**Maclaurin theory with a point core: numerical and analytical results for**

Relation between polar radius and equatorial radius :

Let

*closed analytic expressions for the*

For the Maclaurin spheroid the only contribution to the multipole moments comes from the oblate region :

Then

For a Maclaurin spheroid, is an ellipse:

where

Note

We derive a more general expression for by requiring it to correspond to a level surface. The total potential in the corotating frame is

and we set this equal to the polar potential

thus yielding the following implicit equation for

Define

then to solve the implicit equation for a given value of , we find the value of that gives

or

where .

TEST 1

1. Adopt a value for .
2. From exact Maclaurin theory, compute .
3. From exact Maclaurin theory, compute .
4. Numerically solve for .
5. Results should agree with

This works extremely well, as verified by IDL program *test\_xiofmu3.pro* . The difference between from 4 and from 5 is at the level of the neglected terms in the expansion, with *k* >14, or . This is still well above the numerical noise floor for double-precision IDL calculations (), and far below the limiting precision of Juno gravity data .

TEST 2

1. Adopt values for .
2. Compute from .
3. Set up for numerical quadrature

or

or, write

and compare with analytic answer. Agreement for to within the previous limits!

TEST 3

1. Adopt Test 2 values for and and thus value for
2. Adopt a provisional value
3. Solve for referencing the equatorial potential:

or

with the parameterization that

4. The rest of the calculation of the

is as before. Then compare with analytic answer (below). Agreement for to within the previous limits!

TEST 4 – Jupiter Maclaurin model with a delta-function core

1. Adopt Test 2 values for and and thus value for
2. Adopt a provisional value

etc.

1. Solve for referencing the equatorial potential:

or

all the same as before. However, the calculation of the must be weighted with the (reduced) value of , as follows:

with

where is the delta-function core mass. Rewrite in dimensionless form:

or

Now

Note:

Check analytic expression:

Correct. Confirmed by int\_2d routine.

The crossed-out statement is WRONG!

~~Important result: introduction of the point core does not disturb the Maclaurin result: the surface shape of the object is described by an ellipse with equatorial radius and polar radius . Here is numerical proof:~~

: introduction of the point core DOES disturb the Maclaurin result: the surface shape of the object is NOT described by an ellipse!