Strategic Uncertainty and Time Pressure in Professional Chess

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Motivation

- We are interested in how **time pressure** affects decision making featuring **strategic uncertainty**.
- Previous work focused on online/lab experiments with e.g., bidding in auctions (Haji et al., 2019), bargaining (Karagözoglu and Kocher, 2019), or market entry games (Lindner, 2014). See also Spiliopoulos and Ortmann (2018).

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- We investigate the effect of having more time to decide on move selection by *professional chess players* in prestigious tournaments.
- Why chess? *The* strategic game. Very high proficiency and stakes. Enjoys a tradition in economic research (Dreber et al., 2013; Gerdes and Gränsmark, 2010; González-Díaz and Palacios-Huerta, 2016; Künn et al., 2021; Levitt et al., 2011; Palacios-Huerta and Volij, 2009; Salant and Spenkuch, 2022; Zegners et al., 2020).

Contribution

- We show that in games of FIDE Chess World Cups, time has a small, but consistent effect on strategy selection: with **less remaining time** to make a decision, professional chess players opt for **"safer" moves** more frequently.
- We aim to identify *causal* effects of time with an IV strategy.

➔ Matches to findings that time pressure (sometimes) increases risk aversion in non-strategic settings (Kocher et al., 2019, 2013; Kirchler et al., 2017).
from non-strategic settings that time pressure induces

To fix ideas

Table 1: A simple game

	X	Y	σ^2	min
A	25,20	14,12	30.25	14
B	14,20	25,12	30.25	14
C	**18**,12	**18**,22	**0**	**18**

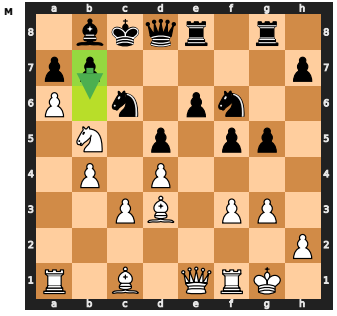
Adapted from Calford (2020)

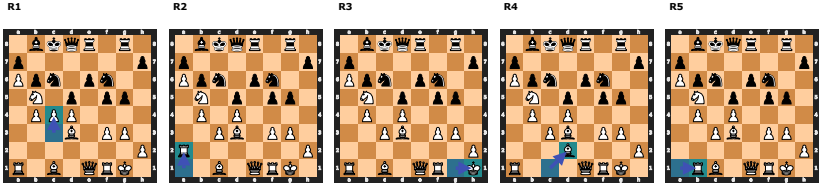
- C is a justifiable strategy if the row player is sufficiently uncertainty (risk + ambiguity) averse (Calford, 2020).
- **Variance**[1] and **min** of outcomes indicators of "safety".

➔ Tradeoff between "best" vs. "safest" strategy.

[1] Note that risk aversion does not *necessarily* imply aversion towards higher variance (Wakker, 2010).

Brkic/Barrientos, FIDE Chess World Cup 2021, after 21. a6





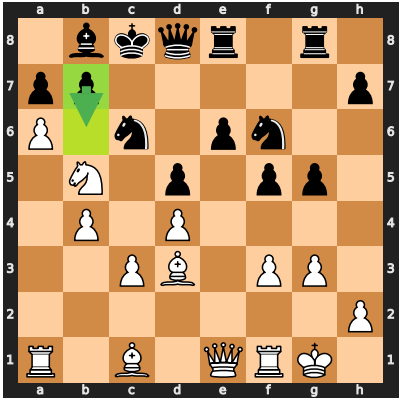
Outcomes in Chess?

- We take two approaches:
  1. (Restricted[2]) engine evaluation of emerging positions (Holdaway and Vul, 2021).
  2. **Difficulty** of emerging positions:
     "I am [...] trying to choose the moves that are most unpleasant for him." - Magnus Carlsen

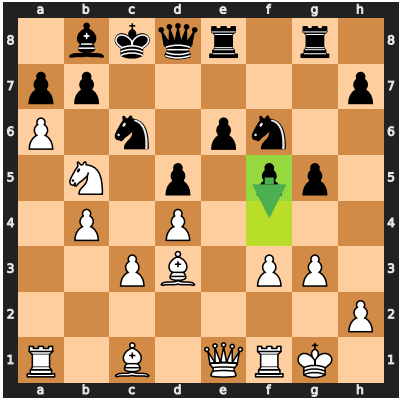
[2]Stockfish depth 15 $\sim$ ELO 2498 - 2623 (Ferreira, 2013).

The Quality of b7b6

Figure 1: Brkic vs Barrientos, FIDE World Cup 2021, Position after 21. a6



Barrientos' move: b7b6



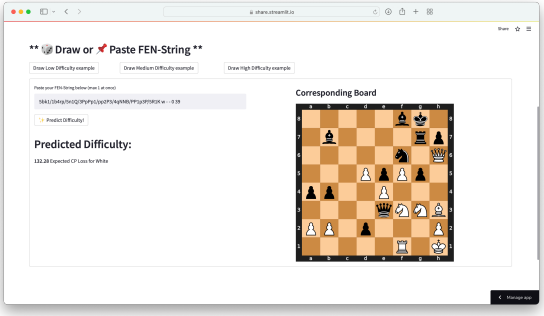
Better alternative: f5f4

Evaluation of b7b6: 115 CP.

