

NAEEM-NOWROUZI-707-HW3

Chapter 7, Problem 1

```
library(faraway)
data(hsb)
hsbm <- na.omit(hsb)

# 1.a) Produce a table showing the proportion of males and females choosing
# the three different programs.
library(tidyverse)

prop.gen <- group_by(hsb, gender, prog) %>% summarise(count = n()) %>%
  group_by(gender) %>% mutate(gtotal = sum(count), proportion =
count/gtotal)
prop.gen

## # A tibble: 6 x 5
## # Groups:   gender [2]
##   gender prog      count gtotal proportion
##   <fct> <fct>      <int> <int>      <dbl>
## 1 female academic    58    109      0.532
## 2 female general    24    109      0.220
## 3 female vocation    27    109      0.248
## 4 male   academic    47     91      0.516
## 5 male   general     21     91      0.231
## 6 male   vocation    23     91      0.253

# We see that within the females, 53% choose academic program type, 22%
# choose general, and 25% choose vocational. And within the males, 52% choose
# academic, 23% choose general, and 25% choose vocational.

# Do the same for SES
prop.ses <- group_by(hsb, ses, prog) %>% summarise(count = n()) %>%
  group_by(ses) %>% mutate(gtotal = sum(count), proportion =
count/gtotal)
prop.ses

## # A tibble: 9 x 5
## # Groups:   ses [3]
##   ses      prog      count gtotal proportion
##   <fct> <fct>      <int> <int>      <dbl>
## 1 high  academic    42     58      0.724
## 2 high  general      9     58      0.155
## 3 high  vocation      7     58      0.121
## 4 low   academic    19     47      0.404
## 5 low   general     16     47      0.340
## 6 low   vocation    12     47      0.255
```

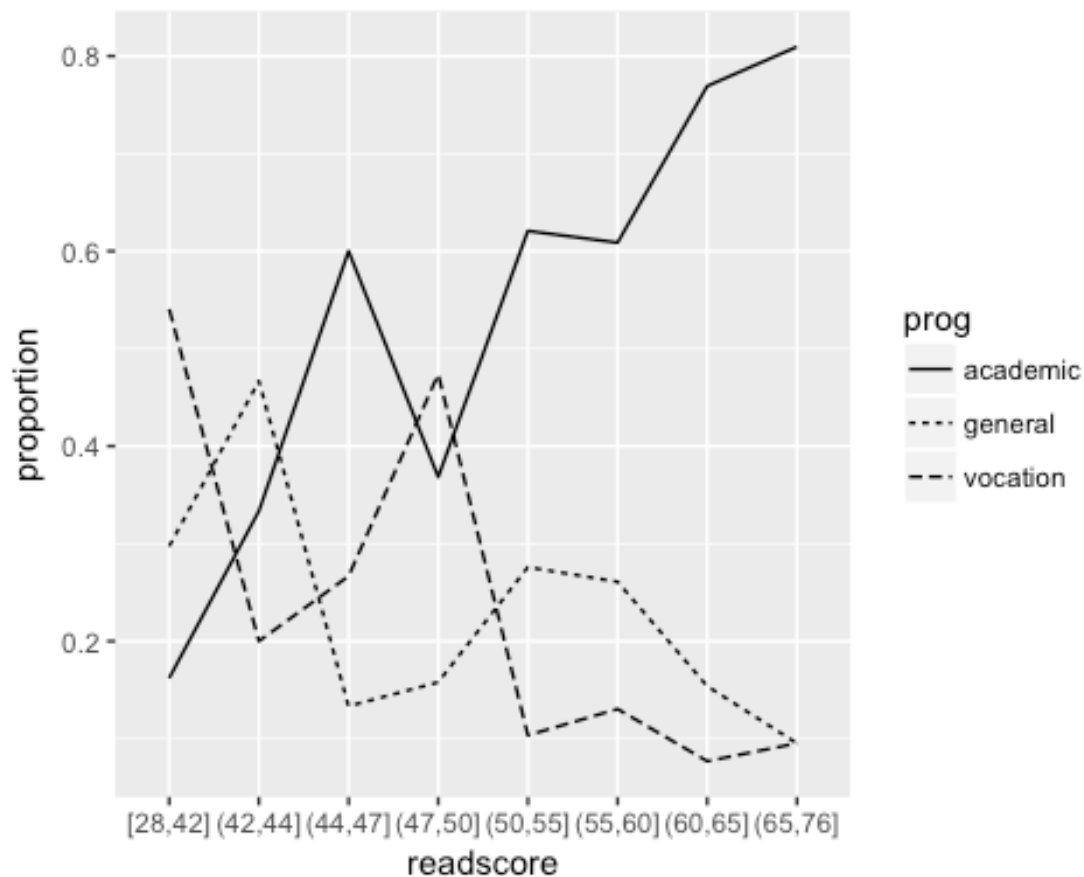
```
## 7 middle academic    44    95    0.463
## 8 middle general     20    95    0.211
## 9 middle vocation    31    95    0.326
```

We observe that in the high socioeconomic class 72% choose the academic program, 16% choose the general program, and 12% choose the vocational program. In the middle class, 46% choose academic, 21% choose general, and 33% choose vocational. In the low class, 40% choose academic, 34% choose general, and 26% choose vocational.

