DEFINITIONS 5

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1. Define what is meant by an elementary matrix and give a specific example of each of the three types of elementary matrices.
An elementary matrix is one that has been created by performing exactly one elementary row operation to I.
Example swap rows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example Multiplying a Row by a Number/scalar:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example Adding Rows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Define the term subspace.

A nonempty set of vectors S is called a subspace if the assumptions that $\vec{u}$ and $\vec{v}$ are in S and α is a scalar imply that
a. $\vec{u} + \vec{v}$ is in S, and
b. $\alpha\vec{u}$ is in S.

3. Define the column space of a matrix and give an example of an mxn matrix whose column space is R^m

The column space of an mxn matrix A is the span of the columns of A, which we denote as col(A). Another way to express this is that it is $\{A\ \vec{x} : \vec{x} \in R^n\}$

Example
Example (using Maple to caculate rref)

$$A = \begin{bmatrix} 1 & -10 & -24 & -42 \\ 1 & -8 & -18 & -32 \\ -2 & 20 & 51 & 87 \end{bmatrix} --> rref(A) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

There are three pivot columns (have leading 1's) which are c_1 , c_2, c_3

$$\text{col A} = \{c_1 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + c_2 \begin{bmatrix} -10 \\ -8 \\ 20 \end{bmatrix} + c_3 \begin{bmatrix} -24 \\ -18 \\ 51 \end{bmatrix} \mid : c_i \in \Re^3\}$$

$$= \text{span} \left(\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -10 \\ -8 \\ 20 \end{bmatrix}, \begin{bmatrix} -24 \\ -18 \\ 51 \end{bmatrix} \right) = \{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x, y, z \in \Re^3\} = \Re^3$$

$$\text{since the rref of} \begin{bmatrix} 1 & -10 & -24 & x \\ 1 & -8 & -18 & y \\ -2 & 20 & 51 & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2z - 8x + 5 \\ 0 & 1 & 0 & -z - (5/2)x + (1/2)y \\ 0 & 0 & 1 & (1/3)z + (2/3)x \end{bmatrix}$$

4. Define the row space of a matrix and give an example of an m x n matrix whose row space is a proper subspace of R^n

The row space of a matrix A is the column space of A^T. It can also be considered as the span of the row vectors of A and is denoted as row(A).

Example

Using the same Matrix A at the example of def 3 and calculate by Maple
Row(A) is already a subspace of $\Re^4$

$$M = Transpose(A) = \begin{bmatrix} 1 & 1 & -2 \\ -10 & -8 & 20 \\ -24 & -18 & 51 \\ -42 & -32 & 87 \end{bmatrix}$$

$$=> rref(M) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

We can see that there are three pivot columns of M are c_1, c_2, c_3

$$\text{span} \left(\begin{bmatrix} 1 \\ -10 \\ -24 \\ -41 \end{bmatrix}, \begin{bmatrix} 1 \\ -8 \\ -18 \\ -32 \end{bmatrix}, \begin{bmatrix} -2 \\ 20 \\ 51 \\ 87 \end{bmatrix} \right) \neq \Re^4. \text{ Hence it is a proper subspace of } \Re^4$$

since the rref of $\begin{bmatrix} 1 & 1 & -2 & x \\ -10 & -8 & 20 & y \\ -24 & -18 & 51 & z \\ -42 & -32 & 87 & t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. Define the null space of a matrix and give a particular example in which you express the null space of your matrix as the span of some vectors.

The null space of an mxn matrix A is null(A) = $\{ \vec{x} \in R^n : A\vec{x} = \vec{0} \}$
Using the Matrix A at the example of def 3
Null space of A = Solution set of Ax = 0

$$Solve[rref(A)|0] = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_4 \\ -2x_4 \\ -x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -1 \\ 1 \end{bmatrix} x_4$

Thus Nullspace of A =

$$Nul(A) = \{x_4 \begin{bmatrix} -2 \\ -2 \\ -1 \\ 1 \end{bmatrix} | : x_4 \in \Re\} = span\left(\begin{bmatrix} -2 \\ -2 \\ -1 \\ 1 \end{bmatrix}\right)$$