From Quality to Difficulty

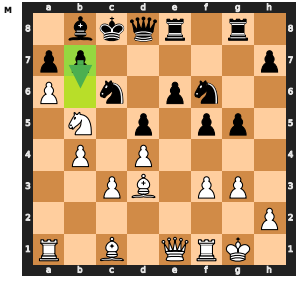
- Idea: $E[q_m] = f(p_m)$. Approximate with a prediction model (McIlroy-Young et al., 2020; Matros and Bilen, 2021).
- We trained a Neural Network on $>$ 100 mio chess moves from professional settings to predict "objective" move quality (Zegners et al., 2020; Strittmatter et al., 2020; Sunde et al., 2022) q_m with piece positions p_m as inputs.

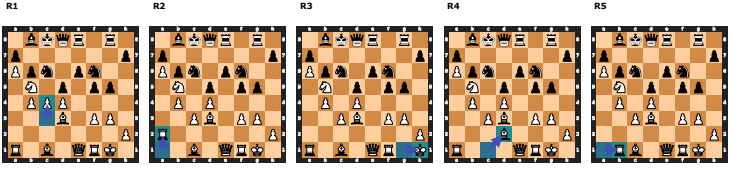
▸ Tensor Representation ▸ Network Architecture ▸ Training



https://nmwitzig.github.io/chess-app

Responses to b7b6





Difficulties: M: 44.92 CP (med. value); R1: 48.92, R2: 39.1, R3: 43.49, R4: 37.32, R5: 37.1, Variance: 19.107, Variance after f5f4: 110.634 (see Fig. 4)

Where do R1-R5 come from?

- Again: restricted Stockfish.
- Teaser: in 91% in WC data, human move among 5 best suggested engine moves. *Robustness-checks based on 3, 4, 5, 6, 7 boards carried out.*

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Difficulty-based measure matches well to classification by Dreber et al. (2013) applied to the WC data. ▸ Openings

Data: FIDE Chess World Cups

- FIDE 2013-2021 World Cups, very prestigious and part of the World Championship cycle. 128 players in each tournament, total >2,000 games and >150,000 moves.
- Each match comprises of 2+ games. In case of a tie, additional games in tighter time control (Classic, Rapid, Slow Blitz, Fast Blitz) are played.

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- Following Howard (2021), we **instrument remaining thinking time** with the interactions of the half-move counter and initial thinking time.
- Intuition: We replace remaining thinking time with the *average* thinking time in that specific situation (e.g., Half-Move 20 in Rapid).

▸ Exclusion Restriction?

Baseline Estimation

- Sample Restrictions:
  - Only half-moves 1-80
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  - Difficulty of a position
  - Quality of a Move
- Game and Half-Move fixed effects.
- Cluster SEs on game level

▸ Descriptives ▸ Empirical Specification

Difficulty: Main Results

	(1) Baseline	(2) Cond. on mean difficulty	(3) Max difficulty
Remaining Time (IV)	0.000093***	0.000124***	0.000020***
	(0.000009)	(0.000009)	(0.000001)
Position Evaluation	-0.000246***	-0.000238***	-0.000035***
	(0.000029)	(0.000028)	(0.000003)
Position Difficulty	0.045358***	0.029953***	0.003308***
	(0.000521)	(0.000567)	(0.000075)
Move Quality	0.000391***	0.000399***	0.000048***
	(0.000054)	(0.000053)	(0.000007)
Avg. Difficulty Resp.		0.578925***	0.809681***
		(0.013799)	(0.001950)
Half-Moves Dummies	Yes	Yes	Yes
Game FE	Yes	Yes	Yes
N	135553	135553	135553
R^2	0.160	0.179	0.896

Difficulty: Robustness Checks

Difficulty results are robust to

- varying the number of relevant responses (3,4,5,6) ▸ No. resps.
- controlling for var(difficulty) of alternative moves ▸ Alternative moves
- including obvious moves ▸ Obvious moves
- controlling for future var(difficulty) ▸ Future

Engine Evaluations: Main Results

	(1) Baseline	(2) Cond. on mean evaluation	(3) Min Evaluation
Remaining Time (IV)	0.000058***	0.000062***	-0.000000**
	(0.000020)	(0.000017)	(0.000000)
Position Evaluation	0.000197***	-0.004037***	0.000041***
	(0.000076)	(0.000251)	(0.000002)
Difficulty	0.039274***	0.030900***	-0.000061***
	(0.001174)	(0.001040)	(0.000008)
Move Quality	0.001830***	0.003785***	-0.000041***
	(0.000186)	(0.000292)	(0.000004)
Avg. Evaluation Resp.		0.006538***	0.000007***
		(0.000175)	(0.000001)
Half-Moves Dummies	Yes	Yes	Yes
Game FE	Yes	Yes	Yes
N	135553	135553	135553
R^2	0.073	0.427	0.502

Interim Conclusion

- **Main Result:** Consistent positive, small effect of time on deciding for "safer" moves.
- This is a novel finding, emphasizing that available thinking time plays a (small) role for strategic decisions also for high-level decision makers.
- Next steps: ▸ Next Steps

Thank you!