1.b) Plot the relationship between program choice and reading score.

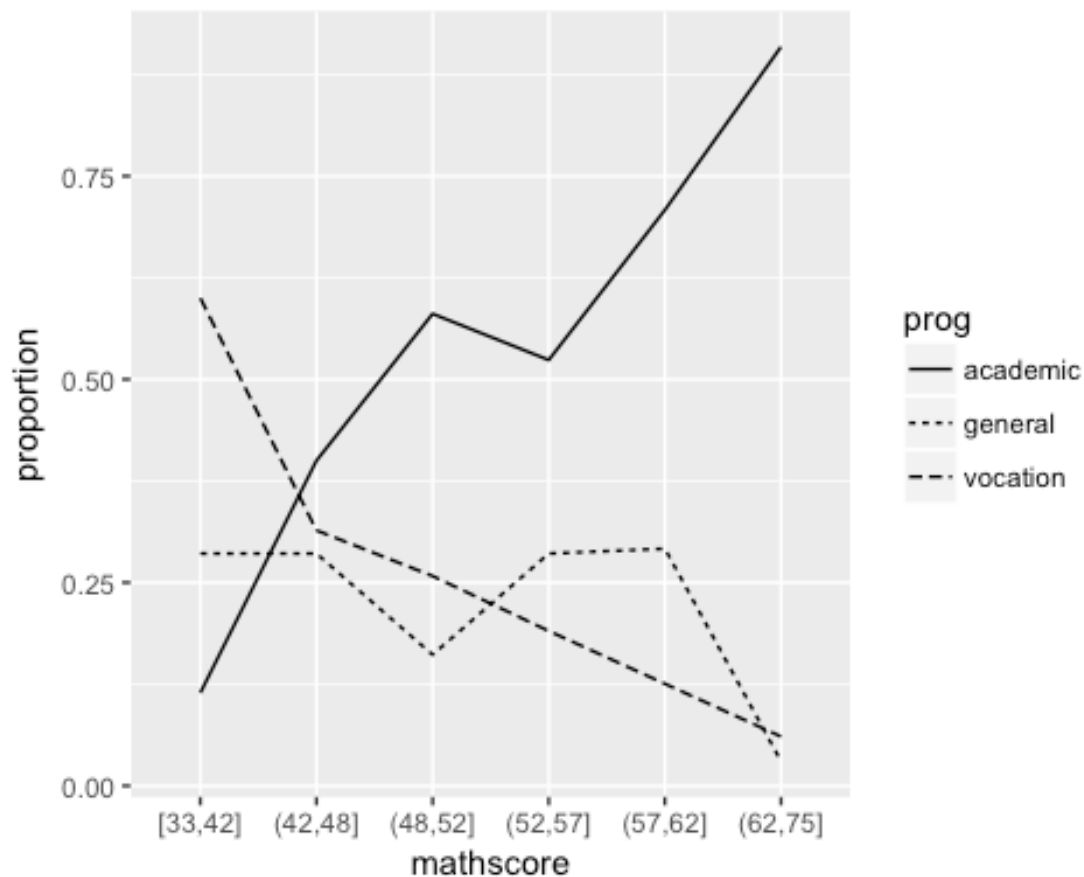
```
progreadd <- mutate(hsbm, readscore = cut_number(read, 8)) %>%
  group_by(readscore, prog) %>% summarise(count = n()) %>%
  group_by(readscore) %>% mutate(tot = sum(count), proportion =
count/tot)

ggplot(progreadd, aes(x = readscore, y = proportion, group = prog, linetype =
prog)) +
  geom_line()
```



```
# Repeat the above plot for math scores
```

```
progmath <- mutate(hsbm, mathscore = cut_number(math, 6)) %>%  
  group_by(mathscore, prog) %>% summarise(count = n()) %>%  
  group_by(mathscore) %>% mutate(tot = sum(count), proportion =  
count/tot)  
  
ggplot(progmath, aes(x = mathscore, y = proportion, group = prog, linetype =  
prog)) +  
  geom_line()
```



We can see in the plots that for both meath and reading scores, as they increase the proportion of academic program choice increases and the other two program type decrease.

```
# 1.c) Compute the correlation matrix for the five subject scores.
```

```
cor(hsb[7:11])
```

```
##           read      write      math  science  socst  
## read      1.0000000 0.5967765 0.6622801 0.6301579 0.6214843  
## write     0.5967765 1.0000000 0.6174493 0.5704416 0.6047932  
## math      0.6622801 0.6174493 1.0000000 0.6307332 0.5444803
```

```
## science 0.6301579 0.5704416 0.6307332 1.0000000 0.4651060
## socst 0.6214843 0.6047932 0.5444803 0.4651060 1.0000000
```

