Part XI

TESTS IN REGRESSION ANALYSIS

Statistical methods embraced by the terms regression analysis and analysis of variance are probably the most well-known and used in practical applications. They are based on the understanding that quantitative responses are often affected by one or a number of regressor variables. The assumed functional relationship between the regressor variables and the response is linear in unknown model parameters. This part deals with statistical tests on these model parameters, where it is either of interest if they are zero, and therefore the respective regressor variable is not relevant for the prediction of the response, or larger, smaller or equal to some pre-specified values. Chapter 16 treats the case of simple linear regression with one regressor variable as well as multiple linear regression with a set of regressor variables. Chapter 17 concentrates on analysis of variance where the effect of solely qualitative variables with finite numbers of possible levels on the response is of interest.

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Tests in regression analysis

Regression analysis investigates and models the relationship between variables. A linear relationship is assumed between a dependent or response variable Y of interest and one or several independent, predictor or regressor variables. We present tests on regression parameters in simple and multiple linear regression analysis. Tests cover the hypothesis on the value of individual regression parameters as well as tests for significance of regression where the hypothesis states that none of the regressor variables has a linear effect on the response.

16.1 Simple linear regression

Simple linear regression relates a response variable Y to the given outcome x of a single regressor variable by assuming the relation $Y = \beta_0 + \beta_1 x + \varepsilon$, which is linear in unknown coefficients or parameters β_0 and β_1. Further ε is an error term which models the deviation of the observed values from the linear relationship. In two-dimensional space this equals a straight line. For this reason simple linear regression is also called *straight line regression*. The value x of the regressor variable is fixed or measured without error. If the regressor variable is a random variable X the model is commonly understood as modeling the response Y conditional on the outcome $X = x$. To analyze if the regressor has an influence on the response Y it is tested if the slope β_1 of the regression line differs from zero. Other tests treat the intercept β_0.

16.1.1 Test on the slope

Description: Tests if the regression coefficient β_1 of a simple linear regression differs from a value β_{10}.

Assumptions:

- A sample of n pairs $(Y_1, x_1), \dots, (Y_n, x_n)$ is given.
- The simple linear regression model for the sample is stated as $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \qquad i = 1, \dots, n.$

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- The error term ε is a random variable which is Gaussian distributed with mean 0 and variance σ^2, that is, $\varepsilon_i \sim N(0, \sigma^2)$ for all $i = 1, \dots n$. It further holds that $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for all $i \neq j$.

Hypotheses:
(A) $H_0 : \beta_1 = \beta_{10}$ vs $H_1 : \beta_1 \neq \beta_{10}$
(B) $H_0 : \beta_1 \leq \beta_{10}$ vs $H_1 : \beta_1 > \beta_{10}$
(C) $H_0 : \beta_1 \geq \beta_{10}$ vs $H_1 : \beta_1 < \beta_{10}$

Test statistic:
$T = \frac{\hat{\beta}_1 - \beta_{10}}{S_{\hat{\beta}_1}}$

with $$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}, \quad S_{\hat{\beta}_1} = \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}}$$

$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$ and $\hat{Y}_i = \overline{Y} - \hat{\beta}_1\overline{x} + \hat{\beta}_1 x_i$

Test decision:
Reject H_0 if for the observed value t of T
(A) $t < t_{\alpha/2,n-2}$ or $t > t_{1-\alpha/2,n-2}$
(B) $t > t_{1-\alpha,n-2}$
(C) $t < t_{\alpha,n-2}$

p-values:
(A) $p = 2P(T \leq (-|t|))$
(B) $p = 1 - P(T \leq t)$
(C) $p = P(T \leq t)$

Annotations:

- The test statistic T follows a t-distribution with $n - 2$ degrees of freedom.
- $t_{\alpha;n-2}$ is the α-quantile of the t-distribution with $n - 2$ degrees of freedom.
- Of special interest is the test problem $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$; the test is then also called a *test for significance of regression*. If H_0 can not be rejected this indicates that there is no linear relationship between x and Y. Either x has no or little effect on Y or the true relationship is not linear (Montgomery 2006, p. 23).
- Alternatively the squared test statistic $F = \left(\frac{\hat{\beta}_1 - \beta_{10}}{S_{\hat{\beta}_1}}\right)^2$ can be used which follows a F-distribution with 1 and $n - 2$ degrees of freedom.

Example: Of interest is the slope of the regression of weight on height in a specific population of students. For this example two hypotheses are tested with (a) $\beta_{10} = 0$ and (b) $\beta_{10} = 0.5$. A dataset of measurements on a random sample of 20 students has been used (dataset in Table A.6).

SAS code

```
* Simple linear regression including test for H0: beta_1=0;
proc reg data=students;
 model weight=height;
run;

* Perform test for H0: beta_1=0.5;
proc reg data=students;
 model weight=height;
 test height=0.5;
run;
quit;
```

SAS output

```
                   Parameter Estimates

                  Parameter   Standard
Variable     DF   Estimate      Error     t Value   Pr > |t|
Intercept     1   -51.81816   35.76340    -1.45      0.1646
height        1     0.67892    0.20645     3.29      0.0041


                  The REG Procedure
                    Model: MODEL1

     Test 1 Results for Dependent Variable weight

                                Mean
Source             DF         Square    F Value    Pr > F
Numerator           1       94.54374       0.75    0.3975
Denominator        18      125.87535
```

Remarks:

- The SAS procedure `proc reg` is the standard procedure for linear regression. It is a powerful procedure and we use here only a tiny part of it.
- For the standard hypothesis $H_0 : \beta_1 = 0$ the `model` *dependent variable= independent variable* statement is sufficient.
- For testing a special hypothesis $H_0 : \beta_1 = \beta_{10}$ you must add the `test` *variable= value* statement. Note, here a F-test is used, which is equivalent to the proposed t-test, because a squared t-distributed random variable with n degrees of freedom is $F(1, n)$-distributed. The p-value stays the same. To get the t-test use the `restrict` *variable= value* statement.
- The `quit;` statement is used to terminate the procedure; `proc reg` is an interactive procedure and SAS then knows not to expect any further input.
- The p-values for the other hypothesis must be calculated by hand. For instance, for $H_0 : \beta_{10} = 0$ the p-value for hypothesis (B) is `1-probt(3.29,18)=0.0020` and for hypothesis (C) `probt(3.29,18)=0.9980`.

R code

```
# Read the data
y<-students$weight
x<-students$height

# Simple linear regression including test for H0: beta_1=0
reg<-summary(lm(y~x))

# Perform test for H0: beta_1=0.5

# Get estimated coefficient
beta_1<-reg$coeff[2,1]

# Get standard deviation of estimated coefficient
std_beta_1<-reg$coeff["x",2]

# Perform the test
t_value<-(beta_1-0.5)/std_beta_1

# Calculate p-value
p_value<-2*pt(-abs(t_value),18)

# Output result
# Simple linear regression
reg

# For hypothesis H0: beta_1=0.5
t_value
p_value
```

R output

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -51.8182    35.7634  -1.449  0.16456
x             0.6789     0.2065   3.288  0.00408 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

> # For hypothesis H0: beta_1=0.5
> t_value
[1] 0.8666546
> p_value
[1] 0.3975371
```

Remarks:

- The function `lm()` performs a linear regression in R. The response variable is placed on the left-hand side of the ~ symbol and the regressor variable on the right-hand side.
- The summary function gets R to return the estimates, p-values, etc. Here we store the values in the object `reg`.

- The standard hypothesis $H_0 : \beta_1 = 0$ is performed by the function `lm()`. The hypothesis $H_0 : \beta_1 = \beta_{10}$ with $\beta_{10} \neq 0$ is not covered by this function but it provides the necessary statistics which we store in above example code in the object `reg`. In the second part of the example we extract the estimated coefficient $\hat{\beta}_1$ with the command `reg$coeff[2,1]` and its estimated standard deviations $S_{\hat{\beta}_1}$ with the command `reg$coeff[2,2]`. These values are then used to perform the test.
- The p-values for the other hypothesis must be calculated by hand. For instance for $H_0 : \beta_{10} = 0$ the p-value for hypothesis (B) is `1-pt(3.29,18)=0.0020` and for hypothesis (C) `pt(3.29,18)=0.9980`.

16.1.2 Test on the intercept

Description: Tests if the regression coefficient β_0 of a simple linear regression differs from a value β_{00}.

Assumptions:
- A sample of n pairs $(Y_1, x_1), \dots, (Y_n, x_n)$ is given.
- The simple linear regression model for the sample is stated as $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \qquad i = 1, \dots, n.$
- The error term ϵ is a random variable which is Gaussian distributed with mean 0 and variance σ^2, that is, $\epsilon_i \sim N(0, \sigma^2)$ for all $i = 1, \dots n$. It further holds that $\mathrm{Cov}(\epsilon_i, \epsilon_j) = 0$ for all $i \neq j$.

Hypotheses:
(A) $H_0 : \beta_0 = \beta_{00}$ vs $H_1 : \beta_0 \neq \beta_{00}$
(B) $H_0 : \beta_0 \leq \beta_{00}$ vs $H_1 : \beta_0 > \beta_{00}$
(C) $H_0 : \beta_0 \geq \beta_{00}$ vs $H_1 : \beta_0 < \beta_{00}$

Test statistic: $T = \frac{\hat{\beta}_0 - \beta_{00}}{S_{\hat{\beta}_0}}$

with $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{x}, \quad \hat{\beta}_1 = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$,

$$S_{\hat{\beta}_0} = \hat{\sigma} \frac{\sqrt{\sum_{i=1}^{n} x_i^2}}{\sqrt{n \sum_{i=1}^{n} (x_i - \overline{x})^2}}, \quad \hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

and $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

Test decision: Reject H_0 if for the observed value t of T

(A) $t < t_{\alpha/2,n-2}$ or $t > t_{1-\alpha/2,n-2}$

(B) $t > t_{1-\alpha,n-2}$

(C)$t < t_{\alpha,n-2}$

p-values: (A) $p = 2P(T \leq (-|t|))$

(B) $p = 1 - P(T \leq t)$

(C) $p = P(T \leq t)$

Annotations:

- The test statistic T follows a t-distribution with $n-2$ degrees of freedom.
- $t_{\alpha;n-2}$ is the α-quantile of the t-distribution with $n-2$ degrees of freedom.
- The hypothesis $\beta_0 = 0$ is used to test if the regression line goes through the origin.

Example: Of interest is the intercept of the regression of weight on height in a specific population of students. For this example two hypotheses are tested with (a) $\beta_{00} = 0$ and (b) $\beta_{00} = 10$. A dataset of measurements on a random sample of 20 students has been used (dataset in Table A.6).

SAS code

```
* Simple linear regression including test for H0: beta_1=0;
proc reg data=students;
 model weight=height;
run;

* Perform test for H0: beta_0=10;
proc reg data=students;
 model weight=height;
 test intercept=10;
run;
```

SAS output

```
                   Parameter Estimates

                  Parameter   Standard
Variable     DF   Estimate      Error    t Value   Pr > |t|
Intercept     1   -51.81816   35.76340   -1.45      0.1646
height        1     0.67892    0.20645    3.29      0.0041
                   The REG Procedure
                     Model: MODEL1
```

```
          Test 1 Results for Dependent Variable weight

                                Mean
Source              DF        Square    F Value    Pr > F
Numerator            1     376.09299       2.99    0.1010
Denominator         18     125.87535
```

Remarks:

- The SAS procedure `proc reg` is the standard procedure for linear regression. It is a powerful procedure and we use here only a tiny part of it.
- For the standard hypothesis $H_0 : \beta_0 = 0$ the `model` *dependent variable*= *independent variable* statement is sufficient.
- For testing a special hypothesis $H_0 : \beta_0 = \beta_{00}$ you must add the `test intercept` *value* statement. Note, here a F-test is used, which is equivalent to the proposed t-test, because a squared t-distributed random variable with n degrees of freedom is $F(1, n)$-distributed. The p-value stays the same. To get the t-test use the `restrict` *variable*= *value* statement.
- The `quit;` statement is used to terminate the procedure; `proc reg` is an interactive procedure and SAS then knows not to expect any further input.
- The p-values for the other hypothesis must be calculate by hand. For instance for $H_0 : \beta_{10} = 0$ the p-value for hypothesis (B) is `1-probt(-51.82,18)=1` and for hypothesis (C) `probt(-51.82,18)=0`.

R code

```
# Read the data
y<-students$weight
x<-students$height

# Simple linear regression
reg<-summary(lm(y~x))

# Perform test for H0: beta_0=10

# Get estimated coefficient
beta_0<-reg$coeff[1,1]

# Get standard deviation of estimated coefficient
std_beta_0<-reg$coeff[1,2]

# Perform the test
t_value<-(beta_0-10)/std_beta_0

# Calculate p-Value
p_value<-2*pt(-abs(t_value),18)
```

```
# Output result
# Simple linear regression
reg

# For hypothesis H0: beta_0=10
t_value
p_value
```

R output

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -51.8182    35.7634  -1.449  0.16456
x             0.6789     0.2065   3.288  0.00408 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

>
> # For hypothesis H0: beta_0=10
> t_value
[1] -1.728531
> p_value
[1] 0.1010077
```

Remarks:

- The function `lm()` performs a linear regression in R. The response variable is placed on the left-hand side of the ~ symbol and the regressor variable on the right-hand side.
- The summary function gets R to return the estimates, p-values, etc. Here we store the values in the object `reg`.
- The standard hypothesis $H_0 : \beta_1 = 0$ is performed by the function `lm()`. The hypothesis $H_0 : \beta_0 = \beta_{00}$ with $\beta_{00} \neq 0$ is not covered by this function but it provides the necessary statistics which we store in the example code in the object `reg`. In the second part of the example we extract the estimated coefficient $\hat{\beta}_0$ with the command `reg$coeff[1,1]` and its estimated standard deviation $S_{\hat{\beta}_0}$ with the command `reg$coeff[1,2]`. These values are then used to perform the test.
- The p-values for the other hypothesis must be calculated by hand. For instance for $H_0 : \beta_{10} = 0$ the p-value for hypothesis (B) is `1-pt(-51.82,18)=1` and for hypothesis (C) `pt(-51.82,18)=0`.

16.2 Multiple linear regression

Multiple linear regression is an extension of the simple linear regression to more than one regressor variable. The response Y is predicted from a set of regressor variables $X_1, \dots, X_p$.

Instead of a straight line a hyperplane is modeled. Again, the values of the regressor variables are either fixed, measured without error or conditioned on (Rencher 1998, chapter 7). Multiple linear regression is based on assuming a relation $Y = \beta_0 + \beta_1 x_1 + \dots + \beta_p x_p + \varepsilon$, which is linear in unknown coefficients or parameters $\beta_0, \dots, \beta_p$. Further ε is an error term which models the deviation of the observed values from the hyperplane. To analyze if individual regressors have an influence on the response Y it is tested if the corresponding parameter differs from zero. Tests for significance of regression test the overall hypothesis that none of the regressor has an influence on Y in the regression model.

16.2.1 Test on an individual regression coefficient

Description: Tests if a regression coefficient β_j of a multiple linear regression differs from a value β_{j0}.

Assumptions:

- A sample of n tuples $(Y_1, x_{11}, \dots x_{1p}), \dots, (Y_n, x_{n1}, \dots x_{np})$ is given.
- The multiple linear regression model for the sample can be written in matrix notation as $\mathbf{Y}{=}\mathbf{X}\beta + \epsilon$ with response vector $\mathbf{Y} = (Y_1 \dots Y_n)'$, unknown parameter vector $\beta = (\beta_0, \beta_1, \dots, \beta_p)'$, random vector of errors ϵ and a matrix with values of the regressors $\mathbf{X}$ (Montgomery *et al*. 2006, p. 68).
- The elements ϵ_i of ϵ follow a Gaussian distribution with mean 0 and variance σ^2, that is, $\epsilon_i \sim N(0, \sigma^2)$ for all $i = 1, \dots n$. It further holds that $\mathrm{Cov}(\epsilon_i, \epsilon_j) = 0$ for all $i \neq j$.

