

DSE6011 - Module 7 - Ch 8 Exercises

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```
library(tree)
library(randomForest)

## randomForest 4.7-1.1
## Type rfNews() to see new features/changes/bug fixes.
library(gbm)

## Warning: package 'gbm' was built under R version 4.3.3
## Loaded gbm 2.2.2
## This version of gbm is no longer under development. Consider transitioning to gbm3, https://github.com
library(tidymodels)

## -- Attaching packages ----- tidymodels 1.2.0 --
## v broom          1.0.5      v recipes          1.0.10
## v dials          1.2.1      v rsample          1.2.1
## v dplyr          1.1.4      v tibble          3.2.1
## v ggplot2        3.5.0      v tidyr           1.3.1
## v infer          1.0.7      v tune            1.2.1
## v modeldata      1.4.0      v workflows       1.1.4
## v parsnip        1.2.1      v workflowsets    1.1.0
## v purrr          1.0.2      v yardstick       1.3.1
## Warning: package 'modeldata' was built under R version 4.3.3
## -- Conflicts ----- tidymodels_conflicts() --
## x dplyr::combine() masks randomForest::combine()
## x purrr::discard() masks scales::discard()
## x dplyr::filter()  masks stats::filter()
## x dplyr::lag()     masks stats::lag()
## x ggplot2::margin() masks randomForest::margin()
## x recipes::step()  masks stats::step()
## * Dig deeper into tidy modeling with R at https://www.tmw.org
library(BART)

## Warning: package 'BART' was built under R version 4.3.3
## Loading required package: nlme
##
## Attaching package: 'nlme'
## The following object is masked from 'package:dplyr':
```

```
##
## collapse
## Loading required package: survival
library(ISLR2)
library(MASS)

##
## Attaching package: 'MASS'
## The following object is masked from 'package:ISLR2':
##
## Boston
## The following object is masked from 'package:dplyr':
##
## select
library(glmnet)

## Loading required package: Matrix
##
## Attaching package: 'Matrix'
## The following objects are masked from 'package:tidyr':
##
## expand, pack, unpack
## Loaded glmnet 4.1-8
```

Exercise 8

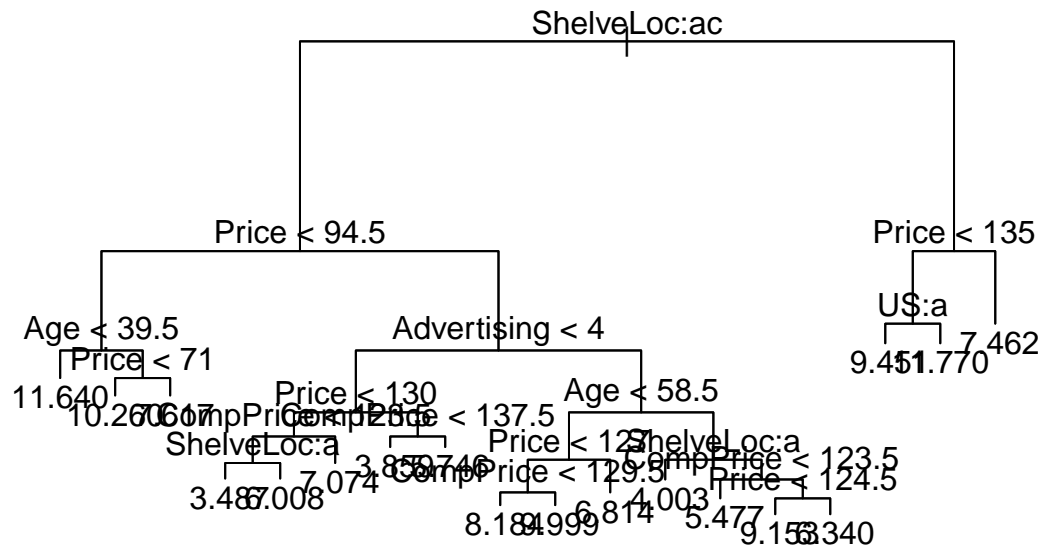
- (a) Split the data set into a training set and a test set.