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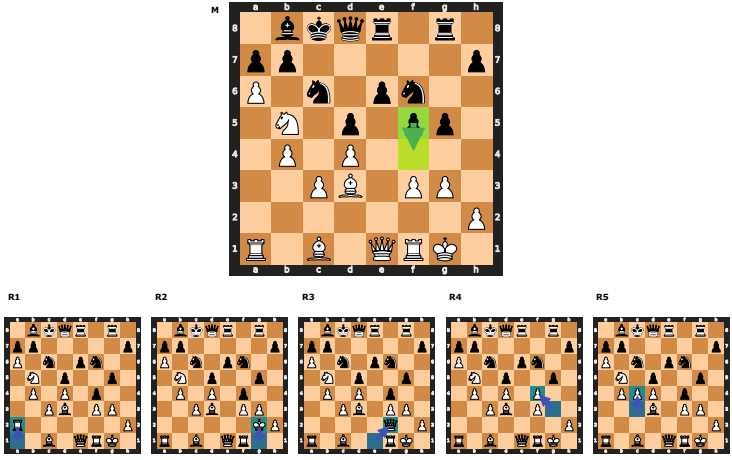
Background *(select.)*

We relate to several empirical literatures:

- The effect of ambiguity and risk preferences on strategies: uncertainty-averse subjects opt for "safe" strategies more often (Calford, 2020). Ambiguity-averse subjects trust less in the trust game (Li et al., 2019).
- The effect of time pressure on economic decision making: time pressure increases risk aversion for losses (Kocher et al., 2013), increases (ambiguity-induced) likelihood insensitivity (Baillon et al., 2018), decreases entry in market entry games (Lindner, 2014), increases disagreements and last-moment-agreements in bargaining-games (Karagözoglu and Kocher, 2019) and leads players to opt for simpler strategies more frequently (Spiliopoulos et al., 2018).
- Using chess to study economic phenomena: men play more risky opening strategies against woman (Dreber et al., 2013; Gerdes and Gränsmark, 2010), age (Zegners et al., 2020), emotions (González-Díaz and Palacios-Huerta, 2016) and remote work (Künn et al., 2021) negatively affect cognitive performance, chess players are complexity-averse in endgames (Salant and Spenkuch, 2022) and play equilibrium play against each other in centipede-games (Palacios-Huerta and Volij, 2009), *contrasted by* Levitt et al. (2011).

Difficulties after f5f4

Figure 2: Responses after Engine Move



Note: Difficulties: R1: 51.38 R2: 61.89 R3: 50.48 R4: 30.6 R5: 60.87, Variance: 110.63

Empirical Specification

- We want to estimate the effect of remaining time on the clock on the risk-taking behavior of players associated with their moves.
- However, we have to account for endogeneity as reversed causality would be prevalent in OLS:

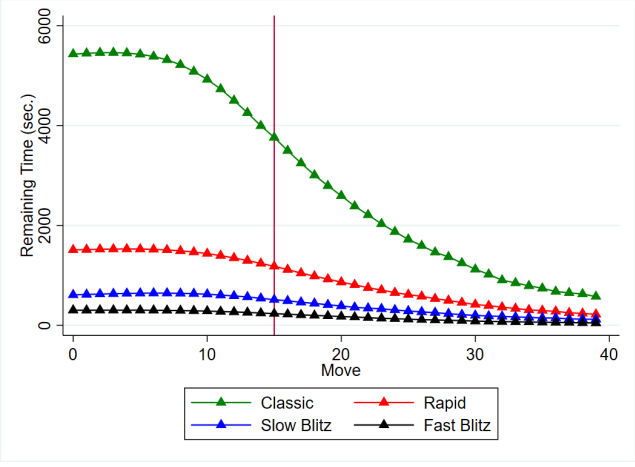
$$\begin{aligned} Risk_{jmgi} = \alpha \cdot Remaining\_Time_{jmgi} + \sum_{k=1}^{80} \phi_k \cdot DHalfMove_k \\ + \boldsymbol{X_{jmgi}}\gamma + \mu_{jmg} + u_{jmgi} \\ j = 1, \ldots, 354 \quad and \quad m = 1, \ldots, 707 \\ g = 1, \ldots, 9 \quad and \quad i = 1, \ldots, 80 \end{aligned} \tag{1}$$

  where j indexes players, m matches, g games and i moves.
- μ_{jmg} denotes interacted player-game-match fixed effects.

- We exploit the tournament setting of the World Cups for identification. Our identifying variation stems from the different playing modes from the tiebreaks
- First Stage:

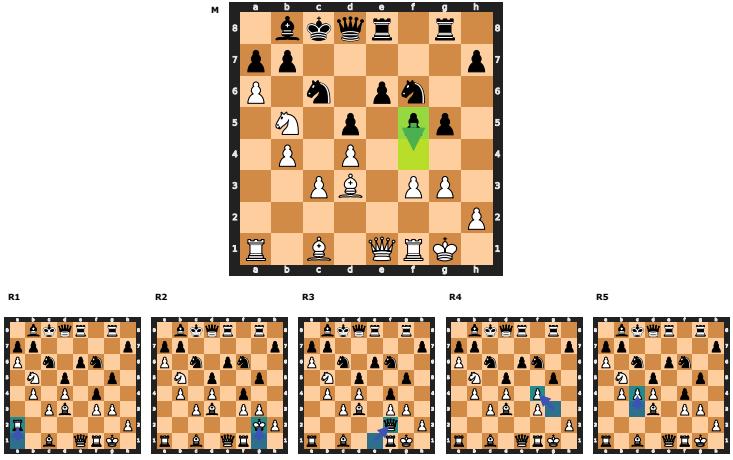
$$
\begin{aligned}
Remaining\_Time_{jmgi} = Initial\_Time_g \cdot \Big(\sum_{k=1}^{80} \psi_k \cdot DHalfMove_k\Big) \\
+ \sum_{k=1}^{80} \phi_k \cdot DHalfMove_k + \boldsymbol{X_{jmgi}}\gamma + \mu_{jmg} + u_{jmgi} \\
+ \boldsymbol{X_{jmgi}}\gamma + \mu_{jmg} + u_{jmgi} \\
j = 1, \ldots, 354 \quad and \quad m = 1, \ldots, 707 \\
g = 1, \ldots, 9 \quad and \quad i = 1, \ldots, 80 \\
Initial\_Time_g \in (Classic,\ Rapid,\ Slow\ Blitz,\ Fast\ Blitz)
\end{aligned}
\tag{2}
$$