1.d)

```
library(nnet)
```

```
multnommod0 <- multinom(prog ~ ., hsbm)
```

```
## # weights: 45 (28 variable)
## initial value 219.722458
## iter 10 value 181.098338
## iter 20 value 154.577078
## iter 30 value 152.478856
## final value 152.478368
## converged
```

```
summary(multnommod0)
```

```
## Call:
## multinom(formula = prog ~ ., data = hsbm)
##
## Coefficients:
## (Intercept) id gendermale raceasian racehispanic
## general 4.263658 -0.007332836 -0.04666403 1.2170225 -0.8702109
## vocation 7.845921 -0.003680462 -0.29724832 -0.7863428 -0.3236628
## racewhite seslow sesmiddle schtyppublic read
## general 0.8609754 1.1547399 0.7430976 0.1384853 -0.05445264
## vocation 0.6223190 0.0728241 1.1897765 1.8285649 -0.04078359
## write math science socst
## general -0.03716360 -0.1037470 0.1065258 -0.01786542
## vocation -0.03220268 -0.1099712 0.0537472 -0.07959798
##
## Std. Errors:
## (Intercept) id gendermale raceasian racehispanic
## general 1.960941 0.007678009 0.4587870 1.064969 0.9286986
## vocation 2.288984 0.008408855 0.5048241 1.476435 0.8924359
## racewhite seslow sesmiddle schtyppublic read write
## general 0.9438010 0.6134530 0.5096129 0.7338284 0.03300204 0.03398842
## vocation 0.9519097 0.7067682 0.5739217 0.9981540 0.03583547 0.03597627
## math science socst
## general 0.03556357 0.03331314 0.02737227
## vocation 0.03885464 0.03445137 0.02963317
##
## Residual Deviance: 304.9567
## AIC: 360.9567
```

The variable math has unusual coefficients. We can see from the correlation matrix and the above plots that math score has the largest effect on the outcome.

1.e)

```
hsbm1 <- mutate(hsb, scoresum = read + write + math + science + socst)
hsbm1 <- hsbm1[-c(7:11)]
multnommod1 <- multinom(prog ~ ., hsbm1)
```

```
## # weights: 33 (20 variable)
## initial value 219.722458
## iter 10 value 167.158173
## iter 20 value 164.141699
## final value 164.130704
## converged
```

```
summary(multnommod1)
```

```
## Call:
## multinom(formula = prog ~ ., data = hsbm1)
##
## Coefficients:
##      (Intercept)          id gendermale raceasian racehispanic
## general      3.227335 -0.003708235  0.24883040  1.0243408   -0.5484976
## vocation      7.112010 -0.003220142 -0.09614882 -0.6015843   -0.1937564
##      racewhite      seslow sesmiddle schtyppublic      scoresum
## general      1.060033  1.0593830  0.6350558    0.3875245 -0.02052599
## vocation      1.098265  0.2517821  1.1874930    1.8098161 -0.04125543
##
## Std. Errors:
##      (Intercept)          id gendermale raceasian racehispanic
## general      1.798815  0.006823237  0.3941480  0.9439661    0.8799224
## vocation      2.157426  0.007659938  0.4364287  1.3769618    0.8411264
##      racewhite      seslow sesmiddle schtyppublic      scoresum
## general      0.8740777  0.5664146  0.4789630    0.6826598  0.005976099
## vocation      0.8970833  0.6797684  0.5566371    0.9568939  0.007225491
##
## Residual Deviance: 328.2614
## AIC: 368.2614
```

We can see that the first model that includes the scores separately has a lower residual deviance and lower AIC, so we conclude that it fits better.

1.f)