```
set.seed(1)
train <- sample(1:nrow(Carseats), 200)
test <- Carseats[-train, ]
```

- b) Fit a regression tree to the training set. Plot the tree, and interpret the results. What test MSE do you obtain?

```
tree.carseats <- tree(Sales ~ ., Carseats, subset = train)
tree.pred <- predict(tree.carseats, test)

plot(tree.carseats)
text(tree.carseats)
```



```
sales.test <- Carseats[-train, "Sales"]
mean((tree.pred - sales.test)^2) #test mse
```

```
## [1] 4.922039
```

c) Use cross-validation in order to determine the optimal level of tree complexity. Does pruning the tree improve the test MSE?

```
cv.sales <- cv.tree(tree.carseats) #18 t nodes gives lowest dev
cv.sales
```

```
## $size
## [1] 18 17 16 15 14 13 12 11 10 8 7 6 5 4 3 2 1
##
## $dev
## [1] 831.3437 852.3639 868.6815 862.3400 862.3400 893.4641 911.2580
## [8] 950.2691 955.2535 1039.1241 1066.6899 1125.0894 1205.5806 1273.2889
## [15] 1302.8607 1349.9273 1620.4687
##
## $k
## [1] -Inf 16.99544 20.56322 25.01730 25.57104 28.01938 30.36962
## [8] 31.56747 31.80816 40.75445 44.44673 52.57126 76.21881 99.59459
## [15] 116.69889 159.79501 337.60153
##
## $method
## [1] "deviance"
##
## attr(,"class")
## [1] "prune" "tree.sequence"
```

```
prune.sales <- prune.tree(tree.carseats, best = 18)
prune.pred <- predict(prune.sales, test)
mean((prune.pred - sales.test)^2)
```

```
## [1] 4.922039
```

Cross validation in the tree shows that the optimal level of tree complexity is with 18, terminal nodes, which is the same as the baseline tree in the previous example. These results give the same test MSE.

d) Use the bagging approach in order to analyze this data. What test MSE do you obtain? Use the

importance() function to determine which variables are most important.

```
set.seed(2)
sales.bag <- randomForest(Sales ~ ., Carseats, subset = train,
                          mtry = 11, importance = TRUE)

## Warning in randomForest.default(m, y, ...): invalid mtry: reset to within valid
## range

yhat.bag <- predict(sales.bag, newdata = test)
mean((yhat.bag - sales.test)^2)

## [1] 2.586535

importance(sales.bag)
```

##		%IncMSE	IncNodePurity
##	CompPrice	25.7444826	170.66690
##	Income	4.7699543	90.24851
##	Advertising	12.3934240	104.10313
##	Population	-2.0752891	59.47636
##	Price	55.2142251	505.28250
##	ShelveLoc	47.2885836	376.66846
##	Age	17.0744805	156.14233
##	Education	2.1151607	45.21764
##	Urban	-0.9618328	8.82340
##	US	5.3624960	18.75708

- e) Use random forests to analyze this data. What test MSE do you obtain? Use the importance() function to determine which variables are most important. Describe the effect of m , the number of variables considered at each split, on the error rate obtained.

```
set.seed(3)
sales.rf <- randomForest(Sales ~ ., Carseats, subset = train,
                         mtry = 4, importance = TRUE) #using p/3 predictors

yhat.rf <- predict(sales.rf, newdata = test)
mean((yhat.rf - sales.test)^2)

## [1] 2.767523

importance(sales.rf)
```

##		%IncMSE	IncNodePurity
##	CompPrice	17.6331422	156.63313
##	Income	4.3070432	123.34376
##	Advertising	10.5447183	105.29972
##	Population	-3.0173206	85.60279
##	Price	41.8071333	434.89549
##	ShelveLoc	39.2121869	319.15366
##	Age	13.4325285	170.17609
##	Education	0.6734260	62.70388
##	Urban	-0.8419159	13.75924
##	US	5.8004774	29.65185

Random forests seems to be giving less accurate results compare to bagging in this scenerio with an MSE of 2.76 whereas bagging gave a test MSE of 2.58. In the rf regression I used $m = 4$ predictors for the random forests to subset as for regression $m = p/3$. Adjusting m affects the bias-variance tradeoff where a lower m value may increase the trees diversity of predictors which may result in underfitting but our test MSE shows that the model seems to be doing pretty well at $m = p/3$.

f) Now analyze the data using BART, and report your results.

