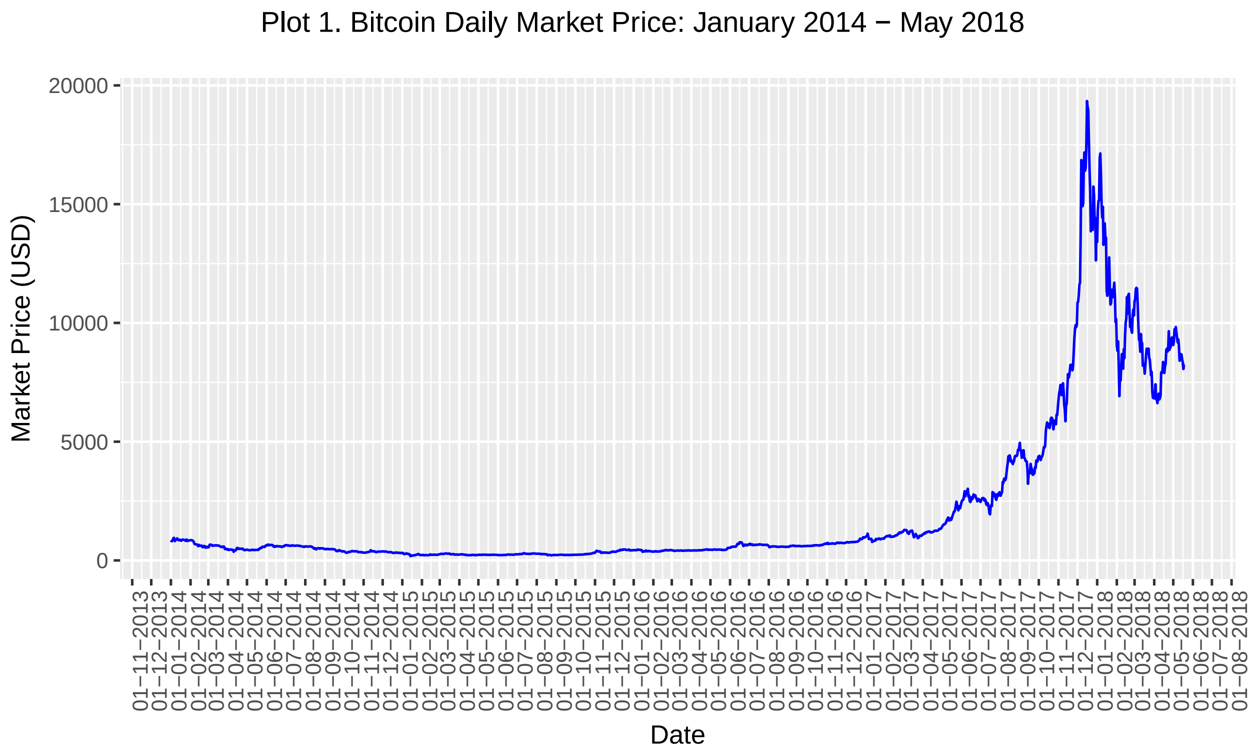
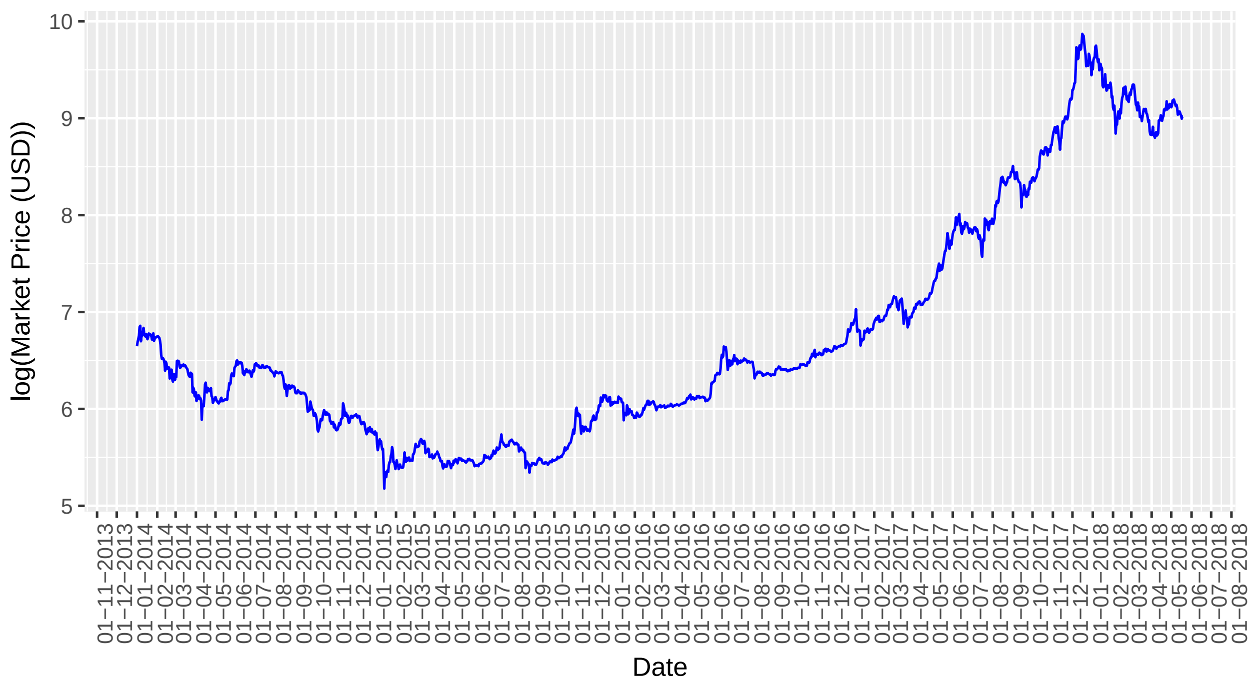
NAEEM NOWROUZI - STAT 715 - FINAL PROJECT

The data set is the historical daily market price of Bitcoin from January 1 2014 to May 18 2018. The daily price includes weekends and holidays as well. Thus the time series object was created with possible seasonality periods of 7,30 and 365.25. We shall attempt to make a 10-day forecast on the daily price. We begin by plotting the time series.

