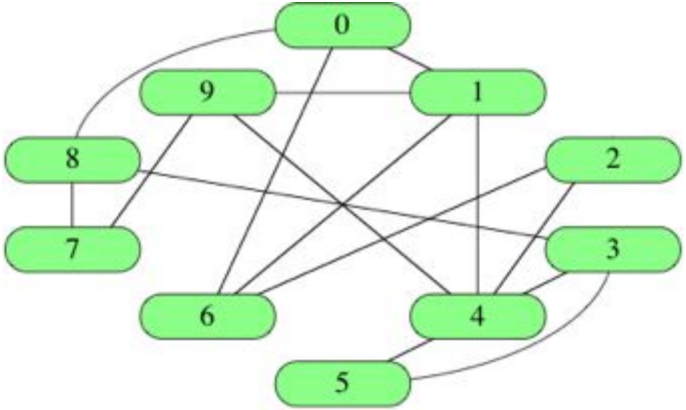
Diskretna matematika 2

Alati i biblioteke

- Python
- networkx
- matplotlib

Predstavljanje grafa u računaru

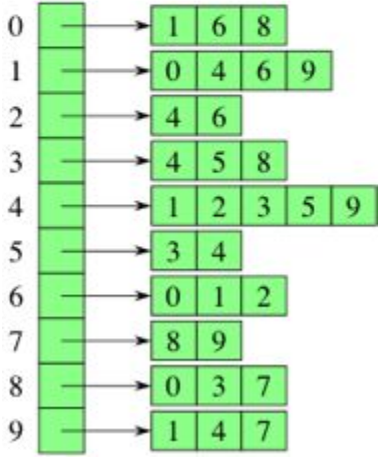
Graf



Matrica susjedstva

	0	1	2	3	4	5	6	7	8	9
0	0	1	0	0	0	0	1	0	1	0
1	1	0	0	0	1	0	1	0	0	1
2	0	0	0	0	1	0	1	0	0	0
3	0	0	0	0	1	1	0	0	1	0
4	0	1	1	1	0	1	0	0	0	1
5	0	0	0	1	1	0	0	0	0	0
6	1	1	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	1
8	1	0	0	1	0	0	0	1	0	0
9	0	1	0	0	1	0	0	1	0	0

Lista susjedstva



Biblioteka networkx

Python biblioteka za rad sa grafovima

- Strukture za predstavljanje
  - Grafova - neusmjerenih grafova bez višestrukih grana
  - Digrafa - usmjerenih grafova
  - Multigrafova - grafova sa višestrukim grandma i petljama
- Standardne algoritme nad grafovima
- Algoritme za analiz grafa
- Poznate grafove, slučajno generisanje grafova
- Crtanje grafa
- …

Instalacija biblioteke

Networkx možete instalirati na svom računaru izvršavanjem naredbe

```
pip3 install networkx
```

Ili možete koristiti google colab za vježbu

https://colab.research.google.com/

Napomena:

Kolokvijum će se realizovati na univerzitetskim računarima bez pristupa internetu, studenti će imati instaliran python, visual studio code i sve potrebne biblioteke.

Networkx biblioteka

```
import networkx as nx
import matplotlib.pyplot as plt
```

Uključujemo biblioteku networkx za rad sa grafovima i biblioteku pyplot iz paketa matplotlib koji služi za vizualizaciju

```
G = nx.Graph()                         #kreiramo prazan neusmjereni graf
G.add_node(1)                          #dodajemo čvor 1
G.add_nodes_from([2, 3])               #dodajemo čvorove 2 i 3
G.add_edge(1, 2)                       #dodajemo granu između čvorova 1 i 2
G.add_edges_from([(1, 3), (2, 3)])     #dodajemo grane (1, 3) i (2, 3)
G.add_edge(5, 6)                       #dodajemo granu između čvorova 5 i 6,
                                       #čvorovi 5 i 6 se automatski dodaju u graf
```

Networkx biblioteka

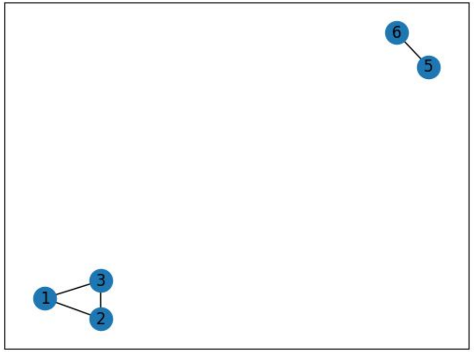
```
print(f'Graf G sadrži {G.number_of_nodes()} čvorova i {G.number_of_edges()} grana.')
```

[16] ✓ 0.0s

```
Graf G sadrži 5 čvorova i 4 grana.
```

```
nx.draw_networkx(G)                      #crtamo graf
plt.show()                               #prikazujemo graf
```