```
library(BART)
x <- Carseats[, 1:11]
y <- Carseats[, "Sales"]
xtrain <- x[train, ]
ytrain <- y[train]

xtest <- x[-train, ]
ytest <- y[-train]
set.seed(1)
bartfit.sales <- gbart(xtrain, ytrain, x.test = xtest)

## *****Calling gbart: type=1
## *****Data:
## data:n,p,np: 200, 15, 200
## y1,yn: 2.781850, 1.091850
## x1,x[n*p]: 10.360000, 1.000000
## xp1,xp[np*p]: 11.220000, 1.000000
## *****Number of Trees: 200
## *****Number of Cut Points: 100 ... 1
## *****burn,nd,thin: 100,1000,1
## *****Prior:beta,alpha,tau,nu,lambda,offset: 2,0.95,0.273474,3,6.80272e-30,7.57815
## *****sigma: 0.000000
## *****w (weights): 1.000000 ... 1.000000
## *****Dirichlet:sparse,theta,omega,a,b,rho,augment: 0,0,1,0.5,1,15,0
## *****printevery: 100
##
## MCMC
## done 0 (out of 1100)
## done 100 (out of 1100)
## done 200 (out of 1100)
## done 300 (out of 1100)
## done 400 (out of 1100)
## done 500 (out of 1100)
## done 600 (out of 1100)
## done 700 (out of 1100)
## done 800 (out of 1100)
## done 900 (out of 1100)
## done 1000 (out of 1100)
## time: 2s
## trcnt,tecnt: 1000,1000

yhat.bart <- bartfit.sales$yhat.test.mean
mean((ytest - yhat.bart)^2)

## [1] 0.1603478
```

Exercise 9

a) Create a training set containing a random sample of 800 observations, and a test set containing the remaining observations.

```
attach(OJ)

set.seed(1)
```

```
train <- sample(1:nrow(OJ), 800)
oj_train <- OJ[train, ]

oj_test <- OJ[-train, ]
```

- b) Fit a tree to the training data, with Purchase as the response and the other variables as predictors. Use the summary() function to produce summary statistics about the tree, and describe the results obtained. What is the training error rate? How many terminal nodes does the tree have?

```
tree.oj <- tree(Purchase ~ ., OJ, subset = train)

summary(tree.oj)
```

```
##
## Classification tree:
## tree(formula = Purchase ~ ., data = OJ, subset = train)
## Variables actually used in tree construction:
## [1] "LoyalCH"      "PriceDiff"    "SpecialCH"    "ListPriceDiff"
## [5] "PctDiscMM"
## Number of terminal nodes: 9
## Residual mean deviance: 0.7432 = 587.8 / 791
## Misclassification error rate: 0.1588 = 127 / 800
```

The training error rate is 0.1588 and there are 9 terminal nodes in the tree model.

- c) Type in the name of the tree object in order to get a detailed text output. Pick one of the terminal nodes, and interpret the information displayed.

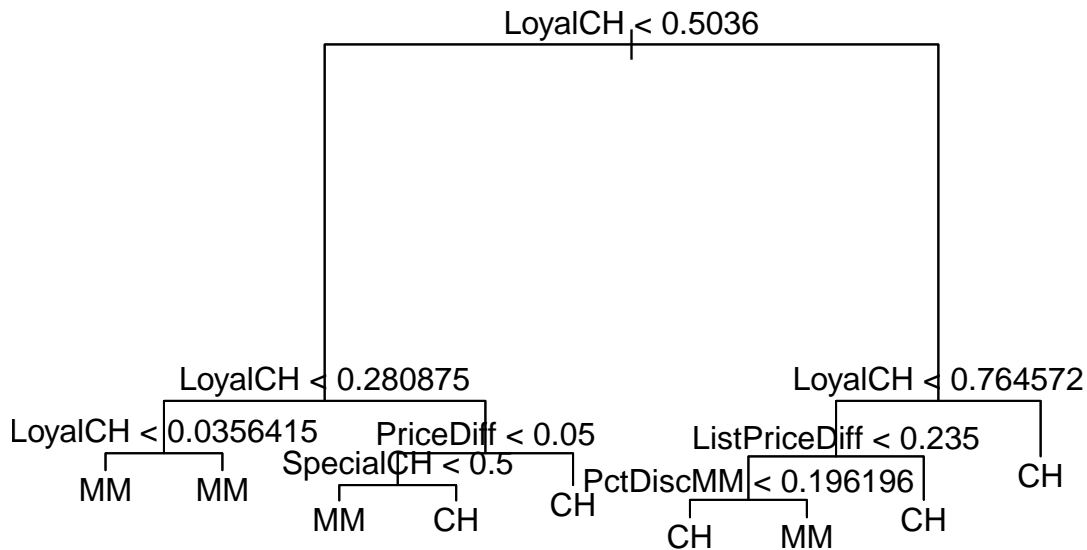
```
tree.oj