Figure 3: Identifying variation



Difficulties after f5f4

Figure 4: Responses after Engine Move



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Measuring Performance in Chess I

Figure 5: How to measure the Performance of a Move in Chess: Centipawns

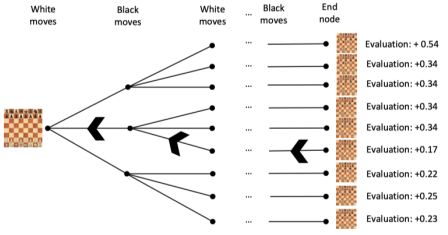


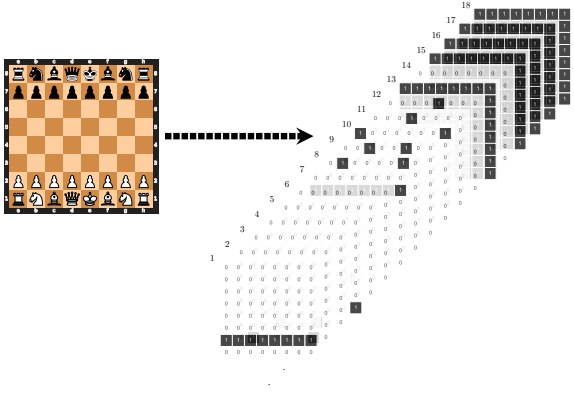
Figure 1: Backward Induction in Chess Engines

Note: Illustration of the decision algorithm built into a chess engine. For a given search depth (number of moves until the end node is reached), the engine calculates evaluations of different alternative moves under the assumption of mutually best response and determines the move that delivers the highest evaluation on the end node. The positions at the end nodes are evaluated using a human-curated evaluation function that considers factors of the chess positions such as number of pawns and pieces on the board, pawn structure, mobility of pieces and king safety.

Source: Zegners et al. (2020)

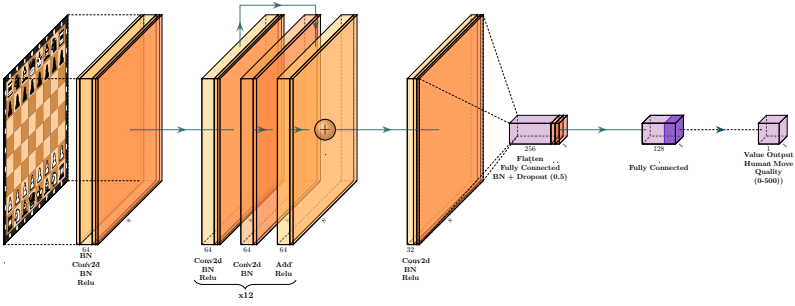
Tensor Representation

Figure 6: Tensor Representation



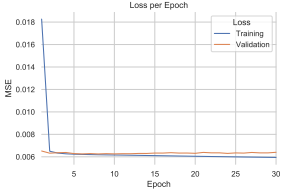
Network Architecture

Figure 7: RCNN Architecture



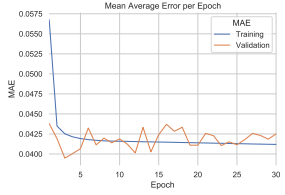
Training Graphs

Figure 8: Epoch and Batchwise Loss during Training

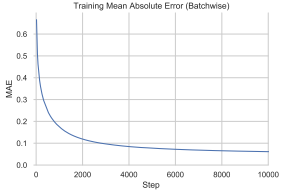


(a) Epoch Loss

(b) Batchwise Loss



(c) Epoch MAE



(d) Batchwise MAE

Model comparison

Table 2: Comparison of Model Performance on hold-out Test Set

Model	MSE	MAE
RCNN	0.00626	0.0416
MLP	0.00631	0.0417
Random Forest Regressor	0.00658	0.0442
Ridge Regression	0.00644	0.0437

Descriptive Statistics

Table 3: Descriptive statistics, stratified by playing mode

	(1) Classic		(2) Rapid		(3) Slow Blitz		(4) Fast Blitz	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
log(var(CP loss))	2.49	1.27	2.57	1.30	2.59	1.30	2.53	1.26
WhiteElo	2663.17	109.97	2678.67	78.36	2686.39	76.01	2705.49	68.16
BlackElo	2661.51	106.60	2677.39	78.38	2685.56	75.75	2697.41	77.69
remaining time	3279.29	1997.27	961.78	537.32	425.63	217.89	190.30	106.70
evaluation of position	13.03	192.70	14.30	236.76	13.01	205.84	16.93	282.05
predicted CP loss	18.29	12.87	19.56	13.86	19.81	13.69	19.41	12.03
blunder magnitude	17.59	88.66	24.22	120.98	26.93	113.95	33.63	183.26
Observations	82965		38334		13040		5368	
Number of players:	354							
Number of games:	2,312							

Exclusion Restriction

Regarding the exclusion restriction:

- Difficult to force tie-break (e.g., in anticipation of other playing modes), especially in half-moves 1-80.
- Level and development of var(difficulty) very similar across playing modes ▸ Trends
- Probability that a game becomes a tiebreak is unrelated to mean(var(difficulty)) cond. on avg. move quality. ▸ Reaching tiebreak
- 51 % of matches are tie-breaks.