Use a stepwise method to reduce the model

```
bestmodel <- step(multnommod0, trace = 0)
```

```
## trying - id
## trying - gender
```

```
## trying - race
## trying - ses
## trying - schtyp
## trying - read
## trying - write
## trying - math
## trying - science
## trying - socst
## # weights: 36 (22 variable)
## initial value 219.722458
## iter 10 value 180.513235
## iter 20 value 156.699696
## final value 155.608810
## converged
## trying - id
## trying - gender
## trying - ses
## trying - schtyp
## trying - read
## trying - write
## trying - math
## trying - science
## trying - socst
## # weights: 33 (20 variable)
## initial value 219.722458
## iter 10 value 172.925326
## iter 20 value 156.065379
## final value 155.776076
## converged
## trying - gender
## trying - ses
## trying - schtyp
## trying - read
## trying - write
## trying - math
## trying - science
## trying - socst
## # weights: 30 (18 variable)
## initial value 219.722458
## iter 10 value 172.662548
## iter 20 value 156.063823
## final value 156.032828
## converged
## trying - ses
## trying - schtyp
## trying - read
## trying - write
## trying - math
## trying - science
## trying - socst
```

```

## # weights: 27 (16 variable)
## initial value 219.722458
## iter 10 value 176.827677
## iter 20 value 156.410686
## final value 156.406678
## converged
## trying - ses
## trying - schtyp
## trying - read
## trying - math
## trying - science
## trying - socst
## # weights: 24 (14 variable)
## initial value 219.722458
## iter 10 value 171.169761
## iter 20 value 157.775586
## final value 157.775540
## converged
## trying - ses
## trying - schtyp
## trying - math
## trying - science
## trying - socst

summary(bestmodel)

## Call:
## multinom(formula = prog ~ ses + schtyp + math + science + socst,
## data = hsbm)
##
## Coefficients:
## (Intercept) seslow sesmiddle schtyppublic math
## general 2.587029 0.87607389 0.6978995 0.6468812 -0.1212242
## vocation 6.687272 -0.01569301 1.2065000 1.9955504 -0.1369641
## science socst
## general 0.08209791 -0.04441228
## vocation 0.03941237 -0.09363417
##
## Std. Errors:
## (Intercept) seslow sesmiddle schtyppublic math
## general 1.686492 0.5758781 0.4930330 0.545598 0.03213345
## vocation 1.945363 0.6690861 0.5571202 0.812881 0.03591701
## science socst
## general 0.02787694 0.02344856
## vocation 0.02864929 0.02586717
##
## Residual Deviance: 315.5511
## AIC: 343.5511

```

```

# Since this method is based on AIC we see that the reduced model with five
variable (ses + schtyp + math + science + socst) has a lower AIC, however,
the deviance is slightly higher, we proceed with this reduced model.

# 1.g)

# Define the observed range of math scores
mathscorlev <- 32:75

# Find the most common levels of the factors in the model and the mean of the
# other predictors in the model.
summary(hsbm$ses)

##      high      low middle
##       58       47      95

summary(hsbm$schtyp)

## private  public
##       32      168

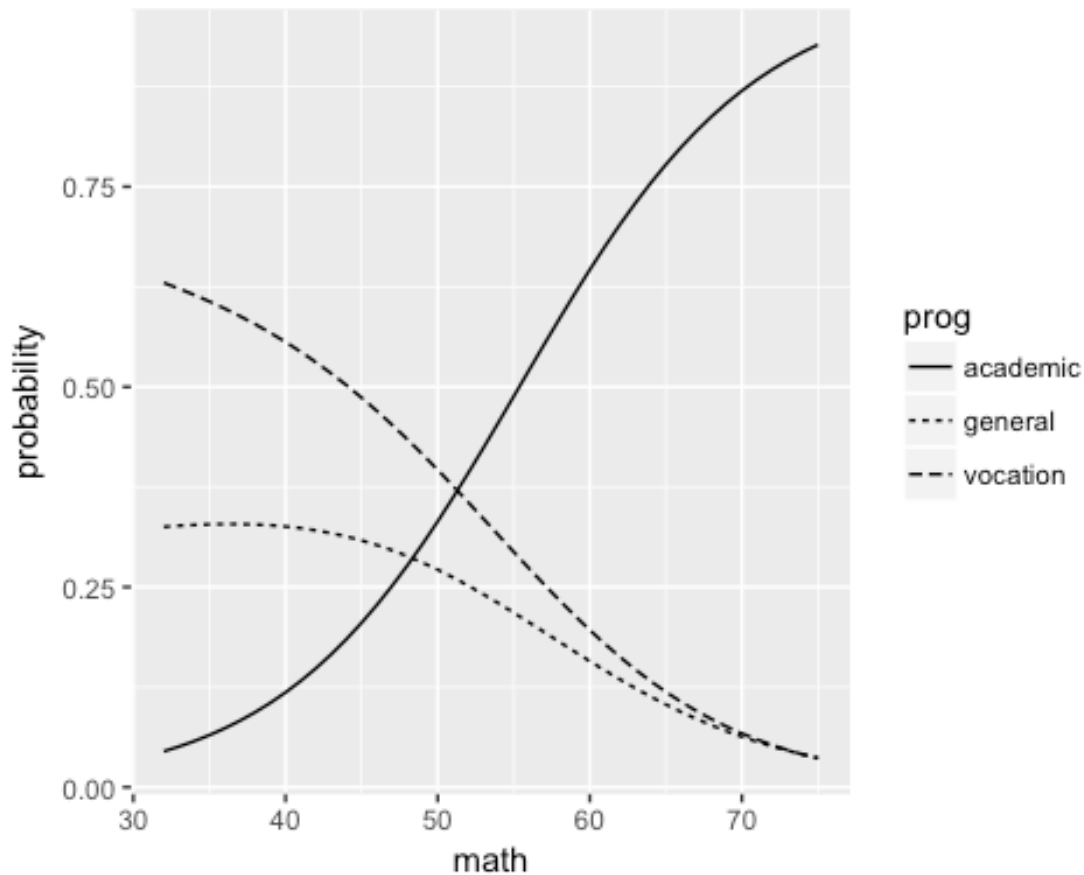
# Get the predicted values for the observed range of math scores.
preds <- data.frame(math=mathscorlev,
                    predict(bestmodel, newdata = data.frame(ses = "middle",
                                                            schtyp = "public", math = mathscorlev, science =
mean(hsbm$science),
                                                            socst = mean(hsbm$socst)),type = "probs"))

library(tidyr)

lpred <- gather(preds, prog, probability, -math)

ggplot(lpred, aes(x = math, y = probability, group = prog, linetype = prog))
+ geom_line()