## node), split, n, deviance, yval, (yprob)
##      * denotes terminal node
##
## 1) root 800 1073.00 CH ( 0.60625 0.39375 )
##    2) LoyalCH < 0.5036 365 441.60 MM ( 0.29315 0.70685 )
##      4) LoyalCH < 0.280875 177 140.50 MM ( 0.13559 0.86441 )
##        8) LoyalCH < 0.0356415 59 10.14 MM ( 0.01695 0.98305 ) *
##        9) LoyalCH > 0.0356415 118 116.40 MM ( 0.19492 0.80508 ) *
##      5) LoyalCH > 0.280875 188 258.00 MM ( 0.44149 0.55851 )
##        10) PriceDiff < 0.05 79 84.79 MM ( 0.22785 0.77215 )
##          20) SpecialCH < 0.5 64 51.98 MM ( 0.14062 0.85938 ) *
##          21) SpecialCH > 0.5 15 20.19 CH ( 0.60000 0.40000 ) *
##        11) PriceDiff > 0.05 109 147.00 CH ( 0.59633 0.40367 ) *
##    3) LoyalCH > 0.5036 435 337.90 CH ( 0.86897 0.13103 )
##      6) LoyalCH < 0.764572 174 201.00 CH ( 0.73563 0.26437 )
##        12) ListPriceDiff < 0.235 72 99.81 MM ( 0.50000 0.50000 )
##          24) PctDiscMM < 0.196196 55 73.14 CH ( 0.61818 0.38182 ) *
##          25) PctDiscMM > 0.196196 17 12.32 MM ( 0.11765 0.88235 ) *
##        13) ListPriceDiff > 0.235 102 65.43 CH ( 0.90196 0.09804 ) *
##    7) LoyalCH > 0.764572 261 91.20 CH ( 0.95785 0.04215 ) *
```

From the information displayed in the tree mode, the second terminal node split is LoyalCH < 0.5036 with 365 observation in the branch. The majority class is “MM” with 70.69% of observation being “MM”,

- d) Create a plot of the tree, and interpret the results.

```
plot(tree.oj)
text(tree.oj, pretty = 0)
```



e) Predict the response on the test data, and produce a confusion matrix comparing the test labels to the predicted test labels. What is the test error rate?

```
oj.pred <- predict(tree.oj, oj_test, type = "class")
table(oj.pred, oj_test$Purchase)
```

```
##
## oj.pred  CH  MM
##      CH 160  38
##      MM   8  64
```

```
(160 + 64) / 270
```

```
## [1] 0.8296296
```

f) Apply the cv.tree() function to the training set in order to determine the optimal tree size.