Hypotheses:
(A) $H_0 : \beta_j = \beta_{j0}$ vs $H_1 : \beta_j \neq \beta_{j0}$
(B) $H_0 : \beta_j \leq \beta_{j0}$ vs $H_1 : \beta_j > \beta_{j0}$
(C) $H_0 : \beta_j \geq \beta_{j0}$ vs $H_1 : \beta_j < \beta_{j0}$

Test statistic: $T = \frac{\hat{\beta}_j - \beta_{j0}}{S_{\hat{\beta}_j}}$
with $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$, $S_{\hat{\beta}_j} = \sqrt{\hat{\sigma}^2 diag_{jj}(\mathbf{X}'\mathbf{X})^{-1}}$,
$\hat{\sigma}^2 = \frac{(\mathbf{Y}-\mathbf{X}\hat{\beta})'(\mathbf{Y}-\mathbf{X}\hat{\beta})}{n}$ and $diag_{jj}(\mathbf{X}'\mathbf{X})^{-1}$ the jjth element of the diagonal of the inverse matrix of $\mathbf{X}'\mathbf{X}$.

Test decision: Reject H_0 if for the observed value t of T
(A) $t < t_{\alpha/2,n-p-1}$ or $t > t_{1-\alpha/2,n-p-1}$
(B) $t > t_{1-\alpha,n-p-1}$
(C) $t < t_{\alpha,n-p-1}$

p-values:
(A) $p = 2P(T \leq (-|t|))$
(B) $p = 1 - P(T \leq t)$
(C) $p = P(T \leq t)$

Annotations:

- The test statistic T follows a t-distribution with $n - p - 1$ degrees of freedom.

- $t_{\alpha;n-p-1}$ is the α-quantile of the t-distribution with $n-p-1$ degrees of freedom.
- Usually it is tested if $\beta_j = 0$. If this hypothesis cannot be rejected it can be concluded that the regressor variable X_j does not add significantly to the prediction of Y, given the other regressor variables X_k with $k \neq j$.
- Alternatively the squared test statistic $F = \left(\frac{\hat{\beta}_j - \beta_{j0}}{S_{\hat{\beta}_j}}\right)^2$ can be used which follows a F-distribution with 1 and $n-p-1$ degrees of freedom. As the test is a partial test of one regressor, the test is also called a *partial F-test*.

Example: Of interest is the effect of sex in a regression of weight on height and sex in a specific population of students. The variable sex needs to be coded as a dummy variable for the regression model. In our example we choose the outcome male as reference, hence the new variable sex takes the value 1 for female students and 0 for male students. We test the hypothesis $\beta_{sex} = 0$. A dataset of measurements on a random sample of 20 students has been used (dataset in Table A.6).

SAS code

```
* Create dummy variable for sex with reference male;
data reg;
 set students;
 if sex=1 then s=0;
 if sex=2 then s=1;
run;

* Perform linear regression;
proc reg data=reg;
 model weight=height s;
run;
quit;
```

SAS output

```
                    Parameter    Standard
Variable      DF     Estimate       Error    t Value    Pr > |t|
Intercept      1    -44.10291    39.97051      -1.10      0.2852
height         1      0.64182     0.22489       2.85      0.0110
s              1     -2.60868     5.46554      -0.48      0.6392
```

Remarks:

- The SAS procedure `proc reg` is the standard procedure for linear regression. It is a powerful procedure and we use here only a tiny part of it.

- For the standard hypothesis $H_0 : \beta_j = 0$ the `model` *dependent variable= independent variables* statement is sufficient. The independent variables are separated by blanks.
- Categorical variables can also be regressors but care must be taken as to which value is the reference value. Here we code sex as the dummy variable, with males as the reference group.
- The `quit;` statement is used to terminate the procedure; `proc reg` is an interactive procedure and SAS then knows not to expect any further input.
- The p-values for the other hypothesis must be calculate by hand. For instance for the variable sex $H_0 : \beta_{20} = 0$ the p-value for hypothesis (B) is `1-probt (-0.48,18)= 0.6815` and for hypothesis (C) `probt(-0.48,18)= 0.3185`.
- For testing a special hypothesis $H_0 : \beta_j = \beta_{j0}$ you must add the `test` *variable= value* statement. Note, here a F-test is used, which is equivalent to the proposed t-test, because a squared t-distributed random variable with n degrees of freedom is $F(1, n)$-distributed. The p-value stays the same. To get the t-test use the `restrict` *variable= value* statement.

R code

```
# Read the data
weight<-students$weight
height<-students$height
sex<-students$sex

# Multiple linear regression
summary(lm(weight~height+factor(sex)))
```

R output

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -44.1029    39.9705  -1.103    0.285
height         0.6418     0.2249   2.854    0.011 *
factor(sex)2  -2.6087     5.4655  -0.477    0.639
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Remarks:

- The function `lm()` performs a linear regression in R. The response variable is placed on the left-hand side of the ~ symbol and the regressor variables on the right-hand side separated by a plus (+).
- Categorical variables can also be regressors, but care must be taken as to which value is the reference value. We use the `factor()` function to tell R that `sex`

is a categorical variable. We see from the output `factor(sex)2` that the effect is for females and therefore the males are the reference. To switch these, recode the values of males and females.

- The summary function gets R to return the estimates, p-values, etc.
- The standard hypothesis $H_0 : \beta_j = 0$ is performed by the function `lm()`. The hypothesis $H_0 : \beta_j = \beta_{j0}$ is not covered by this function but it provides the necessary statistics which can then be used. See Test 16.1.1 on how to do so.
- The p-values for the other hypothesis must be calculated by hand. For instance, for $\beta_2 = 0$ the p-value for hypothesis (B) is `1-pt(-0.477,18)=0.6805` and for hypothesis (C) `pt(-0.47729,18)=0.3196`.

16.2.2 Test for significance of regression

Description: Tests if there is a linear relationship between any of the regressors $X_1, \ldots, X_p$ and the response Y in a linear regression.

Assumptions:
- A sample of n tuples $(Y_1, x_{11}, \ldots x_{1p}), \ldots, (Y_n, x_{n1}, \ldots x_{np})$ is given.
- The multiple linear regression model for the sample can be written in matrix notation as $\mathbf{Y} = \mathbf{X}\beta + \epsilon$ with response vector $\mathbf{Y} = (Y_1 \ldots Y_n)'$, unknown parameter vector $\beta = (\beta_0, \beta_1, \ldots, \beta_p)'$, random vector of errors ϵ and a matrix with values of the regressors $\mathbf{X}$ (Montgomery *et al*. 2006, p.68).
- The elements ϵ_i of ϵ follow a Gaussian distribution with mean 0 and variance σ^2, that is, $\epsilon_i \sim N(0, \sigma^2)$ for all $i = 1, \ldots n$. It further holds that $\mathrm{Cov}(\epsilon_i, \epsilon_j) = 0$ for all $i \neq j$.

Hypotheses: $H_0 : \beta_0 = \beta_1 = \cdots = \beta_p = 0$
vs $H_1 : \beta_j \neq 0$ for at least one $j \in \{1, \ldots, p\}$.

Test statistic:
$$F = \frac{\left[\sum_{i=1}^{n} (Y_i - \overline{Y})^2 - \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2\right] \Big/ p}{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 \Big/ (n-p-1)}$$
where the $\hat{Y}_i$ are calculated through $\hat{\mathbf{Y}} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$.

Test decision: Reject H_0 if for the observed value F_0 of F
$F_0 > f_{1-\alpha;p,n-p-1}$

p-values: $p = 1 - P(F \leq F_0)$

Annotations:
- The test statistic F is $F_{p,n-p-1}$-distributed.
- $f_{1-\alpha;p,n-p-1}$ is the $1-\alpha$-quantile of the F-distribution with p and $n-p-1$ degrees of freedom.

- If the null hypothesis is rejected none of the regressors adds significantly to the prediction of Y. Therefore the test is sometimes called the *overall F-test*.

Example: Of interest is the regression of weight on height and sex in a specific population of students. We test for overall significance of regression, hence the hypothesis $\beta_{height} = \beta_{sex} = 0$. A dataset of measurements on a random sample of 20 students has been used (dataset in Table A.6).

SAS code

```
proc reg data=reg;
 model weight=height sex;
run;
quit;
```

SAS output

```
                   Analysis of Variance

                          Sum of        Mean
Source           DF      Squares      Square  F Value  Pr > F
Model             2   1391.20481   695.60241     5.29  0.0164
Error            17   2235.79519   131.51736
Corrected Total  19   3627.00000
```

Remarks:

- The SAS procedure `proc reg` is the standard procedure for linear regression. It is a powerful procedure and we use here only a tiny part of it.
- Categorical variables can also be regressors, but care must be taken as to which value is the reference value. Here we code sex as the dummy variable, with male as the reference group.
- The `quit;` statement is used to terminate the procedure; `proc reg` is an interactive procedure and SAS then knows not to expect any further input.

R code

```
summary(lm(students$weight~students$height
                                    +factor(students$sex)))
```

R output

```
F-statistic: 5.289 on 2 and 17 DF,  p-value: 0.01637
```

Remarks:

- The function `lm()` performs a linear regression in R. The response variable is placed on the left-hand side of the ∼ symbol and the regressor variables on the right-hand side separated by a plus (+).
- We use the R function `factor()` to tell R that `sex` is a categorical variable.
- The summary function gets R to return parameter estimates, p-values for the overall F-tests, p-values for tests on individual regression parameters, etc.

References

Montgomery D.C., Peck E.A. and Vining G.G. 2006 *Introduction to Linear Regression Analysis*, 4th edn. John Wiley & Sons, Ltd.

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17

Tests in variance analysis

Analysis of variance (ANOVA) in its simplest form analyzes if the mean of a Gaussian random variable differs in a number of groups. Often the factor which determines each group is given by applying different treatments to subjects, for example, in designed experiments in technical applications or in clinical studies. The problem can thereby be seen as comparing group means, which extends the t-test to more than two groups. The underlying statistical model may also be presented as a special case of a linear model. In Section 17.1 we handle the one- and two-way cases of ANOVA. The two-way case extends the treated problem to groups characterized by two factors. In this case it is also of interest if the two factors influence each other in their effect on the measured variable, and hence show an interaction effect. One of the crucial assumptions of an ANOVA is the homogeneity of variance within all groups. Section 17.2 deals with tests to check this assumption.

17.1 Analysis of variance

17.1.1 One-way ANOVA

Description: Tests if the mean of a Gaussian random variable is the same in I groups.

Assumptions:
- Let $Y_{i1}, \dots, Y_{in_i}$, $i \in \{1, \dots, I\}$, be I independent samples of independent Gaussian random variables with the same variance but possibly different group means.
- The sample sizes of the I samples are $n_1, \dots, n_I$ with $n = \sum_{i=1}^{I} n_i$.
- The random variables Y_{ij} can be modeled as $Y_{ij} = \mu_i + e_{ij}$ with $e_{ij} \sim N(0, \sigma^2)$, $\mu_{ij} \in \mathbf{R}$.

Hypotheses: $H_0 : \mu_1 = \cdots = \mu_I$ vs $H_1 : \mu_i \neq \mu_k$ for at least one $i \neq k$.

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Test statistic:

$$F = \frac{\sum_{i=1}^{I} n_i(\overline{Y}_{i+} - \overline{Y}_{++})^2 \Big/ (I-1)}{\sum_{i=1}^{I}\sum_{j=1}^{n_i} (Y_{ij} - \overline{Y}_{i+})^2 \Big/ (n-I)}$$

with $\overline{Y}_{i+} = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}$ and $\overline{Y}_{++} = \frac{1}{n}\sum_{i=1}^{I}\sum_{j=1}^{n_i} Y_{ij}$

Test decision: Reject H_0 if for the observed value F_0 of F
$F_0 > f_{1-\alpha;I-1,n-I}$

p-values: $p = 1 - P(F \leq F_0)$

Annotations:

- The test statistic F is $F_{I-1,n-I}$-distributed (Rencher 1998, chapter 4).
- $f_{1-\alpha;\, I-1,\, n-I}$ is the $(1-\alpha)$-quantile of the F-distribution with $I-1$ and $n-I$ degrees of freedom.
- The numerator of the test statistic is also called MST (mean sum of squares for treatment) and the denominator MSE (mean sum of squares of errors).
- Note that we have presented the one-way model and test for the more general case of an unbalanced design where the sample sizes in the different groups may vary. A balanced design is characterized by an equal number of observations in each group.

Example: To test if the means of the harvest in kilograms of tomatoes in three different greenhouses differ. The dataset contains observations from five fields in each greenhouse (dataset in Table A.12).

SAS code

```
proc anova data = crop;
 class house
 model kg = house;
run;
quit;
```

SAS output

```
Source     DF      Anova SS    Mean Square  F Value    Pr > F
house       2    0.16329333     0.08164667     0.33    0.7262
```

Remarks:

- The SAS procedure `proc anova` is the standard procedure for the analysis of variance with a balanced design as given in this example. For an unbalanced design the procedure `proc glm` should be used (see below).

- By using the `class` statement, SAS treats the variable `house` as a categorical variable.
- The code `model` *dependent variable= independent variable* defines the model.
- The `quit;` statement is used to terminate the procedure; `proc anova` is an interactive procedure and SAS then knows not to expect any further input.
- The program code for `proc glm` is similar:

```
proc glm data = crop;
 class house
 model kg = house;
run;
quit;
```

R code

```
summary(aov(crop$kg~factor(crop$house)))
```

R output

```
                   Df Sum Sq Mean Sq F value Pr(>F)
factor(crop$house)  2 0.1633 0.08165   0.329  0.726
Residuals          12 2.9815 0.24846
```

Remarks:

- The function `aov()` performs an analysis of variance in R. The response variable is placed on the left-hand side of the ~ symbol and the independent variables which define the groups on the right-hand side.
- We use the R function `factor()` to tell R that `house` is a categorical variable.
- The summary function gets R to return the sum of squares, degrees of freedom, p-values, etc.

17.1.2 Two-way ANOVA

Description: Tests if the mean of a Gaussian random variable is the same in groups defined by two factors of interest.

Assumptions:
- Let Y_{ijk}, $i = 1, \ldots, I$, $j = 1, \ldots, J$, $k = 1, \ldots, K$ describe a sample of size $n = IJK$ of independent Gaussian random variables.
- In each of the IJ groups defined by the two factors, we have an equal number of K observations (balanced design).

- Each of the variables Y_{ijk} can be modeled as $Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk}$ with $e_{ijk} \sim N(0, \sigma^2)$, where μ is the overall mean and α_i and β_j are the deviations from it for the first and the second factor and $(\alpha\beta)_{ij}$ describes the interaction between them.