[17] ✓ 0.0s



Networkx biblioteka

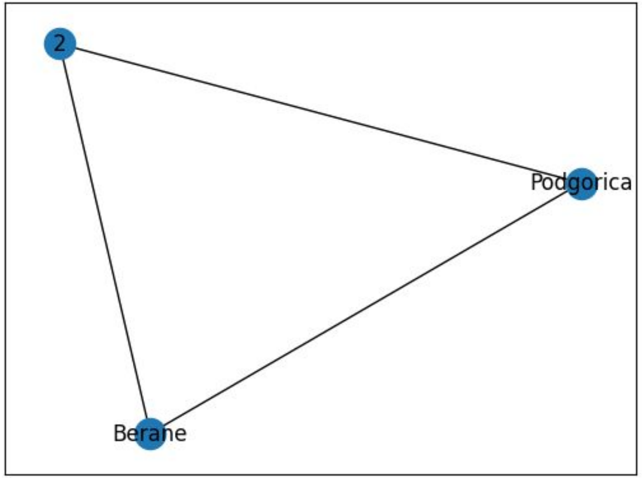
```
Gradovi = nx.Graph()

Gradovi.add_node("Berane")
Gradovi.add_node("Podgorica")
Gradovi.add_node(2)

Gradovi.add_edge("Berane", 2)
Gradovi.add_edge("Berane", "Podgorica")
Gradovi.add_edge("Podgorica", 2)

nx.draw_networkx(Gradovi)
plt.show()
```

[30] ✓ 0.0s



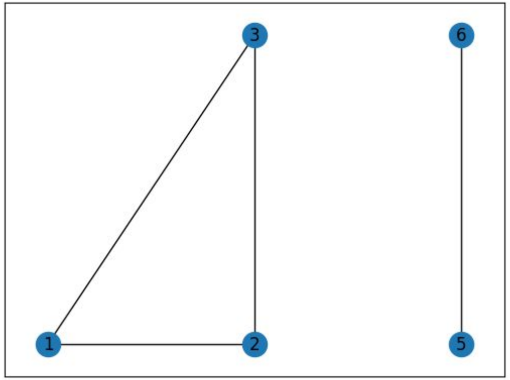
Čvorovi grafa mogu da budu bilo šta, brojevi, riječi, json dokumenti …

Svakoj grani se mogu dodijeliti proizvojljni atributi

Networkx biblioteka

```
pos = {
    1: (0, 0),
    2: (1, 0),
    3: (1, 1),
    5: (2, 0),
    6: (2, 1)
}

nx.draw_networkx(G, pos)          #crtamo graf s pozicijama čvorova
plt.show()                        #prikazujemo graf
```
[18] ✓ 0.0s



Za specificiranje pozicija koristimo dictionary koji za svaki čvor sadrži koordinate na kojima će se on prikazati.

Networkx biblioteka

```
G = nx.complete_graph(5)              #generiše kompletan graf sa 5 čvorova
pos = nx.spring_layout(G)
```

Position nodes using Fruchterman-Reingold force-directed algorithm.

The algorithm simulates a force-directed representation of the network treating edges as springs holding nodes close, while treating nodes as repelling objects, sometimes called an anti-gravity force. Simulation continues until the positions are close to an equilibrium.

```
nx.draw_networkx_nodes(G, pos, nodelist=G.nodes, node_size=700, node_color='skyblue') #crta čvorove
nx.draw_networkx_edges(G, pos, edgelist=G.edges, edge_color='gray', alpha=0.3)        #crta listu grana
nx.draw_networkx_labels(G, pos, font_color='black')                                    #crta labele
```

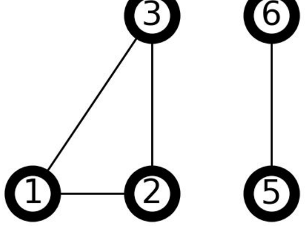
Networkx biblioteka

```
options = {
    "font_size": 32,
    "node_size": 2000,
    "node_color": "white",
    "edgecolors": "black",
    "linewidths": 10, #debljina ivica čvorova
    "width": 2 #debljina linija
}

nx.draw_networkx(G, pos, **options)     #crtamo graf s pozicijama čvorova

ax = plt.gca()                          #uzimamo referencu na objekat koji p
ax.margins(0.2)                         #povećavamo margine grafika
plt.axis("off")                         #isključujemo prikaz osa
plt.show()                              #prikazujemo graf
```

[43] ✓ 0.0s



**options, zamjenjuje vrijednosti iz dictionary u argumente funkcije

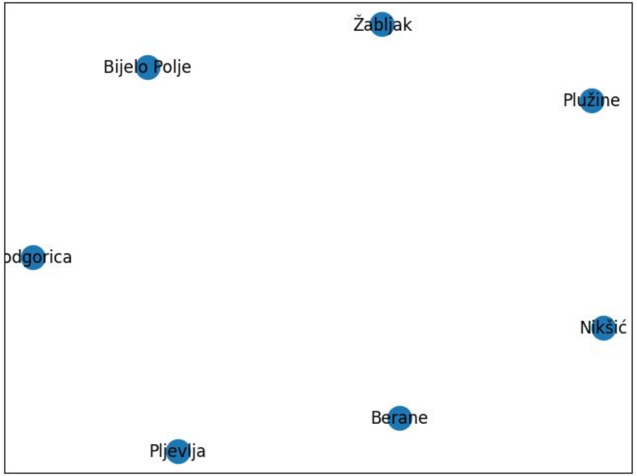
Čvorovi i grane grafa

```
G = nx.Graph()

G.add_node('Berane')
G.add_nodes_from(['Podgorica', 'Nikšić', 'Pljevlja', 'Bijelo Polje'])
G.add_nodes_from(
    [('Plužine', {'broj_stanovnika': 5000}),
     ('Žabljak', {'broj_stanovnika': 4000})])

nx.draw(G, with_labels=True)
plt.axis("on")
plt.show()
```