```
cv_oj <- cv.tree(tree.oj, FUN = prune.misclass)
cv_oj
```

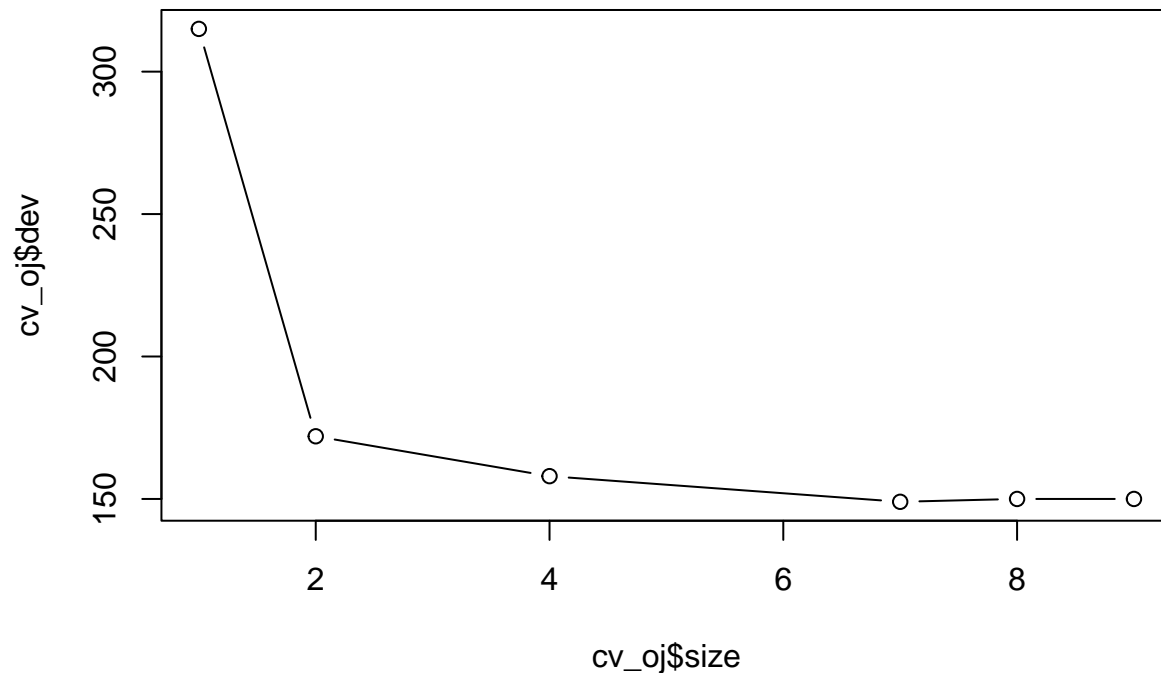
```
## $size
## [1] 9 8 7 4 2 1
##
## $dev
## [1] 150 150 149 158 172 315
##
## $k
## [1] -Inf 0.000000 3.000000 4.333333 10.500000 151.000000
##
## $method
## [1] "misclass"
##
## attr("class")
## [1] "prune" "tree.sequence"
```

From the cross validation on the oj.tree model it is evident that the optimal tree size is one with 7 terminal

nodes since it has the lowest deviance of 149.

g) Produce a plot with tree size on the x-axis and cross-validated classification error rate on the y-axis.

```
plot(cv_oj$size, cv_oj$dev, type = "b")
```



h)

Which tree size corresponds to the lowest cross-validated classification error rate?

A tree with 7 terminal nodes gives the lowest crossvalidated error rate.

i) Produce a pruned tree corresponding to the optimal tree size obtained using cross-validation. If cross-validation does not lead to selection of a pruned tree, then create a pruned tree with five terminal nodes.

```
prune.oj <- prune.tree(tree.oj, best = 7)
prune.pred.oj <- predict(prune.oj, oj_test, type = "class")
table(prune.pred.oj, oj_test$Purchase)
```

```
##
## prune.pred.oj  CH  MM
##              CH 160 36
##              MM   8 66
```

```
(160 + 66) / 270
```

```
## [1] 0.837037
```

j) Compare the training error rates between the pruned and unpruned trees. Which is higher?

The pruned tree gives slightly more accurate predictions with a test MSE of 0.83 which is greater than the unpruned tree with a test MSE of 0.82.

k) Compare the test error rates between the pruned and unpruned trees. Which is higher?

The pruned tree gives slightly more accurate predictions with a test MSE of 0.83 which is greater than the unpruned tree with a test MSE of 0.82.

Exercise 10

- a) Remove the observations for whom the salary information is unknown, and then log-transform the salaries.