Kleibergen-Paap F-statistic: 390.988

Main Results I

Table 2: First Stage IV. Dependent variable: Remaining time before move.

Initial time × Half Move 3	0.918387***
	(0.007328)
Initial time × Half Move 5	0.920552***
	(0.007291)
Initial time × Half Move 7	0.919434***
	(0.007325)
Initial time × Half Move 9	0.917494***
	(0.007333)
Initial time × Half Move 11	0.914296***
	(0.007313)
Initial time × Half Move 13	0.906250***
	(0.007502)
Initial time × Half Move 15	0.892970***
	(0.007378)
Initial time × Half Move 17	0.874283***
	(0.007502)
Initial time × Half Move 19	0.847448***
	(0.007768)
Initial time × Half Move 21	0.818925***
	(0.007995)
Initial time × Half Move 23	0.782511***
	(0.008075)
Initial time × Half Move 25	0.739227***
	(0.008479)
Initial time × Half Move 27	0.691036***
	(0.008816)

Main Results II

Initial time × Half Move 29	0.640801***
	(0.008860)
Initial time × Half Move 31	0.594776***
	(0.009119)
Initial time × Half Move 33	0.548073***
	(0.009148)
Initial time × Half Move 35	0.503605***
	(0.009152)
Initial time × Half Move 37	0.454932***
	(0.009135)
Initial time × Half Move 39	0.413079***
	(0.008935)
Initial time × Half Move 41	0.370107***
	(0.008704)
Initial time × Half Move 43	0.334379***
	(0.008582)
Initial time × Half Move 45	0.299782***
	(0.008304)
Initial time × Half Move 47	0.271382***
	(0.007994)
Initial time × Half Move 49	0.238005***
	(0.007660)
Initial time × Half Move 51	0.204915***
	(0.007323)
Initial time × Half Move 53	0.181102***
	(0.006900)
Initial time × Half Move 55	0.159264***
	(0.006578)
Initial time × Half Move 57	0.140246***

Main Results III

	(0.006297)
Initial time × Half Move 59	0.128413***
	(0.006160)
Initial time × Half Move 61	0.104150***
	(0.005762)
Initial time × Half Move 63	0.091813***
	(0.005599)
Initial time × Half Move 65	0.073644***
	(0.005424)
Initial time × Half Move 67	0.054087***
	(0.004942)
Initial time × Half Move 69	0.047830***
	(0.004466)
Initial time × Half Move 71	0.036170***
	(0.004473)
Initial time × Half Move 73	0.027683***
	(0.004223)
Initial time × Half Move 75	0.015999***
	(0.004045)
Initial time × Half Move 77	0.009468***
	(0.003339)
Initial time × Half Move 4	0.908839***
	(0.007138)
Initial time × Half Move 6	0.910737***
	(0.007123)
Initial time × Half Move 8	0.910889***
	(0.007167)
Initial time × Half Move 10	0.906425***
	(0.007237)

Main Results IV

Initial time × Half Move 12	0.901112***
	(0.007216)
Initial time × Half Move 14	0.890082***
	(0.007255)
Initial time × Half Move 16	0.875895***
	(0.007219)
Initial time × Half Move 18	0.854062***
	(0.007460)
Initial time × Half Move 20	0.826114***
	(0.007685)
Initial time × Half Move 22	0.790965***
	(0.007890)
Initial time × Half Move 24	0.755449***
	(0.008095)
Initial time × Half Move 26	0.708043***
	(0.008416)
Initial time × Half Move 28	0.664678***
	(0.008654)
Initial time × Half Move 30	0.611204***
	(0.008730)
Initial time × Half Move 32	0.566026***
	(0.008899)
Initial time × Half Move 34	0.513846***
	(0.008716)
Initial time × Half Move 36	0.471762***
	(0.008721)
Initial time × Half Move 38	0.423932***
	(0.008601)
Initial time × Half Move 40	0.380298***

Main Results V

	(0.008505)
Initial time × Half Move 42	0.344176***
	(0.008193)
Initial time × Half Move 44	0.312097***
	(0.008066)
Initial time × Half Move 46	0.276712***
	(0.008011)
Initial time × Half Move 48	0.247140***
	(0.007594)
Initial time × Half Move 50	0.218912***
	(0.007418)
Initial time × Half Move 52	0.192913***
	(0.007113)
Initial time × Half Move 54	0.170731***
	(0.006915)
Initial time × Half Move 56	0.149694***
	(0.006624)
Initial time × Half Move 58	0.125840***
	(0.006405)
Initial time × Half Move 60	0.111352***
	(0.006303)
Initial time × Half Move 62	0.094448***
	(0.005843)
Initial time × Half Move 64	0.072211***
	(0.005713)
Initial time × Half Move 66	0.060301***
	(0.005292)
Initial time × Half Move 68	0.047406***
	(0.004910)