```

```
#lpred
```

Clearly, the probability of choosing the academic program type increases rapidly as the math scores get higher, and the other two programs type have a decreased probability of begin chosen as the math score goes higher.

```
# 1.h)
```

```
# Compute a table
```

```
# data.frame(ses = hsbm$ses, schtyp = hsbm$schtyp,
#             predict(bestmodel,newdata = data.frame(ses = hsbm$ses,
#             #             schtyp = hsbm$schtyp, math = mean(hsbm$math), science =
mean(hsbm$science),
#             #             socst = mean(hsbm$socst)), type = "probs"))
```

```
xtabs(predict(bestmodel,newdata = data.frame(ses = hsbm$ses,
schtyp = hsbm$schtyp, math = mean(hsbm$math), science =
mean(hsbm$science),
socst = mean(hsbm$socst)), type = "probs") ~ hsbm$schtyp,
hsbm$ses)
```

```
##
## hsbm$schtyp academic general vocation
## private 24.018832 6.112748 1.868419
## public 84.782164 43.954711 39.263126

# 1.i)

# The student with id 99 is at row 102 in the data set.
predict(bestmodel, newdata = hsb[102,])

## [1] academic
## Levels: academic general vocation

# The predicted value is academic which is wrong, the correct program type is
general.

# 1.j) Construct a table of predicted and observed values.

xtabs(~ predict(bestmodel) + hsbm$prog)

##                hsbm$prog
## predict(bestmodel) academic general vocation
## academic            87      22      17
## general              7      10       4
## vocation            11      13      29

# Compute the correct classification rate

(87 + 10 + 29) / nrow(hsbm)

## [1] 0.63

# We see that 63% of the data are correctly classified, which is not
impressive given that we expect the model to perform worse than this on new
observations.
```

Problem 5, Chapter 7

```
library(faraway)
data("debt")

# Check for NA's in the data and omit them.
debt <- na.omit(debt)
summary(debt)
```

##	incomegp	house	children	singpar
##	Min. :1.000	Min. :1.000	Min. :0.0000	Min. :0.00000
##	1st Qu.:2.000	1st Qu.:2.000	1st Qu.:0.0000	1st Qu.:0.00000
##	Median :3.000	Median :2.000	Median :1.0000	Median :0.00000
##	Mean :3.105	Mean :2.043	Mean :0.9605	Mean :0.05592
##	3rd Qu.:4.000	3rd Qu.:2.000	3rd Qu.:2.0000	3rd Qu.:0.00000
##	Max. :5.000	Max. :3.000	Max. :4.0000	Max. :1.00000

```
##      agegp      bankacc      bsocacc      manage
## Min.   :1.000   Min.   :0.0000   Min.   :0.000   Min.   :1.000
## 1st Qu.:2.000   1st Qu.:1.0000   1st Qu.:0.000   1st Qu.:4.000
## Median :2.000   Median :1.0000   Median :1.000   Median :4.000
## Mean   :2.461   Mean   :0.8421   Mean   :0.625   Mean   :4.207
## 3rd Qu.:3.000   3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:5.000
## Max.   :4.000   Max.   :1.0000   Max.   :1.000   Max.   :5.000
##      ccarduse      cigbuy      xmasbuy      locintrn
## Min.   :1.000   Min.   :0.0000   Min.   :0.000   Min.   :1.500
## 1st Qu.:1.000   1st Qu.:0.0000   1st Qu.:1.000   1st Qu.:3.830
## Median :1.000   Median :0.0000   Median :1.000   Median :4.415
## Mean   :1.701   Mean   :0.3191   Mean   :0.875   Mean   :4.413
## 3rd Qu.:2.000   3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:5.000
## Max.   :3.000   Max.   :1.0000   Max.   :1.000   Max.   :7.000
##      prodebt
## Min.   :1.350
## 1st Qu.:2.710
## Median :3.180
## Mean   :3.199
## 3rd Qu.:3.650
## Max.   :5.470
```

5.a) Declare the response as an ordered factor and make a plot showing the relationship to prodebt.