```
Hitters <- Hitters

Hitters <- Hitters[!is.na(Hitters$Salary), ]

Hitters$log_sal <- log(Hitters$Salary)
```

- b) Create a training set consisting of the first 200 observations, and a test set consisting of the remaining observations.

```
set.seed(2)
hitters.train <- Hitters[1:200, ]
hitters.test <- Hitters[201:nrow(Hitters), ]
```

- c) Perform boosting on the training set with 1,000 trees for a range of values of the shrinkage parameter . Produce a plot with different shrinkage values on the x-axis and the corresponding training set MSE on the y-axis.

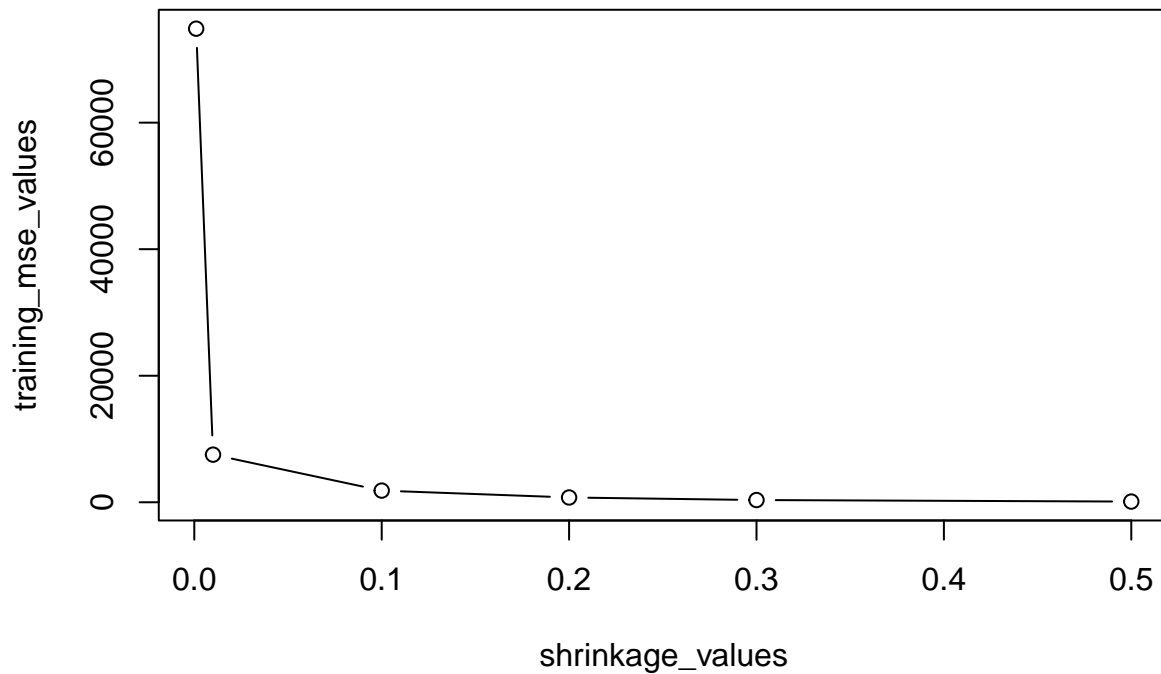
```
set.seed(1)

shrinkage_values <- c(0.001, 0.01, 0.1, 0.2, 0.3, 0.5)

training_mse_values <- numeric(length(shrinkage_values))

for (i in seq_along(shrinkage_values)) {
  hitters.boost <- gbm(Salary ~ ., data = hitters.train,
    distribution = "gaussian",
    n.trees = 1000,
    shrinkage = shrinkage_values[i],
    cv.folds = 0)
  predictions <- predict(hitters.boost, hitters.train, n.trees = 1000)
  training_mse_values[i] <- mean((hitters.train$Salary - predictions)^2)
}