Main Results VI

Initial time × Half Move 70	0.041013***
	(0.005025)
Initial time × Half Move 72	0.029233***
	(0.004387)
Initial time × Half Move 74	0.019990***
	(0.004323)
Initial time × Half Move 76	0.011420***
	(0.004108)
Initial time × Half Move 78	0.002350
	(0.004091)
Half Move 3	-157.206781***
	(14.628152)
Half Move 5	-147.838233***
	(14.420205)
Half Move 7	-145.016737***
	(14.568857)
Half Move 9	-133.527022***
	(14.386017)
Half Move 11	-123.980914***
	(14.312989)
Half Move 13	-110.769751***
	(14.294754)
Half Move 15	-96.636005***
	(14.163071)
Half Move 17	-81.432337***
	(14.119709)
Half Move 19	-55.417715***
	(14.314374)
Half Move 21	-43.960669***

Main Results VII

	(14.471083)
Half Move 23	-29.189256**
	(14.817254)
Half Move 25	-11.538835
	(15.159248)
Half Move 27	9.563680
	(15.637509)
Half Move 29	25.815232*
	(15.662954)
Half Move 31	41.304680**
	(16.097733)
Half Move 33	48.542628***
	(16.304297)
Half Move 35	48.864425***
	(16.198348)
Half Move 37	59.650025***
	(16.158667)
Half Move 39	61.886095***
	(16.131916)
Half Move 41	74.132254***
	(16.138515)
Half Move 43	75.737497***
	(15.792751)
Half Move 45	77.472423***
	(15.542905)
Half Move 47	66.122366***
	(15.036168)
Half Move 49	63.934495***
	(14.543210)

Main Results VIII

Half Move 51	71.827195***
	(14.179295)
Half Move 53	63.730045***
	(13.412469)
Half Move 55	56.544647***
	(13.318733)
Half Move 57	52.898567***
	(12.798669)
Half Move 59	35.841873***
	(12.400184)
Half Move 61	40.695468***
	(12.132041)
Half Move 63	30.620391***
	(11.704394)
Half Move 65	37.418887***
	(11.147566)
Half Move 67	36.831643***
	(10.887880)
Half Move 69	30.199325***
	(10.428615)
Half Move 71	18.899668*
	(9.727593)
Half Move 73	12.424390
	(9.256984)
Half Move 75	3.663131
	(8.337523)
Half Move 77	-2.307812
	(7.480383)
Half Move 4	-131.777389***

Main Results IX

	(14.072278)
Half Move 6	-139.527722***
	(14.441264)
Half Move 8	-137.855664***
	(14.623277)
Half Move 10	-132.398970***
	(14.520985)
Half Move 12	-121.630494***
	(14.458036)
Half Move 14	-104.034523***
	(14.129871)
Half Move 16	-92.280216***
	(13.938844)
Half Move 18	-79.559176***
	(14.186498)
Half Move 20	-62.045574***
	(14.192051)
Half Move 22	-39.493386***
	(14.622877)
Half Move 24	-25.301361*
	(14.882106)
Half Move 26	-6.156945
	(15.275181)
Half Move 28	5.642903
	(15.472063)
Half Move 30	25.813261*
	(15.580989)
Half Move 32	32.770759**
	(15.822742)

Main Results X

Half Move 34	50.519946***
	(15.600233)
Half Move 36	57.054692***
	(15.570671)
Half Move 38	61.598991***
	(15.425019)
Half Move 40	64.891753***
	(15.458871)
Half Move 42	64.141508***
	(15.124056)
Half Move 44	59.854013***
	(15.245669)
Half Move 46	57.194298***
	(15.069795)
Half Move 48	58.672148***
	(14.483561)
Half Move 50	56.244630***
	(14.497180)
Half Move 52	50.698383***
	(13.814388)
Half Move 54	49.978010***
	(13.738445)
Half Move 56	43.369684***
	(13.228169)
Half Move 58	53.685559***
	(13.285139)
Half Move 60	40.638331***
	(12.770757)
Half Move 62	32.353583***

Main Results XI

	(12.309085)
Half Move 64	42.482240***
	(12.090394)
Half Move 66	41.542725***
	(11.507608)
Half Move 68	30.368760***
	(11.031730)
Half Move 70	25.089630**
	(10.908119)
Half Move 72	22.041850**
	(9.963315)
Half Move 74	19.186923*
	(9.883389)
Half Move 76	10.430133
	(8.696995)
Half Move 78	12.833258
	(7.901369)
evaluation of position	0.154998***
	(0.015194)
predicted CP loss	-4.242780***
	(0.348103)
blunder magnitude	-0.096171***
	(0.015644)
Constant	503.864677***
	(16.175124)
Half-Move-Dummies	YES
Kleibergen-Paap F-statistic	390.988

Main Results XII

Observations	135553
R^2	0.945

Note: Standard errors clustered on game level in parentheses.
Interacted player-match-game FE are included.
* p $<$ 0.10, ** p $<$ 0.05, *** p $<$ 0.01

Common Trend Graphic

Figure 9: Evolution of Risk within games of different playing modes

Robustness Checks

Table 5: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7, 6, 5, 4, 3 moves. IV with interacted player-game FE. Controlling for mean(cploss)