Hypotheses: (A) $H_0 : (\alpha\beta)_{11} = \ldots = (\alpha\beta)_{IJ} = 0$
vs $H_1 : (\alpha\beta)_{ij} \neq 0$ for at least one pair (i, j)
(B) $H_0 : \alpha_1 = \ldots = \alpha_I = 0$
vs $H_1 : \alpha_i \neq 0$ for at least one α_i
(C) $H_0 : \beta_1 = \ldots = \beta_J = 0$
vs $H_1 : \beta_j \neq 0$ for at least one β_j

Test statistic:

$$(A)\ F_A = \frac{K \sum_{i=1}^{I} \sum_{j=1}^{J} (\overline{Y}_{ij+} - \overline{Y}_{i++} - \overline{Y}_{+j+} + \overline{Y}_{+++})^2 \big/ (I-1)(J-1)}{\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (Y_{ijk} - \overline{Y}_{ij+})^2 \big/ IJ(K-1)}$$

$$(B)\ F_B = \frac{KJ \sum_{i=1}^{I} (\overline{Y}_{i++} - \overline{Y}_{+++})^2 \big/ (I-1)}{\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (Y_{ijk} - \overline{Y}_{ij+})^2 \big/ IJ(K-1)}$$

$$(C)\ F_C = \frac{KI \sum_{j=1}^{J} (\overline{Y}_{+j+} - \overline{Y}_{+++})^2 \big/ (J-1)}{\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (Y_{ijk} - \overline{Y}_{ij+})^2 \big/ IJ(K-1)}$$

with

$$\overline{Y}_{ij+} = \frac{1}{K} \sum_{k=1}^{K} Y_{ijk} \qquad \overline{Y}_{+++} = \frac{1}{IJK} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} Y_{ijk}$$

$$\overline{Y}_{i++} = \frac{1}{JK} \sum_{j=1}^{J} \sum_{k=1}^{K} Y_{ijk} \qquad \overline{Y}_{+j+} = \frac{1}{IK} \sum_{i=1}^{I} \sum_{k=1}^{K} Y_{ijk}$$

Test decision: Reject H_0 if for the observed value F_0 of F_A, F_B or F_C
(A) $F_0 > f_{1-\alpha;(I-1)(J-1),IJ(K-1)}$
(B) $F_0 > f_{1-\alpha;(I-1),IJ(K-1)}$
(C) $F_0 > f_{1-\alpha;(J-1),IJ(K-1)}$

p-values: $p = 1 - P(F \leq F_0)$

Annotations:

- The test statistic F is F-distributed with $(I-1)(J-1)$ (A), $(I-1)$ (B) or $(J-1)$ degrees of freedom for the nominator and $IJ(K-1)$ degrees of freedom for the denominator (Montgomery and Runger 2007, chapter 14).

- $f_{1-\alpha;r,s}$ is the $1-\alpha$-quantile of the F-distribution with r and s degrees of freedom.
- Hypothesis (A) tests if there is an interaction between the two factors. Hypotheses (A) and (B) are testing the main effects of the two factors.

Example: To test if the means of the harvest in kilograms of tomatoes in three different greenhouses and using five different fertilizers differ. The dataset contains observations from five fields with each fertilizer in each greenhouse (dataset in Table A.12).

SAS code

```
proc anova data= crop;
      class house fertilizer;
      model kg =  house fertilizer;
run;
quit;
```

SAS output

```
                    The ANOVA Procedure

Dependent Variable: kg

Source        DF    Anova SS   Mean Square  F Value  Pr > F
house          2  0.16329333   0.08164667     0.50  0.6268
fertilizer     4  1.66337333   0.41584333     2.52  0.1235
```

Remarks:

- The SAS procedure `proc anova` is the standard procedure for an ANOVA with a balanced design. For an unbalanced design the procedure `proc glm` should be used.
- By using the `class` statement, SAS treats the variables `house` and `fertilizer` as categorical variables.
- The code `model` *dependent variable= independent variables* defines the model. To incorporate an interaction term a star is used, for example, `variable1*variable2`.
- The `quit;` statement is used to terminate the procedure; `proc anova` is an interactive procedure and SAS then knows not to expect any further input.
- The program code for `proc glm` is similar:

```
proc glm data = crop;
 class house fertilizer
 model kg = house fertilizer;
run;
quit;
```

R code

```
kg<-crop$kg
field<-crop$house
fertilizer<-crop$fertilizer

summary(aov(kg~factor(field)+factor(fertilizer)))
```

R output

```
                    Df Sum Sq Mean Sq F value Pr(>F)
factor(house)        2 0.1633  0.0816   0.496  0.627
factor(fertilizer)   4 1.6634  0.4158   2.524  0.123
Residuals            8 1.3181  0.1648
```

Remarks:

- The function `aov()` performs an ANOVA in R. The response variable is placed on the left-hand side of the ∼ symbol and the independent variables which define the groups on the right-hand side separated by a plus (+). To incorporate an interaction term a star is used, for example, `variable1⋆variable2`.
- We use the R function `factor()` to tell R that `house` is a categorical variable.
- The summary function gets R to return the sum of squares, degrees of freedom, p-values, etc.

17.2 Tests for homogeneity of variances

17.2.1 Bartlett test

Description: Tests if the variances of k Gaussian populations differ from each other.

Assumptions:
- Data are measured on an interval or ratio scale.
- Data are randomly sampled from k independent Gaussian distributions.
- The k random variables $X_1, \dots, X_k$ from where the samples are drawn have variances $\sigma_1^2, \dots, \sigma_k^2$.
- Further $(X_{j1}, \dots, X_{jn_j})$ is the j^{th} sample with n_j observations, $j \in \{1, \dots, k\}$.

Hypotheses: $H_0 : \sigma_1^2 = \cdots = \sigma_k^2$ vs $H_1 : \sigma_l \neq \sigma_j$ for at least one $l \neq j$

Test statistic:

$$X^2 = \frac{r\ln\left(\sum_{j=1}^{k}\frac{n_j-1}{r}s_j^2\right) - \sum_{j=1}^{k}(n_j-1)\ln(s_j^2)}{1+\frac{1}{3(k-1)}\left(\left[\sum_{j=1}^{k}\frac{1}{n_j-1)}\right]-\frac{1}{r}\right)}$$

with $s_j^2 = \frac{1}{n_j-1}\sum_{i=1}^{n_j}(X_{ji}-\overline{X}_{j+})^2$, $\overline{X}_{j+} = \frac{1}{n_j}X_{ji}$

and $r = \sum_{j=1}^{k}(n_j-1)$

Test decision: Reject H_0 if for the observed value X_0^2 of X^2
$X_0^2 > \chi^2_{1-\alpha;k-1}$

p-values: $p = 1 - P(X^2 \le X_0^2)$

Annotations:

- The test statistic X^2 is χ^2_{k-1}-distributed (Glaser 1976).
- $\chi^2_{1-\alpha;k-1}$ is the $1-\alpha$-quantile of the χ^2-distribution with $k-1$ degrees of freedom.
- This test was introduced by Maurice Bartlett (1937).
- The Bartlett test is very sensitive to the violation of the Gaussian assumption. If the samples are not Gaussian distributed an alternative is Levene's test (Test 17.2.2).

Example: To test if the variances of the harvest in kilograms of tomatoes in three different greenhouses are the same (dataset in Table A.12).

SAS code

```
proc glm data = crop;
 class house;
 model kg = house;
 means house /hovtest=BARTLETT ;
run;
quit;
```

SAS output

```
                The GLM Procedure

 Bartlett's Test for Homogeneity of kg Variance

  Source         DF    Chi-Square    Pr > ChiSq
  house           2        2.1346        0.3439
```

Remarks:

- The SAS procedure `proc glm` provides the Bartlett test.
- The first lines of code are enabling an ANOVA (see Test 16.2.1).
- The code `means house /hovtest=BARTLETT` lets SAS conduct the Bartlett test.

R code

```
bartlett.test(crop$kg~crop$house)
```

R output

```
 Bartlett test of homogeneity of variances

data:  crop$kg by crop$field
Bartlett's K-squared = 2.1346, df = 2, p-value = 0.3439
```

Remarks:

- The function `bartlett.test()` conducts the Bartlett test.
- The analysis variable is coded on the left-hand side of the ~ and the group variable on the right-hand side.

17.2.2 Levene test

Description: Tests if the variances of k populations differ from each other.

Assumptions:
- Data are measured on an interval or ratio scale.
- Data are randomly sampled from k independent random variables $X_1, \ldots, X_k$ with variances $\sigma_1^2, \ldots, \sigma_k^2$.
- Further $(X_{j1}, \ldots, X_{jn_j})$ is the j^{th} sample with n_j observations, $j \in \{1, \ldots, k\}$.

Hypotheses: $H_0 : \sigma_1^2 = \cdots = \sigma_k^2$ vs $H_1 : \sigma_l \neq \sigma_j$ for at least one $l \neq j$.

Test statistic:

$$L = \frac{\left(\sum_{j=1}^{k}(n_j - 1)\right) \sum_{j=1}^{k} n_j(\overline{Z}_{j+} - \overline{Z}_{++})^2}{(k-1)\sum_{j=1}^{k}\sum_{i=1}^{n_j}(Z_{ji} - \overline{Z}_{j+})^2} \quad \text{with } Z_{ji} = |X_{ji} - \overline{X}_{j+}|,$$

$$\overline{X}_{j+} = \frac{1}{n_j}\sum_{i=1}^{n_j} X_{ji}\,, \quad \overline{Z}_{j+} = \frac{1}{n_j}\sum_{i=1}^{n_j} Z_{ji} \text{ and } \overline{Z}_{++} = \frac{1}{n}\sum_{j=1}^{k}\sum_{i=1}^{n_j} Z_{ji}$$

Test decision: Reject H_0 if for the observed value L_0 of L
$L_0 > f_{1-\alpha;k-1,\sum_{j=1}^{k}(n_j-1)}$

p-values: $p = 1 - P(F \leq L_0)$

Annotations:

- The test statistic L is $F_{k-1,\sum_{j=1}^{k}(n_j-1)}$-distributed.
- $f_{1-\alpha;k-1,\sum_{j=1}^{k}(n_j-1)}$ is the $1-\alpha$-quantile of the F-distribution with $k-1$ and $\sum_{j=1}^{k}(n_j-1)$ degrees of freedom.
- This test was introduced by Howard Levene 1960. In 1974 Morton Brown and Alan Forsythe proposed the use of the median or trimmed mean instead of the mean for calculating the Z_{ij} (Brown and Forsythe 1974). This test is called the *Brown–Forsythe test*.
- This test does not need the assumption of underlying Gaussian distributions and should be used if that assumption is doubtful. If the data are Gaussian distributed Bartlett's test can be used (see Test 17.2.1).

Example: To test if the variances of the harvest in kilograms of tomatoes in three different greenhouses are the same (dataset in Table A.12).

SAS code

```
proc glm data = crop;
 class house;
 model kg = house;
 means house /hovtest=levene (TYPE=ABS) ;
run;
quit;
```

SAS output

```
      Levene's Test for Homogeneity of kg Variance
      ANOVA of Absolute Deviations from Group Means

                    Sum of        Mean
Source      DF      Squares       Square     F Value     Pr > F
house        2       0.2675       0.1337        2.79     0.1012
Error       12       0.5753       0.0479
```

Remarks:

- The SAS procedure `proc glm` provides the Levene test.
- The first lines of code are enabling an ANOVA (see Test 16.2.1).
- The code `means house /hovtest=levene (TYPE=ABS)` lets SAS do the Levene test. In SAS it is also possible to choose the option `(TYPE=SQUARE)` which uses the squared differences.
- The Brown–Forsythe test can be conducted with the option `/hovtest=BF`.

R code

```
# Calculate group means for each field
m<-tapply(crop$kg,crop$house,mean)

# Calculate the Z's
z<-abs(crop$kg-m[crop$house])

# Overall mean of the Z's
z_mean=mean(z)

# Group mean of the Z's
z_gm<-tapply(z,crop$house,mean)

# Make a matrix of the Z's (group in the rows)
z_matrix<-rbind(z[crop$house==1],z[crop$house==2],
                z[crop$house==3])

# Calculate the numerator
nu<-0
for (i in 1:3)
 {
  u<-5*(z_gm[i]-z_mean)^2
  nu<-nu+u
 }

# Calculate the denominator
de<-0
for (j in 1:3)
{
 for (i in 1:5)
  {
   e<-(z_matrix[j,i]-z_gm[j])^2
   de<-de+e
  }
}

# Calculate test statistic and p-value
l<-(12*nu)/(2*de)
p_value<-1-pf(l,2,12)

# Output results
"Levene Test"
l
p_value
```

R output

```
[1] "Levene Test"
> l
       1
2.789499
```

```
> p_value
        1
0.1011865
>
```

Remarks:

- There is no basic R function to calculate Levene's test directly.
- In this example we have $k = 3$ and $\sum_{j=1}^{k}(n_j - 1) = 12$. The respective parts must be adopted if other data are used.
- To apply the Brown–Forsythe test just change the first line of code to `m<-tapply(crop$kg,crop$house,median)`.

References

Bartlett M.S. Properties of sufficiency and statistical tests. *Proceedings of the Royal Statistical Society Series A* **160**, 268–282.

Brown M.B. and Forsythe A.B. 1974 Robust tests for the equality of variances. *Journal of the American Statistical Association* **69**, 364–367.

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Levene H. 1960. *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling* (eds Olkin I et al.), pp. 278–292. Stanford University Press.

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Appendix A

Datasets

Table A.1	Systolic blood pressure (mmHg) of 25 healthy subjects (status=0) and 30 subjects with hypertension (status=1).
Table A.2	Results of a test of the intelligence quotient of 20 subjects before training (IQ1) and after training (IQ2)
Table A.3	Diameters (cm) of workpieces produced by three different machines.
Table A.4	Status of malfunction of 40 workpieces produced by companies A and B, where malfunction=1 indicates a defect.
Table A.5	Number of hospital infections and number of hospitals on the islands of Laputa and Luggnagg with these infections as well as the total number of hospitals on both islands
Table A.6	Body weight (cm) and body height (kg) of 10 male (sex=1) and 10 female (sex=2) students of a biometry and statistic course
Table A.7	Results of 15 coin tosses with heads (side=1) and tails (side=0)
Table A.8	Wheat harvest (in million tons) in Hyboria between 2002 and 2011
Table A.9	Results of inspecting X-rays from 20 patients by two independent reviewers (1=silicosis; 0=no silicosis)
Table A.10	Waiting time (in minutes) at a ticket machine
Table A.11	p-values of 20 t-tests
Table A.12	Crop of tomatoes (in kilograms) of 15 fields in three different green houses with five different fertilizers
Table A.13	Contingency table with the health ratings (poor, fair, and good) of 94 patients determined by two general practitioners. Numbers are absolute values of patients

We accompany each test with a simple example. It shows how to apply these tests in SAS and R. All these datasets are artificial and only for demonstration purpose. Therefore you are welcome to use them for your own purposes, for example, teaching classes but with making reference to our work. The datasets range from medicine, agriculture, gambling to engineering, so we hope this satisfies everybody.

We assume that you are familiar with SAS and R and know how to read data in these programs. Nevertheless, we recommend to download these datasets from our website

`http:\\www.d-taeger.de`

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There you find all datasets as SAS and ASCII files. This can save you a lot of time. Furthermore, all SAS and R codes of each test are stored on this website, either as a SAS file or as an R file. To make the data quickly available we recommend using the file `files.sas` for SAS and `files.R` which you will also find on the website. After running these files the datasets are directly accessible.

In SAS you have to change first the `libname` statement in `files.sas`. This must point to the directory where the datasets are stored on your computer or network directory. So, open `files.sas` and replace *directory* in `libname c "directory";` with the path where you have stored the datasets, for example, `"c:\documents\wileybook\code"`. After running this file the datasets are stored in the *work* library of your SAS session.

Table A.1 Systolic blood pressure (mmHg) of 25 healthy subjects (status=0) and 30 subjects with hypertension (status=1).

No.	Status	mmHg	No.	Status	mmHg
1	0	120	29	1	127
2	0	115	30	1	141
3	0	94	31	1	149
4	0	118	32	1	144
5	0	111	33	1	142
6	0	102	34	1	149
7	0	102	35	1	161
8	0	131	36	1	143
9	0	104	37	1	140
10	0	107	38	1	148
11	0	115	39	1	149
12	0	139	40	1	141
13	0	115	41	1	146
14	0	113	42	1	159
15	0	114	43	1	152
16	0	105	44	1	135
17	0	115	45	1	134
18	0	134	46	1	161
19	0	109	47	1	130
20	0	109	48	1	125
21	0	93	49	1	141
22	0	118	50	1	148
23	0	109	51	1	153
24	0	106	52	1	145
25	0	125	53	1	137
26	1	150	54	1	147
27	1	142	55	1	169
28	1	119			

Dataset name: blood_pressure
Data used in the following test: 2.1.1; 2.1.2; 2.2.1; 2.2.2; 2.2.3; 3.1.1; 3.1.2; 3.2.1; 8.1.1; 8.1.2; 8.2.1; 9.1.1; 9.2.1; 9.1.3; 10.1.1; 11.1.1; 11.1.2; 11.1.3; 11.2.1; 11.2.2

If you use R replace *directory* in `path="directory"`, with the path where the datasets are stored, for example, *c:\\documents\\wileybook\\code*. Note, the double backslashes (\\) are needed here. Now the datasets are available during your R session.