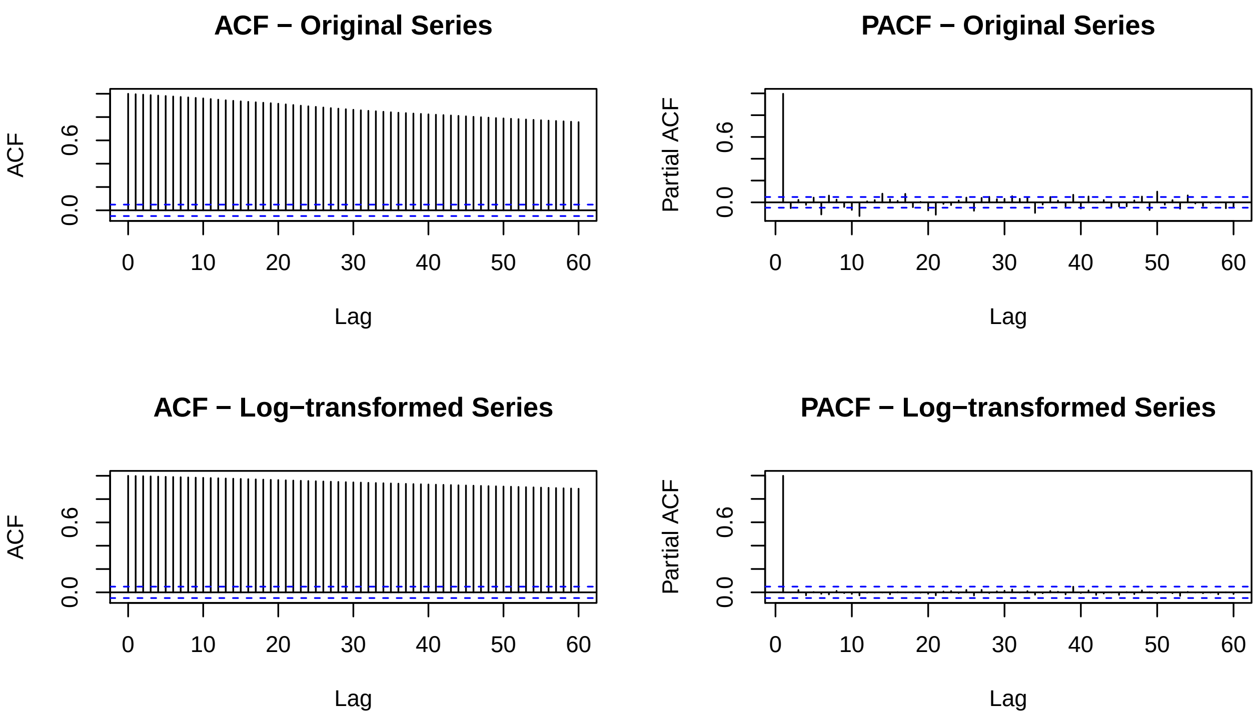


We observe in the plot that the series has relatively mild variability in the begining of our observation in 01-01-2014 until two or three months into the year 2017. The trend looks somewhat constant though there is variability in a smaller scale. The giant upward trend begins in the second half of 2016 and shortly after we can see variability increasing as the level of price increases, with an extremely fast upward movement beginning in late november of 2017 and lasting until mid December when the price has almost doubled at its peak. It then begins to fall back to where it was in late November 2017 with almost the same slope as it was increasing, but with more variablity. It seems safe to assume that the process is non-stationary. We further verify by looking at the ACF and PACF plots of the series. We also first log-transform the series to contain the instability present in the series due to its volatile and explosive growth behavior. We look at the log-transformed series and its ACF and PACFs together with the ACF and PACF of the original series below.

Plot 2. Log−Transformed Bitcoin Daily Market Price: January 2014 − April 2018



Plot 3

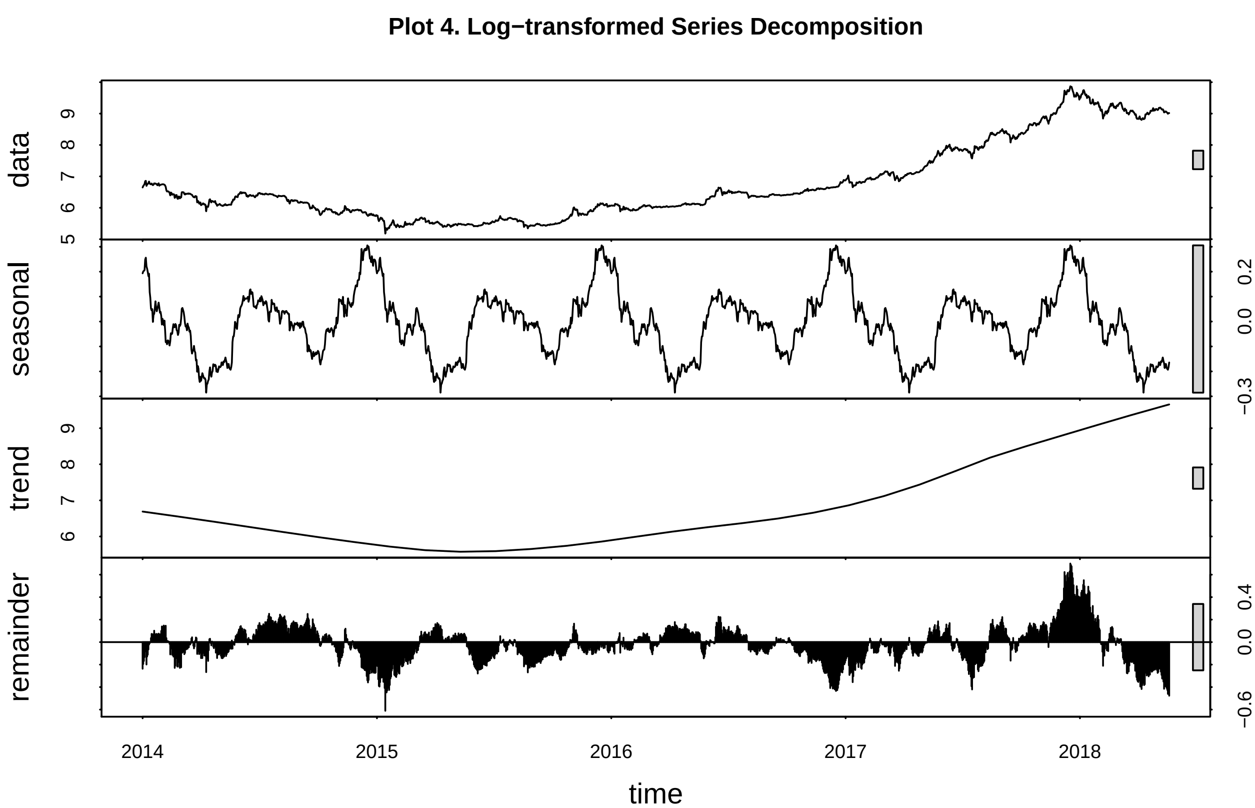


Both of the ACF plots show extreme positive correlations at many lags, decreasing linearly and very slowly. However, this might be the strong effect at lag 1 that is propagating back to the previous lags. The PACF plot confirms this; it shows a significant spike (equal almost to 1) at lag 1 and then suddenly cuts off. These add more evidence against stationarity. We draw a final conclusion by checking the ADF test for stationarity.

```
##
##  Augmented Dickey-Fuller Test
##
## data:  time.series
## Dickey-Fuller = -2.5843, Lag order = 11, p-value = 0.331
## alternative hypothesis: stationary
```

