Biberg model: implementation details

The layout of the algorithm for obtaining the wall and interfacial friction values using Biberg friction model is explained in two alternative ways using pseudo-codes. The first one (Algorithm 1) uses fixed point iteration to minimize the difference between the wall to interface friction ratios for both the phases whereas in the second one (Algorithm 2) a more robust formulation using the no slip condition between the phases. It yields a F(x)=0 type formulation and can be solved using any root search algorithm like Newton- Raphson/ Brent's method .

Algorithm 1: Biberg model implementation: Fixed point iteration

---

1. Take an initial guess τ_{l}, τ_{g} and τ_{lg} from Churchill correlation.
2. Calculate K_{g} and K_{l} using Equation (1).
3. Set a check variable named `convergence` $= 0$ and define a maximum iteration number.

while $\sim$ `convergence` && `iter` $\leq$ `max iter` **do**

4. Calculate R_{g} and R_{l} using Equation (4).
5. Calculate Δ_{il} and Δ_{ig} from Equation (6) by using the values of K_{g}, K_{l} and R_{g}, R_{l} calculated in step 2, 4 and setting $Y = 0$ for the interface.
6. Obtain Λ_{g}, Ψ_{g} , Λ_{l}, Ψ_{l} using Equation (7) and Equation (8) with the values of K_{g}, K_{l} and R_{g}, R_{l} calculated in step 2 and 4.
7. Calculate Λ_{Pois}, $\Lambda_{\mathrm{g,F}}$, $\Lambda_{\mathrm{l,F}}$ from Equation (9).
8. Calculate effective diameter and corresponding Reynold's number for gas and liquid from Equation (10).
9. Calculate λ_{g}, λ_{l}, τ_{g}, τ_{l} and τ_{lg} from Equation (11).
10. Recalculate R_{g} and R_{l} using Equation (4) with updated τ_{g}, τ_{l} and τ_{lg} values.
11. iter $=$ iter $+ 1$.
12. Check the convergence criteria $|R_{\mathrm{g}} - R_{\mathrm{g,old}}| \leq 1e^{-12}$ && $|R_{\mathrm{g}} - R_{\mathrm{g,old}}| \leq 1e^{-12}$ and set *convergence* $= 1$ if that's satisfied.

end

Biberg model: implementation details Contd.

Algorithm 2: Biberg model implementation: implicit approach using no-slip between phases

1. Invoke the function `biberg_noslip` with a variable R_g along with the known values of A_g, A_l, u_g, u_l and CC.
2. This call to `biberg_noslip` returns a univariate function (function of R_g) handle `bib_noslip` as the first output argument in the following steps:
3. Set a check variable named `interface_limit`

while $\sim$ `interface_limit` **do**

4. Calculate K_g and K_l using Equation (1).
5. R_g acts as the input variable to the function call.
6. Calculate Δ_ig from Equation (6) by using the values of K_g calculated in step 4 and variable R_g by setting $Y = 0$ for the interface.
7. Obtain Λ_g, Ψ_g using Equation (7) and Equation (8) with the values of K_g and variable R_g
8. Calculate Λ_Pois, $\Lambda_\mathrm{g,F}$ and $\Lambda_\mathrm{l,F}$ from Equation (9).
9. Calculate effective diameter and corresponding Reynold's number for gas phase from Equation (10).
10. Calculate λ_g and τ_g from Equation (11).
11. Obtain τ_lg from the definition of R_g in Equation (4) using the value of τ_g.
12. Calculate R_l from the obtained value of τ_g and variable R_g using Equation (5).
13. Calculate Δ_il from Equation (6) by using the values of K_l (from step 4) and R_l (calculated in step 12) by setting $Y = 0$ for the interface.
14. Obtain Λ_l, Ψ_l using Equation (7) and Equation (8) with the values of K_l and R_l.
15. Calculate effective diameter and corresponding Reynold's number for liquid phase from Equation (10).
16. Calculate λ_l and τ_l from Equation (11).
17. Calculate the residual in the no-slip condition using Equation (12). This is the univariate function of R_g.
18. Calculate $K_\mathrm{g,min}$ and $K_\mathrm{l,min}$ with the obtained value of τ_lg using Equation (2).
19. Check the smooth interface criteria $K_\mathrm{g,min} \leq K_\mathrm{g}$ && $K_\mathrm{l,min} \leq K_\mathrm{l}$ and set interface_limit = 1 if that's satisfied.