[16] ✓ 0.1s



Čvorovi i grane grafa

```
print(G.nodes['Plužine'])
```

[22] ✓ 0.0s

```
{'broj_stanovnika': 5000}
```

```
G.nodes['Podgorica']['broj_stanovnika'] = 200000
G.nodes.data()
```

[27] ✓ 0.0s

```
NodeDataView({'Berane': {}, 'Podgorica': {'broj_stanovnika': 200000}, 'Nikšić': {}, 'Pljevlja': {}, 'Bijelo Polje': {}, 'Plužine': {'broj_stanovnika': 5000},
```

Čvorovi i grane

```python
PMF = nx.Graph(opis='Prirodno-matematički fakultet')
PMF.add_node("Andrijana", tip="profesor")
PMF.add_nodes_from([
    ("Mina", {"tip": "student", "godina_upisa": 2019}),
    ("Janko", {"tip": "student", "godina_upisa": 2018})]
)

PMF.add_node("Diskretna matematika", broj_casova=3)
PMF.add_nodes_from([
    ("Algebra", {"broj_casova": 4}),
    ("Analiza", {"broj_casova": 4})
])
PMF.add_edges_from([
    ("Andrijana", "Diskretna matematika"),
    ("Mina", "Algebra", {"ocjena": 10}),
    ("Janko", "Diskretna matematika", {"ocjena": 9}),
    ("Janko", "Analiza", {"ocjena": 8})
])
```

```python
PMF.nodes.data()
```

[35] ✓ 0.0s Python

```
NodeDataView({'Andrijana': {'tip': 'profesor'}, 'Mina': {'tip': 'student', 'godina_upisa': 2019}, 'Janko': {'tip': 'student', 'godina_upisa': 2018}, 'Diskretna matematika': {'broj_casova': 3}, 'Algebra': {'broj_casova': 4}, 'Analiza': {'broj_c
```

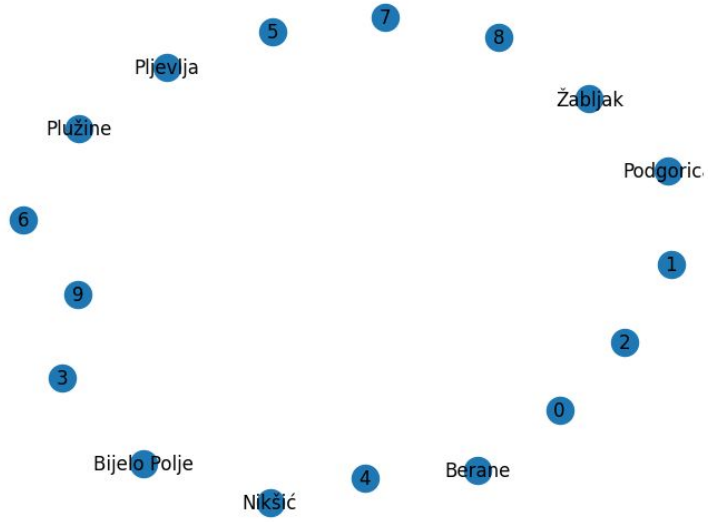
Možemo da dodijelimo atribute grafu, čvoru ili grani.

Čvorovi i grane

```
T = nx.path_graph(10)
G.add_nodes_from(T)
nx.draw(G, with_labels=True)
```

[42] ✓ 0.0s

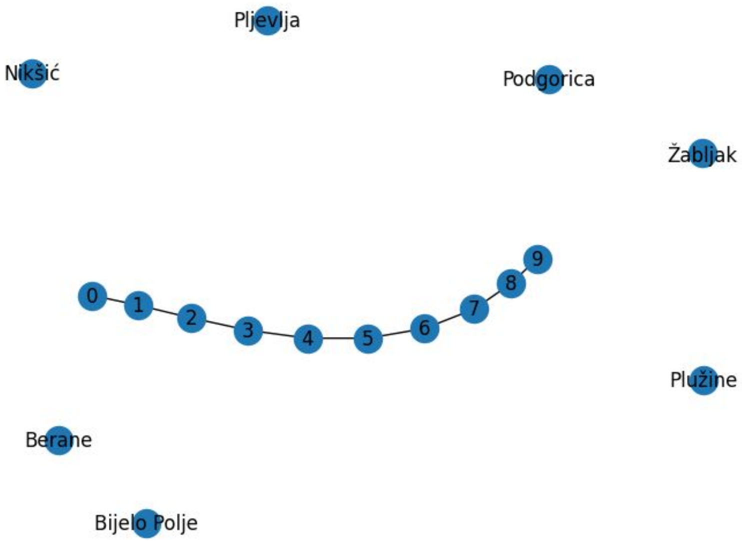
Možemo da dodamo čvorove jednog grafa u drugi graf.