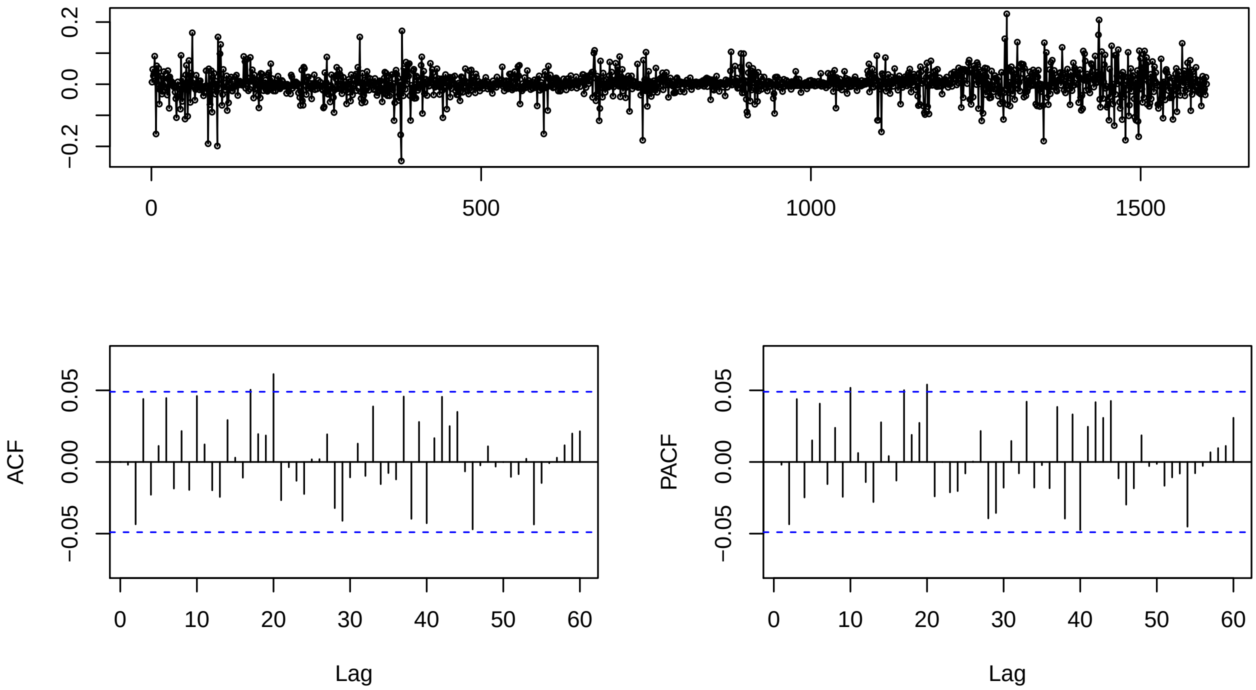
The test's null hypothesis of non-stationarity cannot be rejected since the p-value is large assuming a significance level of 0.05, confirming our assumption of non-stationarity.

We also look at the decomposition plot of the log-transformed series to see the trend, noise and seasonality components. However, no seasonalities were detected in the plot of the original data and so we cannot trust the results in this plot without further analysis for the presense of seasonality in the data. Several different frequencies and seasonal periods were tested and based on the result for the forecasts that we shall see later I proceeded with disregarding seasonality. The results in the following plot are based on a frequency of 365.25(to count for leap years). We see the general upward trend after the first year and some pattern is discernable in the residuals.



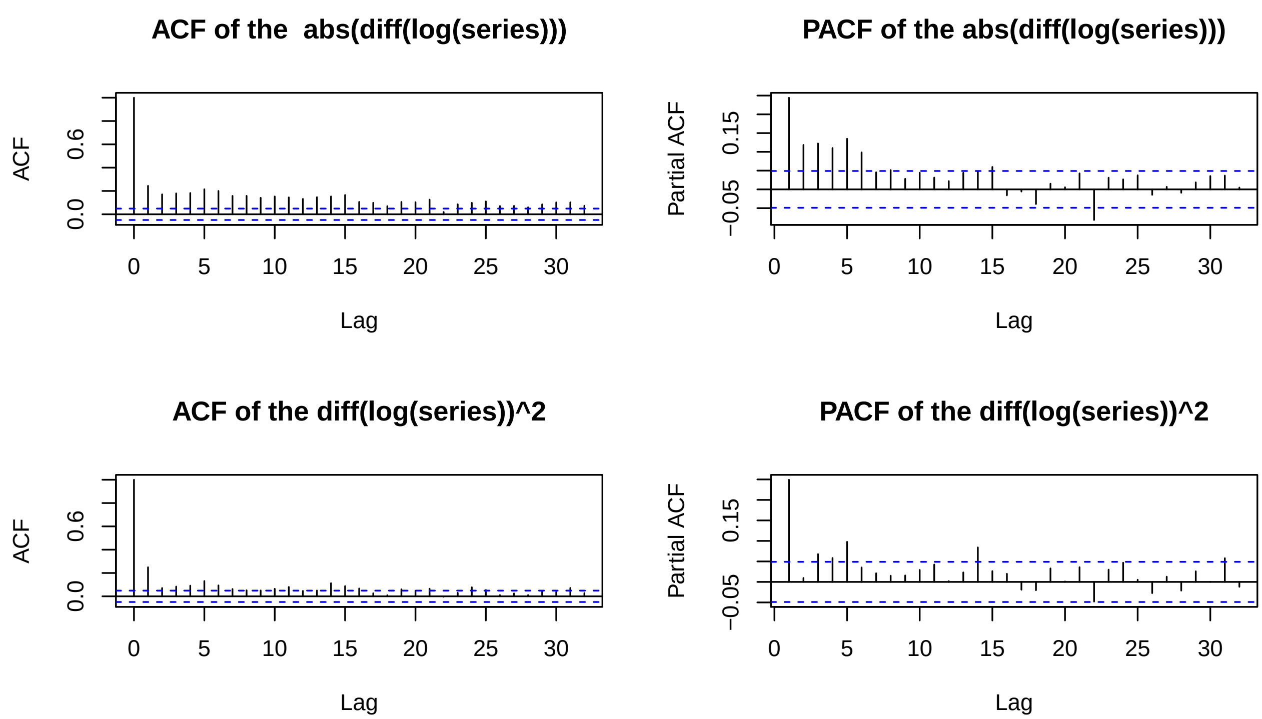
We now attempt at stationarizing the log-transformed series by taking differences. Before that we need to tentatively determine the order of differencing. The ACF plot above showed large correlations at many lags and PACF had a high spike at lag 1. Thus we proceed by differencing once and look at some plots to determine if further differencing is needed.

