DEFINITIONS 9

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1. Define what it means for two matrices to be similar:

   Two (square) matrices A and B are said to be similar if there is an invertible matrix S such that B = $S^{-1}AS$.

   Example:

$$B = \begin{bmatrix} -13 & -8 & -4 \\ 12 & 7 & 4 \\ 24 & 16 & 7 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 1 & 2 \\ -2 & -1 & -3 \\ 1 & -2 & 0 \end{bmatrix}$$

$$\begin{aligned} A &= S^{-1}BS \\ &= \begin{bmatrix} -6 & -4 & -1 \\ -3 & -2 & -1 \\ 5 & 3 & 1 \end{bmatrix} \begin{bmatrix} -13 & -8 & -4 \\ 12 & 7 & 4 \\ 24 & 16 & 7 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -2 & -1 & -3 \\ 1 & -2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{aligned}$$

   Matrix A and B are similar.

2. Define what it means for a matrix to be diagonalizable.

   A (square) matrix is said to be diagonalizable when it?s similar to a diagonal matrix.

   Example:

$$S = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} -1/2 & 3/4 \\ 1/2 & -1/4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

   Since

$$A = S^{-1}.B.S = \begin{bmatrix} 7 & 6 \\ -2 & -1 \end{bmatrix}$$

   A is diagonalizable.

3. Give a condition that will guarantee a matrix is diagonalizable.
   A nxn matrix is diagonalizable if and only if it has n linearly independent eigenvectors.