Investigating the Decay of an Atmospheric Carbon Pulse

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February 22, 2017

Abstract

This report examines a numerical model of the decay of atmospheric carbon dioxide presented by Joos, et al (2013),[2] and utilized by the Fifth Assessment Report by the Intergovernmental Panel on Climate Change (2014).[1] Utilizing three numerical techniques, the bisection method, Newton's method, and the secant method, we verify that the numerical model shows a 40% reduction in an atmospheric carbon pulse over 20 years. Moreover, the model shows that 25% of the initial carbon pulse still remains after 759 years.

1 Introduction

In order to model the climatological impact of anthropogenic gases such as carbon dioxide, we need to quantify the decay of such emissions in the atmosphere. Joos, et al (2013), estimate the decay of a pulse of carbon dioxide in the atmosphere using the following function,

$$f(t) = a_0 + \sum_{i=1}^{3} a_i \cdot e^{-t/\tau_i} \tag{1}$$

where $0 \leq t \leq 1000$ years, $a_0 = 0.2173$, $a_1 = 0.2240$, $a_2 = 0.2824$, $a_3 = 0.2763$, $\tau_1 = 394.4$ years, $\tau_2 = 36.54$ years, and $\tau_3 = 4.304$ years. We have that $f(0) = 1$, which represents 100% of an initial carbon pulse of 100Gt of Carbon. A plot of $f(t)$ and $g(t) = 0.6$ is shown in Figure 1 in the Appendix. The values for each a_i and τ_i are mean values found from a 15-model comparison conducted by Joos, et al (2013).

2 Methods

We begin by defining $f^*(t) = f(t) - 0.6$ so that $f^*(t) = 0$ represents 60% of the initial carbon pulse. For the bisection method, we choose an interval of $[19, 21]$ and divide it in half for the first approximation so that $t_0 = 19 + \frac{21-19}{2} = 20$. We evaluate the product, $f^*(19) * f^*(20)$ to determine whether or not the root occurs in the first half of the interval. If so, we make this the new interval; if not, we take the second half as our new interval. As with the next two methods, this process continues until our approximation is within 10^{-4} years of the previous approximation.

For Newton's Method, we choose an initial approximation, $t_0 = 19$. Then, we let

$$t_{n+1} = t_n - \frac{f^*(t_n)}{f^{*\prime}(t_n)} = t_n - \frac{a_0 + \sum_{i=1}^{3} a_i \cdot e^{-t_n/\tau_i}}{\sum_{i=1}^{3} -\frac{a_i}{\tau_i} \cdot e^{-t_n/\tau_i}}. \tag{2}$$

Lastly, for the secant method, we begin with two initial approximations, $t_0 = 19$, and

$t_1 = 21$, and find the next approximation as follows:

$$t_{n+2} = t_{n+1} - (a_0 + \sum_{i=1}^{3} a_i \cdot e^{-t_{n+1}/\tau_i}) \cdot \left(\frac{t_{n+1} - t_n}{\sum_{i=1}^{3} a_i \cdot (e^{-t_{n+1}/\tau_i} - e^{-t_n/\tau_i})} \right). \tag{3}$$

3 Example

The approximation, t_2, of each method is shown in Figure 2 in the Appendix. Note that the approximation, t_2, is the second iteration of our procedure for both the bisection method and Newton's method, while t_2 is the first iteration for the secant method. For our chosen initial approximations, we have that $t_2 = 19.2500$ for the bisection method, $t_2 \approx 19.3420$ for Newton's method, and $t_2 \approx 19.3559$ for the secant method. The first approximation within our chosen error of 10^{-4} years is 19.3420 years for all three methods. This value represents the approximation t_{14} for the bisection method, t_3 for Newton's Method, and t_4 for the secant method.

When we redefine $f^*(t)$ as $f(t) - 0.25$ to estimate when the carbon pulse will decay to 25% of its original value, we find that all three methods converge on a value of 758.9325 years. While Newton's method and the secant method converge by t_4 and t_6, respectively, the bisection method does not converge until t_{22}.

4 Discussion

In both examples, we find that the bisection method converges more slowly than Newton's method and the secant method. The error for Newton's method and the secant method converge quadratically, while the bisection method converges exponentially. While Joos, et al, propose that the carbon pulse takes 1000 years to reduce to 25%, we find that the model shows a reduction by 75% of the initial carbon pulse after 759 years. However, we have that $f(1000) \approx 0.2350$, very near to 25%, indicating a surprisingly slow rate of decrease over the latter portion of the millenium.

5 References

[1] IPCC, 2013: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Stocker, TF, Qin, D, et al (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 1535 pp.

[2] Joos F, Roth R, et al. Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis. Atmospheric Chemistry and Physics. 2013 Mar 8;13(5):2793-825.

6 Appendix

6.1 Figures

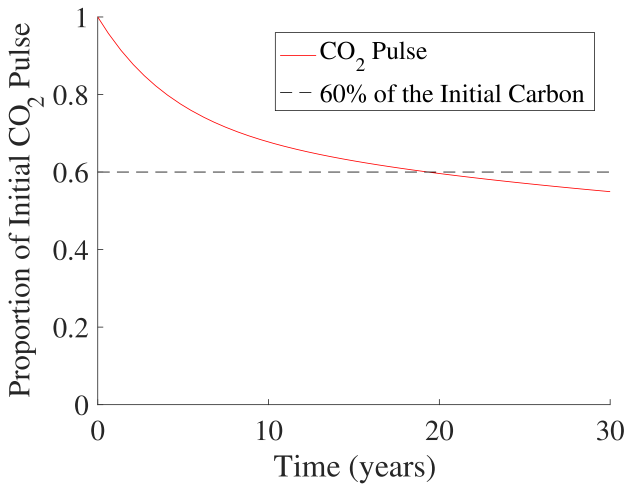

Figure 1: Equation (1) is plotted along side the function, $g(t) = 0.6$, indicating 60% of the initial pulse of carbon.

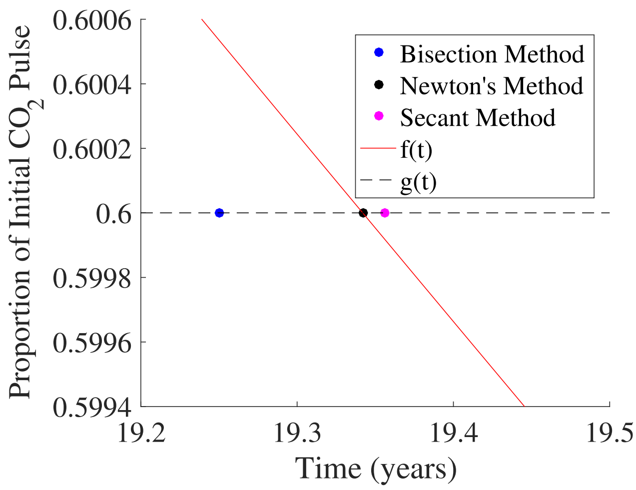

Figure 2: We have that $t_2 = 19.2500$ for the bisection method, $t_2 \approx 19.3420$ for Newton's method, and $t_2 \approx 19.3559$ for the secant method.

6.2 MATLAB Code

MATLAB code is provided on GitHub at https://github.com/nmschroeder.