plot(shrinkage_values, training_mse_values, type = "b")
```



d) Produce a plot with different shrinkage values on the x-axis and the corresponding test set MSE on the y-axis.

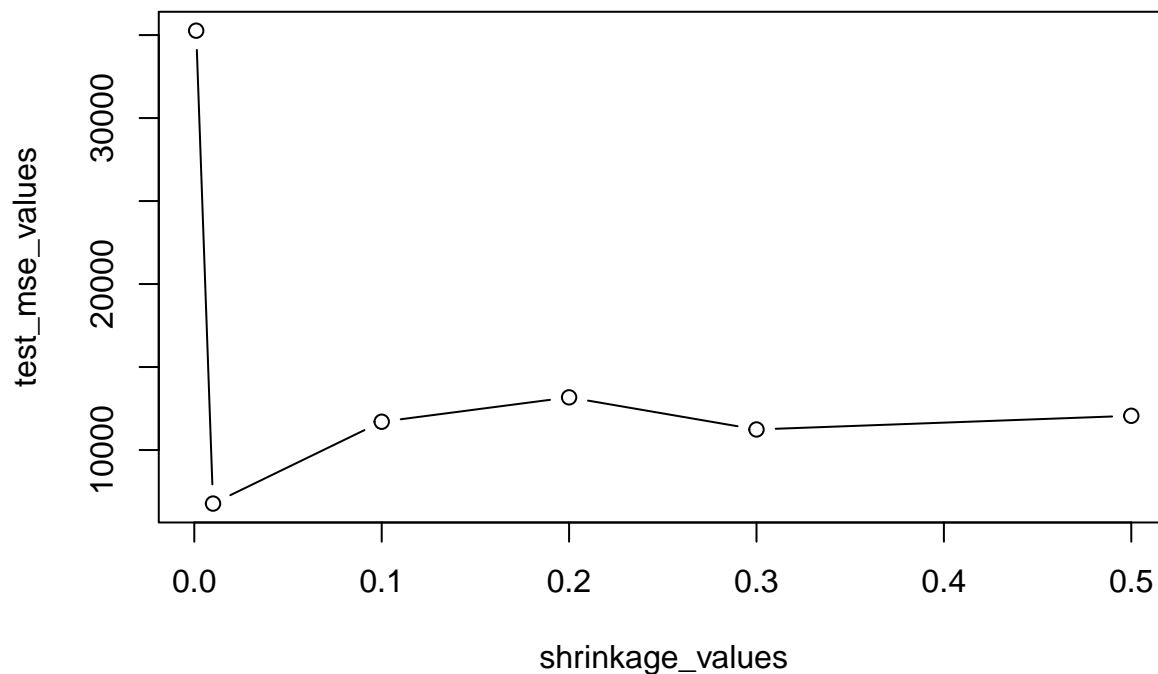
```
set.seed(1)

shrinkage_values <- c(0.001, 0.01, 0.1, 0.2, 0.3, 0.5)

test_mse_values <- numeric(length(shrinkage_values))

for (i in seq_along(shrinkage_values)) {
  hitters.boost <- gbm(Salary ~ ., data = hitters.train,
    distribution = "gaussian",
    n.trees = 1000,
    shrinkage = shrinkage_values[i],
    cv.folds = 0)
  predictions <- predict(hitters.boost, hitters.test, n.trees = 1000)
  test_mse_values[i] <- mean((hitters.test$Salary - predictions)^2)
}

plot(shrinkage_values, test_mse_values, type = "b")
```



e)

Compare the test MSE of boosting to the test MSE that results from applying two of the regression approaches seen in Chapters 3 and 6.

Linear Regression

