_1 \leftarrow {g_1}^{\alpha}, \quad v_0 \leftarrow {g_0}^{\beta}, \quad w_1 \leftarrow {g_1}^{\gamma}, \quad z^{(0)} \leftarrow {g_0}^{\alpha\beta\gamma}, \quad z^{(1)} \leftarrow {g_0}^{\delta}$$

  and flips a bit $b \xleftarrow{\$} \{0, 1\}$. Using the result of the bit flip, the challenger sends $(u_0, u_1, v_0, w_1, z^{(b)})$ to $\mathcal{A}$.

- *$\mathcal{A}$ receives $(u_0, u_1, v_0, w_1, z^{(b)})$ and outputs a bit $\hat{b} \in \{0, 1\}$*

We define the advantage of $\mathcal{A}$ in solving the DBDH problem for e as:

$$\mathrm{DBDHavd}[\mathcal{A}, e] := \left| \frac{1}{2} - \Pr(\hat{b} = b) \right| \tag{A.2}$$

Definition A.2 (Decision BDH assumption [3]). *We say that the decision bilinear Diffie-Hellman assumption holds for the pairing e if for all efficient adversaries $\mathcal{A}$ the quantity* $\mathrm{DBDHavd}[\mathcal{A}, e]$ *is negligible.*

Appendix B

Calculations: performance evaluation

Appendix C

Code

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