The Interval Domain in Homotopy Type Theory

Niels van der Weide and Dan Frumin

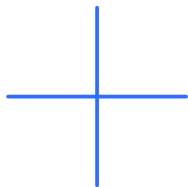# This Talk
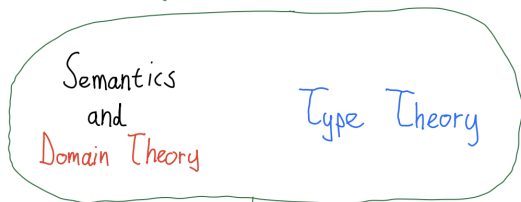
Real numbers, what are they?

Real number: infinite collection of approximations
that become better, better, and better.

Operations on the Real Numbers

But is this all?

Let's recall the program of today

10:15-12:30 Talks
- 10:15-10:30 **Henk Barendregt**: Invited Talk
- 10:30-10:45 **Yves Bertot**: Safe smooth paths between straight line obstacles
- 10:45-11:00 **Lasse Blaauwbroek**, David Cerna, Thibault Gauthier, Jan Jakubuv, Cezary Kaliszyk, Martin Suda and Josef Urban: Learning Guided Automated Reasoning: A Survey
- 11:00-11:15 **Bart Jacobs**: Some Probabilistic Riddles and Some Logical Solutions
- 11:15-11:30 Wieb Bosma and **Henk Don**: Constructing morphisms for arithmetic subsequences of Fibonacci
- 11:30-11:45 **Sebastiaan Terwijn**: Completions in partial combinatory algebra
- 11:45-12:00 **Tonny Hurkens**: Between Brackets
- 12:00-12:15 Harsh Beohar, Sebastian Junges and **Jurriaan Rot**: Relating Apartness and (Branching) Bisimulation Games

12:30-13:30 Lunch

13:30-15:30 Talks
- 13:30-13:45 Franco Barbanera, **Mariangiola Dezani-Ciancaglini**, Ugo De'Liguoro and Betti Venneri: YACC: Yet Another Church Calculus
- 13:45-14:00 **Jan Martens** and Jan Friso Groote: Minimal Depth Distinguishing Formulas without Until for Branching Bisimulation
- 14:00-14:15 **Hans Zantema**: Characterizing morphic sequences
- 14:15-14:30 **Jeroen J.A. Keiren** and Tim Willemse: It's all a Game: Apartness and Bisimilarity
- 14:30-14:45 **Niels van der Weide** and Dan Frumin: The Interval Domain in Homotopy Type Theory
- 14:45-15:00 **Jos Baeten** and Bas Luttik: Sequential Value Passing yields a Kleene Theorem for Processes
- 15:00-15:15 **Frits Vaandrager**: A New Perspective on Conformance Testing Based on Apartness

15:30-16:30 Drinks

Second Theme of Today: Apartness

10:15-12:30 Talks
- ○ 10:15-10:30 **Henk Barendregt**: Invited Talk
- ○ 10:30-10:45 **Yves Bertot**: Safe smooth paths between straight line obstacles
- ○ 10:45-11:00 **Lasse Blaauwbroek**, David Cerna, Thibault Gauthier, Jan Jakubuv, Cezary Kaliszyk, Martin Suda and Josef Urban: Learning Guided Automated Reasoning: A Survey
- ○ 11:00-11:15 **Bart Jacobs**: Some Probabilistic Riddles and Some Logical Solutions
- ○ 11:15-11:30 Wieb Bosma and **Henk Don**: Constructing morphisms for arithmetic subsequences of Fibonacci
- ○ 11:30-11:45 **Sebastiaan Terwijn**: Completions in partial combinatory algebra
- ○ 11:45-12:00 **Tonny Hurkens**: Between Brackets
- ○ 12:00-12:15 Harsh Beohar, Sebastian Junges and **Jurriaan Rot**: Relating Apartness and (Branching) Bisimulation Games

12:30-13:30 Lunch

13:30-15:30 Talks
- ○ 13:30-13:45 Franco Barbanera, **Mariangiola Dezani-Ciancaglini**, Ugo De'Liguoro and Betti Venneri: YACC: Yet Another Church Calculus
- ○ 13:45-14:00 **Jan Martens** and Jan Friso Groote: Minimal Depth Distinguishing Formulas without Until for Branching Bisimulation
- ○ 14:00-14:15 **Hans Zantema**: Characterizing morphic sequences
- ○ 14:15-14:30 **Jeroen J.A. Keiren** and Tim Willemse: It's all a Game: Apartness and Bisimilarity
- ○ 14:30-14:45 **Niels van der Weide** and Dan Frumin: The Interval Domain in Homotopy Type Theory
- ○ 14:45-15:00 **Jos Baeten** and Bas Luttik: Sequential Value Passing yields a Kleene Theorem for Processes
- ○ 15:00-15:15 **Frits Vaandrager**: A New Perspective on Conformance Testing Based on Apartness

15:30-16:30 Drinks
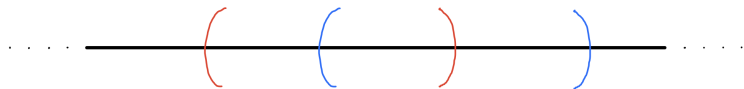
Apartness is a fundamental relation on the real numbers.

