DEFINITIONS 10

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1. Orthonormal Vectors
   Vectors $\vec{v_1}$,...,$\vec{v_n}$ are said to be orthonormal when they are unit vectors and $\vec{v_i} \perp \vec{v_j}$ for all $i \neq j$

   $$\vec{v_i}.\vec{v_j} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

   Example:

   $$\vec{v_1} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \vec{v_1} = \begin{bmatrix} -1 & 1 & 5 \end{bmatrix}$$

   are orthonormal since $\vec{v_1}.\vec{v_2} = 1.(-1) + 1.1 + 0.5 = 0$

2. A square matrix Q is said to be an orthogonal matrix when its columns form an orthonormal basis for R^n

   Ex) 2x2 matrix

   $$S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

   is an orthogonal matrix since its column vectors are linearly independent, orthogonal to each other and span of them equal R^2.

3. For subspace W, the orthogonal complement of W is given by $W^{\perp} = \{\vec{v} : \vec{v} \perp W\}$
   example:

   $$W = span\left(\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}\right)$$
   $$W^{\perp} = span\left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}\right) = null\left(\begin{bmatrix} 1 & 2 & 4 \\ 5 & 1 & 2 \end{bmatrix}\right)$$