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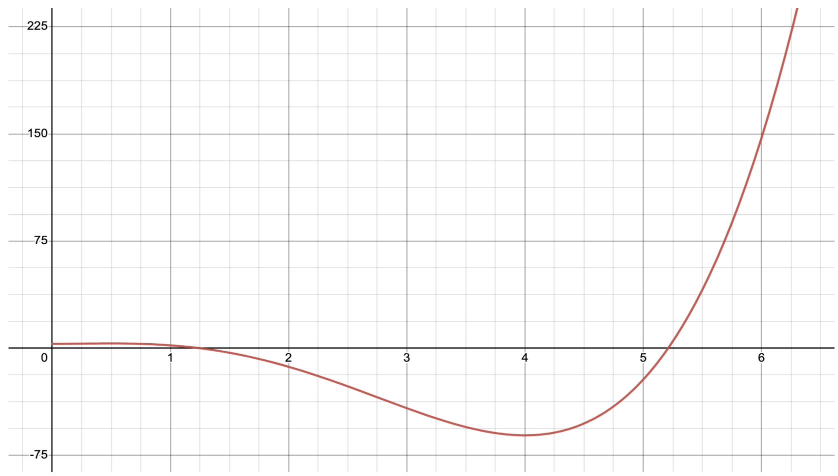
MATH-1210-404-Su22

Math 1210 Signature Assignment: Derivatives

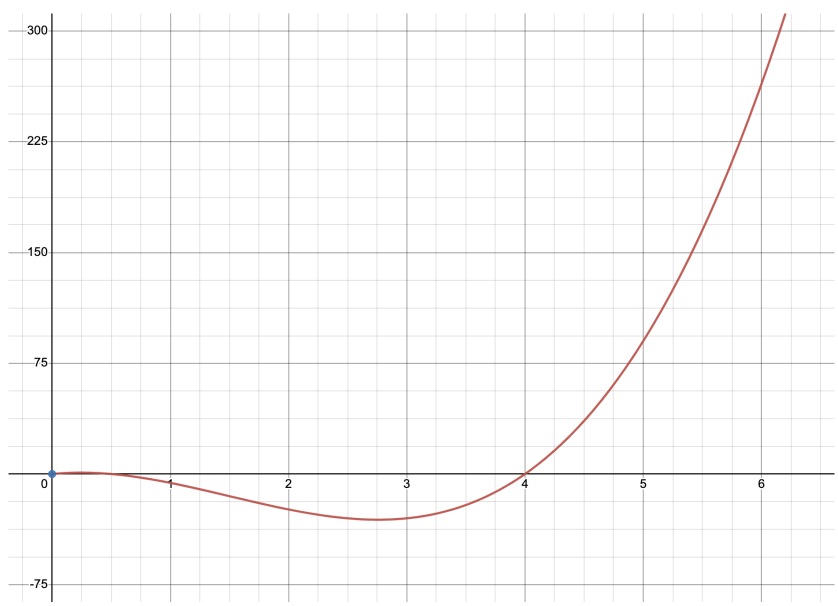
We have a particle that moves according to the following equation where s is in meters and t is in seconds:

$$s(t) = t^4 - 6t^3 + 4t^2 + 3,\ t \geq 0$$



a) If we want to find the velocity of the particle at time t, we would find the equation for the derivative of $s(t)$, called $v(t)$:

$$v(t) = 4t^3 - 18t^2 + 8t$$



The velocity at $t = 0$ would be $v(0) = 4(0)^3 - 18(0)^2 + 8(0) = 0$.

b) We can find when the particle is moving in a positive direction, a negative direction, and is at rest by studying its velocity: A positive velocity indicates a positive direction, a negative velocity indicates a negative direction, and a velocity of 0 indicates that the particle is at rest.

If we factor the velocity equation, we will get:

$$v(t) = 4t^3 - 18t^2 + 8t$$

$$v(t) = 2t(2t^2 - 9t + 4)$$

$$v(t) = 2t(2t - 1)(t - 4)$$

Using this factored form, we can set $v(t) = 0$ which will tell us that the particle is at rest during $t = 0,\ t = 0.5,\ t = 4$.

