1 Liquidity measure

2 Intraday volatility

Volatility is the tendency for prices to change unexpectedly. Prices change in response to new information about values and in response to the demands of impatient traders for liquidity. Fundamental volatility and Trading volatility are two types of volatility at the microscale.

Seventeen different exchanges are represented in the Reuters tick data, among which the top four exchanges by trades—ADF, NAX, DEX, and PSE—are selected for the volatility study.

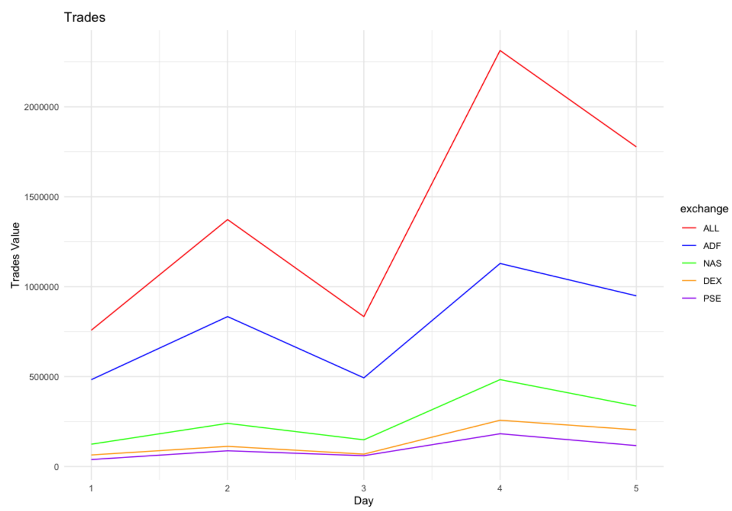

Figure 1: Volume across exchanges

2.1 Roll model

The trade price $p_t = m_t + \epsilon_t$ has two components:

i) the efficient price m_t and

ii) trading noise contribution ϵ_t.

Roll model volatility of the increments of the efficient price $\sigma_u^2 = \gamma_0 + 2\gamma_1$

$\sigma_{day} = \sqrt{n_{trades} * \sigma_u}$

In the formula $\sigma_u^2 = \gamma_0 + 2\gamma_1$, the second term $2\gamma_1$ is negative and subtracts the trading activity contribution from the total volatility.

We obtained the fundamental volatility of the stock price $\sigma_{ann} = \frac{\sigma_u}{\bar{S}} \times \sqrt{n_{trades}} \times \sqrt{252}$

The total volatility is obtained by replacing σ_u^2 with $\gamma_0 = var(\Delta p_t)$, the total price volatility. This gives

$\sigma_{ann}^{total} = \frac{\sqrt{\gamma}_u}{\bar{S}} \times \sqrt{n_{trades}} \times \sqrt{252}$

Autocorrelation at lag 1 suggests a short-term dependency between consecutive differenced price values. For lags beyond 1, the autocorrelation values are within the confidence bounds, implying no statistically significant autocorrelation. The pattern in consistent across all five days.

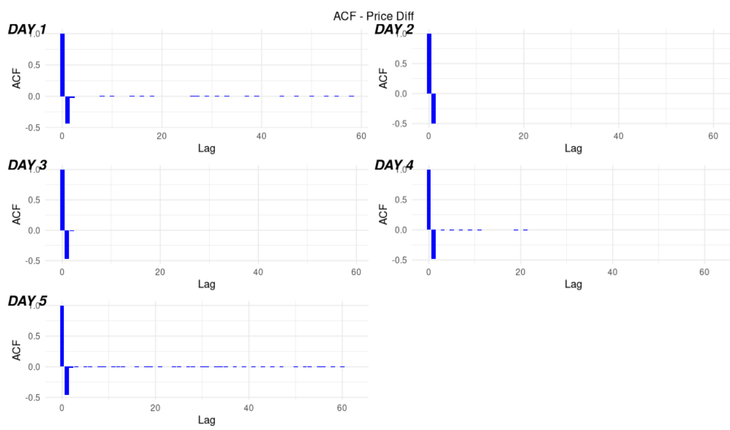

Figure 2: Autocorrelation function (ACF) of price differences.