Čvorovi i grane

```
nx.draw(G, with_labels=True)
```
[47] ✓ 0.0s

Možemo da dodamo grane jednog grafa u drugi graf.




```
G.clear()
```
[48] ✓ 0.0s

Briše sve što se nalazi u grafu.

Svojstva grafa

Iz grafa možemo da dobijemo **spisak čvorova**, **grana**, **stepen čvorova** i **listu susjedstva**.

Dobijamo objekte slične dictionariju koje možemo da kastujemo u listu ili dictionary.

Svojstva grafa

```
G = nx.Graph()
G.add_node(1)
G.add_nodes_from([2, 3])
G.add_edge(1, 2)
G.add_nodes_from('abc')  # add nodes 'a', 'b', 'c'
G.add_node('efg') # add node 'efg'
G.add_edges_from([('a', 'b'), 'ac', 'bc'])
```

[19] ✓ 0.0s

```
list(G.nodes) #dobijemo neki iterable container, i onda da bi dobili listu, moramo da ga castujemo u listu
```

[23] ✓ 0.0s

```
[1, 2, 3, 'a', 'b', 'c', 'efg']
```

Gen
Start Chat to Gener

```
list(G.edges)
```

[24] ✓ 0.0s

```
[(1, 2), ('a', 'b'), ('a', 'c'), ('b', 'c')]
```

Svojstva grafa

```
dict(G.adj)
```

[28] ✓ 0.0s

```
{1: AtlasView({2: {}}),
 2: AtlasView({1: {}}),
 3: AtlasView({}),
 'a': AtlasView({'b': {}, 'c': {}}),
 'b': AtlasView({'a': {}, 'c': {}}),
 'c': AtlasView({'a': {}, 'b': {}}),
 'efg': AtlasView({})}
```

```
list(G.adj['a'])
```

[32] ✓ 0.0s

```
['b', 'c']
```

Svojstva grafa

```
[35]
dict(G.degree)
✓ 0.0s
```

```
{1: 1, 2: 1, 3: 0, 'a': 2, 'b': 2, 'c': 2, 'efg': 0}
```

```
[36]
G.degree['a']
✓ 0.0s
```

```
2
```

Uklanjanje čvorova i grana

```
G.remove_node('a')
G.remove_nodes_from([1, 2])
G.remove_edge('b', 'c')
G.remove_edges_from([('a', 'b'), ('b', 'c')])
```

[47] ✓ 0.0s

Uklanjanjem čvora uklanjanju se i sve njegove grane. Ako čvor/grana nije u grafu dobićemo exception prilikom poziva metode remove_node / remove_edge. Metode remove_nodes from i remove_edges_from ne bacaju exception.

Vizualizacija grafa

```
# Perform DFS and construct the DFS tree
   dfs_edges = list(nx.dfs_edges(graph, source=start_node))
   dfs_tree = nx.DiGraph(dfs_edges)

   # Plot settings
   fig, axes = plt.subplots(1, 2, figsize=(12, 6))

   # Draw original graph
   ax = axes[0]
   pos = nx.spring_layout(graph)  # Layout for node positioning
   nx.draw(graph, pos, ax=ax, with_labels=True, node_color='lightblue', edge_color='gray')
   ax.set_title("Original Graph")

   # Draw DFS tree
   ax = axes[1]
   nx.draw(dfs_tree, pos, ax=ax, with_labels=True, node_color='lightgreen', edge_color='red', arrows=True)
   ax.set_title("DFS Tree")

   plt.show()
```

Vizualizacija grafa

```
G = nx.Graph()
G.add_edges_from([
    ('a', 'b', {'weight': 5}),
    ('c', 'b', {'weight': 2}),
    ('e', 'b', {'weight': 5}),
    ('a', 'e', {'weight': 3})
])

pos = nx.spring_layout(G)
nx.draw_networkx_nodes(G, pos, nodelist=G.nodes)
nx.draw_networkx_labels(G, pos, font_color='black')
nx.draw_networkx_edges(G, pos, edgelist=G.edges)
nx.draw_networkx_edge_labels(G, pos, {e: v['weight'] for e, v in G.edges.items()}, label_pos=0.7,
verticalalignment='bottom')
```

DFS algoritam

```
dict(nx.traversal.dfs_edges(G, 'a'))
```

1. Napisati program koji u prvom redu učitava broj čvorova (n) i broj grana (m) neusmjerenog grafa G.
   U narednih m redova se učitavaju po dva broja koji predstavljaju indekse čvorova jedne grane.
   U m+2 liniji se učitava indeks još jednog čvora u.
   Potrebno je ispitati da li je:
   a. Sve čvorove koji se nalaze u istoj komponenti povezanosti kao čvor u.
   b. Koliko komponenti povezanosti sadrži graf G.
   c. Štampati najveću komponentu povezanosti.
   d. Da li je graf G bipartitan.
   e. Da li G sadrži cikluse.
   f. Prikazati graf G i stablo DFS obilaska grafa G.