You will find the dataset names as well as the number which refers to the tests in this book in the footnotes to the tables. The variable names correspond to the column names in the tables. Please note, R is case sensitive. This may lead to errors. In the case of any questions you may contact us at: `book@d-taeger.de`.

Table A.2 Results of a test of the intelligence quotient of 20 subjects before training (IQ1) and after training (IQ2).

No.	IQ1	IQ2	No.	IQ1	IQ2
1	127	137	11	88	98
2	98	108	12	96	106
3	105	115	13	110	120
4	83	93	14	87	97
5	133	143	15	88	98
6	90	100	16	88	100
7	107	117	17	105	115
8	98	108	18	95	111
9	91	101	19	79	89
10	100	110	20	106	116

Dataset name: iq
Data used in the following tests: 2.2.4; 2.2.5; 3.2.2; 8.2.2; 14.2.2

Table A.3 Diameters (cm) of workpieces produced by three different machines.

No.	Machine	Diameter	No.	Machine	Diameter
1	1	10.36	16	2	8.95
2	1	9.37	17	2	9.90
3	1	8.61	18	2	10.16
4	1	11.19	19	2	8.48
5	1	8.86	20	2	8.00
6	1	10.43	21	3	12.88
7	1	9.59	22	3	8.38
8	1	11.30	23	3	8.10
9	1	9.17	24	3	13.09
10	1	9.86	25	3	8.85
11	2	6.75	26	3	7.17
12	2	11.82	27	3	10.60
13	2	15.58	28	3	9.43
14	2	11.12	29	3	10.06
15	2	6.54	30	3	7.40

Dataset name: workpieces
Data used in the following test: 8.3.1

Table A.4 Status of malfunction of 40 workpieces produced by companies A and B, where malfunction=1 indicates a defect.

No.	Company	Malfunction	No.	Company	Malfunction
1	A	1	21	B	0
2	A	1	22	B	0
3	A	0	23	B	0
4	A	1	24	B	0
5	A	0	25	B	1
6	A	1	26	B	0
7	A	0	27	B	0
8	A	1	28	B	0
9	A	0	29	B	0
10	A	1	30	B	0
11	A	0	31	B	1
12	A	0	32	B	0
13	A	1	33	B	1
14	A	0	34	B	1
15	A	0	35	B	0
16	A	1	36	B	0
17	A	1	37	B	0
18	A	1	38	B	0
19	A	0	39	B	0
20	A	1	40	B	0

Dataset name: malfunction
Data used in the following tests: 4.1.1; 4.2.1; 4.2.2; 14.1.1; 14.1.2; 14.1.3; 14.3.1; 14.3.2

Table A.5 Number of hospital infections and number of hospitals on the islands of Laputa and Luggnagg with these infections as well as the total number of hospitals on both islands.

Infections	Laputa	Luggnagg	Total
0	0	1	1
1	2	1	3
2	4	3	7
3	6	5	11
4	5	4	9
5	3	3	6
6	0	5	5

Dataset name: infections
Data used in the following tests: 5.1.1; 5.1.2

Table A.6 Body weight (cm) and body height (kg) of 10 male (sex=1) and 10 female (sex=2) students of a biometry and statistic course.

No.	Height	Weight	Sex	No.	Height	Weight	Sex
1	197	93	1	11	167	59	2
2	165	59	1	12	176	70	2
3	179	71	1	13	161	57	2
4	191	78	1	14	168	60	2
5	177	72	1	15	164	66	2
6	153	61	1	16	181	67	2
7	169	72	1	17	182	71	2
8	178	29	1	18	143	46	2
9	184	85	1	19	169	53	2
10	177	75	1	20	175	66	2

Dataset name: students
Data used in the following tests: 7.1.1; 7.1.2; 7.1.3; 7.2.1; 15.1.1; 15.1.2; 15.1.3; 16.1.1; 16.1.2; 16.2.1; 16.2.2

Table A.7 Results of 15 coin tosses with heads (side=1) and tails (side=0).

Toss	Side	Toss	Side	Toss	Side
1	1	6	0	11	1
2	1	7	1	12	1
3	0	8	0	13	0
4	1	9	0	14	1
5	0	10	0	15	0

Dataset name: coin
Data used in the following test: 13.1.1

Table A.8 Wheat harvest (in million tons) in Hyboria between 2002 and 2011.

Year	Harvest	Year	Harvest
2002	488	2007	496
2003	158	2008	302
2004	262	2009	391
2005	457	2010	377
2006	140	2011	220

Dataset name: harvest
Data used in the following test: 13.1.2; 13.2.1; 13.2.2

Table A.9 Results of inspecting X-rays from 20 patients by two independent reviewers (1=silicosis; 0=no silicosis).

Patient	Reviewer1	Reviewer2	Patient	Reviewer1	Reviewer2
1	1	1	11	1	1
2	0	1	12	0	0
3	0	0	13	0	0
4	1	1	14	1	0
5	1	0	15	1	0
6	1	1	16	0	1
7	0	0	17	0	0
8	0	0	18	1	1
9	1	0	19	1	1
10	0	1	20	0	0

Dataset name: silicosis
Data used in the following test: 14.2.1

Table A.10 Waiting time (in minutes) at a ticket machine.

No.	Time	No.	Time	No.	Time
1	8.3	5	11.7	9	0.9
2	7.9	6	12.8	10	15.2
3	7.4	7	2.4		
4	0.6	8	0.8		

Dataset name: waiting
Data used in the following tests: 12.1.1; 12.1.2; 12.1.3; 15.2.1

Table A.11 p-values of 20 t-tests.

No.	pvalue	No.	pvalue	No.	pvalue	No.	pvalue
1	0.9502	6	0.5679	11	0.7327	16	0.1574
2	0.3859	7	0.4772	12	0.3858	17	0.9634
3	0.7718	8	0.7148	13	0.3056	18	0.0284
4	0.5159	9	0.0834	14	0.1298	19	0.2220
5	0.9057	10	0.8021	15	0.3189	20	0.7318

Dataset name: pvalues
Data used in the following test: 15.2.2

Table A.12 Crop of tomatoes (in kilograms) of 15 fields in three different greenhouses with five different fertilizers.

No.	kg	House	Fertilizer	No.	kg	House	Fertilizer
1	0.51	1	1	9	0.06	2	4
2	0.25	1	2	10	0.42	2	5
3	0.64	1	3	11	1.13	3	1
4	0.22	1	4	12	0.43	3	2
5	1.05	1	5	13	0.22	3	3
6	0.99	2	1	14	0.25	3	4
7	0.40	2	2	15	1.81	3	5
8	0.94	2	3				

Dataset name: crop
Data used in the following tests: 17.1.1; 17.1.2; 17.2.1; 17.2.2

Table A.13 Contingency table with the health ratings (poor, fair, and good) of 94 patients determined by two general practitioners. Numbers are absolute values of patients.

GP1 / GP2	Poor	Fair	Good
Poor	10	8	12
Fair	13	14	6
Good	1	10	20

Dataset name: none
Data used in the following test: 14.2.3

Appendix B

Tables

Table B.1 Critical values $u_{1-\alpha}$ of the standard Gaussian distribution.

α	0.005	0.01	0.025	0.05	0.10	0.20
	2.5758	2.3263	1.9600	1.6449	1.2816	0.8416

[a] $u_{1-\alpha}$ is the $(1-\alpha)$-quantile of the standard normal distribution with $u_{1-\alpha} = -u_\alpha$
Calculation in SAS: `probit(1-α)` Calculation in R: `qnorm(1-α)`

Statistical Hypothesis Testing with SAS and R, First Edition. Dirk Taeger and Sonja Kuhnt.

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Table B.2 Critical values $t_{1-\alpha;\nu}$ of the t-distribution with ν degrees of freedom.

ν	α					
	0.005	0.01	0.025	0.05	0.10	0.20
1	63.6567	31.8205	12.7062	6.3138	3.0777	1.3764
2	9.9248	6.9646	4.3027	2.9200	1.8856	1.0607
3	5.8409	4.5407	3.1824	2.3534	1.6377	0.9785
4	4.6041	3.7469	2.7764	2.1318	1.5332	0.9410
5	4.0321	3.3649	2.5706	2.0150	1.4759	0.9195
6	3.7074	3.1427	2.4469	1.9432	1.4398	0.9057
7	3.4995	2.9980	2.3646	1.8946	1.4149	0.8960
8	3.3554	2.8965	2.3060	1.8595	1.3968	0.8889
9	3.2498	2.8214	2.2622	1.8331	1.3830	0.8834
10	3.1693	2.7638	2.2281	1.8125	1.3722	0.8791
11	3.1058	2.7181	2.2010	1.7959	1.3634	0.8755
12	3.0545	2.6810	2.1788	1.7823	1.3562	0.8726
13	3.0123	2.6503	2.1604	1.7709	1.3502	0.8702
14	2.9768	2.6245	2.1448	1.7613	1.3450	0.8681
15	2.9467	2.6025	2.1314	1.7531	1.3406	0.8662
16	2.9208	2.5835	2.1199	1.7459	1.3368	0.8647
17	2.8982	2.5669	2.1098	1.7396	1.3334	0.8633
18	2.8784	2.5524	2.1009	1.7341	1.3304	0.8620
19	2.8609	2.5395	2.0930	1.7291	1.3277	0.8610
20	2.8453	2.5280	2.0860	1.7247	1.3253	0.8600
21	2.8314	2.5176	2.0796	1.7207	1.3232	0.8591
22	2.8188	2.5083	2.0739	1.7171	1.3212	0.8583
23	2.8073	2.4999	2.0687	1.7139	1.3195	0.8575
24	2.7969	2.4922	2.0639	1.7109	1.3178	0.8569
25	2.7874	2.4851	2.0595	1.7081	1.3163	0.8562
26	2.7787	2.4786	2.0555	1.7056	1.3150	0.8557
27	2.7707	2.4727	2.0518	1.7033	1.3137	0.8551
28	2.7633	2.4671	2.0484	1.7011	1.3125	0.8546
29	2.7564	2.4620	2.0452	1.6991	1.3114	0.8542
30	2.7500	2.4573	2.0423	1.6973	1.3104	0.8538
40	2.7045	2.4233	2.0211	1.6839	1.3031	0.8507
50	2.6778	2.4033	2.0086	1.6759	1.2987	0.8489
60	2.6603	2.3901	2.0003	1.6706	1.2958	0.8477
70	2.6479	2.3808	1.9944	1.6669	1.2938	0.8468
80	2.6387	2.3739	1.9901	1.6641	1.2922	0.8461
90	2.6316	2.3685	1.9867	1.6620	1.2910	0.8456
100	2.6259	2.3642	1.9840	1.6602	1.2901	0.8452
250	2.5956	2.3414	1.9695	1.6510	1.2849	0.8431
500	2.5857	2.3338	1.9647	1.6479	1.2832	0.8423
∞	2.5758	2.3263	1.9600	1.6449	1.2816	0.8416

[a] $t_{1-\alpha;\nu}$ is the $(1-\alpha)$-quantile of the t-distribution with ν degrees of freedom and $t_{1-\alpha;\nu} = -t_{\alpha;\nu}$ Calculation in SAS: `tinv(1-α,ν)` Calculation in R: `qt(1-α,ν)`

Table B.3 Critical upper-tail values $\chi^2_{1-\alpha;\nu}$ of the χ^2-distribution with ν degrees of freedom.

ν	α					
	0.005	0.01	0.025	0.05	0.10	0.20
1	7.8794	6.6349	5.0239	3.8415	2.7055	1.6424
2	10.5966	9.2103	7.3778	5.9915	4.6052	3.2189
3	12.8382	11.3449	9.3484	7.8147	6.2514	4.6416
4	14.8603	13.2767	11.1433	9.4877	7.7794	5.9886
5	16.7496	15.0863	12.8325	11.0705	9.2364	7.2893
6	18.5476	16.8119	14.4494	12.5916	10.6446	8.5581
7	20.2777	18.4753	16.0128	14.0671	12.0170	9.8032
8	21.9550	20.0902	17.5345	15.5073	13.3616	11.0301
9	23.5894	21.6660	19.0228	16.919	14.6837	12.2421
10	25.1882	23.2093	20.4832	18.307	15.9872	13.4420
11	26.7568	24.7250	21.9200	19.6751	17.2750	14.6314
12	28.2995	26.2170	23.3367	21.0261	18.5493	15.8120
13	29.8195	27.6882	24.7356	22.3620	19.8119	16.9848
14	31.3193	29.1412	26.1189	23.6848	21.0641	18.1508
15	32.8013	30.5779	27.4884	24.9958	22.3071	19.3107
16	34.2672	31.9999	28.8454	26.2962	23.5418	20.4651
17	35.7185	33.4087	30.1910	27.5871	24.7690	21.6146
18	37.1565	34.8053	31.5264	28.8693	25.9894	22.7595
19	38.5823	36.1909	32.8523	30.1435	27.2036	23.9004
20	39.9968	37.5662	34.1696	31.4104	28.4120	25.0375
21	41.4011	38.9322	35.4789	32.6706	29.6151	26.1711
22	42.7957	40.2894	36.7807	33.9244	30.8133	27.3015
23	44.1813	41.6384	38.0756	35.1725	32.0069	28.4288
24	45.5585	42.9798	39.3641	36.4150	33.1962	29.5533
25	46.9279	44.3141	40.6465	37.6525	34.3816	30.6752
26	48.2899	45.6417	41.9232	38.8851	35.5632	31.7946
27	49.6449	46.9629	43.1945	40.1133	36.7412	32.9117
28	50.9934	48.2782	44.4608	41.3371	37.9159	34.0266
29	52.3356	49.5879	45.7223	42.5570	39.0875	35.1394
30	53.6720	50.8922	46.9792	43.7730	40.2560	36.2502
40	66.7660	63.6907	59.3417	55.7585	51.8051	47.2685
50	79.4900	76.1539	71.4202	67.5048	63.1671	58.1638
60	91.9517	88.3794	83.2977	79.0819	74.3970	68.9721
70	104.2149	100.4252	95.0232	90.5312	85.5270	79.7146
80	116.3211	112.3288	106.6286	101.8795	96.5782	90.4053
90	128.2989	124.1163	118.1359	113.1453	107.5650	101.0537
100	140.1695	135.8067	129.5612	124.3421	118.4980	111.6667
250	311.3462	304.9396	295.6886	287.8815	279.0504	268.5986
500	585.2066	576.4928	563.8515	553.1268	540.9303	526.4014

[a] $\chi^2_{1-\alpha;\nu}$ is the upper $(1-\alpha)$-quantile of the χ^2-distribution with ν degrees of freedom Calculation in SAS: `cinv(1-α,ν)` Calculation in R: `qchisq(1-α,ν)`

Table B.4 Critical lower-tail values $\chi^2_{\alpha;\nu}$ of the χ^2-distribution with ν degrees of freedom.