The ACF of trade signs shows a slow, gradual decay over the lags, suggesting a persistent autocorrelation in the trade signs.

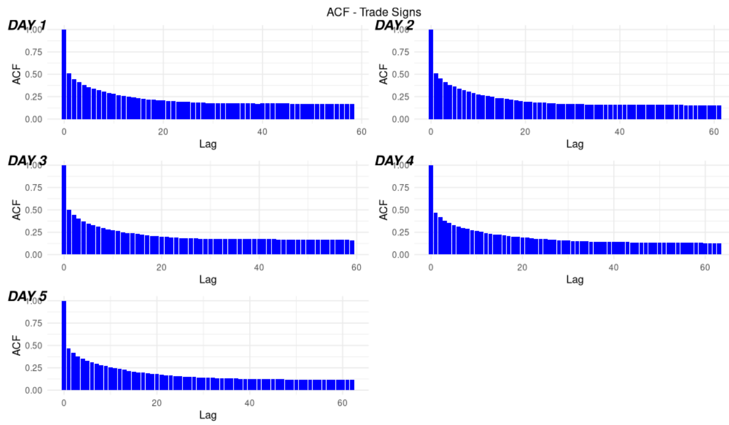

Figure 3: Autocorrelation function (ACF) of trade signs.

For most days, the mid prices closely track the trade prices, indicating a relatively efficient market with minimal divergence between the actual trades and the theoretical mid prices. Sudden spikes in the trade price (red line) suggest high volatility during specific periods.

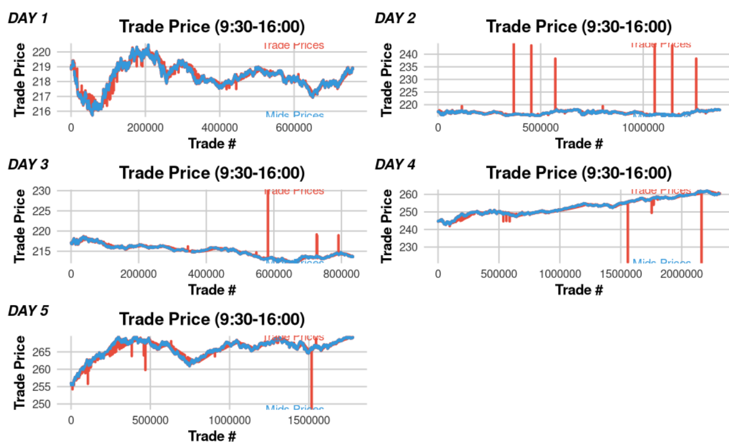

Figure 4: Price plot over all days.

Across all days, most trades have relatively low volumes (a consistent baseline). However, sporadic spikes in trade size suggest periods of high trading activity or large block trades. The volume baseline remains relatively constant across the observed days, with no major shifts in trading patterns except for the spikes. Spikes may indicate either large institutional trades or block trades, reaction to news or market events during speicifc times of the day.

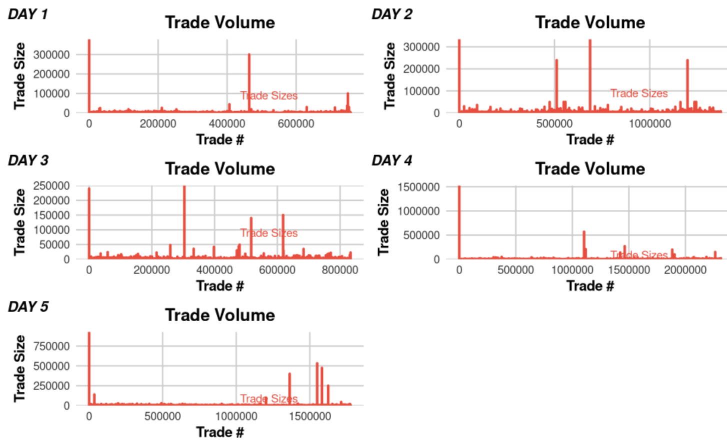

Figure 5: Volume plot over all days.

