Math 215, Spring 2018 - Assignment # 9

Name: Hieu Tran

11.1 **Exercise 8:** Define a relation on $\mathbb{Z}$ as xRy if $|x-y|<1$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why. What familiar relation is this?

Proof. Relation R is reflexive, because if $n \in \mathbb{Z}$, then $|n-n| = 0 < 1$, so nRn.
Relation R is symmetric, because if $m, n \in \mathbb{Z}$ such that mRn, then $|n-m| = |m-n| \leq 1$, and so nRm.
Relation R is transitive, because $|x-y| < 1$ for $x, y \in \mathbb{Z}$ when $x = y$. Thus, suppose aRb and bRc, then we must have $a = b = c$. So $|a-c| = 0 < 1$, that means aRc. Therefore, R is transitive. □

11.1 **Exercise 12:** Prove that the relation $|$ (divides) on the set $\mathbb{Z}$ is reflexive and transitive.

Proof. Relation $|$ is reflexive because $x|x\ \forall x \in \mathbb{Z}$.
We will show that $|$ is transitive. Suppose $x|y$ and $y|z$. That means $y = ax$ and $z = by$ for some integers a, b. Thus we have $z = by = b(ax) = (ba)x$, that means $x|z$. Therefore $|$ is reflexive. □

11.2 **Exercise 6:** There are five different equivalence relations on the set $A = \{a, b, c\}$. Describe them all. Diagrams will suffice.

Proof. □

11.2 **Exercise 8:** Define a relation R on $\mathbb{Z}$ as xRy if and only if $x^2 + y^2$ is even. Prove R is an equivalence relation. Describe its equivalence classes.

Proof. To show that R is an equivalance relation, we must show that R is reflexive, symmetric, and transitive.
If $x \in \mathbb{Z}$, then $x^2 + x^2 = 2x^2$ and since $x^2 \in Z$, $x^2 + x^2$ is even, we have xRx and so R is reflexive.
Suppose that $x, y \in \mathbb{Z}$ such that xRy. Then we have $x^2 + y^2 = y^2 + x^2$ is even. So yRx, and R is symmetric.
Suppose that $x, y, z \in \mathbb{Z}$ such that xRy and yRz. Then $x^2 + y^2$ is even and $y^2 + z^2$ is even. So, $x^2 + y^2 = 2a$ and $y^2 + z^2 = 2b$ for some integers a and b. Then we have that $x^2 = 2ay^2$ and $z^2 = 2b - y^2$, so $x^2 + z^2 = 2a - y^2 + 2b - y^2 = 2(a + b - y^2)$. Since $a + by \in \mathbb{Z}$, $x^2 + y^2$ is even and so xRz. Therefore, R is transitive.
The distinct equivalence classes of R are:
$[0] = \{x \in \mathbb{Z} :$ x is even$\}$
$[1] = \{x \in \mathbb{Z} :$ x is odd$\}$ □

11.3 **Exercise 6:** List all the partitions of the set $A = \{a, b, c\}$.

Proof. $\{\{a\}, \{b\}, \{c\}\}, \{\{a\}, \{b, c\}\}, \{\{a, b\}, \{c\}\}, \{\{a, c\}, b\}, \{\{a, b, c\}\}$ $\square$

11.4 **Exercise 2:** Write the addition and multiplication tables for $\mathbb{Z}_3$.

Proof. Addition table for $\mathbb{Z}_3$:

$+$	$[0]$	$[1]$	$[2]$
$[0]$	$[0]$	$[1]$	$[2]$
$[1]$	$[1]$	$[2]$	$[0]$
$[2]$	$[2]$	$[0]$	$[1]$

Multiplication table for $\mathbb{Z}_3$:

$\cdot$	$[0]$	$[1]$	$[2]$
$[0]$	$[0]$	$[0]$	$[0]$
$[1]$	$[0]$	$[1]$	$[2]$
$[2]$	$[0]$	$[2]$	$[1]$

$\square$

11.4 **Exercise 6:** Suppose $[a], [b] \in \mathbb{Z}_6$ and $[a] \cdot [b] = [0]$. Is it necessarily true that either $[a] = [0]$ or $[b] = [0]$?.

Proof. No because we have $[2], [3] \in \mathbb{Z}_6$ and $[2] \cdot [3] = [0]$ but both $[2] \neq [0]$ and $[3] \neq [0]$. $\square$