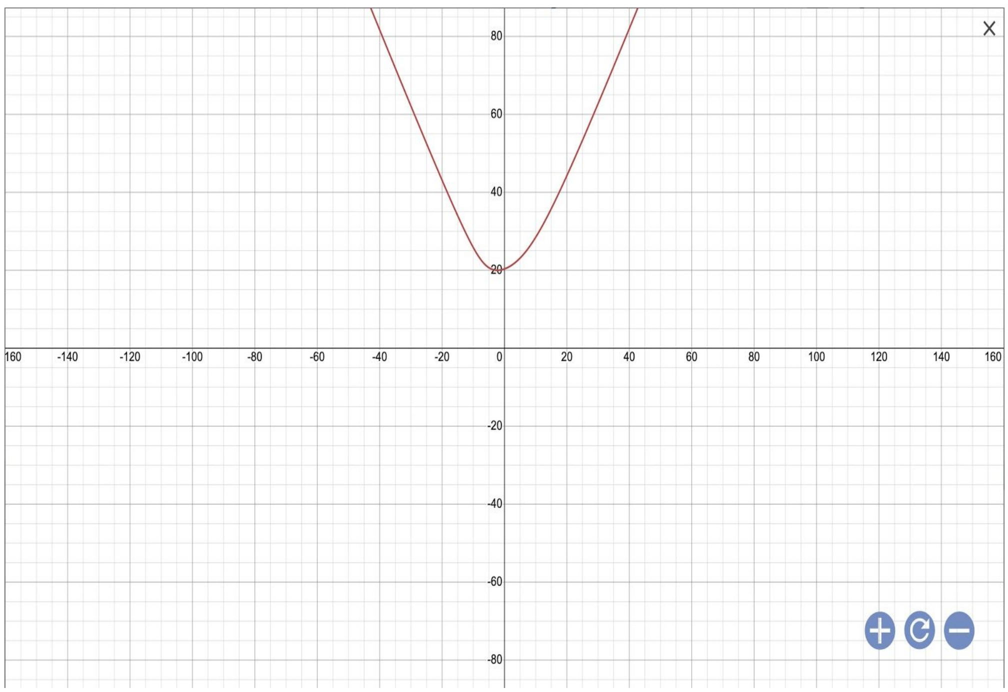
- (Please show your workings). Over all real numbers, find the minimum value of a positive real number, y such that

  $y = sqrt((x+6)^2 + 25) + sqrt((x-6)^2 + 121)

  I have presented my workings in the images below

  (GRAPH SHOWING PROOF OF ANSWER)



To find minimum value of y

equate $\frac{dy}{dx}$ to 0

$$\therefore \frac{d}{dx}\left(\sqrt{(x+6)^2+25} + \sqrt{(x-6)^2+121}\right.$$

Using the sum/difference rule where $(f \pm g)' = f' \pm g'$

$$\frac{d}{dx}\left(\sqrt{(x+6)^2+25} + \sqrt{(x-6)^2+121} = \frac{d}{dx}\left(\sqrt{(x+6^2)+25}\right. + \frac{d}{dx}\left(\sqrt{(x-6)^2+121}\right)\right.$$

$$= \frac{d}{dx}\left(\sqrt{x^2+12x+36+25}\right) + \frac{d}{dx}\left(\sqrt{x^2-12x+157}\right)$$

Taking first part using chain rule

$$\frac{d}{dx}\left(\sqrt{x^2+12x+61}\right) = \frac{du}{dx}$$

where $u = (x^2+12x+61)^{1/2}$

and $v = x^2+12x+61$

$\therefore u = v^{1/2}$

$$\frac{du}{dx} = \frac{du}{dv} \times \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{1}{2}v^{-1/2} = \frac{1}{2\sqrt{v}}$$

$$\frac{dv}{dx} = 2x+12$$

$$\frac{du}{dx} = \frac{1}{2(x^2+12x+61)^{1/2}} \times 2(x+6)$$

$$= \frac{x+6}{\sqrt{x^2+12x+61}}$$

Taking second part using chain rule

$$\frac{d}{dx}\left(\sqrt{x^2-12x+157}\right) = \frac{du}{dx}$$

where $u = (x^2-12x+157)^{1/2}$

and $v = x^2-12x+157$

$\therefore u = v^{1/2}$

$$dy/dx = du/dy \times \frac{dv}{dx}$$

$$du/dv = 1/2\, v^{-1/2} = 1/2\sqrt{v}$$

$$\frac{dv}{dx} = 2(x-6)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2-12x+157}} \times 2(x-6)$$

$$= \frac{x-6}{\sqrt{x^2-12x+157}}$$

$$\frac{dy}{dx} = \frac{x+6}{\sqrt{x^2+12x+61}} + \frac{x-6}{\sqrt{x^2-12x+157}} = 0$$

$$\frac{x+6}{\sqrt{x^2+12x+61}} = \frac{6-x}{\sqrt{x^2-12x+157}}$$ -> Square both sides

$$\frac{(x+6)^2}{x^2+12x+61} = \frac{(6-x)^2}{x^2-12x+157}$$

$$(6-x)^2(x^2+12x+61) = (x+6)^2(x^2-12x+157)$$

$$[x^2-12x+36][x^2+12x+61] = [x^2+12x+36][x^2-12x+157]$$

$$x^4 + 12\cancel{x}x^3 + 61x^2 - 12\cancel{x}x^3 - 144x^2 + 1884x + 36x^2 - 432x + 2196 =$$

$$x^4 - 12\cancel{x}x^3 + 157x^2 + 12\cancel{x}x^3 - 144x^2 + 1884x + 36x^2 - 432x + 5652$$

CONTD

$$x^4 - 47x^2 - 300x + 2196 = x^4 + 49x^2 + 1452x + 5652$$

$$96x^2 + 1752x + 3456 = 0$$

- Solve the quadratic

$$x = \frac{-1752 \pm \sqrt{1752^2 - (4 \times 96 \times 3456)}}{2(96)}$$

$$x = -\frac{9}{4}, \quad x = -16$$

Taking the higher value for y min

$$y = \sqrt{(-9/4 + 6)^2 + 25} + \sqrt{(-9/4 - 6)^2 + 121}$$

$$= \sqrt{(15/4)^2 + 25} + \sqrt{(-33/4)^2 + 121}$$

$$y = \sqrt{\frac{225}{16} + 25} + \sqrt{\frac{1089}{16} + 121}$$

$$= \sqrt{625/16} + \sqrt{3025/16}$$

$$= \frac{25}{4} + \frac{55}{4}$$

$$= 80/4$$

$$= \underline{20}$$

TESTING WITH -16

$$y = \sqrt{(-16+6)^2 + 25} + \sqrt{(-16-6)^2 + 121}$$

$$= \sqrt{100+25} + \sqrt{484 + 121}$$

$$= \sqrt{125} + \sqrt{605}$$

$$= 35.8$$

↓

HIGHER VALUE AND NOT MINIMUM

minimum value of y that fulfills all conditions = $\underline{\underline{20}}$