PCI Planning

PROBLEM FORMULATION

Parameter	Range	Description	Status
G	$G = 180$	the number of squares in grid	constant
N	$N \in \{1, \ldots, 2004\}$	the number of sites	constant
$\mathcal{S}$	$\mathcal{S} = \{s_{1,1}, s_{1,2}, s_{1,3}, \ldots, s_{N,1}, s_{N,2}, s_{N,3}\}$	the set of site sectors	constant
D	$D = 5km$	the threshold distance for sites around the center of a square	constant
K	$K = 504$	the number of possible PCI values: $\lvert\{0, 1, \ldots, K-1\}\rvert$	constant
$\mathbf{L} = [l_k]_{K\times 1}$	$l_1 = 1, \ldots, l_K = K$	the vector that contains PCI values in increasing order. For example, $\mathbf{L} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$, for $K = 3$.	constant
$\mathcal{S}_g$	$g \in [G]$ $\mathcal{S}_g \subset \mathcal{S}$	the set of site sectors within distance D of square center g which transmit signal to square center g.	constant
$\mathbf{P} = [p_{g,n,i}]_{G\times N\times 3}$	$g \in [G]$ $n \in [N]$ $i \in \{1, 2, 3\}$	**power matrix**: The received power of sector i of site n, i.e. $s_{n,i}$, in square center g. When $p_{g,n,i} = 0$, either site sector $s_{n,i}$ is further than D from square center g, or, site sector $s_{n,i}$ is within distance D of g, but the direction of transition in $s_{n,i}$ is such that g does not receive signal from $s_{n,i}$. For sites (n) where the coverage areas of sectors are non-overlapping, if $p_{g,n,i} \neq 0$, then $p_{g,n,i+1 \text{ mode } 3} = 0$ and $p_{g,n,i+2 \text{ mode } 3} = 0$. For sites where the coverage areas of sectors are overlapping, two (or three) of $p_{g,n,i} \neq 0$, $p_{g,n,i+1 \text{ mode } 3} = 0$ and $p_{g,n,i+2 \text{ mode } 3} = 0$ can be non-zero.	constant
$\mathbf{M} = [m_{k,i}]_{K\times 3}$	$k \in [K]$ $i \in \{1, 2, 3\}$	**mode matrix**: $m_{k,i} = 1$ if k mode $3 = i$.	constant.
$\mathbf{S} = [s_{i,j}]_{3\times 3}$	$\mathbf{S} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$	**sector matrix**: this matrix will be used to describe the condition that sectors of a site should have consecutive PCI values.	constant.
$\mathbf{X} = [x_{n,i,k}]_{N\times 3\times K}$	$n \in [N]$ $i \in \{1, 2, 3\}$ $k \in [K]$ $x_{n,i,k} \in \{0, 1\}$	**allocation matrix**: If $x_{n,i,k} = 1$, then the PCI allocated to sector $s_{n,i}$ is k. In other words, PCI value allocated to sector $s_{n,i}$ is $\mathbf{X}[n, i, :] \times \mathbf{L}$. The matrix $\mathbf{XL}$ is a $N \times 3$ matrix whose n, i element is the PCI value of sector $s_{n,i}$, and its n'th row contains the three PCI values of site n.	determined by PCI planning.
$\mathbf{Y} = [y_{n,i,h}]_{N\times 3\times 3}$	$n \in [N]$ $i \in \{1, 2, 3\}$ $h \in \{0, 1, 2\}$ $y_{n,i,h} \in \{0, 1\}$	**mode allocation matrix**: If $y_{n,i,h} = 1$, then the mode 3 PCI allocated to sector $s_{n,i}$ is $h \in \{0, 1, 2\}$.	determined by PCI planning.

Parameter	Range	Description	Status
$\mathbf{U} = [u_{n,i,n',j}]_{N\times 3\times N\times 3}$	$n, n' \in [N]$ $i, j \in \{1,2,3\}$ $u_{n,i,n',j} \in \{0,1\}$	**similarity matrix**: $u_{n,i,n',j} = 1$ if PCI allocated to sectors $s_{n,i}$ and $s_{n',j}$ are the same.	determined by PCI planning.
$\mathbf{V} = [v_{n,i,n',j}]_{N\times 3\times N\times 3}$	$n, n' \in [N]$ $i, j \in \{1,2,3\}$ $v_{n,i,n',j} \in \{0,1\}$	**mode similarity matrix**: $v_{n,i,n',j} = 1$ if mode 3 of PCI allocated to sectors $s_{n,i}$ and $s_{n',j}$ are the same.	determined by PCI planning.