Plot 5.
Residuals, ACF and PACF of the differenced log−transformed series



The plot of the residuals has the appearance of a stationary process, with a mean that appears to be constant around 0. However, the series exhibits significant volatility clustering as larger fluctuations cluster together, particularly in the begining and in the later parts of the series. This indicates that the conditional variance is changing, implying that we may not have independence and identical distribution. We can check this with the ACF and PACF for the absolute and squared differences, since if the differences are iid, then so are the absolute and squared ones.

Plot 6



We see that there are significant correlations for multiple lags in all four plots, suggesting that the differences are not independent and identically distributed. But we may still have uncorrelatedness, which is necessary for a series to be white noise. We check the ACF and PACF plots above of the differenced series and none of them show anything concerning as there are no statistically significant correlations at any lag. Our empirical conclusion based on these plots is that the differenced log-transformed series is stationary. We check the ADF test as well, and it confirms our conclusion. These suggest that a white-noise model works. We may now proceed to finding appropriate models for our series.

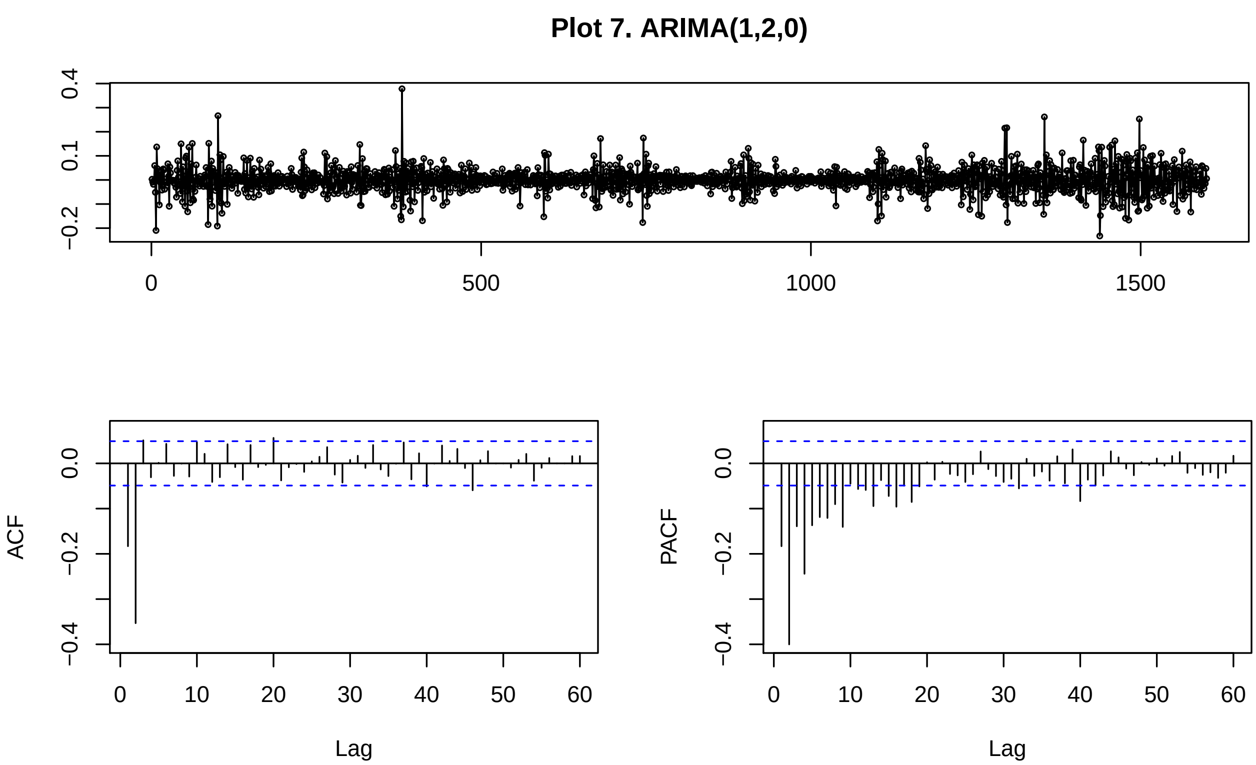
```
##
##  Augmented Dickey-Fuller Test
##
## data:  diff(time.series)
## Dickey-Fuller = -9.9479, Lag order = 11, p-value = 0.01
## alternative hypothesis: stationary
```

We need to specify the orders of AR and MA for our ARIMA model by looking at the ACF and PACF plots in plot 5 of the differenced series. However, neither of them really give us a clue. They neither have any significant lags, particularly at lag 1 they are both very close to 0, nor do they show any strong patterns, such as cutting off at a particular lag. Although the very small value at lag 1 is negative in both, suggesting an MA term for our model. Since there is no conclusive judgement, we will try a few combinations manually and by using the auto.arima function.

```
## Series: price$Close.Price
## ARIMA(1,2,0)
```