ν	α					
	0.005	0.01	0.025	0.05	0.10	0.20
1	3.93^{-5}	0.0002	0.0010	0.0039	0.0158	0.0642
2	0.0100	0.0201	0.0506	0.1026	0.2107	0.4463
3	0.0717	0.1148	0.2158	0.3518	0.5844	1.0052
4	0.2070	0.2971	0.4844	0.7107	1.0636	1.6488
5	0.4117	0.5543	0.8312	1.1455	1.6103	2.3425
6	0.6757	0.8721	1.2373	1.6354	2.2041	3.0701
7	0.9893	1.2390	1.6899	2.1673	2.8331	3.8223
8	1.3444	1.6465	2.1797	2.7326	3.4895	4.5936
9	1.7349	2.0879	2.7004	3.3251	4.1682	5.3801
10	2.1559	2.5582	3.2470	3.9403	4.8652	6.1791
11	2.6032	3.0535	3.8157	4.5748	5.5778	6.9887
12	3.0738	3.5706	4.4038	5.2260	6.3038	7.8073
13	3.5650	4.1069	5.0088	5.8919	7.0415	8.6339
14	4.0747	4.6604	5.6287	6.5706	7.7895	9.4673
15	4.6009	5.2293	6.2621	7.2609	8.5468	10.3070
16	5.1422	5.8122	6.9077	7.9616	9.3122	11.1521
17	5.6972	6.4078	7.5642	8.6718	10.0852	12.0023
18	6.2648	7.0149	8.2307	9.3905	10.8649	12.8570
19	6.8440	7.6327	8.9065	10.1170	11.6509	13.7158
20	7.4338	8.2604	9.5908	10.8508	12.4426	14.5784
21	8.0337	8.8972	10.2829	11.5913	13.2396	15.4446
22	8.6427	9.5425	10.9823	12.3380	14.0415	16.3140
23	9.2604	10.1957	11.6886	13.0905	14.8480	17.1865
24	9.8862	10.8564	12.4012	13.8484	15.6587	18.0618
25	10.5197	11.5240	13.1197	14.6114	16.4734	18.9398
26	11.1602	12.1981	13.8439	15.3792	17.2919	19.8202
27	11.8076	12.8785	14.5734	16.1514	18.1139	20.7030
28	12.4613	13.5647	15.3079	16.9279	18.9392	21.5880
29	13.1211	14.2565	16.0471	17.7084	19.7677	22.4751
30	13.7867	14.9535	16.7908	18.4927	20.5992	23.3641
40	20.7065	22.1643	24.4330	26.5093	29.0505	32.3450
50	27.9907	29.7067	32.3574	34.7643	37.6886	41.4492
60	35.5345	37.4849	40.4817	43.1880	46.4589	50.6406
70	43.2752	45.4417	48.7576	51.7393	55.3289	59.8978
80	51.1719	53.5401	57.1532	60.3915	64.2778	69.2069
90	59.1963	61.7541	65.6466	69.126	73.2911	78.5584
100	67.3276	70.0649	74.2219	77.9295	82.3581	87.9453
250	196.1606	200.9386	208.0978	214.3916	221.8059	231.0128
500	422.3034	429.3875	439.9360	449.1468	459.9261	473.2099

[a] $\chi^2_{\alpha;\nu}$ is the lower α-quantile of the χ^2-distribution with ν degrees of freedom Calculation in SAS: `cinv(α, ν)` Calculation in R: `qchisq(α, ν)`

Table B.5 Critical values $F_{1-\alpha;\nu_1,\nu_2}$ of the F-distribution with ν_1 and ν_2 degrees of freedom for α=0.025.

ν_1	ν_2															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	647.7890	38.5063	17.4434	12.2179	10.0070	8.8131	8.0727	7.5709	7.2093	6.9367	6.7241	6.5538	6.4143	6.2979	6.1995	6.1151
2	799.5000	39.0000	16.0441	10.6491	8.4336	7.2599	6.5415	6.0595	5.7147	5.4564	5.2559	5.0959	4.9653	4.8567	4.7650	4.6867
3	864.1630	39.1655	15.4392	9.9792	7.7636	6.5988	5.8898	5.4160	5.0781	4.8256	4.6300	4.4742	4.3472	4.2417	4.1528	4.0768
4	899.5833	39.2484	15.1010	9.6045	7.3879	6.2272	5.5226	5.0526	4.7181	4.4683	4.2751	4.1212	3.9959	3.8919	3.8043	3.7294
5	921.8479	39.2982	14.8848	9.3645	7.1464	5.9876	5.2852	4.8173	4.4844	4.2361	4.0440	3.8911	3.7667	3.6634	3.5764	3.5021
6	937.1111	39.3315	14.7347	9.1973	6.9777	5.8198	5.1186	4.6517	4.3197	4.0721	3.8807	3.7283	3.6043	3.5014	3.4147	3.3406
7	948.2169	39.3552	14.6244	9.0741	6.8531	5.6955	4.9949	4.5286	4.1970	3.9498	3.7586	3.6065	3.4827	3.3799	3.2934	3.2194
8	956.6562	39.3730	14.5399	8.9796	6.7572	5.5996	4.8993	4.4333	4.1020	3.8549	3.6638	3.5118	3.3880	3.2853	3.1987	3.1248
9	963.2846	39.3869	14.4731	8.9047	6.6811	5.5234	4.8232	4.3572	4.0260	3.7790	3.5879	3.4358	3.3120	3.2093	3.1227	3.0488
10	968.6274	39.3980	14.4189	8.8439	6.6192	5.4613	4.7611	4.2951	3.9639	3.7168	3.5257	3.3736	3.2497	3.1469	3.0602	2.9862
11	973.0252	39.4071	14.3742	8.7935	6.5678	5.4098	4.7095	4.2434	3.9121	3.6649	3.4737	3.3215	3.1975	3.0946	3.0078	2.9337
12	976.7079	39.4146	14.3366	8.7512	6.5245	5.3662	4.6658	4.1997	3.8682	3.6209	3.4296	3.2773	3.1532	3.0502	2.9633	2.889
13	979.8368	39.4210	14.3045	8.7150	6.4876	5.3290	4.6285	4.1622	3.8306	3.5832	3.3917	3.2393	3.1150	3.0119	2.9249	2.8506
14	982.5278	39.4265	14.2768	8.6838	6.4556	5.2968	4.5961	4.1297	3.7980	3.5504	3.3588	3.2062	3.0819	2.9786	2.8915	2.8170
15	984.8668	39.4313	14.2527	8.6565	6.4277	5.2687	4.5678	4.1012	3.7694	3.5217	3.3299	3.1772	3.0527	2.9493	2.8621	2.7875
16	986.9187	39.4354	14.2315	8.6326	6.4032	5.2439	4.5428	4.0761	3.7441	3.4963	3.3044	3.1515	3.0269	2.9234	2.836	2.7614
17	988.7331	39.4391	14.2127	8.6113	6.3814	5.2218	4.5206	4.0538	3.7216	3.4737	3.2816	3.1286	3.0039	2.9003	2.8128	2.7380
18	990.3490	39.4424	14.1960	8.5924	6.3619	5.2021	4.5008	4.0338	3.7015	3.4534	3.2612	3.1081	2.9832	2.8795	2.7919	2.7170
19	991.7973	39.4453	14.1810	8.5753	6.3444	5.1844	4.4829	4.0158	3.6833	3.4351	3.2428	3.0896	2.9646	2.8607	2.7730	2.6980
20	993.1028	39.4479	14.1674	8.5599	6.3286	5.1684	4.4667	3.9995	3.6669	3.4185	3.2261	3.0728	2.9477	2.8437	2.7559	2.6808
21	994.2856	39.4503	14.1551	8.5460	6.3142	5.1538	4.4520	3.9846	3.6520	3.4035	3.2109	3.0575	2.9322	2.8282	2.7403	2.6651
22	995.3622	39.4525	14.1438	8.5332	6.3011	5.1406	4.4386	3.9711	3.6383	3.3897	3.1970	3.0434	2.9181	2.8139	2.7260	2.6507
23	996.3462	39.4544	14.1336	8.5216	6.2891	5.1284	4.4263	3.9587	3.6257	3.3770	3.1843	3.0306	2.9052	2.8009	2.7128	2.6374
24	997.2492	39.4562	14.1241	8.5109	6.2780	5.1172	4.4150	3.9472	3.6142	3.3654	3.1725	3.0187	2.8932	2.7888	2.7006	2.6252
25	998.0808	39.4579	14.1155	8.5010	6.2679	5.1069	4.4045	3.9367	3.6035	3.3546	3.1616	3.0077	2.8821	2.7777	2.6894	2.6138
26	998.8490	39.4594	14.1074	8.4919	6.2584	5.0973	4.3949	3.9269	3.5936	3.3446	3.1516	2.9976	2.8719	2.7673	2.6790	2.6033
27	999.5609	39.4609	14.1000	8.4834	6.2497	5.0884	4.3859	3.9178	3.5845	3.3353	3.1422	2.9881	2.8623	2.7577	2.6692	2.5935
28	1000.2225	39.4622	14.0930	8.4755	6.2416	5.0802	4.3775	3.9093	3.5759	3.3267	3.1334	2.9793	2.8534	2.7487	2.6602	2.5844
29	1000.8388	39.4634	14.0866	8.4681	6.2340	5.0724	4.3697	3.9014	3.5679	3.3186	3.1253	2.9710	2.8451	2.7403	2.6517	2.5758
30	1001.4144	39.4646	14.0805	8.4613	6.2269	5.0652	4.3624	3.8940	3.5604	3.3110	3.1176	2.9633	2.8372	2.7324	2.6437	2.5678
40	1005.5981	39.4729	14.0365	8.4111	6.1750	5.0125	4.3089	3.8398	3.5055	3.2554	3.0613	2.9063	2.7797	2.6742	2.5850	2.5085
50	1008.1171	39.4779	14.0099	8.3808	6.1436	4.9804	4.2763	3.8067	3.4719	3.2214	3.0268	2.8714	2.7443	2.6384	2.5488	2.4719
60	1009.8001	39.4812	13.9921	8.3604	6.1225	4.9589	4.2544	3.7844	3.4493	3.1984	3.0035	2.8478	2.7204	2.6142	2.5242	2.4471
70	1011.0040	39.4836	13.9793	8.3458	6.1074	4.9434	4.2386	3.7684	3.4330	3.1818	2.9867	2.8307	2.7030	2.5966	2.5064	2.4291
80	1011.9079	39.4854	13.9697	8.3349	6.0960	4.9318	4.2268	3.7563	3.4207	3.1694	2.9740	2.8178	2.6900	2.5833	2.4930	2.4154
90	1012.6115	39.4868	13.9623	8.3263	6.0871	4.9227	4.2175	3.7469	3.4111	3.1596	2.9641	2.8077	2.6797	2.5729	2.4824	2.4047
100	1013.1748	39.4879	13.9563	8.3195	6.0800	4.9154	4.2101	3.7393	3.4034	3.1517	2.9561	2.7996	2.6715	2.5646	2.4739	2.3961
250	1016.2218	39.4939	13.9238	8.2823	6.0413	4.8758	4.1696	3.6981	3.3613	3.1089	2.9124	2.7552	2.6263	2.5186	2.4273	2.3487

Table B.5 (*continued*)

v_1	v_2															
	17	18	19	20	21	22	23	24	25	26	27	28	29	30	40	50
1	6.0420	5.9781	5.9216	5.8715	5.8266	5.7863	5.7498	5.7166	5.6864	5.6586	5.6331	5.6096	5.5878	5.5675	5.4239	5.3403
2	4.6189	4.5597	4.5075	4.4613	4.4199	4.3828	4.3492	4.3187	4.2909	4.2655	4.2421	4.2205	4.2006	4.1821	4.0510	3.9749
3	4.0112	3.9539	3.9034	3.8587	3.8188	3.7829	3.7505	3.7211	3.6943	3.6697	3.6472	3.6264	3.6072	3.5894	3.4633	3.3902
4	3.6648	3.6083	3.5587	3.5147	3.4754	3.4401	3.4083	3.3794	3.3530	3.3289	3.3067	3.2863	3.2674	3.2499	3.1261	3.0544
5	3.4379	3.3820	3.3327	3.2891	3.2501	3.2151	3.1835	3.1548	3.1287	3.1048	3.0828	3.0626	3.0438	3.0265	2.9037	2.8327
6	3.2767	3.2209	3.1718	3.1283	3.0895	3.0546	3.0232	2.9946	2.9685	2.9447	2.9228	2.9027	2.8840	2.8667	2.7444	2.6736
7	3.1556	3.0999	3.0509	3.0074	2.9686	2.9338	2.9023	2.8738	2.8478	2.8240	2.8021	2.7820	2.7633	2.7460	2.6238	2.5530
8	3.0610	3.0053	2.9563	2.9128	2.8740	2.8392	2.8077	2.7791	2.7531	2.7293	2.7074	2.6872	2.6686	2.6513	2.5289	2.4579
9	2.9849	2.9291	2.8801	2.8365	2.7977	2.7628	2.7313	2.7027	2.6766	2.6528	2.6309	2.6106	2.5919	2.5746	2.4519	2.3808
10	2.9222	2.8664	2.8172	2.7737	2.7348	2.6998	2.6682	2.6396	2.6135	2.5896	2.5676	2.5473	2.5286	2.5112	2.3882	2.3168
11	2.8696	2.8137	2.7645	2.7209	2.6819	2.6469	2.6152	2.5865	2.5603	2.5363	2.5143	2.4940	2.4752	2.4577	2.3343	2.2627
12	2.8249	2.7689	2.7196	2.6758	2.6368	2.6017	2.5699	2.5411	2.5149	2.4908	2.4688	2.4484	2.4295	2.4120	2.2882	2.2162
13	2.7863	2.7302	2.6808	2.6369	2.5978	2.5626	2.5308	2.5019	2.4756	2.4515	2.4293	2.4089	2.3900	2.3724	2.2481	2.1758
14	2.7526	2.6964	2.6469	2.6030	2.5638	2.5285	2.4966	2.4677	2.4413	2.4171	2.3949	2.3743	2.3554	2.3378	2.2130	2.1404
15	2.7230	2.6667	2.6171	2.5731	2.5338	2.4984	2.4665	2.4374	2.4110	2.3867	2.3644	2.3438	2.3248	2.3072	2.1819	2.1090
16	2.6968	2.6404	2.5907	2.5465	2.5071	2.4717	2.4396	2.4105	2.3840	2.3597	2.3373	2.3167	2.2976	2.2799	2.1542	2.0810
17	2.6733	2.6168	2.5670	2.5228	2.4833	2.4478	2.4157	2.3865	2.3599	2.3355	2.3131	2.2924	2.2732	2.2554	2.1293	2.0558
18	2.6522	2.5956	2.5457	2.5014	2.4618	2.4262	2.3940	2.3648	2.3381	2.3137	2.2912	2.2704	2.2512	2.2334	2.1068	2.0330
19	2.6331	2.5764	2.5265	2.4821	2.4424	2.4067	2.3745	2.3452	2.3184	2.2939	2.2713	2.2505	2.2313	2.2134	2.0864	2.0122
20	2.6158	2.5590	2.5089	2.4645	2.4247	2.3890	2.3567	2.3273	2.3005	2.2759	2.2533	2.2324	2.2131	2.1952	2.0677	1.9933
21	2.6000	2.5431	2.4930	2.4484	2.4086	2.3728	2.3404	2.3109	2.2840	2.2594	2.2367	2.2158	2.1965	2.1785	2.0506	1.9759
22	2.5855	2.5285	2.4783	2.4337	2.3938	2.3579	2.3254	2.2959	2.2690	2.2443	2.2216	2.2006	2.1812	2.1631	2.0349	1.9599
23	2.5721	2.5151	2.4648	2.4201	2.3801	2.3442	2.3116	2.2821	2.2551	2.2303	2.2076	2.1865	2.1671	2.1490	2.0203	1.9451
24	2.5598	2.5027	2.4523	2.4076	2.3675	2.3315	2.2989	2.2693	2.2422	2.2174	2.1946	2.1735	2.1540	2.1359	2.0069	1.9313
25	2.5484	2.4912	2.4408	2.3959	2.3558	2.3198	2.2871	2.2574	2.2303	2.2054	2.1826	2.1615	2.1419	2.1237	1.9943	1.9186
26	2.5378	2.4806	2.4300	2.3851	2.3450	2.3088	2.2761	2.2464	2.2192	2.1943	2.1714	2.1502	2.1306	2.1124	1.9827	1.9066
27	2.5280	2.4706	2.4200	2.3751	2.3348	2.2986	2.2659	2.2361	2.2089	2.1839	2.1609	2.1397	2.1201	2.1018	1.9718	1.8955
28	2.5187	2.4613	2.4107	2.3657	2.3254	2.2891	2.2563	2.2265	2.1992	2.1742	2.1512	2.1299	2.1102	2.0919	1.9615	1.8850
29	2.5101	2.4527	2.4019	2.3569	2.3165	2.2802	2.2473	2.2174	2.1901	2.1651	2.1420	2.1207	2.1010	2.0827	1.9519	1.8752
30	2.5020	2.4445	2.3937	2.3486	2.3082	2.2718	2.2389	2.2090	2.1816	2.1565	2.1334	2.1121	2.0923	2.0739	1.9429	1.8659
40	2.4422	2.3842	2.3329	2.2873	2.2465	2.2097	2.1763	2.1460	2.1183	2.0928	2.0693	2.0477	2.0276	2.0089	1.8752	1.7963
50	2.4053	2.3468	2.2952	2.2493	2.2081	2.1710	2.1374	2.1067	2.0787	2.0530	2.0293	2.0073	1.9870	1.9681	1.8324	1.7520
60	2.3801	2.3214	2.2696	2.2234	2.1819	2.1446	2.1107	2.0799	2.0516	2.0257	2.0018	1.9797	1.9591	1.9400	1.8028	1.7211
70	2.3619	2.3030	2.2509	2.2045	2.1629	2.1254	2.0913	2.0603	2.0319	2.0058	1.9817	1.9595	1.9388	1.9195	1.7810	1.6984
80	2.3481	2.2890	2.2368	2.1902	2.1485	2.1108	2.0766	2.0454	2.0169	1.9907	1.9665	1.9441	1.9232	1.9039	1.7644	1.6810
90	2.3372	2.2780	2.2257	2.1790	2.1371	2.0993	2.0650	2.0337	2.0051	1.9787	1.9544	1.9319	1.9110	1.8915	1.7512	1.6671
100	2.3285	2.2692	2.2167	2.1699	2.1280	2.0901	2.0557	2.0243	1.9955	1.9691	1.9447	1.9221	1.9011	1.8816	1.7405	1.6558
250	2.2804	2.2205	2.1673	2.1199	2.0773	2.0388	2.0038	1.9718	1.9425	1.9155	1.8905	1.8674	1.8459	1.8258	1.6802	1.5917