A. *constants and givens*

$$G, N, \mathcal{S}, D, K, \mathbf{L}, \mathcal{S}_g, \mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{S}$$

B. *objective function*

$$\begin{aligned}\min \sum_{g} \quad & \omega_1 \sum_{\substack{n,n'\in[N] \\ i,j\in\{1,2,3\}}} u_{n,i,n',j}(p_{g,n,i} - p_{g,n',j})^2 + \\ & \omega_2 \sum_{\substack{n,n'\in[N] \\ i,j\in\{1,2,3\}}} v_{n,i,n',j}(p_{g,n,i} - p_{g,n',j})^2\end{aligned}$$

C. *constraints*

$$\sum_{k\in[K]} x_{n,i,k} = 1, \quad n \in [N], i \in \{1,2,3\} \tag{1}$$

$$u_{n,i,n',j} = \sum_{k\in[K]} x_{n,i,k} x_{n',j,k}, \quad n, n' \in [N], i, j \in \{1,2,3\} \tag{2}$$

$$y_{n,i,h} = \sum_{k\in[K]} x_{n,i,k} m_{k,h}, \quad n \in [N], h \in \{0,1,2\} \tag{3}$$

$$v_{n,i,n',j} = \sum_{h\in\{0,1,2\}} y_{n,i,h} y_{n',j,h}, \quad n, n' \in [N], i, j \in \{1,2,3\} \tag{4}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ & \vdots & \\ 0 & 0 & 0 \end{bmatrix} < |\mathbf{XLS}| \leq \begin{bmatrix} 2 & 2 & 2 \\ & \vdots & \\ 2 & 2 & 2 \end{bmatrix} \tag{5}$$

- constraint (1): Each site sector has exactly one PCI value.
- constraint (2): The elements of similarity matrix are determined from the elements of the allocation matrix. If for two site sectors $s_{n,i}$ and $s_{n',j}$, both $x_{n,i,k}$ and $x_{n',j,k}$ are 1, then $u_{n,i,n',j} = 1$.

- constraint (3): The elements of mode allocation matrix are determined from allocation matrix, with the help of mode matrix. Recall that mode matrix stores the mod 3 value of each PCI value.
- constraint (4): Just like similarity matrix is derived from allocation matrix, mode similarity matrix is derived from mode allocation matrix.
- constraint (5): The matrix $\mathbf{XLS}$ is a $N \times 3$ matrix whose element $n, 1$ is the PCI value of sector $s_{n,1}$ minus the PCI value of sector $s_{n,2}$. Similarly, the element $n, 2$ of this matrix is the PCI value of sector $s_{n,1}$ minus the PCI value of sector $s_{n,3}$. Finally, the element $n, 3$ of this matrix is the PCI value of sector $s_{n,2}$ minus the PCI value of sector $s_{n,3}$. This is in accordance with columns of sector matrix. This constraint states that PCI values of sectors in a site should be consecutive. In other words, each $s_{n,i}$, $n \in [N]$, $i \in \{1, 2, 3\}$ can be one of $-2, -1, 1, 2$.

Alternative constraints instead of (5)

There are two alternatves for constraint (5):

- PCI values of sectors in a site are not necessarily consecutive, but should be different:

$$\begin{bmatrix} 0 & 0 & 0 \\ & \vdots & \\ 0 & 0 & 0 \end{bmatrix} < |\mathbf{XLS}| \tag{6}$$

- PCI values of sectors in a site are consecutive in a predetermined order. For each site, PCI of sector 2 is PCI of sector 1 plus 1, and PCI of sector 3 is PCI of sector 2 plus 1.