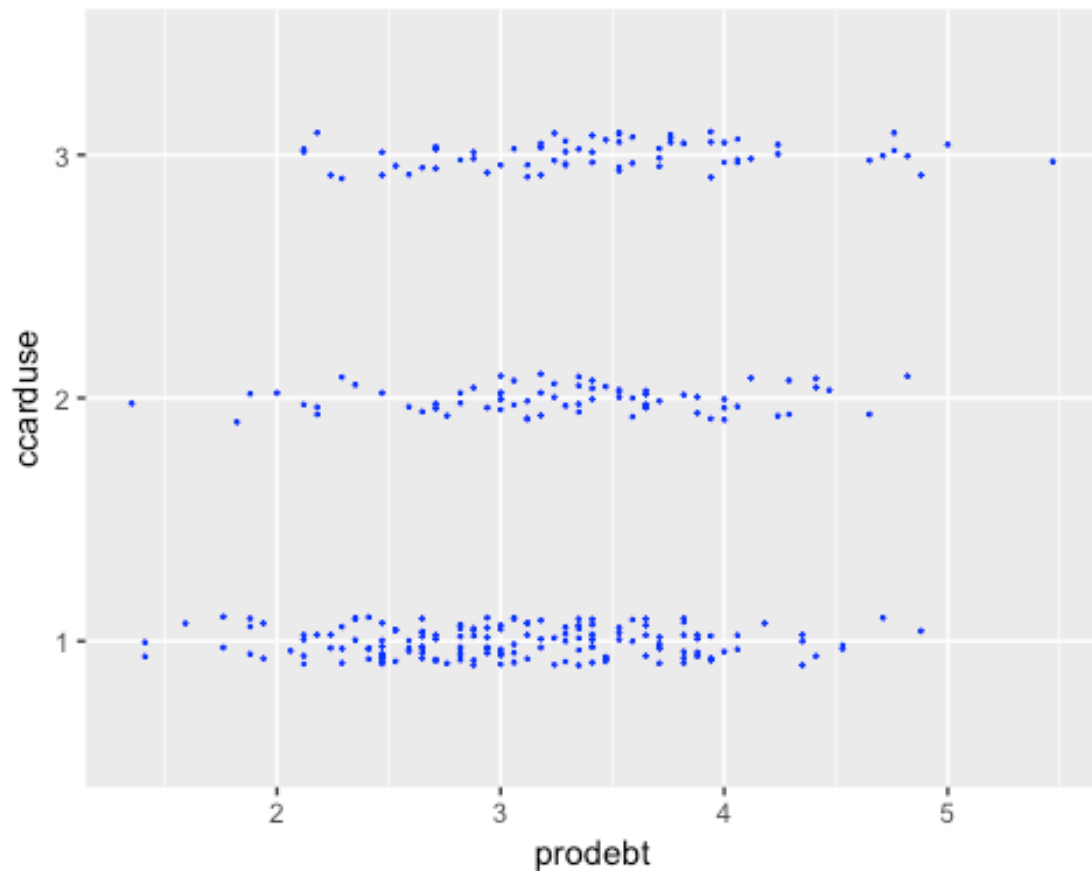
```
# Declare the response variable ccarduse as ordered.
debt$ccarduse <- factor(debt$ccarduse, ordered = TRUE)
```

```
# Verify that the response is indeed ordered.
is.ordered(debt$ccarduse)
```

```
## [1] TRUE
```

Create the plot

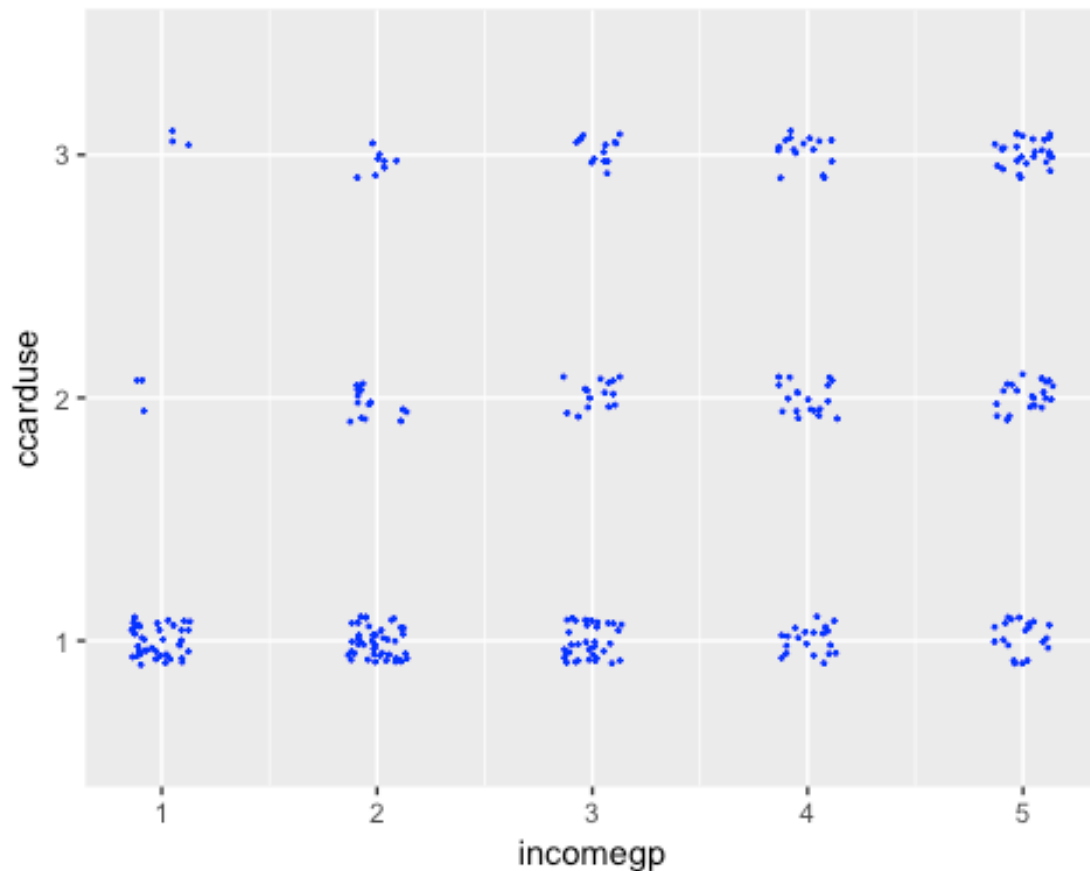
```
library(ggplot2)
ggplot(debt, aes(x = prodebt, y = ccarduse)) +
  geom_jitter(width = 0, height = 0.1, size = 0.15, color = "blue")
```