```
hiiter.lm <- lm(Salary ~ ., data = hitters.train)

lm.pred <- predict(hiiter.lm, hitters.test)

lm.mse <- mean((hitters.test$Salary - lm.pred)^2)
lm.mse
```

```
## [1] 25671.77
```

Ridge Regression

```
x_train <- model.matrix(Salary ~ ., hitters.train)[, -1]
y_train <- hitters.train$Salary
x_test <- model.matrix(Salary ~ ., hitters.test)[, -1]
y_test <- hitters.test$Salary

hitters.ridge <- glmnet(x_train, y_train, alpha = 0)

ridge_cv <- cv.glmnet(x_train, y_train, alpha = 0)
ridge_best_lambda <- ridge_cv$lambda.min
ridge_pred <- predict(hitters.ridge, s = ridge_best_lambda, newx = x_test)

ridge_mse <- mean((y_test - ridge_pred)^2)
ridge_mse
```

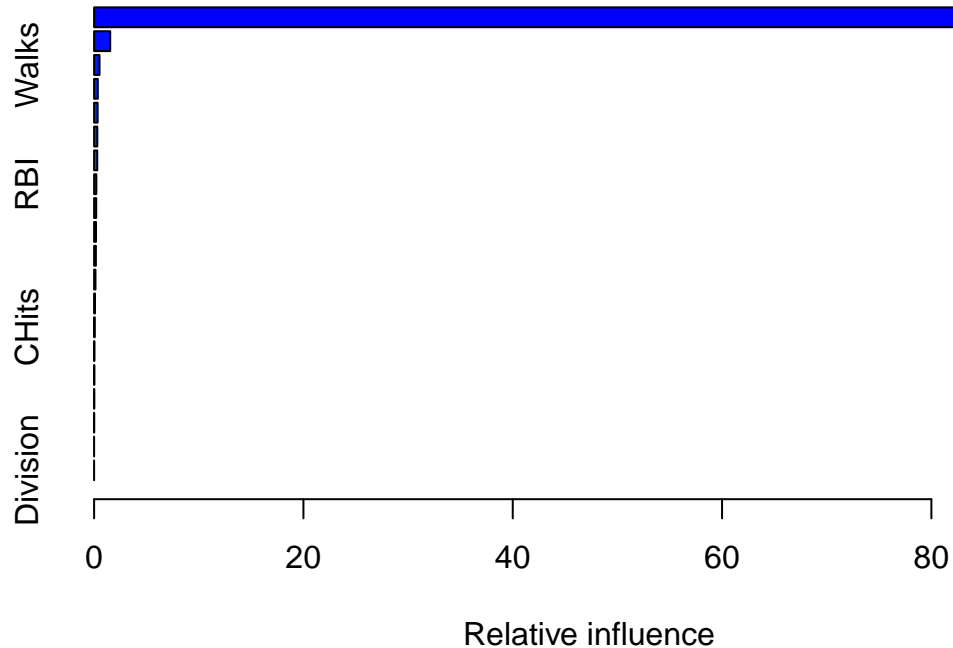
```
## [1] 22435.18
```

f) Which variables appear to be the most important predictors in the boosted model?

```
set.seed(1)
hitters_boost <- gbm(Salary ~ ., data = hitters.train,
```

```
distribution = "gaussian",
n.trees = 1000,
shrinkage = 0.01)
```

```
summary(hitters_boost)
```



```
##          var      rel.inf
## log_sal    log_sal 95.544435081
## CHmRun     CHmRun  1.539665915
## Walks      Walks  0.517006328
## PutOuts    PutOuts 0.354206461
## Hits       Hits   0.335534795
## HmRun      HmRun   0.305691009
## CRBI       CRBI    0.299709909
## RBI        RBI     0.213106408
## CWalks     CWalks  0.202228969
## AtBat      AtBat   0.181865959
## CAtBat     CAtBat  0.170949297
## CRuns      CRuns   0.130009679
## Runs       Runs    0.084030327
## CHits      CHits   0.059506972
## Assists    Assists 0.025810291
## Errors     Errors  0.020435896
## Years      Years   0.012497069
## NewLeague  NewLeague 0.003309634
## League     League  0.000000000
## Division   Division 0.000000000
```

log_sal and CHmRun seem to be the most important variables.

g) Now apply bagging to the training set. What is the test set MSE for this approach?

```
set.seed(2)
hitters_bag <- randomForest(Salary ~ ., hitters.train,
```

```
      mtry = 20, importance = TRUE)
hitters.yhat<- predict(hitters.bag, newdata = hitters.test)
mean((hitters.yhat - hitters.test$Salary)^2)
```

```
## [1] 398.7028
```