Table B.5 (*continued*)

ν_1	ν_2					
	60	70	80	90	100	250
1	5.2856	5.2470	5.2184	5.1962	5.1786	5.0849
2	3.9253	3.8903	3.8643	3.8443	3.8284	3.7439
3	3.3425	3.3090	3.2841	3.2649	3.2496	3.1687
4	3.0077	2.9748	2.9504	2.9315	2.9166	2.8373
5	2.7863	2.7537	2.7295	2.7109	2.6961	2.6175
6	2.6274	2.5949	2.5708	2.5522	2.5374	2.4591
7	2.5068	2.4743	2.4502	2.4316	2.4168	2.3384
8	2.4117	2.3791	2.3549	2.3363	2.3215	2.2429
9	2.3344	2.3017	2.2775	2.2588	2.2439	2.1650
10	2.2702	2.2374	2.2130	2.1942	2.1793	2.0999
11	2.2159	2.1829	2.1584	2.1395	2.1245	2.0447
12	2.1692	2.1361	2.1115	2.0925	2.0773	1.9971
13	2.1286	2.0953	2.0706	2.0515	2.0363	1.9555
14	2.0929	2.0595	2.0346	2.0154	2.0001	1.9187
15	2.0613	2.0277	2.0026	1.9833	1.9679	1.8861
16	2.0330	1.9992	1.9741	1.9546	1.9391	1.8567
17	2.0076	1.9736	1.9483	1.9288	1.9132	1.8303
18	1.9846	1.9504	1.9250	1.9053	1.8897	1.8062
19	1.9636	1.9293	1.9037	1.8840	1.8682	1.7842
20	1.9445	1.9100	1.8843	1.8644	1.8486	1.7640
21	1.9269	1.8922	1.8664	1.8464	1.8305	1.7454
22	1.9106	1.8758	1.8499	1.8298	1.8138	1.7282
23	1.8956	1.8606	1.8346	1.8144	1.7983	1.7122
24	1.8817	1.8466	1.8204	1.8001	1.7839	1.6973
25	1.8687	1.8334	1.8071	1.7867	1.7705	1.6834
26	1.8566	1.8212	1.7947	1.7743	1.7579	1.6704
27	1.8453	1.8097	1.7831	1.7626	1.7461	1.6581
28	1.8346	1.7989	1.7722	1.7516	1.7351	1.6466
29	1.8246	1.7888	1.7620	1.7412	1.7247	1.6357
30	1.8152	1.7792	1.7523	1.7315	1.7148	1.6254
40	1.7440	1.7069	1.6790	1.6574	1.6401	1.5465
50	1.6985	1.6604	1.6318	1.6095	1.5917	1.4945
60	1.6668	1.6279	1.5987	1.5758	1.5575	1.4573
70	1.6433	1.6038	1.5740	1.5507	1.5320	1.4291
80	1.6252	1.5851	1.5549	1.5312	1.5122	1.4069
90	1.6108	1.5702	1.5396	1.5156	1.4963	1.3889
100	1.5990	1.5581	1.5271	1.5028	1.4833	1.3739
250	1.5317	1.4880	1.4546	1.4282	1.4067	1.2821

[a] $F_{1-\alpha;\nu_1,\nu_2}$ is the $1-\alpha$-quantile of the F-distribution with ν_1 (numerator) and ν_2 (denominator) degrees of freedom Calculation in SAS: `finv(`$1-\alpha;\nu_1,\nu_2$`)` Calculation in R: `qf(`$1-\alpha;\nu_1,\nu_2$`)`

Table B.6 Critical values $F_{1-\alpha;\nu_1,\nu_2}$ of the F-distribution with ν_1 and ν_2 degrees of freedom for α=0.05.

ν_1	ν_2															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	161.4476	18.5128	10.1280	7.7086	6.6079	5.9874	5.5914	5.3177	5.1174	4.9646	4.8443	4.7472	4.6672	4.6001	4.5431	4.4940
2	199.5000	19.0000	9.5521	6.9443	5.7861	5.1433	4.7374	4.4590	4.2565	4.1028	3.9823	3.8853	3.8056	3.7389	3.6823	3.6337
3	215.7073	19.1643	9.2766	6.5914	5.4095	4.7571	4.3468	4.0662	3.8625	3.7083	3.5874	3.4903	3.4105	3.3439	3.2874	3.2389
4	224.5832	19.2468	9.1172	6.3882	5.1922	4.5337	4.1203	3.8379	3.6331	3.4780	3.3567	3.2592	3.1791	3.1122	3.0556	3.0069
5	230.1619	19.2964	9.0135	6.2561	5.0503	4.3874	3.9715	3.6875	3.4817	3.3258	3.2039	3.1059	3.0254	2.9582	2.9013	2.8524
6	233.9860	19.3295	8.9406	6.1631	4.9503	4.2839	3.8660	3.5806	3.3738	3.2172	3.0946	2.9961	2.9153	2.8477	2.7905	2.7413
7	236.7684	19.3532	8.8867	6.0942	4.8759	4.2067	3.7870	3.5005	3.2927	3.1355	3.0123	2.9134	2.8321	2.7642	2.7066	2.6572
8	238.8827	19.3710	8.8452	6.0410	4.8183	4.1468	3.7257	3.4381	3.2296	3.0717	2.9480	2.8486	2.7669	2.6987	2.6408	2.5911
9	240.5433	19.3848	8.8123	5.9988	4.7725	4.0990	3.6767	3.3881	3.1789	3.0204	2.8962	2.7964	2.7144	2.6458	2.5876	2.5377
10	241.8817	19.3959	8.7855	5.9644	4.7351	4.0600	3.6365	3.3472	3.1373	2.9782	2.8536	2.7534	2.6710	2.6022	2.5437	2.4935
11	242.9835	19.4050	8.7633	5.9358	4.7040	4.0274	3.6030	3.3130	3.1025	2.9430	2.8179	2.7173	2.6347	2.5655	2.5068	2.4564
12	243.9060	19.4125	8.7446	5.9117	4.6777	3.9999	3.5747	3.2839	3.0729	2.9130	2.7876	2.6866	2.6037	2.5342	2.4753	2.4247
13	244.6898	19.4189	8.7287	5.8911	4.6552	3.9764	3.5503	3.2590	3.0475	2.8872	2.7614	2.6602	2.5769	2.5073	2.4481	2.3973
14	245.3640	19.4244	8.7149	5.8733	4.6358	3.9559	3.5292	3.2374	3.0255	2.8647	2.7386	2.6371	2.5536	2.4837	2.4244	2.3733
15	245.9499	19.4291	8.7029	5.8578	4.6188	3.9381	3.5107	3.2184	3.0061	2.8450	2.7186	2.6169	2.5331	2.4630	2.4034	2.3522
16	246.4639	19.4333	8.6923	5.8441	4.6038	3.9223	3.4944	3.2016	2.9890	2.8276	2.7009	2.5989	2.5149	2.4446	2.3849	2.3335
17	246.9184	19.4370	8.6829	5.8320	4.5904	3.9083	3.4799	3.1867	2.9737	2.8120	2.6851	2.5828	2.4987	2.4282	2.3683	2.3167
18	247.3232	19.4402	8.6745	5.8211	4.5785	3.8957	3.4669	3.1733	2.9600	2.7980	2.6709	2.5684	2.4841	2.4134	2.3533	2.3016
19	247.6861	19.4431	8.6670	5.8114	4.5678	3.8844	3.4551	3.1613	2.9477	2.7854	2.6581	2.5554	2.4709	2.4000	2.3398	2.2880
20	248.0131	19.4458	8.6602	5.8025	4.5581	3.8742	3.4445	3.1503	2.9365	2.7740	2.6464	2.5436	2.4589	2.3879	2.3275	2.2756
21	248.3094	19.4481	8.6540	5.7945	4.5493	3.8649	3.4349	3.1404	2.9263	2.7636	2.6358	2.5328	2.4479	2.3768	2.3163	2.2642
22	248.5791	19.4503	8.6484	5.7872	4.5413	3.8564	3.4260	3.1313	2.9169	2.7541	2.6261	2.5229	2.4379	2.3667	2.3060	2.2538
23	248.8256	19.4523	8.6432	5.7805	4.5339	3.8486	3.4179	3.1229	2.9084	2.7453	2.6172	2.5139	2.4287	2.3573	2.2966	2.2443
24	249.0518	19.4541	8.6385	5.7744	4.5272	3.8415	3.4105	3.1152	2.9005	2.7372	2.6090	2.5055	2.4202	2.3487	2.2878	2.2354
25	249.2601	19.4558	8.6341	5.7687	4.5209	3.8348	3.4036	3.1081	2.8932	2.7298	2.6014	2.4977	2.4123	2.3407	2.2797	2.2272
26	249.4525	19.4573	8.6301	5.7635	4.5151	3.8287	3.3972	3.1015	2.8864	2.7229	2.5943	2.4905	2.4050	2.3333	2.2722	2.2196
27	249.6309	19.4587	8.6263	5.7586	4.5097	3.8230	3.3913	3.0954	2.8801	2.7164	2.5877	2.4838	2.3982	2.3264	2.2652	2.2125
28	249.7966	19.4600	8.6229	5.7541	4.5047	3.8177	3.3858	3.0897	2.8743	2.7104	2.5816	2.4776	2.3918	2.3199	2.2587	2.2059
29	249.9510	19.4613	8.6196	5.7498	4.5001	3.8128	3.3806	3.0844	2.8688	2.7048	2.5759	2.4718	2.3859	2.3139	2.2525	2.1997
30	250.0951	19.4624	8.6166	5.7459	4.4957	3.8082	3.3758	3.0794	2.8637	2.6996	2.5705	2.4663	2.3803	2.3082	2.2468	2.1938
40	251.1432	19.4707	8.5944	5.7170	4.4638	3.7743	3.3404	3.0428	2.8259	2.6609	2.5309	2.4259	2.3392	2.2664	2.2043	2.1507
50	251.7742	19.4757	8.5810	5.6995	4.4444	3.7537	3.3189	3.0204	2.8028	2.6371	2.5066	2.4010	2.3138	2.2405	2.1780	2.1240
60	252.1957	19.4791	8.5720	5.6877	4.4314	3.7398	3.3043	3.0053	2.7872	2.6211	2.4901	2.3842	2.2966	2.2229	2.1601	2.1058
70	252.4973	19.4814	8.5656	5.6793	4.4220	3.7298	3.2939	2.9944	2.7760	2.6095	2.4782	2.3720	2.2841	2.2102	2.1472	2.0926
80	252.7237	19.4832	8.5607	5.6730	4.4150	3.7223	3.2860	2.9862	2.7675	2.6008	2.4692	2.3628	2.2747	2.2006	2.1373	2.0826
90	252.9000	19.4846	8.5569	5.6680	4.4095	3.7164	3.2798	2.9798	2.7609	2.5939	2.4622	2.3556	2.2673	2.1931	2.1296	2.0748
100	253.0411	19.4857	8.5539	5.6641	4.4051	3.7117	3.2749	2.9747	2.7556	2.5884	2.4566	2.3498	2.2614	2.1870	2.1234	2.0685
250	253.8043	19.4917	8.5375	5.6425	4.3811	3.6861	3.2479	2.9466	2.7264	2.5583	2.4256	2.3179	2.2287	2.1536	2.0893	2.0336

Table B.6 (*continued*)