Iscrtavanje grafa

```
def visualize_graph_and_dfs_tree(graph, start_node):
    # Perform DFS and construct the DFS tree
    dfs_edges = list(nx.dfs_edges(graph, source=start_node))
    dfs_tree = nx.DiGraph(dfs_edges)

    # Plot settings
    fig, axes = plt.subplots(1, 2, figsize=(12, 6))

    # Draw original graph
    ax = axes[0]
    pos = nx.spring_layout(graph)  # Layout for node positioning
    nx.draw(graph, pos, ax=ax, with_labels=True, node_color='lightblue', edge_color='gray')
    ax.set_title("Original Graph")

    # Draw DFS tree
    ax = axes[1]
    nx.draw(dfs_tree, pos, ax=ax, with_labels=True, node_color='lightgreen', edge_color='red', arrows=True)
    ax.set_title("DFS Tree")

    plt.show()
```

Iscrtavanje grafa

```
G = nx.Graph()
edges = [('A', 'B'), ('A', 'C'), ('B', 'D'), ('C', 'D'),
       ('D', 'E'), ('E', 'F'), ('E', 'G'), ('G', 'H'), ('F', 'H')]
G.add_edges_from(edges)
d_edges = list(nx.dfs_edges(G, source='A')) # Grane koje DFS obidje

plt.figure(figsize=(10, 10))
pos = nx.spring_layout(G)

nx.draw_networkx_nodes(G, pos, nodelist=G.nodes, node_size=700, node_color='skyblue')
nx.draw_networkx_edges(G, pos, edgelist=G.edges, edge_color='gray', alpha=0.3)
nx.draw_networkx_labels(G, pos, font_color='black')
nx.draw_networkx_edges(G, pos, edgelist=d_edges, edge_color='red')

plt.title('Iscrtavanje v2')
plt.show()
```

Grafički nizovi

2. Napisati program koji provjerava da li je niz čvorova

$\{d_1, d_2, d_3, \ldots, d_{n-1}, d_n\}$, grafički.

Teorema: Niz $\{d_1, d_2, d_3, \ldots, d_{n-1}, d_n\}$ je grafički akko je grafički niz $\{d_2-1, d_3-1, \ldots, d_{d1-1}-1, d_{d1}-1, d_{d1+1}-1, \ldots, d_{n-1}, d_n\}$.

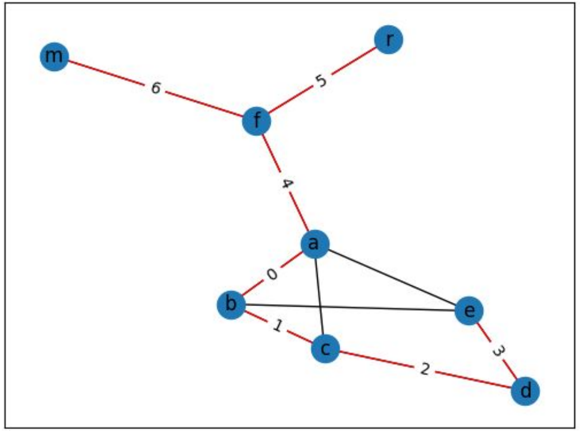
DFS algoritam

In Depth First Search (or DFS) for a graph, we traverse all adjacent vertices one by one. When we traverse an adjacent vertex, we completely finish the traversal of all vertices reachable through that adjacent vertex

- Koristi se za provjeru da li su dva čvora u istoj komponenti povezanosti
- Koliko imamo komponenti povezanosti
- Nalaženje artikulacionih čvorova
- Nalaženje mostova
- Nalaženje ciklusa u grafu
- ...

DFS algoritam (dfs_edges)

Vraća listu grana redosljedom kako su obiđene DFS algoritmom.




```
[21] dfs_edges = list(nx.traversal.dfs_edges(g, 'a'))
```

```
pos = nx.spring_layout(g)
nx.draw_networkx_nodes(g.nodes, pos)
nx.draw_networkx_edges(g, pos, g.edges)
nx.draw_networkx_edges(g, pos, dfs_edges, edge_color='red')
nx.draw_networkx_labels(g, pos)
nx.draw_networkx_edge_labels(g, pos, {e: i for i, e in enumerate(dfs_edges)})
```