We can find which direction the particle moves by plugging t values in between these zeros into the equation (being cognizant that $t \geq 0$):

$$v(0.25) = 4(0.25)^3 - 18(0.25)^2 + 8(0.25) = 0.625 - 1.125 + 2 = 1.5$$

$$v(1) = 4(1)^3 - 18(1)^2 + 8(1) = 4 - 18 + 8 = -6$$

$$v(5) = 4(5)^3 - 18(5)^2 + 8(5) = 500 - 450 + 40 = 90$$

Therefore, the particle is moving in a positive direction in the intervals $(0, 0.5) \cup (4, \infty)$. It is moving in a negative direction in the interval $(0.5, 4)$.

c) We can find the total distance traveled by the particle in the first 6 seconds by finding the points in time in which the particle is moving left, when it's at rest, and when it's moving right, and then adding the absolute values of the difference in the particle's positions at each of these points:

From $t = 0$ to $t = 0.5$, the particle is moving in a positive direction and the distance traveled by the particle is $|s(0) - s(0.5)|$:

$$|s(0) - s(0.5)| = |3 - 3.313| = 0.313$$

We will calculate the rest of the distances in this manner, and then sum the results:

$$|s(0.5) - s(4)| = |3.313 + 61| = 64.313$$

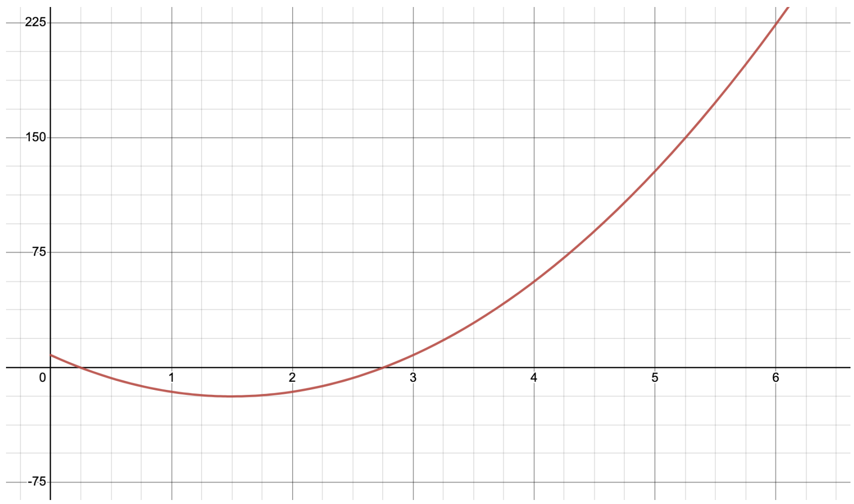
$$|s(4) - s(6)| = |-61 - 147| = 208$$

$$0.313 + 64.313 + 208 = 272.626$$

Therefore, the total distance traveled by the particle in the first 6 seconds was 272.626 meters.

d) If we want to find the acceleration equation of the particle, we must find the derivative of the velocity function, called $a(t)$:

$$a(t) = 12t^2 - 36t + 8$$



At $t = 0$, the acceleration is $a(0) = 12(0)^2 - 36(0) + 8 = 8$.

e) The particle is accelerating at positive values of $a(t)$ while it is decelerating at negative values. While we can try to find zeros by factoring, $a(t)$ does not factor nicely. Instead, let's use the quadratic formula to find the zero values of t:

$$t = \frac{36 \pm \sqrt{36^2 - 4(12)(8)}}{2(12)}$$

$$t = \frac{36 \pm \sqrt{1296 - 384}}{24}$$

$$t = \frac{36 \pm \sqrt{912}}{24}$$

$$t = \frac{36 \pm \sqrt{912}}{24}, \sqrt{912} \approx 30.2$$

$$t = \frac{36 \pm 30.2}{24} = 0.242,\ 2.758$$

Therefore, the particle is accelerating between $(0, 0.242) \cup (2.758, \infty)$ and decelerating between $(0.242, 2.758)$.

Reflection

This assignment builds off of straightforward formulas and derivatives, but I still struggled to understand some concepts such as total distance traveled (we can't just calculate a single value of $s(t)$ to find total distance traveled because we might have both positive and negative movement directions). I found it challenging to provide explanations for all of the calculations instead of just writing formulas on paper, but I think it enhanced my understanding of the process to try and use words to explain my work.

This assignment digs deep into the theory and calculations of particle movement, which don't seem that exciting on the surface, but can be especially relative to the field of Computer Science. If I want to create a program (such as a video game) to simulate particle movement, I can use the position, velocity, and acceleration functions to create a much more accurate simulation than if I tried to write equations by hand. When moving a particle from point a to point b in a computer program, you generally set the starting and ending position along with a delay time (which the system averages to determine the speed that the particle moves pixel-by-pixel) but you can replace this delay by using velocity and acceleration formulas to customize the speed that the particle will move from one pixel to the next.