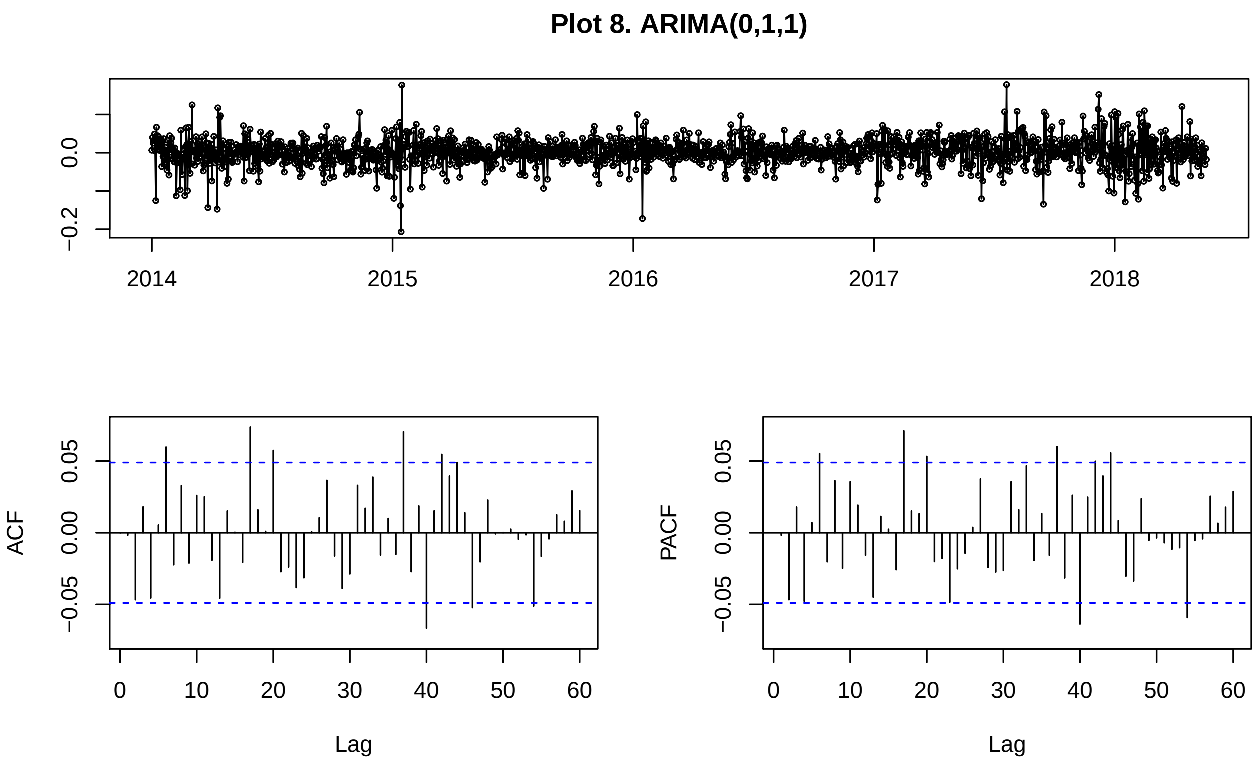
```
## Box Cox transformation: lambda= 0
##
## Coefficients:
##           ar1
##       -0.4791
## s.e.   0.0220
##
## sigma^2 estimated as 0.002479:  log likelihood=2526.68
## AIC=-5049.37   AICc=-5049.36   BIC=-5038.62
##
## Training set error measures:
##                     ME     RMSE      MAE        MPE     MAPE     MASE
## Training set -4.738063 303.1124 102.7682 -0.1271202 3.359203 1.267271
##                    ACF1
## Training set -0.1174611
```

The auto.arima function suggests an ARIMA(1,2,0) model as the best model, contrary to both of our specifications of first order differencing and addition of an MA term. The AR coefficient given is -0.4791 with a standard error of 0.0220 and a sigma^2 of about 0.0025. The RMSE is quite high at 303.1. We check the residulas, ACF and PAC plots.



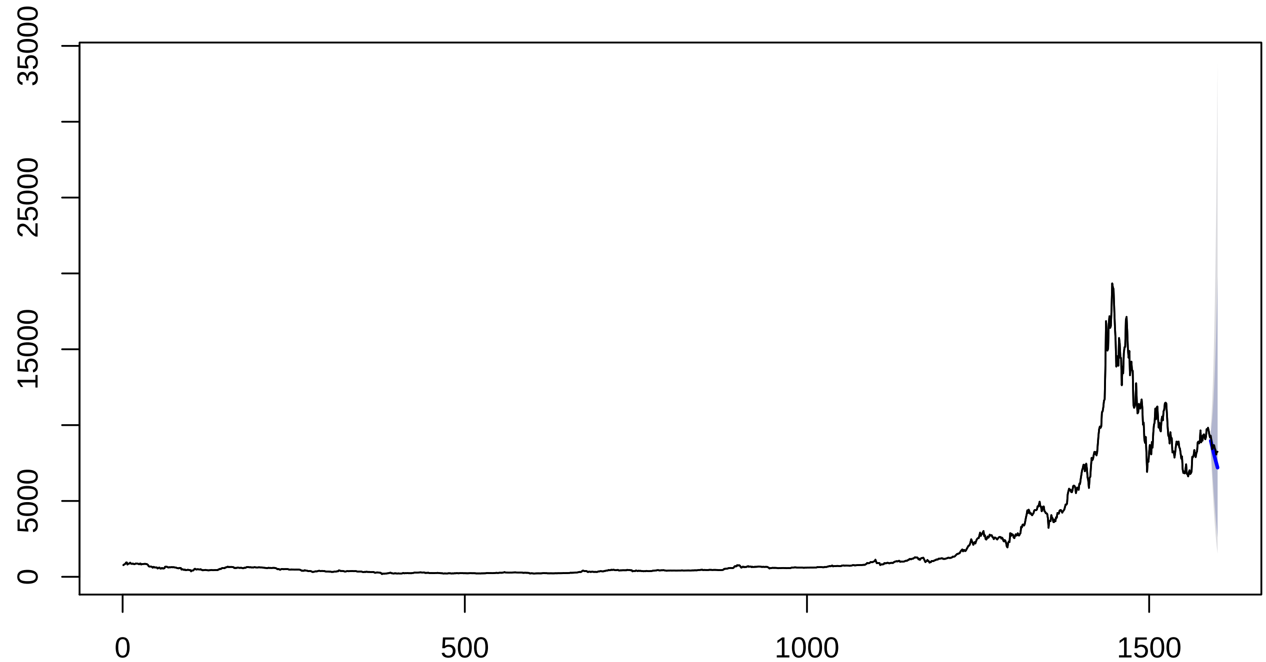
The residuals plot exhibits some volatility cluster, overal the mean seem to be constant around zero, with possible one or two outliers. The ACF plot of the residuals show significant negative autocorrelation at lags 1 and 2, and the PACF plot shows negative spikes at the first several lags, cutting off only after about lag 20. These suggests that the model does not fit the data very well and we can model the residuals. Below we look at the results and plots for the model with our specifications, i.e., ARIMA(0,1,1).

```
## Series: price$deseasoned
## ARIMA(0,1,1)
## Box Cox transformation: lambda= 0
##
## Coefficients:
##           ma1
##       -0.0128
## s.e.   0.0262
##
## sigma^2 estimated as 0.001257:  log likelihood=3071.4
## AIC=-6138.8   AICc=-6138.79   BIC=-6128.04
##
## Training set error measures:
##                    ME    RMSE      MAE       MPE    MAPE       MASE
## Training set 5.778008 182.113 66.75521 0.1102086 2.53937 0.03156411
##                    ACF1
## Training set 0.02530663
```