end

20. This residual function can be solved for R_g using any root search algorithm like Newton- Raphson/ Brent's method.
21. Call the function `biberg_noslip` again with the known value of R_g to obtain the friction values τ_g, τ_l and τ_lg.

Appendix: relevant equations for Biberg model

- **Interfacial turbulence parameter**

$$\left.\begin{aligned}
&\kappa = 0.40\\
&B = 5.5\\
&K_{\mathrm{g,B0}} = \frac{0.065\rho_{\mathrm{g}}(u_{\mathrm{g}}-u_{\mathrm{l}})^2}{(\rho_{\mathrm{l}}-\rho_{\mathrm{g}})g\cos\phi h_{\mathrm{g}}};\\
&K_{\mathrm{l,B0}} = 10\sqrt{\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{l}}}}\left|\frac{(u_{\mathrm{g}}-u_{\mathrm{l}})}{u_{\mathrm{l}}}\right|.
\end{aligned}\right\}\tag{1}$$

$$\left.\begin{aligned}
K_{\mathrm{g,min}} &= \frac{\mu_{\mathrm{g}}}{\rho_{\mathrm{g}}}\frac{\exp(-\kappa B)}{\sqrt{\frac{|\tau_{\mathrm{lg}}|}{\rho_{\mathrm{g}}}}h_{\mathrm{g}}};\\
K_{\mathrm{l,min}} &= \frac{\mu_{\mathrm{l}}}{\rho_{\mathrm{l}}}\frac{\exp(-\kappa B)}{\sqrt{\frac{|\tau_{\mathrm{lg}}|}{\rho_{\mathrm{l}}}}h_{\mathrm{l}}}.
\end{aligned}\right\}\tag{2}$$

$$\left.\begin{aligned}
K_{\mathrm{g,B}} &= \max(K_{\mathrm{g,B0}}, K_{\mathrm{g,min}});\\
K_{\mathrm{l,B}} &= \max(K_{\mathrm{l,B0}}, K_{\mathrm{l,min}}).
\end{aligned}\right\}\tag{3}$$

- **Shear ratio**

$$\left. R_{\mathrm{g}} = \frac{\tau_{\mathrm{lg}}}{\tau_{\mathrm{g}}} \quad ; \qquad R_{\mathrm{l}} = -\frac{\tau_{\mathrm{lg}}}{\tau_{\mathrm{l}}}. \right\}\tag{4}$$

$$\left. G = \frac{(\rho_{\mathrm{l}}-\rho_{\mathrm{g}})\,Ag\sin\phi}{R_{\mathrm{g}}\tau_{\mathrm{g}}} \quad ; \qquad R_{\mathrm{l}} = -\frac{\alpha_{\mathrm{g}}R_{\mathrm{g}}P_{\mathrm{l}}}{\alpha_{\mathrm{l}}P_{\mathrm{g}} + R_{\mathrm{g}}\left(P_{\mathrm{lg}} - \alpha_{\mathrm{g}}\alpha_{\mathrm{l}}G\right)}. \right\}\tag{5}$$