	(1) 7 moves	(2) 6 moves	(3) 5 moves	(4) 4 moves	(5) 3 moves
remaining time	0.000023***	0.000024***	0.000023***	0.000025***	0.000023***
	(0.000008)	(0.000008)	(0.000008)	(0.000008)	(0.000008)
mean (cp loss) based on 7 moves	0.041663***				
	(0.000919)				
evaluation of position	-0.000154***	-0.000169***	-0.000189***	-0.000214***	-0.000251***
	(0.000028)	(0.000029)	(0.000030)	(0.000032)	(0.000031)
difficulty	0.024094***	0.027057***	0.031138***	0.036335***	0.041625***
	(0.000621)	(0.000656)	(0.000692)	(0.000738)	(0.000796)
move quality	0.000339***	0.000357***	0.000373***	0.000396***	0.000450***
	(0.000047)	(0.000051)	(0.000055)	(0.000060)	(0.000064)
mean (cp loss) based on 6 moves		0.036441***			
		(0.000897)			
mean (cp loss) based on 5 moves			0.028970***		
			(0.000886)		
mean (cp loss) based on 4 moves				0.019402***	
				(0.000886)	
mean (cp loss) based on 3 moves					0.006675***
					(0.000898)
Half-Move-Dummies	YES	YES	YES	YES	YES
Observations	135553	135553	135553	135553	135553
R^2	0.224	0.203	0.177	0.153	0.124

Note: Sample: World Cup Games from 2013 – 2021

Robustness Checks

Table 6: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7 moves. IV with interacted player-game FE. Controlling for future risk.

	(1) IV	(2) IV	(3) IV	(4) IV	(5) IV
remaining time	0.000020***	0.000017***	0.000017***	0.000017***	0.000017***
	(0.000006)	(0.000006)	(0.000006)	(0.000006)	(0.000006)
evaluation of position	-0.000192***	-0.000173***	-0.000172***	-0.000212***	-0.000256***
	(0.000023)	(0.000026)	(0.000027)	(0.000028)	(0.000031)
difficulty	0.039912***	0.038798***	0.038978***	0.039391***	0.039754***
	(0.000535)	(0.000536)	(0.000548)	(0.000558)	(0.000577)
move quality	0.000358***	0.000359***	0.000421***	0.000533***	0.000690***
	(0.000051)	(0.000070)	(0.000086)	(0.000094)	(0.000106)
log(var(CP loss)) in t+1	0.263058***	0.238580***	0.235872***	0.233419***	0.230563***
	(0.003328)	(0.003193)	(0.003229)	(0.003254)	(0.003266)
log(var(CP loss)) in t+2		0.064506***	0.058075***	0.056155***	0.055564***
		(0.002951)	(0.003031)	(0.003095)	(0.003124)
log(var(CP loss)) in t+3			0.013335***	0.014052***	0.014965***
			(0.002842)	(0.002975)	(0.002984)
log(var(CP loss)) in t+4				-0.005772**	-0.003556
				(0.002766)	(0.002891)
log(var(CP loss)) in t+5					-0.011472***
					(0.002825)
Half-Move-Dummies	YES	YES	YES	YES	YES
Observations	133438	131090	128613	126000	123256
R^2	0.248	0.246	0.242	0.236	

Note: Sample: World Cup Games from 2013 – 2021

Robustness Checks

Table 7: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7 moves. IV with interacted player-game FE. Controlling for variance of alternative moves.

	(1) IV	2 (IV)
remaining time	0.000027***	0.000026***
	(0.000009)	(0.000008)
evaluation of position	-0.000276***	-0.000235***
	(0.000029)	(0.000029)
difficulty	0.047226***	0.049840***
	(0.000602)	(0.000608)
move quality	0.000380***	0.000396***
	(0.000051)	(0.000053)
leave-out mean of log(var(CP loss))	0.001007***	
	(0.000054)	
mean of log(var(CP loss)) after first best response		0.000259***
		(0.000044)
mean of log(var(CP loss)) after second best response		0.000253***
		(0.000046)
mean of log(var(CP loss)) after third best response		0.000038
		(0.000045)
mean of log(var(CP loss)) after fourth best response		0.000013
		(0.000050)
mean of log(var(CP loss)) after fifth best response		0.000137***
		(0.000047)
mean of log(var(CP loss)) after sixth best response		0.000090*
		(0.000048)
mean of log(var(CP loss)) after seventh best response		0.000171***
		(0.000051)
Half-Move Dummies	YES	YES
Observations	135553	132854
R^2	0.194	0.204

Robustness Checks

Table 8: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7 moves. IV with interacted player-game FE. Different subsamples

	(1) at least 30s remaining time	(2) only must win games	(3) drawn games	(4) lost and won games
remaining time	0.000026***	0.000056***	0.000004	0.000034***
	(0.000009)	(0.000020)	(0.000013)	(0.000012)
evaluation of position	-0.000218***	-0.000369***	-0.000498***	-0.000213***
	(0.000031)	(0.000114)	(0.000057)	(0.000029)
difficulty	0.051244***	0.047712***	0.056093***	0.046556***
	(0.000618)	(0.001308)	(0.000956)	(0.000767)
quality of move	0.000439***	0.000262	0.000599**	0.000347***
	(0.000058)	(0.000161)	(0.000245)	(0.000052)
Half-Move-Dummies	YES	YES	YES	YES
Observations	133796	16948	67139	68414
R^2	0.186	0.209	0.173	0.208

Note: Sample: World Cup Games from 2013 – 2021

Robustness Checks

Table 9: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7 moves. IV with interacted player-game FE. Different subsamples

	(1) With players below 2400 Elo and with obvious moves	(2) With obvious moves	(3) with players below 2400 Elo
remaining time	0.000030***	0.000031***	0.000028***
	(0.000009)	(0.000009)	(0.000009)
evaluation of position	-0.000210***	-0.000200***	-0.000225***
	(0.000028)	(0.000028)	(0.000029)
predicted CP loss	0.046268***	0.046290***	0.051824***
	(0.000510)	(0.000517)	(0.000599)
blunder magnitude	0.000289***	0.000297***	0.000383***
	(0.000038)	(0.000040)	(0.000050)
Half-Move-Dummies	YES	YES	YES
Observations	160325	157554	137486
R^2	0.177	0.176	0.192

Note: Sample: World Cup Games from 2013 – 2021

Robustness Checks

Table 10: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7 moves. IV with interacted player-game FE. Excluding playing modes.