Nalazenje komponenti povezanosti

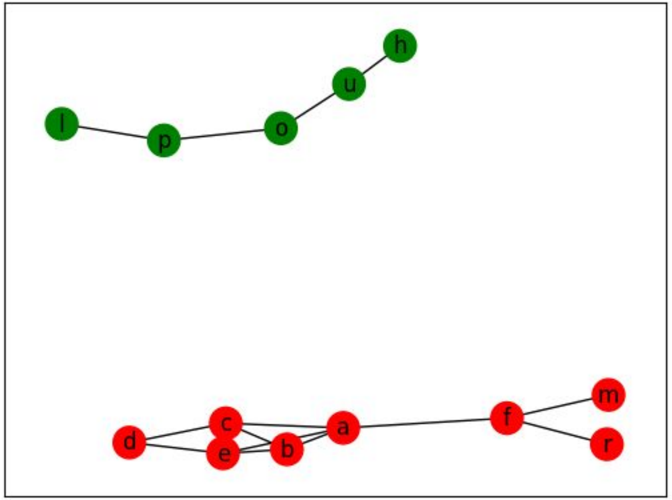
```
[29] components = {n:-1 for n in g.nodes}

     cnt = 0
     for n, c in components.items():
       if c == -1:
         dfs_edges = list(nx.dfs_edges(g, n))

         for u, v in dfs_edges:
           components[u] = components[v] = cnt

         cnt += 1

     print(components)
```

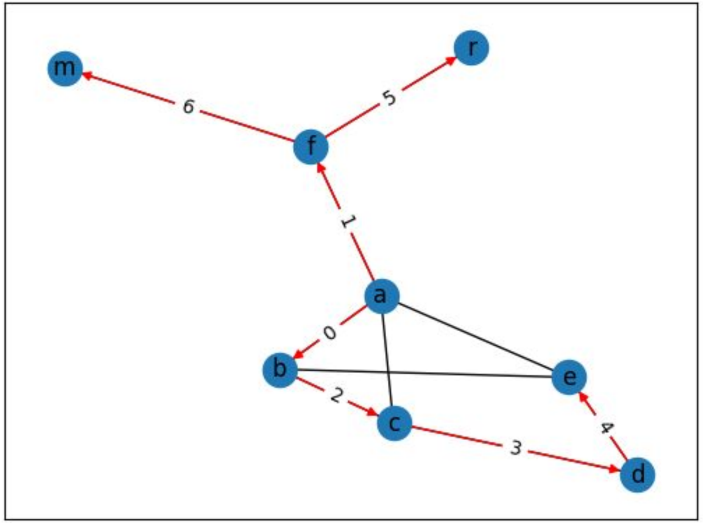



```
{'a': 0, 'b': 0, 'c': 0, 'd': 0, 'e': 0, 'f': 0, 'r': 0, 'm': 0, 'h': 1, 'u': 1, 'o': 1, 'p': 1, 'l': 1}
```

```
colors = ['red', 'green']
colors = [colors[components[n]] for n in g.nodes]
nx.draw_networkx_nodes(g, pos, node_color=colors, cmap=plt.cm.rainbow)
nx.draw_networkx_edges(g, pos, edgelist=g.edges)
nx.draw_networkx_labels(g, pos)
plt.show()
```

DFS algoritam (dfs_tree)

Vraća stablo sa čvorovima usmjerenim grandma u smjeru kako se je DFS algoritmom kretao.




```
[26] dfs_tree = nx.dfs_tree(g, 'a')
```

```
[36] nx.draw_networkx_nodes(g.nodes, pos)
     nx.draw_networkx_edges(g, pos, g.edges)
     nx.draw_networkx_labels(g, pos)
     nx.draw_networkx_edges(dfs_tree, pos, dfs_tree.edges, edge_color='red')
     nx.draw_networkx_edge_labels(g, pos, {e: i for i, e in enumerate(dfs_tree.edges)})
```

BFS algoritam

bfs_edges - vraća grane redosljedom kojim su obiđene u BFS algoritmu

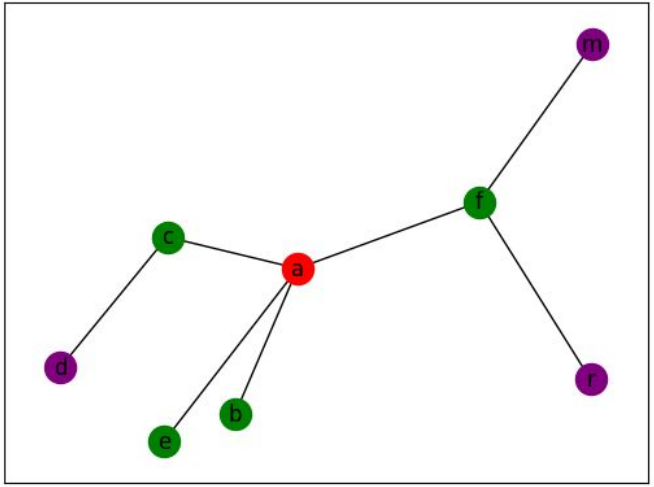
bfs_layers - vraća slojeve u bfs obilasku

```
bfs_edges = list(nx.bfs_edges(g, 'a'))
bfs_layers = list(nx.bfs_layers(g, 'a'))

colors = ['red', 'green', 'purple']

for i, layer in enumerate(bfs_layers):
    nx.draw_networkx_nodes(g, pos, layer, node_color=colors[i])
    nx.draw_networkx_labels(g, pos, {l: l for l in layer})

nx.draw_networkx_edges(g, pos, bfs_edges)
```



