DEFINITIONS 8

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1. Definition of determinant:
   i) $det([a_{11}]) = a_{11}$
   ii)$det(A) = \sum_{j=1}^{n} a_{1j}(-1)^{1+j}det(M_{ij})$

2. The adjoint of A is the transpose of the matrix of cofactors of A. (adj(A))

3. The characteristic polynomial of A is the $det(A - \lambda I)$

4. The algebraic multiplicity of an eigenvalue of A is the number of times its factor occurs in $det(A - \lambda I)$

$$A = \begin{bmatrix} 4 & 10 & 0 & -2 & 3 & 1 & -5 \\ -3 & 5 & -5 & 7 & 12 & -2 & 5 \\ 5 & 5 & 0 & 0 & 5 & -5 & 0 \\ -9 & 5 & -7 & 3 & 3 & 11 & 0 \\ 10 & 0 & 6 & -6 & -6 & -6 & -5 \\ 6 & 10 & 1 & -1 & 4 & -2 & -5 \\ 3 & 12 & -4 & 6 & 16 & -7 & 3 \end{bmatrix}$$

a) By using Maple, we calculate: $rref(A) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0 & 0 & 0 & -0.5 \\ 0 & 0 & 1 & 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$=> det(rref(A)) = 0$

Hence, $det(A) = 0$

b) By using Maple, we have

$$adj(A) = \begin{bmatrix} 600 & 600 & -600 & 0 & 600 & -600 & 0 \\ 600 & 600 & -600 & 0 & 600 & -600 & 0 \\ 600 & 600 & -600 & 0 & 600 & -600 & 0 \\ 600 & 600 & -600 & 0 & 600 & -600 & 0 \\ -600 & -600 & 600 & 0 & -600 & 600 & 0 \\ 600 & 600 & -600 & 0 & 600 & -600 & 0 \\ 1200 & 1200 & -1200 & 0 & 1200 & -1200 & 0 \end{bmatrix}$$

c) The characteristic polynomial of A: $\lambda(\lambda-5)^2(\lambda-3)(\lambda+2)^3$

d) The eigenvalues of A: $0, 5, 3, -2$

e) The bases for the eigenspaces associated with the eigenvalues of A:

$$E_0 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 2 \end{bmatrix} \right\}, E_3 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right\}, E_5 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}, E_{-2} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

f) For $\lambda = 0$, the geometric multiplicity is: 1 and the algebraic multiplicity is: 1
For $\lambda = 3$, the geometric multiplicity is: 1 and the algebraic multiplicity is: 1
For $\lambda = 5$, the geometric multiplicity is: 2 and the algebraic multiplicity is: 2
For $\lambda = -2$, the geometric multiplicity is: 1 and the algebraic multiplicity is: 3