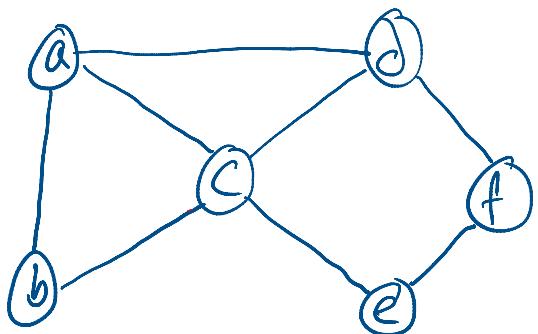


## Week 13: Graphs

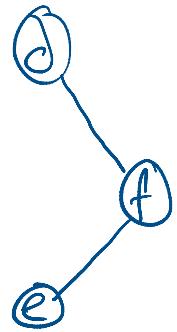
Friday, December 4, 2020 9:50 AM

ADT → nodes  
→ vertices



a: b, c, d  
b: a, c  
c: a, b, d, e  
d: a, c, f  
e: c, f  
f: d, e

|   | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| a | 0 | 1 | 1 | 1 | 0 | 0 |
| b | 1 | 0 | 1 | 0 | 0 | 0 |
| c | 1 | 1 | 0 | 1 | 1 | 0 |
| d | 1 | 0 | 1 | 0 | 0 | 1 |
| e | 0 | 0 | 1 | 0 | 0 | 1 |
| f | 0 | 0 | 0 | 1 | 1 | 0 |



\* undirected graph:  
symmetrical matrix

## Types of Graphs:

- undirected
  - can go both directions on an edge
- directed
  - each edge has direction
- weighted / unweighted
  - edges have a "cost" associated with using them

## Data Structure Representations

→ adjacency matrix

- rows / columns for each node
- true if there is an edge
- b/n 2 nodes

→ adjacency list

- table / list of nodes
- each node has a list of nodes adjacent to it

\* hash map

- \* much more space efficient

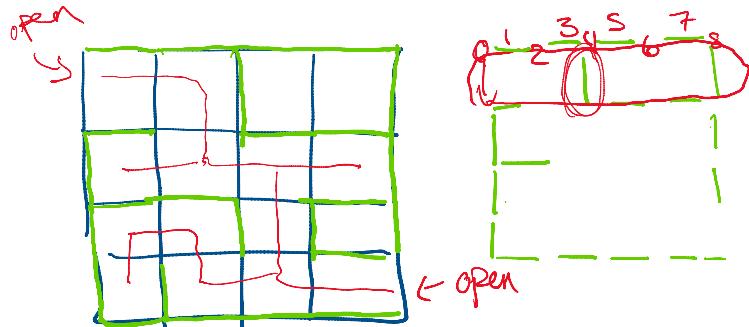
## Operations

→ neighbors ( $v$ ):  
 → adjacency list:  $O(\deg(v))$   
 → matrix:  $O(n)$

→ check if 2 nodes ( $u, v$ ) are adjacent  
 → list:  $O(\deg(v))$   
 → matrix:  $O(1)$   
 $m(u, v)$

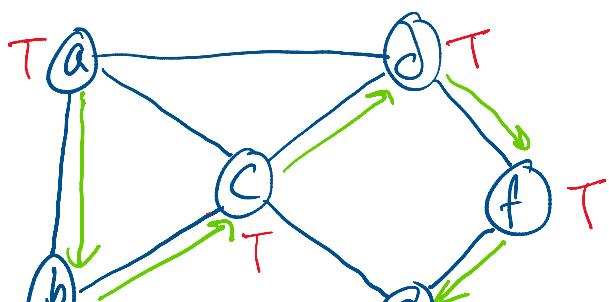
→ add node to graph  
 → list:  $O(1)$  or  $O(\deg(v))$   
 → matrix:  $O(n)$

→ remove node  
 → list:  $O(\deg(v))$   
 → matrix:  $O(n)$



open: is there an opening in the row

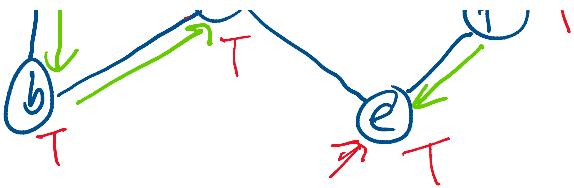
copen: is there an opening in the column?



DFS( $G$ ):

for each node  $v$ :  
 $\text{visited}[v] \leftarrow F$  \*

for each node  $v$ :  
 if  $\text{visited}[v]$ : \*  
 $\text{explore}(G, v)$  \*



## Search methods

### Depth-First Search

- Stack
- Starts at a node
- visits a neighbor
- then a neighbor of neighbor
- keeps following a path until it hits a deadend
- back tracks

if  $\text{!visited}[v]$ :  
explore  $(G, v)$

explore  $(G, v)$ :

$\text{visited}[v] \leftarrow T$

for each neighbor

$u$  of  $v$ :

if  $\text{!visited}[u]$ :

\* explore  $(G, u)$

order: a, b, c, d, f, e

- \* usually does not find best path
- \* will find a solution if it exists

### Breadth-First

- queue
- starts at beginning node
- visits all neighbors
- chooses 1 neighbor &
- visits all of its neighbors

\* level - search