The red line (representing "ALL") generally shows the highest roll values across all days, indicating that the combined market volume or activity across all exchanges is consistently the highest.

There is a dip in roll values for Day 3 across all exchanges, suggesting reduced market activity or lower trading volumes.

By Day 5, all exchanges (ADF, NAS, DEX, PSE) converge to similar roll values, reflecting consistent trading behavior.

A notable drop on Day 3 might indicate a day of lower market participation, possibly due to external events or lack of significant trading catalysts.

On Day 5, roll values across exchanges align, suggesting synchronized trading activity across markets.The convergence of roll values across exchanges by Day 5 indicates increased market synchronization, likely due to alignment in trading strategies or external factors driving uniform behavior.

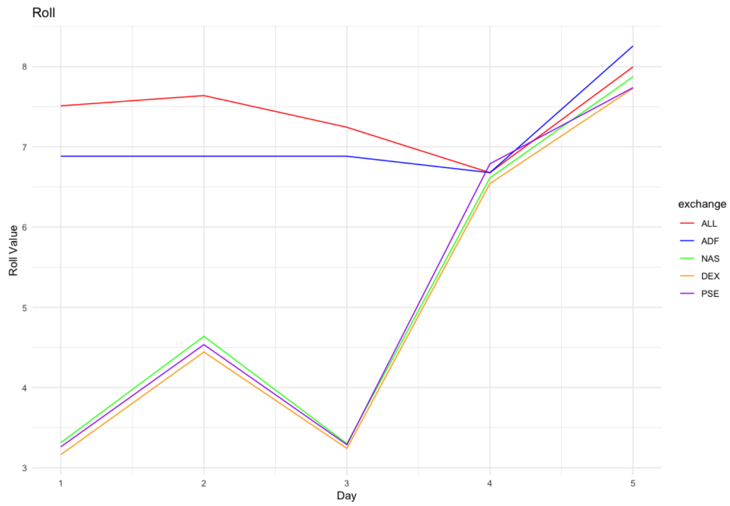

Figure 6: σ_{day}^{roll} across exchanges

2.1.1 Roll model estimate - across different exchanges

Roll model parameters γ_0 γ_1 σ_u^2 σ_{ann} σ_{ann}^{total} are calculated using aut-correlation at lag-0 and lag-1 with
$c_{roll} = \sqrt{-\gamma_1}$
and $prices = mid.price + c_{improved} * tradeSigns$

Day-5 shows c=0.010721, which shows spread esitmate is 2 $\times$ c=0.021442 cents

```
td <- getTradeDirection(tqdata)
pr <- as.numeric(tqdata$PRICE)
dpr <- diff(pr)
deps <- diff(td)
mids <- (as.numeric(tqdata$OFR) + as.numeric(tqdata$BID))/2
dm <- diff(mids)
(fit.lm <- lm(dpr ~ dm + deps))
fit.lm$coeff[3]
```

Parameter	ALL	ADF	NAS	DEX	PSE
γ_0	0.000553	0.000804	0.000094	0.000174	0.000278
γ_1	-0.000239	-0.000353	-0.000003	-0.000009	-0.000002
σ_u^2	0.000074	0.000098	0.000088	0.000156	0.000274
σ_{ann}	119.219577	109.254274	52.592650	50.237125	51.767872
σ_{ann}^{total}	325.135041	312.769808	54.247270	53.177290	52.134473
c_{roll}	0.015475	0.018782	0.001681	0.003063	0.001395
$c_{improved}$	0.012005	0.013175	0.007682	0.009389	0.008555

Table 1: Day 1

Parameter	ALL	ADF	NAS	DEX	PSE
γ_0	0.006223	0.010163	0.000101	0.000196	0.000241
γ_1	-0.003090	-0.005056	-0.000006	-0.000010	-0.000003
σ_u^2	0.000042	0.000051	0.000090	0.000176	0.000235
σ_{ann}	121.249242	103.788033	73.631340	70.529922	71.988734
σ_{ann}^{total}	1467.618732	1461.371830	78.054486	74.356083	72.965617
c_{roll}	0.055592	0.071105	0.002355	0.003131	0.001791
$c_{improved}$	0.012128	0.013327	0.007795	0.009612	0.008996

