Math 215, Spring 2018 - Assignment # 9

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Section 11.1 8 **Statement:** Define a relation on Z as xRy if |x-y| < 1. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why. What familiar relation is this?

Proof. i) Show that R is reflective:
For any x ∈ Z, then $|x - x| = 0 < 1$. Then xRx

ii) Show that R is symmetric
For any x,y ∈ Z
Assume that xRy that is |x-y| < 1
Now |x-y| = |y-x| < 1. Then yRx

iii) Show that R is transitive
For any x,y,z ∈ Z
We have: |x-y| < 1 x, y Z when x = y.
Assume that xRy and yRz, that is |x-y| < 1 and |y-z| < 1.
Then we must have x = y = z. So |x-z| = 0 < 1, that means xRz. Therefore, R is transitive.

□

Section 11.1 12 **Statement:** Prove that the relation | (divides) on the set Z is reflexive and transitive. (Use Example 11.8 as a guide if you are unsure of how to proceed.)

Proof. For any x ∈ Z, x = 1x, so x|x, hence the relation is reflexive. Suppose x, y, z ∈ Z are such that x|y and y|z. Then there exist integers a and b such that xa = y and yb = z. Then ab ∈ Z and x(ab) = z, so x|z. We conclude the relation is transitive. □

Section 11.2 6 **Statement:** There are five different equivalence relations on the set A = {a,b,c}. Describe them all. Diagrams will suffice.

Proof. R = {(a,a), (b,b), (c,c)}

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R = {(a, a), (b,b), (c, c), (a,c), (c,a), (b,a), (a,b) (b,c), (c,b)}

□

Section 11.2 8 **Statement:** Define a relation R on Z as xR y if and only if $x^2 + y^2$ is even. Prove R is an equivalence relation. Describe its equivalence classes.

Proof. We must show that R is reflexive, symmetric and transitive.
i) Show that R is reflexive; i.e., show that ∀ x ∈ Z, xRx:
Let x ∈ Z. Then $x^2 + x^2 = 2x^2$, which is even, so xRx.

ii) Show that R is symmetric; i.e., show that ∀ x, y ∈ Z, if xRy then yRx.
Let x, y ∈ Z
Assume xRy that is $x^2 + y^2$ is even
Now $y^2 + x^2 = x^2 + y^2$. So $y^2 + x^2$ is even and yRx.

iii) Show that R is transitive; i.e., show that ∀ x, y, z ∈ Z, if xRy and yRz then xRz. Let x, y, z ∈ Z. Assume xRy and yRz, that is that $x^2+y^2 = 2s$ and $y^2 + z^2 = 2t, s, t \in Z$. So $x^2 = 2^s y^2$ and $z^2 = 2^t y^2$, with s, t ∈ Z
And $x^2 + z^2 = 2^s y^2 + 2^t y^2 = 2s + 2t2y^2 = 2(s + ty^2) wheres + ty^2 \in Z$
So $x^2 + z^2$ is even. Therefore xRz
This proves that R is an equivalence relation on Z.
= {x : xR1} = {x : x^2 + 1 is even} = {x : x^2 is odd } = {x : x is odd }
= {x : xR2} = {x : x^2 + 4 is even} = {x : x^2 is even} = {x : x is even} So these are the two distinct equivalence classes □

Section 11.3 2 **Statement:** List all the partitions of the set A = {a,b,c}. Compare your answer to the answer to Exercise 6 of Section 11.2.

Proof. {{a}, {b}, {c}}
{{a, b}, {c}}
{{a}, {b, c}}
{{a, c}, {b}}
{{a, b, c}}

□

Section 11.4 2 **Statement:** Write the addition and multiplication tables for $\mathbb{Z}_3$.

Proof. Below are table of addition and multiplication for $\mathbb{Z}_3$

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

.	0	1	2
0	0	0	0
1	0	1	1
2	0	2	1

□

Section 11.4 6 **Statement:** Suppose [a],[b] $\in \mathbb{Z}_6$ and [a][b] = [0]. Is it necessarily true that either [a] = [0] or [b] = [0]?

Proof. Here is the multiplication table of $\mathbb{Z}_6$

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	0	4
5	0	5	4	3	4	1

As we can see, if [a][b] = [0], then possibly, [a] = [2] and [b] = 3. It means that it is not necessarily true that either [a] = [0] or [b] = [0] if [a][b] = [0].

$\square$