	(1) Without fast blitz games	(2) Without classic games
remaining time	0.000026***	0.000120*
	(0.000009)	(0.000068)
evaluation of position	-0.000224***	-0.000306***
	(0.000030)	(0.000037)
difficulty	0.051058***	0.049437***
	(0.000621)	(0.000887)
move quality	0.000392***	0.000429***
	(0.000057)	(0.000067)
Half-Move-Dummies	YES	YES
Observations	130342	55099
R^2	0.189	0.197

Note: Sample: World Cup Games from 2013 – 2021

Robustness Checks

Table 11: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7 moves. Controlling for blunder in last move.

	(1) interacted player-game FE	(2) IV with interacted player-game FE.
evaluation of position	-0.000195***	-0.000202***
	(0.000029)	(0.000029)
difficulty	0.050588***	0.050401***
	(0.000598)	(0.000605)
move quality	0.000366***	0.000362***
	(0.000052)	(0.000052)
blunder in previous move	0.200035***	0.200530***
	(0.012108)	(0.012130)
remaining time		0.000026***
		(0.000009)
Half-Move-Dummies	YES	YES
Observations	140099	135553
R^2	0.339	0.190

Note: Sample: World Cup Games from 2013 – 2021. Blunder defined as move with at least 50 CP loss.

Robustness Checks

Table 12: Robustness Check: Dependent variable: Log(1+var(cploss)), based on 7 moves. OLS regressions. Comparing risk in different classes of openings.

	(1)	(2)	(3)	(4)	(5)	(6)
opening solid for both players	-0.171***					
	(0.007)					
opening solid for one player, neutral for the other player		-0.022***				
		(0.008)				
opening neutral for both players			0.069***			
			(0.011)			
opening solid for one player, aggressive for the other player				0.172***		
				(0.015)		
opening aggressive for one player, neutral for the other player					0.238***	
					(0.012)	
opening aggressive for both players						0.338***
						(0.019)
Constant	2.603***	2.531***	2.517***	2.515***	2.503***	2.514***
	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)
Observations	140159	140159	140159	140159	140159	140159
R^2	0.004	0.000	0.000	0.001	0.003	0.002

Note: Sample: World Cup Games from 2013 – 2021.

Robustness Checks

Table 13: Robustness Check: Dependent variable: Log(1+var(cploss)). Comparing the gain domain and the loss domain.

	(1)	(2)	(3)	(4)
white has advantage (threshold 100 CP)	-0.581267***			
	(0.021516)			
white has advantage (threshold 1 CP)		-0.458205***		
		(0.012427)		
black has advantage (threshold 100 CP)			-0.289589***	
			(0.021017)	
black has advantage (threshold 1 CP)				-0.013385
				(0.011149)
Constant	3.140166***	2.791676***	3.052616***	2.540876***
	(0.018245)	(0.011374)	(0.012807)	(0.005489)
Observations	21871	76234	20378	73022
R^2	0.033	0.021	0.009	0.000

Note: Sample: World Cup Games from 2013 – 2021.

Robustness Checks

Table 14: Robustness check: Dependent variable: Log(1+mean(cploss)), based on 7 moves.

	(1) No FE	(2) interacted player-game FE	(3) interacted player-game FE	(4) IV interacted player-game FE
remaining time	-0.000111***	-0.000180***	-0.000006***	-0.000000
	(0.000002)	(0.000002)	(0.000002)	(0.000003)
evaluation of position			-0.000028***	-0.000029***
			(0.000006)	(0.000006)
difficulty			0.021527***	0.021542***
			(0.000234)	(0.000234)
move quality			0.000005	0.000004
			(0.000009)	(0.000009)
Half-Move-Dummies	NO	NO	YES	YES
Observations	135610	135610	135553	135553
R^2	0.164	0.406	0.779	0.728

Note: Sample: World Cup Games from 2013 – 2021

Reaching Tiebreak

Table 15: LPM on the match-level. Dependent variable: Reaching the tiebreak.

	(1)	(2)	(3)
average log(1 + var(cploss)) in classical games	-0.203***	0.056	0.046
	(0.039)	(0.060)	(0.059)
average difficulty in classical games		-0.030***	-0.027***
		(0.007)	(0.006)
average move quality in classical games		-0.002**	-0.002**
		(0.001)	(0.001)
rating difference			-0.001***
			(0.000)
Constant	0.857***	0.897***	0.940***
	(0.085)	(0.090)	(0.089)
Observations	707	707	707
R^2	0.037	0.080	0.112

Next steps

- Validate difficulty measure with survey among chess players.
- Investigate difficulty manipulation for opponent.
- Relax assumption of belief of equal likelihood of responses. *E.g., weighting boards according to geometric series*