v_1	v_2															
	17	18	19	20	21	22	23	24	25	26	27	28	29	30	40	50
1	4.4513	4.4139	4.3807	4.3512	4.3248	4.3009	4.2793	4.2597	4.2417	4.2252	4.2100	4.1960	4.1830	4.1709	4.0847	4.0343
2	3.5915	3.5546	3.5219	3.4928	3.4668	3.4434	3.4221	3.4028	3.3852	3.3690	3.3541	3.3404	3.3277	3.3158	3.2317	3.1826
3	3.1968	3.1599	3.1274	3.0984	3.0725	3.0491	3.0280	3.0088	2.9912	2.9752	2.9604	2.9467	2.9340	2.9223	2.8387	2.7900
4	2.9647	2.9277	2.8951	2.8661	2.8401	2.8167	2.7955	2.7763	2.7587	2.7426	2.7278	2.7141	2.7014	2.6896	2.6060	2.5572
5	2.8100	2.7729	2.7401	2.7109	2.6848	2.6613	2.6400	2.6207	2.6030	2.5868	2.5719	2.5581	2.5454	2.5336	2.4495	2.4004
6	2.6987	2.6613	2.6283	2.5990	2.5727	2.5491	2.5277	2.5082	2.4904	2.4741	2.4591	2.4453	2.4324	2.4205	2.3359	2.2864
7	2.6143	2.5767	2.5435	2.5140	2.4876	2.4638	2.4422	2.4226	2.4047	2.3883	2.3732	2.3593	2.3463	2.3343	2.2490	2.1992
8	2.5480	2.5102	2.4768	2.4471	2.4205	2.3965	2.3748	2.3551	2.3371	2.3205	2.3053	2.2913	2.2783	2.2662	2.1802	2.1299
9	2.4943	2.4563	2.4227	2.3928	2.3660	2.3419	2.3201	2.3002	2.2821	2.2655	2.2501	2.236	2.2229	2.2107	2.1240	2.0734
10	2.4499	2.4117	2.3779	2.3479	2.3210	2.2967	2.2747	2.2547	2.2365	2.2197	2.2043	2.1900	2.1768	2.1646	2.0772	2.0261
11	2.4126	2.3742	2.3402	2.3100	2.2829	2.2585	2.2364	2.2163	2.1979	2.1811	2.1655	2.1512	2.1379	2.1256	2.0376	1.9861
12	2.3807	2.3421	2.3080	2.2776	2.2504	2.2258	2.2036	2.1834	2.1649	2.1479	2.1323	2.1179	2.1045	2.0921	2.0035	1.9515
13	2.3531	2.3143	2.2800	2.2495	2.2222	2.1975	2.1752	2.1548	2.1362	2.1192	2.1035	2.0889	2.0755	2.0630	1.9738	1.9214
14	2.3290	2.2900	2.2556	2.2250	2.1975	2.1727	2.1502	2.1298	2.1111	2.0939	2.0781	2.0635	2.0500	2.0374	1.9476	1.8949
15	2.3077	2.2686	2.2341	2.2033	2.1757	2.1508	2.1282	2.1077	2.0889	2.0716	2.0558	2.0411	2.0275	2.0148	1.9245	1.8714
16	2.2888	2.2496	2.2149	2.1840	2.1563	2.1313	2.1086	2.0880	2.0691	2.0518	2.0358	2.0210	2.0073	1.9946	1.9037	1.8503
17	2.2719	2.2325	2.1977	2.1667	2.1389	2.1138	2.0910	2.0703	2.0513	2.0339	2.0179	2.0030	1.9893	1.9765	1.8851	1.8313
18	2.2567	2.2172	2.1823	2.1511	2.1232	2.0980	2.0751	2.0543	2.0353	2.0178	2.0017	1.9868	1.9730	1.9601	1.8682	1.8141
19	2.2429	2.2033	2.1683	2.1370	2.1090	2.0837	2.0608	2.0399	2.0207	2.0032	1.9870	1.9720	1.9581	1.9452	1.8529	1.7985
20	2.2304	2.1906	2.1555	2.1242	2.0960	2.0707	2.0476	2.0267	2.0075	1.9898	1.9736	1.9586	1.9446	1.9317	1.8389	1.7841
21	2.2189	2.1791	2.1438	2.1124	2.0842	2.0587	2.0356	2.0146	1.9953	1.9776	1.9613	1.9462	1.9322	1.9192	1.8260	1.7709
22	2.2084	2.1685	2.1331	2.1016	2.0733	2.0478	2.0246	2.0035	1.9842	1.9664	1.9500	1.9349	1.9208	1.9077	1.8141	1.7588
23	2.1987	2.1587	2.1233	2.0917	2.0633	2.0377	2.0144	1.9932	1.9738	1.9560	1.9396	1.9244	1.9103	1.8972	1.8031	1.7475
24	2.1898	2.1497	2.1141	2.0825	2.0540	2.0283	2.0050	1.9838	1.9643	1.9464	1.9299	1.9147	1.9005	1.8874	1.7929	1.7371
25	2.1815	2.1413	2.1057	2.0739	2.0454	2.0196	1.9963	1.9750	1.9554	1.9375	1.9210	1.9057	1.8915	1.8782	1.7835	1.7273
26	2.1738	2.1335	2.0978	2.0660	2.0374	2.0116	1.9881	1.9668	1.9472	1.9292	1.9126	1.8973	1.8830	1.8698	1.7746	1.7183
27	2.1666	2.1262	2.0905	2.0586	2.0299	2.0040	1.9805	1.9591	1.9395	1.9215	1.9048	1.8894	1.8751	1.8618	1.7663	1.7097
28	2.1599	2.1195	2.0836	2.0517	2.0229	1.9970	1.9734	1.9520	1.9323	1.9142	1.8975	1.8821	1.8677	1.8544	1.7586	1.7017
29	2.1536	2.1131	2.0772	2.0452	2.0164	1.9904	1.9668	1.9453	1.9255	1.9074	1.8907	1.8752	1.8608	1.8474	1.7513	1.6942
30	2.1477	2.1071	2.0712	2.0391	2.0102	1.9842	1.9605	1.9390	1.9192	1.9010	1.8842	1.8687	1.8543	1.8409	1.7444	1.6872
40	2.1040	2.0629	2.0264	1.9938	1.9645	1.9380	1.9139	1.8920	1.8718	1.8533	1.8361	1.8203	1.8055	1.7918	1.6928	1.6337
50	2.0769	2.0354	1.9986	1.9656	1.9360	1.9092	1.8848	1.8625	1.8421	1.8233	1.8059	1.7898	1.7748	1.7609	1.6600	1.5995
60	2.0584	2.0166	1.9795	1.9464	1.9165	1.8894	1.8648	1.8424	1.8217	1.8027	1.7851	1.7689	1.7537	1.7396	1.6373	1.5757
70	2.0450	2.0030	1.9657	1.9323	1.9023	1.8751	1.8503	1.8276	1.8069	1.7877	1.7700	1.7535	1.7382	1.7240	1.6205	1.5580
80	2.0348	1.9927	1.9552	1.9217	1.8915	1.8641	1.8392	1.8164	1.7955	1.7762	1.7584	1.7418	1.7264	1.7121	1.6077	1.5445
90	2.0268	1.9846	1.9470	1.9133	1.8830	1.8555	1.8305	1.8076	1.7866	1.7672	1.7493	1.7326	1.7171	1.7027	1.5975	1.5337
100	2.0204	1.9780	1.9403	1.9066	1.8761	1.8486	1.8234	1.8005	1.7794	1.7599	1.7419	1.7251	1.7096	1.6950	1.5892	1.5249
250	1.9849	1.9418	1.9035	1.8691	1.8381	1.8099	1.7843	1.7608	1.7391	1.7191	1.7006	1.6834	1.6674	1.6524	1.5425	1.4748

Table B.6 (*continued*)

ν_1	ν_2					
	60	70	80	90	100	250
1	4.0012	3.9778	3.9604	3.9469	3.9361	3.8789
2	3.1504	3.1277	3.1108	3.0977	3.0873	3.0319
3	2.7581	2.7355	2.7188	2.7058	2.6955	2.6407
4	2.5252	2.5027	2.4859	2.4729	2.4626	2.4078
5	2.3683	2.3456	2.3287	2.3157	2.3053	2.2501
6	2.2541	2.2312	2.2142	2.2011	2.1906	2.1350
7	2.1665	2.1435	2.1263	2.1131	2.1025	2.0463
8	2.0970	2.0737	2.0564	2.0430	2.0323	1.9756
9	2.0401	2.0166	1.9991	1.9856	1.9748	1.9174
10	1.9926	1.9689	1.9512	1.9376	1.9267	1.8687
11	1.9522	1.9283	1.9105	1.8967	1.8857	1.8271
12	1.9174	1.8932	1.8753	1.8613	1.8503	1.7910
13	1.8870	1.8627	1.8445	1.8305	1.8193	1.7595
14	1.8602	1.8357	1.8174	1.8032	1.7919	1.7315
15	1.8364	1.8117	1.7932	1.7789	1.7675	1.7065
16	1.8151	1.7902	1.7716	1.7571	1.7456	1.6841
17	1.7959	1.7708	1.7520	1.7375	1.7259	1.6638
18	1.7784	1.7531	1.7342	1.7196	1.7079	1.6453
19	1.7625	1.7371	1.7180	1.7033	1.6915	1.6283
20	1.7480	1.7223	1.7032	1.6883	1.6764	1.6127
21	1.7346	1.7088	1.6895	1.6745	1.6626	1.5983
22	1.7222	1.6962	1.6768	1.6618	1.6497	1.585
23	1.7108	1.6846	1.6651	1.6499	1.6378	1.5726
24	1.7001	1.6738	1.6542	1.6389	1.6267	1.5610
25	1.6902	1.6638	1.6440	1.6286	1.6163	1.5502
26	1.6809	1.6543	1.6345	1.6190	1.6067	1.5400
27	1.6722	1.6455	1.6255	1.6100	1.5976	1.5305
28	1.6641	1.6372	1.6171	1.6015	1.5890	1.5215
29	1.6564	1.6294	1.6092	1.5935	1.5809	1.513
30	1.6491	1.6220	1.6017	1.5859	1.5733	1.5049
40	1.5943	1.5661	1.5449	1.5284	1.5151	1.443
50	1.5590	1.5300	1.5081	1.4910	1.4772	1.4019
60	1.5343	1.5046	1.4821	1.4645	1.4504	1.3723
70	1.5160	1.4857	1.4628	1.4448	1.4303	1.3499
80	1.5019	1.4711	1.4477	1.4294	1.4146	1.3322
90	1.4906	1.4594	1.4357	1.4171	1.4020	1.3178
100	1.4814	1.4498	1.4259	1.4070	1.3917	1.3058
250	1.4285	1.3945	1.3684	1.3477	1.3308	1.2318

[a] $F_{1-\alpha;\nu_1,\nu_2}$ is the $1-\alpha$-quantile of the F-distribution with ν_1 (numerator) and ν_2 (denominator) degrees of freedom
Calculation in SAS: `finv` $(1-\alpha;\nu_1,\nu_2)$
Calculation in R: `qf` $(1-\alpha;\nu_1,\nu_2)$

Table B.7 Critical values $F_{1-\alpha;\nu_1,\nu_2}$ of the F-distribution with ν_1 and ν_2 degrees of freedom for α=0.10.

ν_1	ν_2															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	39.8635	8.5263	5.5383	4.5448	4.0604	3.7759	3.5894	3.4579	3.3603	3.2850	3.2252	3.1765	3.1362	3.1022	3.0732	3.0481
2	49.5000	9.0000	5.4624	4.3246	3.7797	3.4633	3.2574	3.1131	3.0065	2.9245	2.8595	2.8068	2.7632	2.7265	2.6952	2.6682
3	53.5932	9.1618	5.3908	4.1909	3.6195	3.2888	3.0741	2.9238	2.8129	2.7277	2.6602	2.6055	2.5603	2.5222	2.4898	2.4618
4	55.8330	9.2434	5.3426	4.1072	3.5202	3.1808	2.9605	2.8064	2.6927	2.6053	2.5362	2.4801	2.4337	2.3947	2.3614	2.3327
5	57.2401	9.2926	5.3092	4.0506	3.4530	3.1075	2.8833	2.7264	2.6106	2.5216	2.4512	2.3940	2.3467	2.3069	2.2730	2.2438
6	58.2044	9.3255	5.2847	4.0097	3.4045	3.0546	2.8274	2.6683	2.5509	2.4606	2.3891	2.3310	2.2830	2.2426	2.2081	2.1783
7	58.9060	9.3491	5.2662	3.9790	3.3679	3.0145	2.7849	2.6241	2.5053	2.4140	2.3416	2.2828	2.2341	2.1931	2.1582	2.1280
8	59.4390	9.3668	5.2517	3.9549	3.3393	2.9830	2.7516	2.5893	2.4694	2.3772	2.3040	2.2446	2.1953	2.1539	2.1185	2.0880
9	59.8576	9.3805	5.2400	3.9357	3.3163	2.9577	2.7247	2.5612	2.4403	2.3473	2.2735	2.2135	2.1638	2.1220	2.0862	2.0553
10	60.1950	9.3916	5.2304	3.9199	3.2974	2.9369	2.7025	2.5380	2.4163	2.3226	2.2482	2.1878	2.1376	2.0954	2.0593	2.0281
11	60.4727	9.4006	5.2224	3.9067	3.2816	2.9195	2.6839	2.5186	2.3961	2.3018	2.2269	2.1660	2.1155	2.0729	2.0366	2.0051
12	60.7052	9.4081	5.2156	3.8955	3.2682	2.9047	2.6681	2.5020	2.3789	2.2841	2.2087	2.1474	2.0966	2.0537	2.0171	1.9854
13	60.9028	9.4145	5.2098	3.8859	3.2567	2.8920	2.6545	2.4876	2.3640	2.2687	2.1930	2.1313	2.0802	2.0370	2.0001	1.9682
14	61.0727	9.4200	5.2047	3.8776	3.2468	2.8809	2.6426	2.4752	2.3510	2.2553	2.1792	2.1173	2.0658	2.0224	1.9853	1.9532
15	61.2203	9.4247	5.2003	3.8704	3.2380	2.8712	2.6322	2.4642	2.3396	2.2435	2.1671	2.1049	2.0532	2.0095	1.9722	1.9399
16	61.3499	9.4289	5.1964	3.8639	3.2303	2.8626	2.6230	2.4545	2.3295	2.2330	2.1563	2.0938	2.0419	1.9981	1.9605	1.9281
17	61.4644	9.4325	5.1929	3.8582	3.2234	2.8550	2.6148	2.4458	2.3205	2.2237	2.1467	2.0839	2.0318	1.9878	1.9501	1.9175
18	61.5664	9.4358	5.1898	3.8531	3.2172	2.8481	2.6074	2.4380	2.3123	2.2153	2.1380	2.0750	2.0227	1.9785	1.9407	1.9079
19	61.6579	9.4387	5.1870	3.8485	3.2117	2.8419	2.6008	2.4310	2.3050	2.2077	2.1302	2.0670	2.0145	1.9701	1.9321	1.8992
20	61.7403	9.4413	5.1845	3.8443	3.2067	2.8363	2.5947	2.4246	2.2983	2.2007	2.1230	2.0597	2.0070	1.9625	1.9243	1.8913
21	61.8150	9.4437	5.1822	3.8405	3.2021	2.8312	2.5892	2.4188	2.2922	2.1944	2.1165	2.0530	2.0001	1.9555	1.9172	1.8840
22	61.8829	9.4458	5.1801	3.8371	3.1979	2.8266	2.5842	2.4135	2.2867	2.1887	2.1106	2.0469	1.9939	1.9490	1.9106	1.8774
23	61.9450	9.4478	5.1781	3.8339	3.1941	2.8223	2.5796	2.4086	2.2816	2.1833	2.1051	2.0412	1.9881	1.9431	1.9046	1.8712
24	62.0020	9.4496	5.1764	3.8310	3.1905	2.8183	2.5753	2.4041	2.2768	2.1784	2.1000	2.0360	1.9827	1.9377	1.8990	1.8656
25	62.0545	9.4513	5.1747	3.8283	3.1873	2.8147	2.5714	2.3999	2.2725	2.1739	2.0953	2.0312	1.9778	1.9326	1.8939	1.8603
26	62.1030	9.4528	5.1732	3.8258	3.1842	2.8113	2.5677	2.3961	2.2684	2.1697	2.0909	2.0267	1.9732	1.9279	1.8891	1.8554
27	62.1480	9.4542	5.1718	3.8235	3.1814	2.8082	2.5643	2.3925	2.2646	2.1657	2.0869	2.0225	1.9689	1.9235	1.8846	1.8508
28	62.1897	9.4556	5.1705	3.8213	3.1788	2.8053	2.5612	2.3891	2.2611	2.1621	2.0831	2.0186	1.9649	1.9194	1.8804	1.8466
29	62.2286	9.4568	5.1693	3.8193	3.1764	2.8025	2.5582	2.3860	2.2578	2.1586	2.0795	2.0149	1.9611	1.9155	1.8765	1.8426
30	62.2650	9.4579	5.1681	3.8174	3.1741	2.8000	2.5555	2.3830	2.2547	2.1554	2.0762	2.0115	1.9576	1.9119	1.8728	1.8388
40	62.5291	9.4662	5.1597	3.8036	3.1573	2.7812	2.5351	2.3614	2.2320	2.1317	2.0516	1.9861	1.9315	1.8852	1.8454	1.8108
50	62.6881	9.4712	5.1546	3.7952	3.1471	2.7697	2.5226	2.3481	2.2180	2.1171	2.0364	1.9704	1.9153	1.8686	1.8284	1.7934
60	62.7943	9.4746	5.1512	3.7896	3.1402	2.7620	2.5142	2.3391	2.2085	2.1072	2.0261	1.9597	1.9043	1.8572	1.8168	1.7816
70	62.8703	9.4769	5.1487	3.7855	3.1353	2.7564	2.5082	2.3326	2.2017	2.1000	2.0187	1.9520	1.8963	1.8490	1.8083	1.7729
80	62.9273	9.4787	5.1469	3.7825	3.1316	2.7522	2.5036	2.3277	2.1965	2.0946	2.0130	1.9461	1.8903	1.8428	1.8019	1.7664
90	62.9717	9.4801	5.1454	3.7801	3.1286	2.7489	2.5000	2.3239	2.1924	2.0903	2.0086	1.9416	1.8855	1.8379	1.7969	1.7612
100	63.0073	9.4812	5.1443	3.7782	3.1263	2.7463	2.4971	2.3208	2.1892	2.0869	2.0050	1.9379	1.8817	1.8340	1.7929	1.7570
250	63.1996	9.4872	5.1379	3.7677	3.1136	2.7319	2.4814	2.3040	2.1714	2.0682	1.9855	1.9175	1.8606	1.8122	1.7705	1.7340