Table 2: Day 2

Parameter	ALL	ADF	NAS	DEX	PSE
γ_0	0.001355	0.002234	0.000080	0.000164	0.000177
γ_1	-0.000646	-0.001071	-0.000003	-0.000006	-0.000001
σ_u^2	0.000063	0.000092	0.000073	0.000152	0.000180
σ_{ann}	114.984459	107.055343	52.409247	51.459669	52.188730
σ_{ann}^{total}	533.660641	526.873995	54.597910	53.418574	51.822663
c_{roll}	0.025421	0.032723	0.001769	0.002429	0.001120
$c_{improved}$	0.010721	0.011722	0.007268	0.008897	0.007830

Table 3: Day 3

Parameter	ALL	ADF	NAS	DEX	PSE
γ_0	0.003904	0.007744	0.000143	0.000236	0.000295
γ_1	-0.001899	-0.003771	-0.000012	-0.000026	-0.000012
σ_u^2	0.000106	0.000202	0.000118	0.000185	0.000271
σ_{ann}	248.546880	239.987100	120.140460	109.610159	111.551916
σ_{ann}^{total}	1508.654637	1484.461833	131.880548	123.852757	116.345054
c_{roll}	0.043579	0.061407	0.003485	0.005063	0.003447
$c_{improved}$	0.015987	0.017723	0.010703	0.012761	0.012863

Table 4: Day 4

Parameter	ALL	ADF	NAS	DEX	PSE
γ_0	0.002313	0.004137	0.000187	0.000293	0.000449
γ_1	-0.001069	-0.001939	-0.000011	-0.000026	-0.000004
σ_u^2	0.000174	0.000259	0.000166	0.000240	0.000441
σ_{ann}	279.512621	249.103036	118.465563	111.111515	113.708563
σ_{ann}^{total}	1017.850330	994.706778	125.962374	122.613997	114.663811
c_{roll}	0.032699	0.044029	0.003288	0.005114	0.001929
$c_{improved}$	0.018419	0.020353	0.012339	0.014418	0.014067

Table 5: Day 5

2.2 Realized Variance

This method uses trade prices sampled at lag q. The daily volatility estimate at lag q is given by

$$\sigma^2_{Day}(q)(q\Delta) = Var(\Delta p_q), \Delta p_q := p_{t+q} - p_t$$

Typically a lag of 5 minutes is sufficient for the trading noise term ϵ_t to average out. The required lag can be estimated visually from the signature plot, which is a graphical representation of $\sigma Day(q)$ vs q. This plot should have a plateau where the daily volatility is independent of lag.

Sigature plots is stable across all 5 days. Considering all exchanges $q_{5min} = 63.29$ is seen to be highest on day 5. Figure 8 shows us that by day 5 the q_{5min} hit its peak. $q_{5min} = 68.16$ on ADF is higher compared to ALL exchanges combined, possible reason could be due to better pricing and spread available on ADF.