- **Model coefficients for calculating effective diameter**

$$\begin{aligned}
\Delta(Y,R,K) = {}& \ln(1-Y) + \frac{(K^3+R^3)\ln[Y+K(1-Y)]}{|R|^{5/2}-K^3} + \frac{(R+\sqrt{|R|})\sqrt[3]{|R|}\ln[Y+|R|^{5/6}(1-Y)]}{3(K-|R|^{5/6})}\\
&-\frac{(R+\sqrt{|R|})(K+2|R|^{5/6})\sqrt[3]{|R|}\ln[Y^2-(1-Y)(Y-(1-Y)|R|^{5/6})|R|^{5/6}]}{6(K^2+|R|^{5/6}K+|R|^{5/3})}\\
&+\frac{K(R+\sqrt{|R|})\sqrt[3]{|R|}}{\sqrt{3}(K^2+|R|^{5/6}K+|R|^{5/3})}\tan^{-1}\left[\frac{2(Y-1)|R|^{5/3}+(2Y-1)|R|^{5/6}+2Y)}{\sqrt{3}|R|^{5/6}}\right];
\end{aligned}\tag{6}$$

$$\begin{aligned}
\Lambda(R,K) = {}& \frac{5(R+\sqrt{|R|})(|R|^{5/2}+K^2+K)R^2\ln(|R|)}{6(K^3-|R|^{5/2})(|R|^{5/2}-1)}\\
&-\frac{K(K^3+R^3)\ln(K)}{(K-1)(K^3-|R|^{5/2})} + \frac{(R+\sqrt{|R|})\sqrt[3]{|R|}}{\sqrt{3}(|R|^{5/3}+|R|^{5/6}+1)(K^2+|R|^{5/6}K+|R|^{5/3})}\\
&\left\{(K-|R|^{5/3})\tan^{-1}\left[3^{-1/2}(1+2|R|^{-5/6})\right] - \left[K+(K+1)|R|^{5/6}\right]|R|^{5/6}\tan^{-1}\left[3^{-\frac{1}{2}}(2|R|^{5/6}+1)\right]\right\};
\end{aligned}\tag{7}$$

$$\Psi(R,K) = \frac{K(R+\sqrt{|R|})\sqrt[3]{|R|}}{\sqrt{3}(K^2+|R|^{5/6}K+|R|^{5/3})}\tan^{-1}\left(\frac{1+\frac{2}{|R|^{5/6}}}{\sqrt{3}}\right). \tag{8}$$

- **Correlations for free surface flow and Poiseuille flow**

$$\left.\begin{aligned} K_{\mathrm{l,F}} &= 0.56;\\ \Lambda_{\mathrm{Pois}} &= \frac{2}{27}(2\sqrt{3}\pi+9);\\ \Lambda_{\mathrm{g,F}} &= 0;\\ \Lambda_{\mathrm{l,F}} &= \frac{K_{\mathrm{l,F}}\ln(K_{\mathrm{l,F}})}{1-K_{\mathrm{l,F}}}. \end{aligned}\right\} \tag{9}$$

- **Effective pipeflow diameter**

$$\left.\begin{aligned} F_{\mathrm{g}} &= \frac{\Lambda_{\mathrm{g}}+\Psi_{\mathrm{g}}-\Lambda_{g,F}}{\Lambda_{\mathrm{Pois}}-\Lambda_{g,F}};\\ F_{\mathrm{l}} &= \frac{\Lambda_{\mathrm{l}}+\Psi_{\mathrm{l}}-\Lambda_{l,F}}{\Lambda_{\mathrm{Pois}}-\Lambda_{l,F}};\\ D_{\mathrm{eff,g}} &= 4\frac{A_{\mathrm{g}}}{P_{\mathrm{g}}}\left(\frac{P_{\mathrm{g}}}{P_{\mathrm{g}}+P_{\mathrm{lg}}}\right)^{F_{\mathrm{g}}};\\ D_{\mathrm{eff,l}} &= 4\frac{A_{\mathrm{l}}}{P_{\mathrm{l}}}\left(\frac{P_{\mathrm{l}}}{P_{\mathrm{l}}+P_{\mathrm{lg}}}\right)^{F_{\mathrm{l}}};\\ Re_{\mathrm{g}} &= \frac{\rho_{\mathrm{g}}D_{\mathrm{eff,g}}|u_{\mathrm{g}}|}{\mu_{\mathrm{g}}};\\ Re_{\mathrm{l}} &= \frac{\rho_{\mathrm{l}}D_{\mathrm{eff,l}}|u_{\mathrm{l}}|}{\mu_{\mathrm{l}}}. \end{aligned}\right\} \tag{10}$$