In this plot we can see a more pronounced relationship between the lowest frequency of credit card use, i.e., never, and debt attitude, particularly with lower quantities of the debt attitude score, i.e., less favorable to debt. However, for higher levels of frequency of card use, 2 and 3, the data points seem to be similarly scattered, slightly denser in the mid-range of the debt attitude scores.

Then response against the income group.

```
ggplot(debt, aes(x = incomegp, y = ccarduse)) +  
  geom_jitter(width = 0.14, height = 0.1, size = 0.24, color = "blue")
```



In this plot as well, a more pronounced relationship can be seen between the lowest level of the card use frequency, and income group, particularly the lower levels of income group. Another slightly pronounced relationship between the third level of card use frequency and the fifth level of the income group is visible.

5.b) Fit a Proportional Odds model for credit card use to all other variables.

```
library(MASS)
```

```
##
```

```
## Attaching package: 'MASS'
```

```
## The following object is masked from 'package:dplyr':
```

```
##
```

```
## select
```

Fit the Proportional Odds model

```
pomod <- polr(ccarduse ~ ., debt )
```

Look at the coefficients for largest t-values.

```
summary(pomod)$coef
```

```
##
## Re-fitting to get Hessian

##           Value Std. Error   t value
## incomegp  0.47131302  0.1060967  4.4422968
## house     0.11600148  0.2323630  0.4992251
## children -0.07872411  0.1250325 -0.6296291
## singpar   0.88171828  0.5971140  1.4766330
## agegp     0.20568368  0.1576103  1.3050145
## bankacc   2.10269577  0.5933918  3.5435203
## bsocacc   0.47321630  0.2671328  1.7714643
## manage    0.18179169  0.1652902  1.0998331
## cigbuy    -0.73545858  0.2980681 -2.4674178
## xmasbuy   0.47014289  0.4129631  1.1384622
## locintrn  0.11881236  0.1423979  0.8343685
## prodebt   0.61046374  0.1822466  3.3496579
## 1|2       7.96937466  1.4751711  5.4023391
## 2|3       9.39436162  1.5051079  6.2416532
```

The two most significant predictors (having largest t-values) are the "incomegp" and "bankacc", the later having a t-value that is close to that of the "prodebt" variable.

bankacc is obviously significantly influential on ccarduse since it is very unlikely to have a credit card without a bank account. The coefficient for bankacc is about 2.1 which means that the the odds of moving from ccarduse level of 1 to 2/3 or from 1/2 to 3 increase by a factor of $\exp(2.1) = 8.16$ when the bankacc is equal to 1. The coefficient for incomegp is about 0.47. That is, the odds of moving from ccarduse level of 1 to 2/3 or from 1/2 to 3 increase by a factor of $\exp(0.47) = 1.60$ when income goes to next level.

The least significant predictor is security of housing tenure with a t-value of 0.499.

5.c) Fit a Proportional Odds model using the least significant predictor.

```
pomodhouse <- polr(ccarduse ~ house, debt)
summary(pomodhouse)$coef
```

```
##
## Re-fitting to get Hessian

##           Value Std. Error   t value
## house  0.5626874  0.1770438  3.178238
## 1|2    1.2821108  0.3870200  3.312776
## 2|3    2.3980930  0.4032797  5.946476
```

We see that the t-value for the variable "house" in this model is significantly larger than in the full model with a lower standard error. The t-value is close to that of "bankacc" and "prodebt" in the full model, so it is much more significant than the full model suggests.