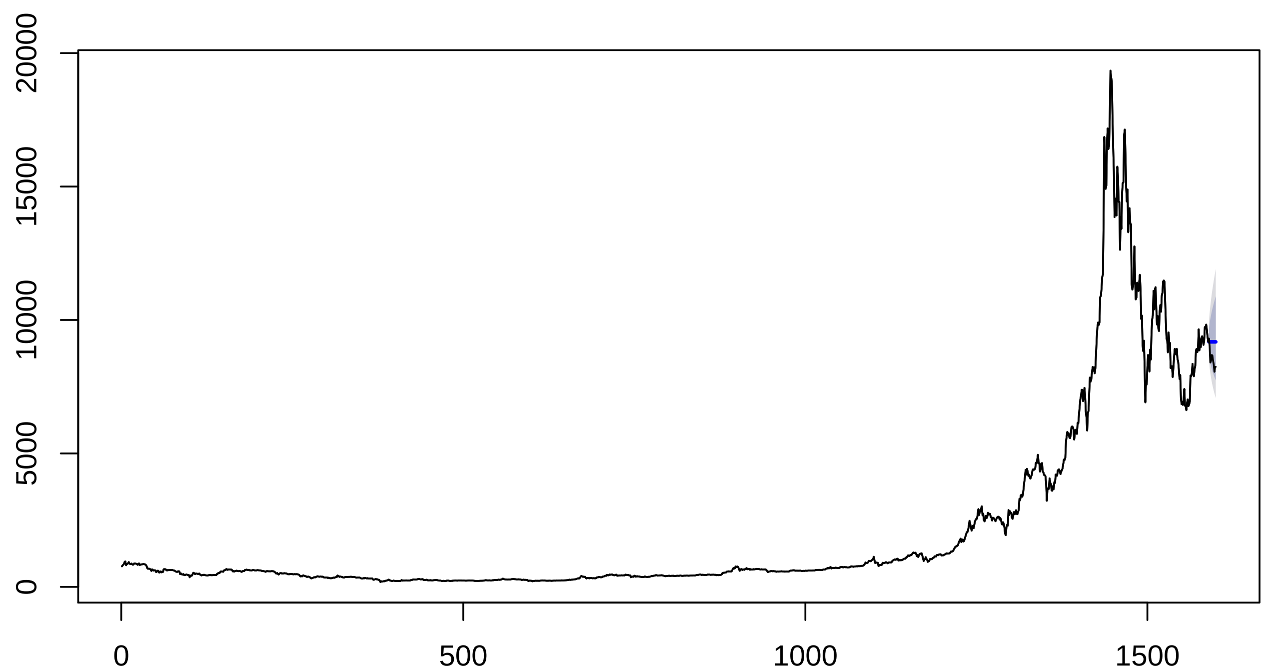
There are no big differences between the two residual plots, maybe one more outlier in hte later model. However, the ACF and PACF plots look much better, with almost no significant lags at all. We will compare the forecast results from the two models for a 11-day period(since after trying different periods this one gave the best result).

Plot 9. Forecasts From ARIMA(1,2,0)



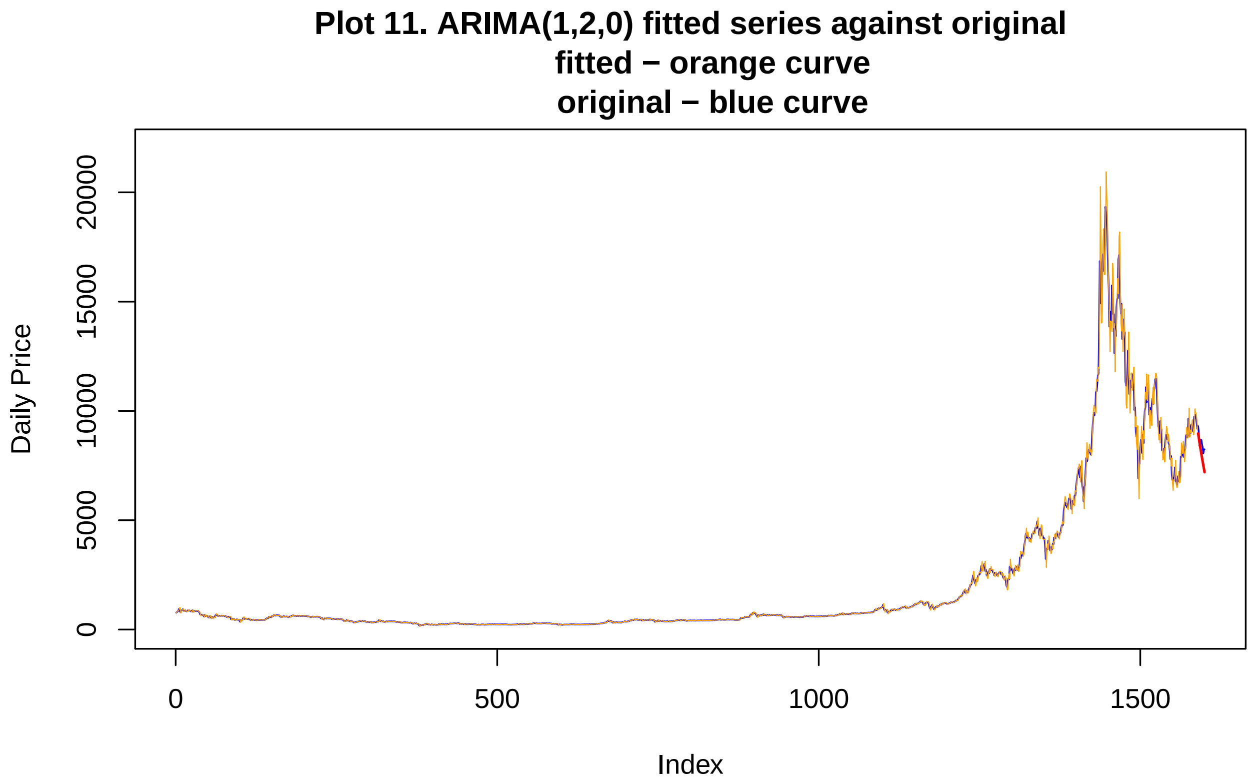
ARIMA(1,2,0) train error = 302.9371634, ARIMA(1,2,0) test error = 588.3209929