BFS algoritam

Napisati program koji provjerava da li je graf bipartitan

```
def bipartite(g):
  bfs_layers = list(nx.bfs_layers(g, 'a'))
  parity = {n: -1 for n in g.nodes}

  for n in g.nodes:
    if parity[n] == -1:
      bfs_layers = list(nx.bfs_layers(g, 'a'))

      for i, nodes in enumerate(bfs_layers):
        for node in nodes:
          parity[node] = i % 2


  for u, v in g.edges:
    if parity[u] == parity[v]:
      return False

  return True

bipartite(g)
```

DFS algoritam

```
print(nx.is_connected(G))
print(nx.is_bipartite(G))
print(nx.cycle_basis(G))
print(list(nx.simple_cycles(G)))
```

Čvorna povezanost grafa

```
print(nx.all_pairs_node_connectivity(G))
```

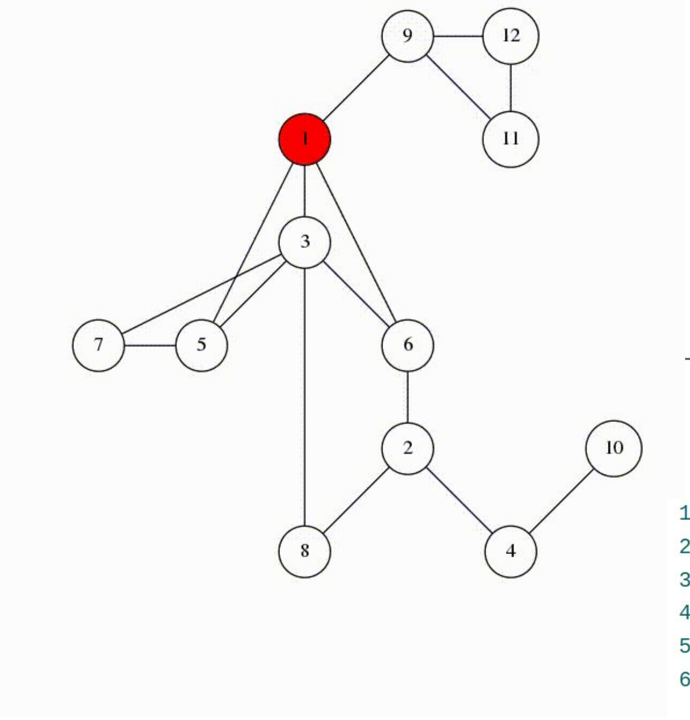
Za svaki par čvorova (u, v) dobijamo minimalan broj čvorova koje treba ukloniti iz grafa G da bi čvorovi u i v bili u različitim komponentama.

Nalaženje mostova i artikulacionih čvorova

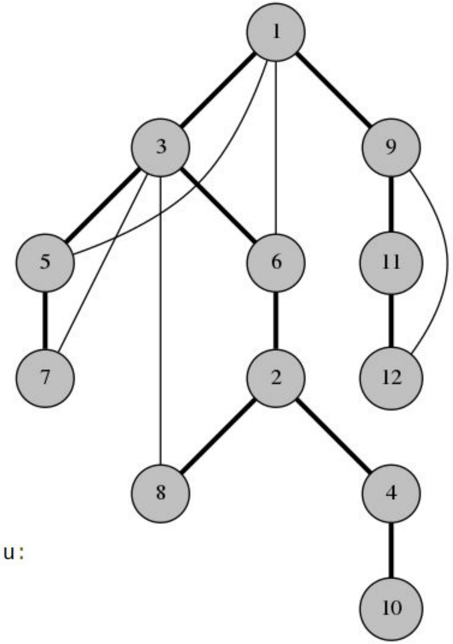
Grana uv je most ako se uklanjanjem te grane povećava broj povezanih komponenti u grafu

Čvor u je artkulacioni čvor ako se uklanjem tog čvora povećava broj poezanih komponenti u grafu

Klasifikacija grana prilikom DFS algoritma



DFS algorithm




```
function visit(u):
    mark u as visited
    for each vertex v among the neighbours of u:
        if v is not visited:
            mark the edge uv
            call visit(v)
```

Klasifikacija grana prilikom DFS algoritma

- **Tree Edge**: It is an edge which is present in the tree obtained after applying DFS on the graph. All the Green edges are tree edges.
- **Back edge**: It is an edge (u, v) such that v is the ancestor of node u but is not part of the DFS tree. Edge from **6 to 2** is a back edge. <u>Presence of back edge indicates a cycle in directed graph</u>.

Nalaženje mostova i artikulacionih čvorova

Observation 1. The back-edges of the graph all connect a vertex with its descendant in the spanning tree. **This is why DFS tree is so useful.**

Suppose that there is an edge uv, and without loss of generality the depth-first traversal reaches u while v is still unexplored. Then:

- if the depth-first traversal goes to v from u using uv, then uv is a span-edge;
- if the depth-first traversal doesn't go to v from u using uv, then v was already visited when the traversal looked at it at step 4. Thus it was explored while exploring one of the other neighbours of u, which means that v is a descendant of u in the DFS tree.