5.d) Use stepwise AIC to reduce the full model.

```
reducedpomod <- step(pomod, trace = 0)
```

```
summary(reducedpomod)
```

```
##
```

```
## Re-fitting to get Hessian
```

```
## Call:
```

```
## polr(formula = ccarduse ~ incomegp + agegp + bankacc + bsocacc +
```

```
##   cigbuy + prodebt, data = debt)
```

```
##
```

```
## Coefficients:
```

```
##           Value Std. Error t value
```

```
## incomegp  0.4589    0.1007   4.555
```

```
## agegp     0.2696    0.1352   1.993
```

```
## bankacc   2.0816    0.5753   3.618
```

```
## bsocacc   0.5048    0.2591   1.949
```

```
## cigbuy    -0.7677    0.2922  -2.627
```

```
## prodebt   0.5635    0.1755   3.211
```

```
##
```

```
## Intercepts:
```

```
##           Value Std. Error t value
```

```
## 1|2  5.9944  0.9961    6.0178
```

```
## 2|3  7.3948  1.0276    7.1961
```

```
##
```

```
## Residual Deviance: 517.5895
```

```
## AIC: 533.5895
```

The qualitative effect of the predictor, as discussed above, is that the odds of moving to the next level of ccarduse increases by a factor of $\exp(\text{coefficient})$ when the corresponding predictor is increased by one unit or moves to the next level. We see that the predictor "house" is dropped from the model, but we saw above that this predictor is significant when we use only that in the one-predictor model.

5.e) Compute the median value of the predictors in the reduced model.

```
median.inc <- median(debt$incomegp)
```

```
median.age <- median(debt$agegp)
```

```
median.bank <- median(debt$bankacc)
```

```
median.bsoc <- median(debt$bsocacc)
```

```
median.cig <- median(debt$cigbuy)
```

```
median.pdebt <- median(debt$prodebt)
```

Compute the predicted probabilities at the median values for smokers

```
predict(reducedpomod, data.frame(incomegp = median.inc, agegp = median.age,
```

```

bankacc = median.bank, bsocacc = median.bsoc, cigbuy = debt$cigbuy[debt$cig[]
== 1], prodebt = median.pdebt), type = "probs")[1,]

##           1           2           3
## 0.6149076 0.2513666 0.1337258

# Compute the predicted probabilities at the median values for non-smokers
predict(reducedpomod, data.frame(incomegp = median.inc, agegp = median.age,
bankacc = median.bank, bsocacc = median.bsoc, cigbuy = debt$cigbuy[debt$cig[]
== 0], prodebt = median.pdebt), type = "probs")[1,]

##           1           2           3
## 0.4256250 0.3247658 0.2496092

# The highest probability in both cases is for the first level of ccarduse. So
both groups have a higher probability of never using their ccards and a lower
probability of using their cards regularly, while non-smokers have a higher
probability of regularly using their cards than smokers do.

# 5.f) Fit a Proportional Hazards model

phmod <- polr(ccarduse ~ incomegp + agegp + bankacc + bsocacc +
              cigbuy + prodebt, data = debt, method = "cloglog")
summary(phmod)

##
## Re-fitting to get Hessian

## Call:
## polr(formula = ccarduse ~ incomegp + agegp + bankacc + bsocacc +
##       cigbuy + prodebt, data = debt, method = "cloglog")
##
## Coefficients:
##              Value Std. Error t value
## incomegp  0.2454    0.05950   4.125
## agegp     0.1936    0.08224   2.354
## bankacc   0.9984    0.23658   4.220
## bsocacc   0.3087    0.15704   1.966
## cigbuy   -0.3120    0.15789  -1.976
## prodebt   0.3418    0.10872   3.143
##
## Intercepts:
##      Value   Std. Error t value
## 1|2  3.0002   0.5307    5.6536
## 2|3  3.8261   0.5424    7.0541
##
## Residual Deviance: 527.372
## AIC: 543.372

# Recompute the two sets of probabilities from the previous part.

```



```

predict(phmod, data.frame(incomegp = median.inc, agegp = median.age,
                           bankacc = median.bank, bsocacc =
median.bsoc,
                           cigbuy = debt$cigbuy[debt$cig[] == 1],
                           prodebt = median.pdebt), type = "probs")[1,]

```

```

##           1           2           3
## 0.5571469 0.2872181 0.1556350

```

```

predict(phmod, data.frame(incomegp = median.inc, agegp = median.age,
                           bankacc = median.bank, bsocacc =
median.bsoc,
                           cigbuy = debt$cigbuy[debt$cig[] == 0],
                           prodebt = median.pdebt), type = "probs")[1,]

```

```

##           1           2           3
## 0.4491074 0.2946605 0.2562321

```

Using the Proportional Hazards model seems to make the two sets of probabilities more similar to each other, making it almost equally likely for both smokers and nonsmokers to never, often, or regularly use their creditcards. The non-smokers, however, are still more likely to use their credit cars regularly than are smokers.