Real Numbers and Sameness
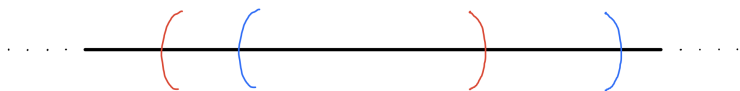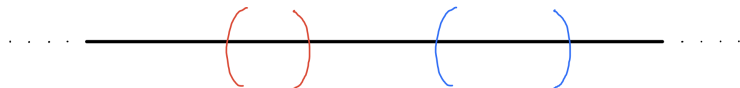
When are numbers x and y the same?

Real Numbers and Sameness

When are numbers x and y the same?

Real Numbers and Sameness

When are numbers x and y the same?

Key insight:

- Sameness requires infinite evidence
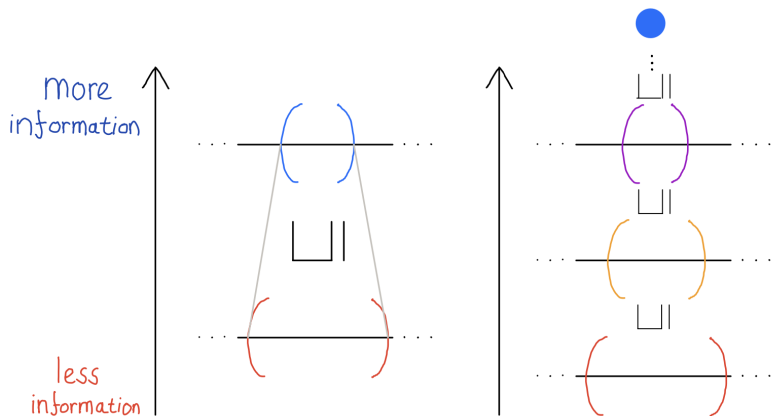
- Apartness requires finite evidence

Apartness, sharp elements, and the Scott topology on domains

by

Tom de Jong

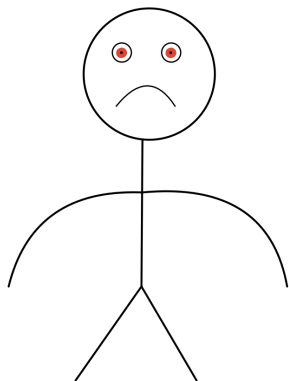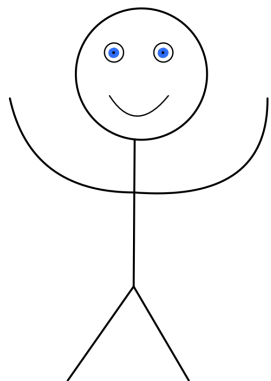This paper taught us; domains have an intrinsic notion of apartness

Domain Theory: A Theory of Approximation

Domain:
- collection of elements
- ordered by how much information they have
- consistent pieces of information can be combined

What does it mean to be different?

Spot the differences! Find properties satisfied by one and not the other.

What does it mean to be different? (Leibniz)

Leibniz: two objects are the same if they have the same properties

Leibniz: two objects are different if there is a property that is satisfied by only one of them.

Apartness in domains
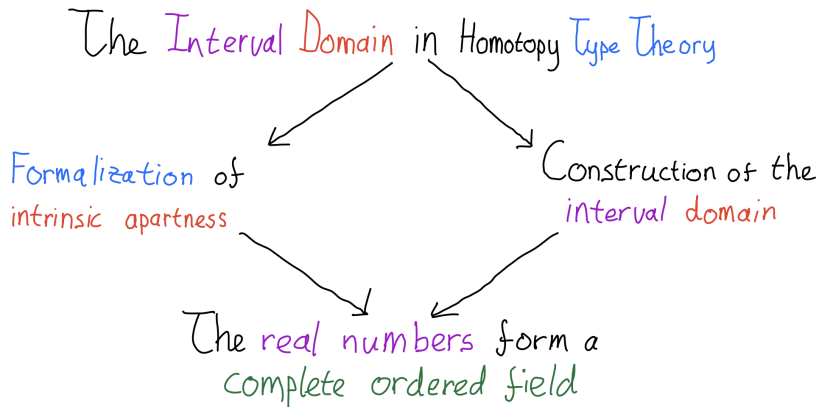
elements x and y
in a domain   intrinsically apart

two ~~objects~~ are ~~different~~ if there

is a ~~property~~ ~~satisfied by only one of them~~.

Scott open
subset O   x ∈ O and y ∉ O or vice versa

So... what is the paper about?

The Interval Domain in Homotopy Type Theory

Formalization of
intrinsic apartness

Construction of the
interval domain

The real numbers form a
complete ordered field

Happy sixtieth birthday Herman!

from Dan & Niels