Nalaženje mostova i artikulacionih čvorova

Observation 2. A span-edge *uv* is a bridge if and only if there exists no back-edge that connects a descendant of *uv* with an ancestor of *uv*. In other words, a span-edge *uv* is a bridge if and only if there is no back-edge that "passes over" *uv*.

Removing the edge uv splits the spanning tree to two disconnected parts: the subtree of uv and the rest of the spanning tree. If there is a back-edge between these two components, then the graph is still connected, otherwise uv is a bridge. The only way a back-edge can connect these components is if it connects a descendant of uv with an ancestor of uv.

Observation 3. A back-edge is never a bridge.

Nalaženje mostova i artikulacionih čvorova

This gives rise to the classical bridge-finding algorithm. Given a graph G:

1. find the DFS tree of the graph;
2. for each span-edge uv, find out if there is a back-edge "passing over" uv, if there isn't, you have a bridge.

Let dp[u] be the number of back-edges passing over the edge between u and its parent. Then,

$$\mathrm{dp}[u] = (\#\text{ of back-edges going up from } u) - (\#\text{ of back-edges going down from } u) + \sum_{v \text{ is a child of } u} \mathrm{dp}[v]$$

The edge between u and its parent is a bridge if and only if dp[u]=0.

Zadatak

Napisati program koji orijentiše grane neusmjerenog grafa G tako da dobijeni usmjereni graf bude jako povezan. Usmjeren graf G je jako povezan ako postoji put između svaka dva čvora

Hint: Ako graf sadrži mostove tada rješenje ne postoji.

Zadatak

A *cactus* is a graph where every edge (or sometimes, vertex) belongs to at most one simple cycle. The first case is called an *edge cactus*, the second case is a *vertex cactus*. Cacti have a simpler structure than general graphs, as such it is easier to solve problems on them than on general graphs. But only on paper: cacti and cactus algorithms can be very annoying to implement if you don't think about what you are doing.

You are given a connected vertex cactus with N vertices. Answer queries of the form "how many distinct simple paths exist from vertex p to vertex q?".

Još zadataka: https://codeforces.com/blog/entry/68138

Tarjanov algoritam

Čvor U je artikulacioni čvor ako:

1. If there is NO way to get to a node V with **strictly** smaller discovery time than the discovery time of U following the DFS traversal, then U is an articulation point. (it has to be **strictly** because if it is equal it means that U is the root of a cycle in the DFS traversal which means that U is still an *articulation point*).
2. If U is the root of the DFS tree and it has at least 2 children subgraphs disconnected from each other, then U is an articulation point.

Zadaci:
https://onlinejudge.org/index.php?option=online judge page=show_problem&problem=251

https://onlinejudge.org/index.php?option=onlinejudge&page=show_problem&problem=551#google_vignette

https://onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&page=show_problem&problem=737

https://onlinejudge.org/index.php?option=onlinejudge&page=show_problem&problem=1140

https://onlinejudge.org/index.php?option=onlinejudge&page=show_problem&problem=1706

Ojlerov graf

Flerijev algoritam

1. Izabere se proizvoljan čvor w_0 i stavi $W_0 = \{w_0\}$
2. Ako je staza $W_i = w_0 e_1 w_1 \dots e_i w_i$ već izabrana, grana $e_{i+1} \in E(G) - \{e_1, \dots, e_i\}$ bira se prema sledećim uslovima:
   (a) e_{i+1} je incidentna sa w_i
   (b) e_{i+1} nije most u $G - \{e_1, \dots, e_i\}$, osim ako nema drugog izbora.
3. Povratak na korak 2. Ako je nemoguće - STOP.

Teorema *Ako je multigraf G Ojlerov, svaka staza konstruisana Flerijevim algoritmom je zatvorena Ojlerova staza.*

- **Problem kineskog poštara:** U poštanskoj zgradi poštar preuzima poštu, odlazi je razdijeliti po adresama, a zatim se vraća u Poštu. Svakom ulicom neke oblasti mora proći bar jednom. Želi odabrati što kraću zatvorenu šetnju.

Ojlerov graf (Hierholzer's algorithm)

1. Start with an empty stack and an empty circuit (eulerian path).
    - If all vertices have even degree: choose any of them. This will be the current vertex.
    - If there are exactly 2 vertices having an odd degree: choose one of them. This will be the current vertex.
    - Otherwise no Euler circuit or path exists.

Repeat step 2 until the current vertex has no more neighbors and the stack is empty.

2. If current vertex has no neighbors:
    - Add it to circuit,
    - Remove the last vertex from the stack and set it as the current one.

Otherwise:

- Add the vertex to the stack,
- Take any of its neighbors, remove the edge between selected neighbor and that vertex, and set that neighbor as the current vertex.

TO BE CONTINUED