Plot 10. Forecasts From ARIMA(0,1,1)



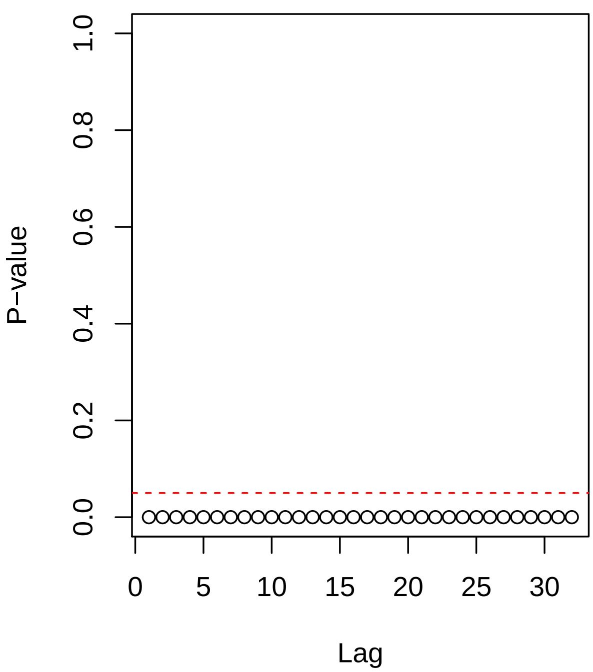
ARIMA(0,1,1) train error = 244.5637036, ARIMA(0,1,1) test error = 730.4861658

We see that the model suggested by auto.arima performs better as the test error(sqrt(MSE)) is smaller. Although it is still quite high at 588.321, it predicts the direction and overal slop correctly. However, the confidence interval in this case is much larger than the (0,1,1) model. The forecast from the ARIMA(0,1,1) is almost simply a straight line, that is the mean. The training errors are smaller at about 303 for the ARIMA(1,2,0) model and 244 for the ARIMA(0,1,1) model. We lastly look at the fitted series by our best model against the original series, including the forecasted part.

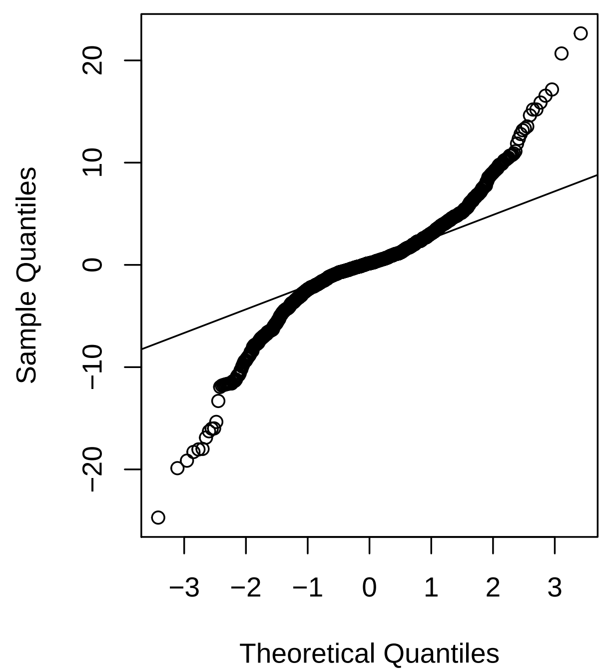


Next we want to determine if there are ARCH effects in the series. We look at the ACF and PACF plots of the squared and absolute returns (differences) back in plot 6. All of the ACF and PACF plots exhibit significant correlations at multiple lags. Thus, the residuals do not seem to be independent and so we can try to fit a GARCH model in order to model the volatility clustering of the series. We confirm this using the McLeod-Li test below. We see that the p-values for all lags are significant, suggesting presence of ARCH effects. The qq-norm plot also indicates non-normality.

Plot13. Mc. Leod−Li Test



Plot 14. Normal Q−Q Plot

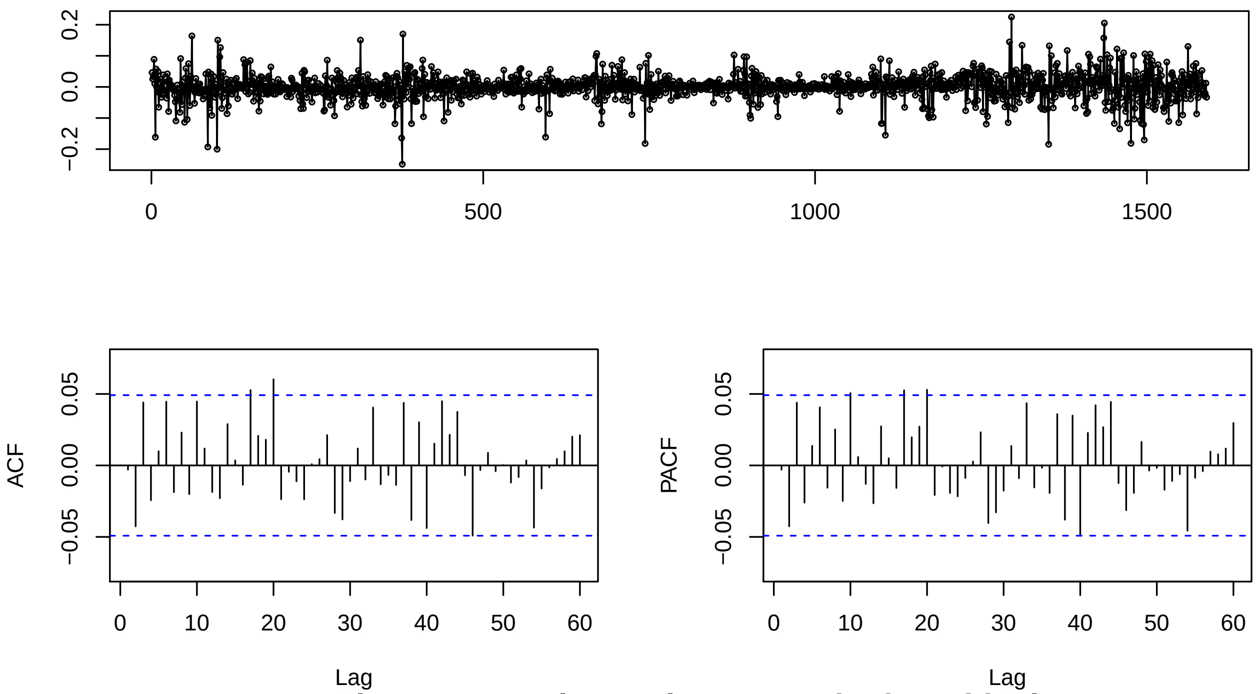


We would now like to determine the orders of our GARCH model. We check the EACF table for the squared differences and it is not very clear, it could be suggesting a (1,2) or a (2,2) GARCH model. The same table for the absolute differences has a more clear cut shape and suggests a (1,2) model. We will try both.