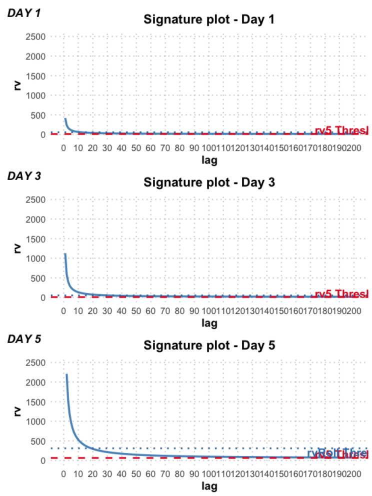

Figure 7: Signature plot

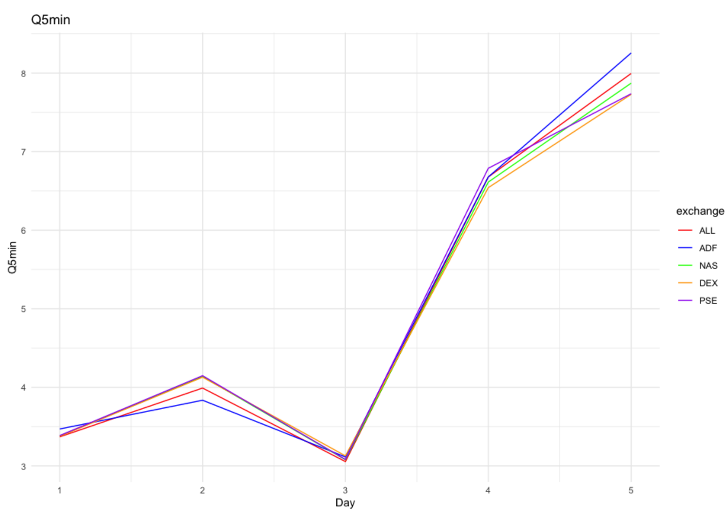

Figure 8: q_{5min} *plot across exchanges*

2.2.1 Roll Model vs Realized Variance

Type	ALL		ADF		NAS		DEX		PSE	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	419.49	20.48	388.19	19.70	11.33	3.37	11.22	3.35	10.79	3.28
q_2	237.95	15.43	217.78	14.76	11.33	3.37	10.62	3.26	10.70	3.27
q_{5min}	11.35	3.37	12.04	3.47	11.43	3.38	11.40	3.38	11.47	3.39
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	56.40	7.51	47.37	6.88	10.98	3.31	10.01	3.16	10.63	3.26
Trades	758112		483079		124082		64315		38855	

Table 6: Day 1

Type	ALL		ADF		NAS		DEX		PSE	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	8547.23	92.45	8474.62	92.06	24.18	4.92	21.94	4.68	21.13	4.60
q_2	4302.79	65.60	4258.68	65.26	22.83	4.78	20.84	4.56	20.85	4.57
q_{5min}	15.92	3.99	14.71	3.84	17.12	4.14	17.05	4.13	17.21	4.15
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	58.34	7.64	42.75	6.88	21.51	4.64	19.74	4.44	20.56	4.53
Trades	1373398		833862		240038		112176		87602	

Table 7: Day 2

Type	ALL		ADF		NAS		DEX		PSE	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	1130.13	33.62	1101.57	33.19	11.83	3.44	11.32	3.37	10.66	3.26
q_2	591.30	24.32	573.52	23.95	11.36	3.37	10.92	3.30	10.73	3.28
q_{5min}	9.33	3.05	9.69	3.11	9.48	3.08	9.78	3.13	9.50	3.08
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	52.47	7.24	45.48	6.88	10.90	3.30	10.51	3.24	10.81	3.29
Trades	833824		493149		148489		69071		60175	

Table 8: Day 3

Type	**ALL**		**ADF**		**NAS**		**DEX**		**PSE**	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	9031.90	95.04	8744.54	93.51	69.02	8.31	60.87	7.80	53.72	7.33
q_2	4638.52	68.11	4486.54	66.98	63.15	7.95	54.28	7.37	51.55	7.18
q_{5min}	44.59	6.68	44.59	6.68	43.73	6.61	42.79	6.54	46.08	6.79
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	245.14	15.66	228.55	6.88	57.28	7.57	47.68	6.90	49.38	7.03
Trades	2313397		1129191		483448		257414		182361	

Table 9: Day 4

Type	**ALL**		**ADF**		**NAS**		**DEX**		**PSE**	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	4111.19	64.12	3926.35	62.66	62.96	7.93	59.66	7.72	52.17	7.22
q_2	2210.60	47.02	3926.35	62.66	59.32	7.70	54.33	7.37	51.74	7.19
q_{5min}	63.92	8.00	68.16	8.26	61.97	7.87	59.75	7.73	59.86	7.74
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	310.03	17.61	246.24	15.69	55.69	7.46	48.99	7.00	51.31	7.16
Trades	1777544		949185		336278		203923		116282	

Table 10: Day 5

3 PIN measure