Table B.7 (*continued*)

v_1	v_2															
	17	18	19	20	21	22	23	24	25	26	27	28	29	30	40	50
1	3.0262	3.0070	2.9899	2.9747	2.9610	2.9486	2.9374	2.9271	2.9177	2.9091	2.9012	2.8938	2.8870	2.8807	2.8354	2.8087
2	2.6446	2.6239	2.6056	2.5893	2.5746	2.5613	2.5493	2.5383	2.5283	2.5191	2.5106	2.5028	2.4955	2.4887	2.4404	2.4120
3	2.4374	2.4160	2.3970	2.3801	2.3649	2.3512	2.3387	2.3274	2.3170	2.3075	2.2987	2.2906	2.2831	2.2761	2.2261	2.1967
4	2.3077	2.2858	2.2663	2.2489	2.2333	2.2193	2.2065	2.1949	2.1842	2.1745	2.1655	2.1571	2.1494	2.1422	2.0909	2.0608
5	2.2183	2.1958	2.1760	2.1582	2.1423	2.1279	2.1149	2.1030	2.0922	2.0822	2.0730	2.0645	2.0566	2.0492	1.9968	1.9660
6	2.1524	2.1296	2.1094	2.0913	2.0751	2.0605	2.0472	2.0351	2.0241	2.0139	2.0045	1.9959	1.9878	1.9803	1.9269	1.8954
7	2.1017	2.0785	2.0580	2.0397	2.0233	2.0084	1.9949	1.9826	1.9714	1.9610	1.9515	1.9427	1.9345	1.9269	1.8725	1.8405
8	2.0613	2.0379	2.0171	1.9985	1.9819	1.9668	1.9531	1.9407	1.9292	1.9188	1.9091	1.9001	1.8918	1.8841	1.8289	1.7963
9	2.0284	2.0047	1.9836	1.9649	1.9480	1.9327	1.9189	1.9063	1.8947	1.8841	1.8743	1.8652	1.8568	1.8490	1.7929	1.7598
10	2.0009	1.9770	1.9557	1.9367	1.9197	1.9043	1.8903	1.8775	1.8658	1.8550	1.8451	1.8359	1.8274	1.8195	1.7627	1.7291
11	1.9777	1.9535	1.9321	1.9129	1.8956	1.8801	1.8659	1.8530	1.8412	1.8303	1.8203	1.8110	1.8024	1.7944	1.7369	1.7029
12	1.9577	1.9333	1.9117	1.8924	1.8750	1.8593	1.8450	1.8319	1.8200	1.8090	1.7989	1.7895	1.7808	1.7727	1.7146	1.6802
13	1.9404	1.9158	1.8940	1.8745	1.8570	1.8411	1.8267	1.8136	1.8015	1.7904	1.7802	1.7708	1.7620	1.7538	1.6950	1.6602
14	1.9252	1.9004	1.8785	1.8588	1.8412	1.8252	1.8107	1.7974	1.7853	1.7741	1.7638	1.7542	1.7454	1.7371	1.6778	1.6426
15	1.9117	1.8868	1.8647	1.8449	1.8271	1.8111	1.7964	1.7831	1.7708	1.7596	1.7492	1.7395	1.7306	1.7223	1.6624	1.6269
16	1.8997	1.8747	1.8524	1.8325	1.8146	1.7984	1.7837	1.7703	1.7579	1.7466	1.7361	1.7264	1.7174	1.7090	1.6486	1.6128
17	1.8889	1.8638	1.8414	1.8214	1.8034	1.7871	1.7723	1.7587	1.7463	1.7349	1.7243	1.7146	1.7055	1.6970	1.6362	1.6000
18	1.8792	1.8539	1.8314	1.8113	1.7932	1.7768	1.7619	1.7483	1.7358	1.7243	1.7137	1.7039	1.6947	1.6862	1.6249	1.5884
19	1.8704	1.8450	1.8224	1.8022	1.7840	1.7675	1.7525	1.7388	1.7263	1.7147	1.7040	1.6941	1.6849	1.6763	1.6146	1.5778
20	1.8624	1.8368	1.8142	1.7938	1.7756	1.7590	1.7439	1.7302	1.7175	1.7059	1.6951	1.6852	1.6759	1.6673	1.6052	1.5681
21	1.8550	1.8294	1.8066	1.7862	1.7678	1.7512	1.7360	1.7222	1.7095	1.6978	1.6870	1.6770	1.6677	1.6590	1.5965	1.5592
22	1.8482	1.8225	1.7997	1.7792	1.7607	1.7440	1.7288	1.7149	1.7021	1.6904	1.6795	1.6695	1.6601	1.6514	1.5884	1.5509
23	1.8420	1.8162	1.7932	1.7727	1.7541	1.7374	1.7221	1.7081	1.6953	1.6835	1.6726	1.6625	1.6531	1.6443	1.5810	1.5432
24	1.8362	1.8103	1.7873	1.7667	1.7481	1.7312	1.7159	1.7019	1.6890	1.6771	1.6662	1.6560	1.6465	1.6377	1.5741	1.5361
25	1.8309	1.8049	1.7818	1.7611	1.7424	1.7255	1.7101	1.6960	1.6831	1.6712	1.6602	1.6500	1.6405	1.6316	1.5677	1.5294
26	1.8259	1.7999	1.7767	1.7559	1.7372	1.7202	1.7047	1.6906	1.6776	1.6657	1.6546	1.6444	1.6348	1.6259	1.5617	1.5232
27	1.8213	1.7951	1.7719	1.7510	1.7322	1.7152	1.6997	1.6855	1.6725	1.6605	1.6494	1.6391	1.6295	1.6206	1.5560	1.5173
28	1.8169	1.7907	1.7674	1.7465	1.7276	1.7106	1.6950	1.6808	1.6677	1.6556	1.6445	1.6342	1.6246	1.6156	1.5507	1.5118
29	1.8128	1.7866	1.7632	1.7422	1.7233	1.7062	1.6906	1.6763	1.6632	1.6511	1.6399	1.6295	1.6199	1.6109	1.5458	1.5067
30	1.8090	1.7827	1.7592	1.7382	1.7193	1.7021	1.6864	1.6721	1.6589	1.6468	1.6356	1.6252	1.6155	1.6065	1.5411	1.5018
40	1.7805	1.7537	1.7298	1.7083	1.6890	1.6714	1.6554	1.6407	1.6272	1.6147	1.6032	1.5925	1.5825	1.5732	1.5056	1.4648
50	1.7628	1.7356	1.7114	1.6896	1.6700	1.6521	1.6358	1.6209	1.6072	1.5945	1.5827	1.5718	1.5617	1.5522	1.4830	1.4409
60	1.7506	1.7232	1.6988	1.6768	1.6569	1.6389	1.6224	1.6073	1.5934	1.5805	1.5686	1.5575	1.5472	1.5376	1.4672	1.4242
70	1.7418	1.7142	1.6896	1.6674	1.6474	1.6292	1.6125	1.5973	1.5833	1.5703	1.5582	1.5470	1.5366	1.5269	1.4555	1.4119
80	1.7351	1.7073	1.6826	1.6603	1.6401	1.6218	1.6051	1.5897	1.5755	1.5625	1.5503	1.5390	1.5285	1.5187	1.4465	1.4023
90	1.7298	1.7019	1.6771	1.6547	1.6344	1.6160	1.5991	1.5837	1.5694	1.5563	1.5441	1.5327	1.5221	1.5122	1.4394	1.3947
100	1.7255	1.6976	1.6726	1.6501	1.6298	1.6113	1.5944	1.5788	1.5645	1.5513	1.5390	1.5276	1.5169	1.5069	1.4336	1.3885
250	1.7020	1.6734	1.6479	1.6249	1.6041	1.5851	1.5677	1.5517	1.5370	1.5233	1.5106	1.4988	1.4877	1.4773	1.4007	1.3528

Table B.7 (*continued*)

ν_1	ν_2					
	60	70	80	90	100	250
1	2.7911	2.7786	2.7693	2.7621	2.7564	2.7257
2	2.3933	2.3800	2.3701	2.3625	2.3564	2.3239
3	2.1774	2.1637	2.1535	2.1457	2.1394	2.1058
4	2.0410	2.0269	2.0165	2.0084	2.0019	1.9675
5	1.9457	1.9313	1.9206	1.9123	1.9057	1.8704
6	1.8747	1.8600	1.8491	1.8406	1.8339	1.7978
7	1.8194	1.8044	1.7933	1.7846	1.7778	1.7409
8	1.7748	1.7596	1.7483	1.7395	1.7324	1.6949
9	1.7380	1.7225	1.7110	1.7021	1.6949	1.6567
10	1.7070	1.6913	1.6796	1.6705	1.6632	1.6244
11	1.6805	1.6645	1.6526	1.6434	1.6360	1.5965
12	1.6574	1.6413	1.6292	1.6199	1.6124	1.5723
13	1.6372	1.6209	1.6086	1.5992	1.5916	1.5509
14	1.6193	1.6028	1.5904	1.5808	1.5731	1.5319
15	1.6034	1.5866	1.5741	1.5644	1.5566	1.5149
16	1.5890	1.5721	1.5594	1.5496	1.5418	1.4995
17	1.5760	1.5589	1.5461	1.5362	1.5283	1.4855
18	1.5642	1.5470	1.5340	1.5240	1.5160	1.4727
19	1.5534	1.5360	1.5230	1.5128	1.5047	1.4610
20	1.5435	1.5259	1.5128	1.5025	1.4943	1.4501
21	1.5343	1.5166	1.5034	1.4930	1.4848	1.4401
22	1.5259	1.5080	1.4947	1.4842	1.4759	1.4308
23	1.5180	1.5000	1.4866	1.4761	1.4677	1.4221
24	1.5107	1.4926	1.4790	1.4684	1.4600	1.4140
25	1.5039	1.4857	1.4720	1.4613	1.4528	1.4064
26	1.4975	1.4791	1.4653	1.4546	1.4460	1.3993
27	1.4915	1.4730	1.4591	1.4483	1.4397	1.3926
28	1.4859	1.4673	1.4533	1.4424	1.4337	1.3862
29	1.4806	1.4618	1.4478	1.4368	1.4280	1.3802
30	1.4755	1.4567	1.4426	1.4315	1.4227	1.3745
40	1.4373	1.4176	1.4027	1.3911	1.3817	1.3304
50	1.4126	1.3922	1.3767	1.3646	1.3548	1.3009
60	1.3952	1.3742	1.3583	1.3457	1.3356	1.2795
70	1.3822	1.3608	1.3444	1.3316	1.3212	1.2632
80	1.3722	1.3503	1.3337	1.3206	1.3100	1.2503
90	1.3642	1.3420	1.3251	1.3117	1.3009	1.2397
100	1.3576	1.3352	1.3180	1.3044	1.2934	1.2310
250	1.3197	1.2953	1.2765	1.2614	1.2491	1.1763

[a] $F_{1-\alpha;\nu_1,\nu_2}$ is the $1-\alpha$-quantile of the F-distribution with ν_1 (numerator) and ν_2 (denominator) degrees of freedom Calculation in SAS: `finv(1 −` $\alpha;\nu_1,\nu_2$`)` Calculation in R: `qf(1 −` $\alpha;\nu_1,\nu_2$`)`

Glossary

α	The significance level α of a statistical test.
$\Phi(\boldsymbol{x})$	Distribution function of the standard normal distribution: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-t^2/2} \partial t.$
$\chi^2_{\alpha;\boldsymbol{n}}$	The α-quantile of the χ^2-distribution with n degrees of freedom (Table B.3 and Table B.4).
$\boldsymbol{t}_{\alpha;\boldsymbol{n}}$	The α-quantile of the t-distribution with n degrees of freedom (Table B.2).
$\boldsymbol{z}_{\alpha}$	The α-quantile of the standard normal distribution (Table B.1): $z_\alpha = \Phi^{-1}(\alpha)$.
$\lvert\boldsymbol{x}\rvert$	The absolute value of x.
$\overline{\boldsymbol{x}}$	The sample mean of a sample $x_1, \ldots, x_n$: $\overline{x} = \frac{1}{n} \sum\limits_{i=1}^{n} x_i$.
Continuity correction	A continuity correction is often applied when approximating the cumulative probability function $P(X \geq x)$ of a discrete random variable by the standard normal distribution function. Usually a correction factor of 0.5 is used such that $P(X \geq x) \approx \Phi\left(\frac{x-E(X)+0.5}{\sqrt{Var(X)}}\right)$.
Empirical distribution function (EDF)	Let $x_{(1)}, \ldots, x_{(n)}$ be a descending ordered sample, then the EDF is defined as: $$F(x) = \begin{cases} 0 & \text{for all } x < x_{(1)} \\ k/n & \text{for } x_i \leq x < x_{(i+1)}, k = 1, \ldots, n-1 \\ 1 & \text{for all } x \geq x_{(n).} \end{cases}$$
$\boldsymbol{F}_{\boldsymbol{n}_1;\, \boldsymbol{n}_2}$	Distribution function of the F-distribution with n_1 and n_2 degrees of freedom.

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$\boldsymbol{f}_{\alpha;\boldsymbol{n}_1,\boldsymbol{n}_2}$	The α-quantile of the F-distribution with n_1 and n_2 degrees of freedom (Tables B.5–B.7).
$\boldsymbol{H}_0$	The null hypothesis of a test problem.
$\boldsymbol{H}_1$	The alternative hypothesis of a test problem.
$\boldsymbol{n}$	The sample size of a sample $x_1, \ldots, x_n$.
Ranks	Let $x_1, \ldots, x_n$ be a sample. The ordered sample (from the lowest to the highest value) is $x_{(1)}, \ldots, x_{(n)}$. Then j of $x_{(j)}$ is the `r`ank of the corresponding value x_j. For example, let $4, 5, 2, 9, 3$ be a sample of size 5, then the ordered sample is: $2, 3, 4, 5, 9$. The rank of the sample value 2 is 1, the rank of sample value 3 is 2, and the rank of sample value 9 is 5.
Run	Let n_1 observations of random variable X_1 and n_2 observations of random variable X_2 be given. Assume that both samples are combined and (if at least ordinal) are arranged in increasing or time of occurrence order. A run is a group of successive observations generated from the same random variable. The same idea can be applied if the observations are coming from a binary random variable. For example, a coin is tossed 10 times; the result of these tosses are either (H)eads or (T)ails. The observed sequence is: $\underline{HH}\ \underline{T}\ \underline{HH}\ \underline{TTT}\ \underline{H}$. This sequence has five runs, namely HH, T, HH, TTT, H.
Mid ranks	This is a way of dealing with tied values, which are identical values in an ordered sequence. The same rank is assigned to these values, namely the mean of their ranks. For example, let $4, 2, 4, 4, 5$ be a sample. It is unclear if the observations 1, 3, or 4 will get the ranks 2, 3, or 4. The arithmetic mean of the ranks of the tied values is $(2 + 3 + 4)/3 = 3$, so each value 4 will get the mid rank 3. The rank vector is $(1, 3, 3, 3, 5)$ while the sum of ranks is still 15.
p-value	The probability of observing a sample as discrepant with the null hypothesis H_0 as the observed sample under the null hypothesis.
Ties	If one or more observations in a sample have the same value they are called tied values.
$1\!\!1_A\{x\}$	Characteristic function: $1\!\!1_A\{x\} = \begin{cases} 1 & \\ & \textit{if} \\ 0 & \end{cases} \begin{matrix} x \in A \\ \\ x \notin A \end{matrix}$

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