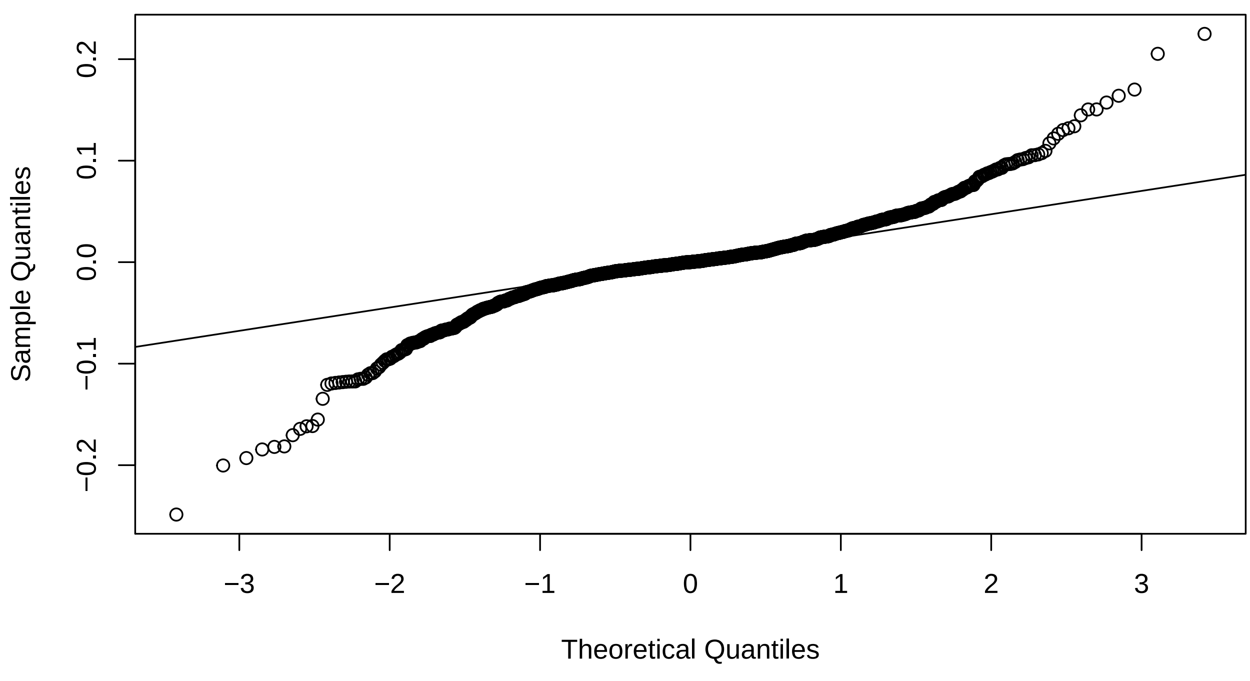
```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  o  o  x
## 1 o x o o x o o o o o o  o  o  x
## 2 x x o o x o o o o o o  o  o  x
## 3 x x o o x o o o o o o  o  o  x
## 4 x x x x o o o o o o o  o  o  x
## 5 x x x x x o o o o o o  o  o  x
## 6 x x o o o x x o o o o  o  o  o
## 7 x x o o o x x o o o o  o  o  o
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  x  x
## 1 x x o o o o o o o o o  o  o  o
## 2 x x o o o o o o o o o  o  o  o
## 3 x x o o o o o o o o o  o  o  o
## 4 x x o x o o o o o o o  o  o  o
## 5 x x x x x o o o o o o  o  o  o
## 6 x x x x x x o o o o o  o  o  o
## 7 x x x o x x x o o o o  o  o  o
```

Plot 15. GARCH(1,2) Residuals, ACF and PACF



Plot 16. Normal−QQ Plot GARCH(1,2) Residuals




```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 2), data = diff(time.series.log)[1:1590],
##     cond.dist = "norm", include.mean = TRUE, trace = FALSE)
```

```
##
## Mean and Variance Equation:
##  data ~ garch(1, 2)
## <environment: 0x7f93245aa150>
##  [data = diff(time.series.log)[1:1590]]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       beta2
## 1.5248e-03  3.1113e-05  2.0801e-01  2.2058e-01  5.7988e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.525e-03   6.973e-04    2.187 0.028770 *
## omega  3.111e-05   9.188e-06    3.386 0.000708 ***
## alpha1 2.080e-01   2.839e-02    7.326 2.38e-13 ***
## beta1  2.206e-01   5.832e-02    3.783 0.000155 ***
## beta2  5.799e-01   5.374e-02   10.791  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  3080.759    normalized:  1.937584
##
## Description:
##  Wed May 23 18:22:09 2018 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2676.028  0
##  Shapiro-Wilk Test  R    W      0.9230152 0
##  Ljung-Box Test     R    Q(10)  31.77388  0.0004366955
##  Ljung-Box Test     R    Q(15)  35.46736  0.002108688
##  Ljung-Box Test     R    Q(20)  42.62561  0.002290324
##  Ljung-Box Test     R^2  Q(10)  7.372013  0.6899195
##  Ljung-Box Test     R^2  Q(15)  11.85956  0.6896265
##  Ljung-Box Test     R^2  Q(20)  16.18601  0.7050198
##  LM Arch Test       R    TR^2   8.484277  0.7462339
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.868879 -3.851987 -3.868899 -3.862605
```

We see that the residuals plot of the GARCH(1,2) model does not look very different from our ARIMA models. However, the ACF and PACF plots af the residuals are better, without significant autocorrelations at any lag. p-values in the summary are small as well. Unfortunately, I encountered issues in forecasting using the GARCH model and in performing the gBox test(perhaps because I used a different package to fit the

GARCH model, since the garch function from the Tseries package did not work properly), and so I am not including them here. The plots we went over suggested that a white noise model is appropriate for this series, specially the ACF and PACF of the absolute and squared differences. The Mc.Leod-Li test also suggested the presence of an ARCH effect in our time series that we hoped to be able to model. However, more accurate results require a more in depth analysis of the series and the pattern that may be hidden in the residuals.