- **Wall and interfacial shear stress**

$$\left.\begin{aligned} \frac{1}{\sqrt{\lambda_{g0}}} &= -1.8\log_{10}\left(\frac{6.9}{Re_{\mathrm{g}}}+\left(\frac{k_s}{3.7D}\right)^{1.11}\right);\\ \frac{1}{\sqrt{\lambda_{l0}}} &= -1.8\log_{10}\left(\frac{6.9}{Re_{\mathrm{g}}}+\left(\frac{k_s}{3.7D}\right)^{1.11}\right);\\ \frac{1}{\sqrt{\lambda_{\mathrm{g}}}} &= \frac{\frac{5.02}{Re_{\mathrm{g}}\sqrt{\lambda_{g0}}}-4.6\left(\frac{2.51}{Re_{\mathrm{g}}\sqrt{\lambda_{g0}}}+\frac{k_s}{3.7D}\right)\log_{10}\left(\frac{2.51}{Re_{\mathrm{g}}\sqrt{\lambda_{g0}}}+\frac{k_s}{3.7D}\right)}{\frac{5.02}{Re_{\mathrm{g}}}+2.3\left(\frac{2.51}{Re_{\mathrm{g}}\sqrt{\lambda_{g0}}}+\frac{k_s}{3.7D}\right)};\\ \frac{1}{\sqrt{\lambda_{\mathrm{l}}}} &= \frac{\frac{5.02}{Re_{\mathrm{l}}\sqrt{\lambda_{l0}}}-4.6\left(\frac{2.51}{Re_{\mathrm{l}}\sqrt{\lambda_{l0}}}+\frac{k_s}{3.7D}\right)\log_{10}\left(\frac{2.51}{Re_{\mathrm{l}}\sqrt{\lambda_{l0}}}+\frac{k_s}{3.7D}\right)}{\frac{5.02}{Re_{\mathrm{l}}}+2.3\left(\frac{2.51}{Re_{\mathrm{l}}\sqrt{\lambda_{l0}}}+\frac{k_s}{3.7D}\right)};\\ \tau_{\mathrm{g}} &= \frac{1}{8}\lambda_{\mathrm{g}}\rho_{\mathrm{g}}|u_{\mathrm{g}}|u_{\mathrm{g}};\\ \tau_{\mathrm{l}} &= \frac{1}{8}\lambda_{\mathrm{l}}\rho_{\mathrm{l}}|u_{\mathrm{l}}|u_{\mathrm{l}};\\ |\tau_{\mathrm{lg}}| &= \frac{\kappa^2\rho_{\mathrm{g}}\rho_{\mathrm{l}}|R_{\mathrm{g}}R_{\mathrm{l}}|(u_{\mathrm{g}}-u_{\mathrm{l}})^2}{\left[\mathrm{sgn}(u_{\mathrm{g}})(\Lambda_{\mathrm{g}}-\Delta_{ig})\sqrt{\rho_{\mathrm{l}}|R_{\mathrm{l}}|}-\mathrm{sgn}(u_{\mathrm{l}})(\Lambda_{\mathrm{l}}-\Delta_{il})\sqrt{\rho_{\mathrm{g}}|R_{\mathrm{g}}|}\right]^2}. \end{aligned}\right\} \tag{11}$$

- **Residual in no-slip condition**

$$\left.\mathrm{residual\_noslip} = \mathrm{sgn}(u_{\mathrm{g}})\frac{\sqrt{\frac{|\tau_{\mathrm{g}}|}{\rho_{\mathrm{g}}}}}{\kappa}(\Lambda_{\mathrm{g}}-\Delta_{ig})-\mathrm{sgn}(u_{\mathrm{l}})\frac{\sqrt{\frac{|\tau_{\mathrm{l}}|}{\rho_{\mathrm{l}}}}}{\kappa}(\Lambda_{\mathrm{l}}-\Delta_{il})\right\} \tag{12}$$