D. unknowns

$$\mathbf{X},\ \mathbf{Y},\ \mathbf{U},\ \mathbf{V}$$

E. final formulation (binary quadratic programming)

$$\min \sum_{g} \quad \omega_1 \sum_{\substack{n,n' \in [N] \\ i,j \in \{1,2,3\}}} \left(\sum_{k \in [K]} x_{n,i,k} x_{n',j,k} \ \frac{\min\left(p_{g,n,i}, p_{g,n',j}\right)}{\max\left(p_{g,n,i}, p_{g,n',j}\right)} \right) +$$

$$\omega_2 \sum_{\substack{n,n' \in [N] \\ i,j \in \{1,2,3\}}} \left(\sum_{h \in \{0,1,2\}} \Big(\sum_{k \in [K]} x_{n,i,k} m_{k,h} \Big) \Big(\sum_{k \in [K]} x_{n',j,k} m_{k,h} \Big) \ \frac{\min\left(p_{g,n,i}, p_{g,n',j}\right)}{\max\left(p_{g,n,i}, p_{g,n',j}\right)} \right)$$

such that:

$$\sum_{k \in [K]} x_{n,i,k} = 1, \quad n \in [N], i \in \{1, 2, 3\}$$

$$\begin{bmatrix} -2 & -2 & -2 \\ & \vdots & \\ -2 & -2 & -2 \end{bmatrix} \leq \mathbf{XLS} \leq \begin{bmatrix} 2 & 2 & 2 \\ & \vdots & \\ 2 & 2 & 2 \end{bmatrix}$$

F. matrix form

In order to better understand the above optimization problem, we use pair indices (n, i) for site n and sector i, and denote that with ni for simplicity. Therefore, the objective function is

$$\begin{aligned} \min \sum_{g} \quad & \omega_1 \sum_{\substack{ni \\ nj'}} \left(\sum_{k\in[K]} x_{ni,k} x_{n'j,k} \ \frac{\min(p_{g,ni}, p_{g,n'j})}{\max(p_{g,ni}, p_{g,n'j})} \right) + \\ & \omega_2 \sum_{\substack{ni \\ nj}} \left(\sum_{h\in\{0,1,2\}} \big(\sum_{k\in[K]} x_{ni,k} m_{k,h} \big) \big(\sum_{k\in[K]} x_{n'j,k} m_{k,h} \big) \ \frac{\min(p_{g,ni}, p_{g,n'j})}{\max(p_{g,ni}, p_{g,n'j})} \right) \end{aligned}$$

Let us define the matrix $\mathbf{Q}_g := [\frac{\min(p_{g,ni}, p_{g,n'j})}{\max(p_{g,ni}, p_{g,n'j})}]_{(N\times 3)\times(N\times 3)}$. With this definition, the objective function becomes

$$\begin{aligned} \min \sum_{g} \quad & \omega_1 \sum_{\substack{ni \\ nj'}} \left(\sum_{k\in[K]} x_{ni,k} x_{n'j,k} \ q_{ni,n'j} \right) + \\ & \omega_2 \sum_{\substack{ni \\ nj}} \left(\sum_{h\in\{0,1,2\}} \big(\sum_{k\in[K]} x_{ni,k} m_{k,h} \big) \big(\sum_{k\in[K]} x_{n'j,k} m_{k,h} \big) \ q_{ni,n'j} \right) \end{aligned}$$

Now we can express the objective function in matrix form.

$$\min \sum_{g} \quad \omega_1 \text{Tr}(\mathbf{X}^T \mathbf{Q}_g \mathbf{X}) + \omega_2 \text{Tr}(\mathbf{M}^T \mathbf{X}^T \mathbf{Q}_g \mathbf{X} \mathbf{M})$$

The difference between the first and second terms is that in the second term, we used $\mathbf{XM}$ instead of $\mathbf{X}$ since mode 3 is required. In summary, we have

$$\min \sum_{g} \quad \omega_1 \text{Tr}(\mathbf{X}^T \mathbf{Q}_g \mathbf{X}) + \omega_2 \text{Tr}(\mathbf{M}^T \mathbf{X}^T \mathbf{Q}_g \mathbf{X} \mathbf{M}) \tag{7}$$

such that: (8)

$$\mathbf{X} \mathbf{1}_K = \mathbf{1}_{N\times 3}, \tag{9}$$

$$\begin{bmatrix} -2 & -2 & -2 \\ & \vdots & \\ -2 & -2 & -2 \end{bmatrix} \leq \mathbf{XLS} \leq \begin{bmatrix} 2 & 2 & 2 \\ & \vdots & \\ 2 & 2 & 2 \end{bmatrix} \tag{10}$$