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Math 1210 Signature Assignment: Mean Value Theorem

1. The Mean Value Theorem states that if f is continuous on the interval $[a,\ b]$, and if it is differentiable on $(a,\ b)$, then there must be a tangent slope equal to the secant slope somewhere in that interval. There exists c in $(a,\ b)$ such that $f'(c)\ =\ \frac{f(b)-f(a)}{b-a}$.

2. Let's look at the function $f(x)\ =\frac{x+2}{x+5}$ on the interval $[-3, 7]$: We know this function is undefined where the denominator is 0, which only occurs at $x\ =\ -5$. This is outside our interval, therefore our function is continuous. Our function is also differentiable on $(-3, 7)$ because it is continuous and there are no asymptotes on our interval.

3. Let's find the average rate of change of $f(x)$ on $[-3, 7]$. We can do this by first finding the points at $f(-3)$ and $f(7)$:

$$f(-3)\ =\frac{-3+2}{-3+5}=\frac{-1}{2}=\frac{-2}{4}$$

$$f(7)\ =\frac{7+2}{7+5}=\frac{9}{12}=\frac{3}{4}$$

We can find the average rate of change between these two points by finding the slope of the line that connects them:

$$m\ =\frac{\frac{-2}{4}-\frac{3}{4}}{-3-7}=\frac{-5}{4}*\frac{1}{-10}=\frac{5}{40}=\frac{1}{8}$$

Now let's use the point-slope form to find the equation of the line that connects these two points:

$$y-\frac{3}{4}=\frac{1}{8}(x\ -\ 7)$$

$$y=\frac{1}{8}x-\frac{7}{8}+\frac{6}{8}$$

$$y=\frac{1}{8}x-\frac{1}{8}$$

4. In order to find all values of c that satisfy $f'(c)\ =\ \frac{f(b)-f(a)}{b-a}$, we must find the derivative of $f(x)$:

$$f'(x) = \frac{(x+5)*1-(x+2)*1}{(x+5)^2} = \frac{x+5-x-2}{(x+5)^2} = \frac{3}{(x+5)^2}$$

Now we equate this to the slope of our interval:

$$\frac{3}{(x+5)^2} = \frac{1}{8}\left(\frac{3}{3}\right)$$

$$\frac{3}{(x+5)^2} = \frac{3}{24}$$

$$(x + 5)^2 = 24$$

$$x + 5 = \pm\sqrt{24}$$

$$x = -5 \pm \sqrt{24}$$

Our interval excludes values less than -3, so we can evaluate our result as $x = -5 + \sqrt{24}$. This is the one value of c that satisfies $f'(c) = \frac{f(b)-f(a)}{b-a}$ on our interval. If we plug this value into our original equation, it will give us the y-value for this point:

$$f(-5 + \sqrt{24}) = \frac{-5+\sqrt{24}+2}{-5+\sqrt{24}+5} = \frac{-3+\sqrt{24}}{\sqrt{24}}$$

Lastly, let's find the equation of the tangent line at this point using the point-slope form again:

$$y - \frac{-3+\sqrt{24}}{\sqrt{24}} = \frac{1}{8}[x - (-5 + \sqrt{24})]$$

$$y = \frac{1}{8}x + \frac{5}{8} - \frac{\sqrt{24}}{8} + \frac{-3+\sqrt{24}}{\sqrt{24}}$$

In order to combine the terms for the y-intercept, let's change the last term so it has the same denominator as the other two terms:

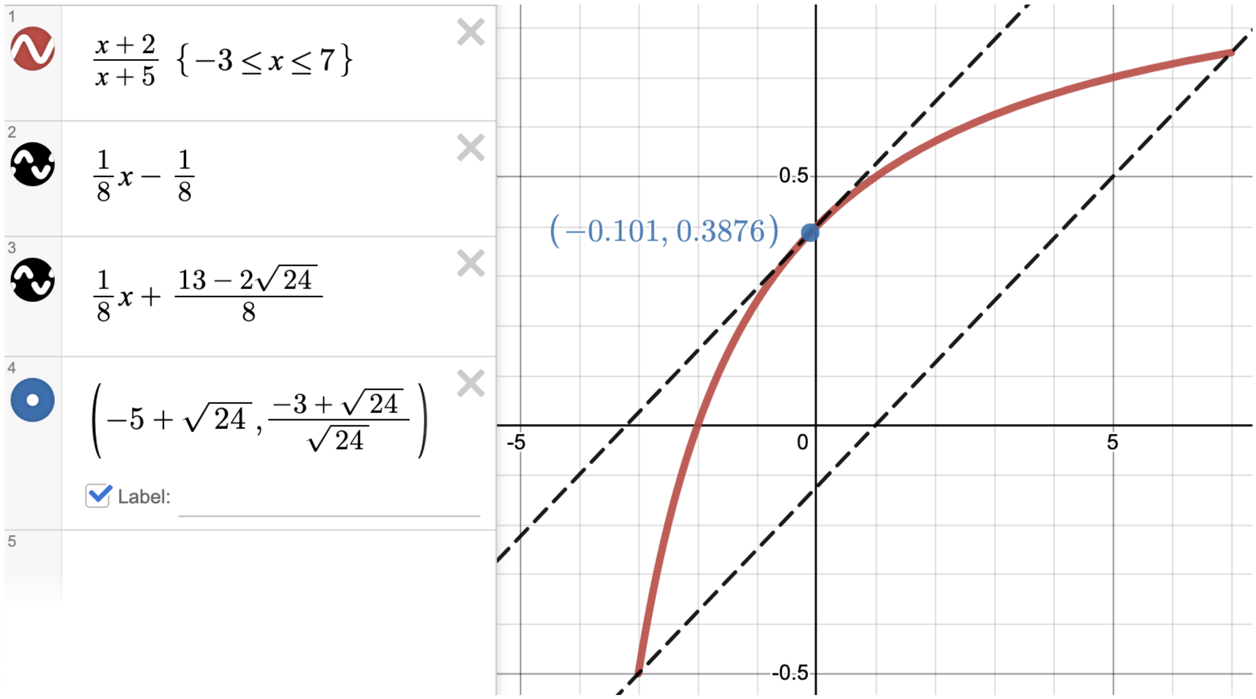
$$\frac{-3+\sqrt{24}}{\sqrt{24}}\left(\frac{\sqrt{24}}{\sqrt{24}}\right) = \frac{-3\sqrt{24}+24}{24} = \frac{-\sqrt{24}+8}{8}$$

$$\frac{5}{8} - \frac{\sqrt{24}}{8} - \frac{\sqrt{24}}{8} + \frac{8}{8} = \frac{13-2\sqrt{24}}{8}$$

The final equation of the line of the tangent slope at $f(c)$ is:

$$y = \frac{1}{8}x + \frac{13-2\sqrt{24}}{8}$$

5. We can illustrate these equations using a graph:



6. The definition of the Mean Value Theorem is very technical, but what the definition boils down to is that, if something is traveling from point a to point b, regardless of what path it takes (as long as it is a smooth path without any jumps or sharp corners), there will be at least one point in time where it is moving in the exact same direction as the direct path from a to b. It means that you cannot travel to b from a without ever facing in the direction of a to b.

   For a real-world example, consider the directions an airplane faces during a flight: Because of external factors (the directions runways are facing, wind speed and direction, and other air traffic), airplanes will never travel in a directly straight line from origin to destination. However, if your flight path never includes a tangent slope equal to the secant slope from origin to destination anywhere in the path, you know that there is a problem with your flight plan.

   I struggled a bit with this assignment because some of the final equations contained roots, and I'm the kind of person who always wants math to end in nice numbers. But I reviewed my work multiple times and am satisfied that I completed the calculations to the best of my ability. I really enjoy working through a problem and waiting until after I've completed all of the calculations to graph the equations. It helps me learn how to look for problems in my calculations before the graph even comes up, and it just feels good to see the graph after I've finished a problem correctly.