

Worksheet 4

Part 1

A small 25 W light bulb has an efficiency of 20%. How many photons are approximately emitted per second? Assume in the calculations that we only use average photons of wavelength 500 nm.

25W energy of the entire lightbulb. Efficiency is 20% so 20% of 25W is converted to light. So, 5W is converted to light.

Energy of a photon is given by:

$$E = hc/\lambda$$

therefore a single photon has energy of:

$$E = 6.626 \cdot 10^{-34} \cdot 2.9979 \cdot 10^8 / 500 \cdot 10^{-9} = 3.97 \cdot 10^{-30} \text{ W}$$

So, number of photons emitted per second is:

$$5 / 3.97 \cdot 10^{-30} = 1.26 \cdot 10^{28} \text{ photons}$$

Part 2

A light bulb (2.4 V and 0.7 A), which is approximately sphere-shaped with a diameter of 1 cm, emits light equally in all directions. Find the following entities (ideal conditions assumed)

- Radiant flux
- Radiant intensity
- Radiant exitance
- Emitted energy in 5 minutes **Use W for Watt, J for Joule, m for meter, s for second and sr for steradian.**

Radiant flux is equivalent to power, so it is $2.4\text{V} \cdot 0.7\text{A} = 1.68\text{W}$

Radiant intensity - to find it, first we need to calculate the area of the sphere (light bulb). in this case:

$$A = 4 \cdot \pi \cdot r^2 = 4 \cdot \pi \cdot (0.01/2)^2 = 3.14 \cdot 10^{-4} \text{ m}^2$$

calculating solid angle yields results in steradians:

$$\Omega = A / r^2 = 3.14 \cdot 10^{-4} / (0.01/2)^2 = 12.56 \text{ sr}$$

Radiant intensity is then:

$$I = P / \Omega = 1.68 / 12.56 = 0.134 \text{ W/sr}$$

Radiant exitance is the total power emitted per unit area. In this case, it is:

$$M = P / A = 1.68 / 3.14 \cdot 10^{-4} = 5.35 \cdot 10^3 \text{ W/m}^2$$

Emitted energy in 5 minutes is:

$$E = P \cdot t = 1.68\text{W} \cdot 5\text{min} \cdot 60\text{s} = 504 \text{ J}$$

Part 3

The light bulb from above is observed by an eye, which has an opening of the pupil of 6 mm and a distance of 1 m from the light bulb. Find the irradiance received in the eye.

Light that hits the opening of the pupil is a circle with diameter of 6mm. The area of the circle is:

$$A = \pi \cdot r^2 = \pi \cdot (0.006/2)^2 = 2.83 \cdot 10^{-5} \text{ m}^2$$

Considering the distance of 1m, the solid angle is:

$$\Omega = A / \text{distance}^2 = 2.83 \cdot 10^{-5} / 1^2 = 2.83 \cdot 10^{-5} \text{ sr}$$

Given this, we can calculate intensity over the solid angle (considering the entire sphere):

$$I = P / \Omega = 1.68 / 4\pi = 1.68 / 12.56 = 0.134 \text{ W/sr}$$

Irradiance at the eye given the intensity,distance and the area of the eye is:

$$E = I / (\text{distance}^2) = 0.134 / 1 = 0.134 \text{ W/m}^2$$

Part 4

A 200 W spherically shaped light bulb (20% efficiency) emits red light of wavelength 650 nm equally in all directions. The light bulb is placed 2 m above a table. Calculate the irradiance at the table. Photometric quantities can be calculated from radiometric ones based on the equation Photometric= Radiometric· 685· V(λ) in which V(λ) is the luminous efficiency curve. At 650 nm, the luminous efficiency curve has a value of 0.1. Calculate the illuminance.

First, we need to calculate the radiant flux of the light bulb. Efficiency is 20%, so 20% of 200W is converted to light:

$$P = 200 \cdot 0.2 = 40\text{W}$$

Considering the light is emitted equally in all directions over a sphere, the radiant intensity is:

$$I = P / 4\pi = 40 / 4\pi = 3.18 \text{ W/sr}$$

considering that the table is 2m away right below the light bulb ($\cos\theta = 1$), the irradiance at the table is:

$$E = I / (\text{distance}^2) = 3.18 / 4 = 0.795 \text{ W/m}^2$$

to calculate Photometric quantities, we use the formula given above:

$$E = 0.795 \cdot 685 \cdot 0.1 = 54.5 \text{ lx}$$

Part 5

In a simple arrangement the luminous intensity of an unknown light source is determined from a known light source. The light sources are placed 1 m from each other and illuminate a double sided screen placed between the light sources. The screen is moved until both sides are equally illuminated as observed by a photometer. At the position of match, the screen is 35 cm from the known source with luminous intensity $I_s = 40 \text{ lm/sr} = 40 \text{ cd}$ and 65 cm from the unknown light source. What is the luminous intensity I_x of the unknown source?

Given that the screen is equally illuminated by both sources, we assume that the illuminance is the same on both sides of the screen. The illuminance is given by:

$$E = P \cdot \cos\theta / (4\pi \cdot \text{distance}^2)$$

$$E_1 = E_2$$

here we assume the $\cos\theta$ is 1, the 4π is cancelled out, the relation can be simplified to:

$$I_s / \text{distance}_1^2 = I_x / \text{distance}_2^2$$

$$I_x = I_s \cdot \text{distance}_2^2 / \text{distance}_1^2 = 40 \cdot (0.65)^2 / (0.35)^2 = 137.96 \text{ cd}$$

Part 6

The radiance L from a diffuse light source (emitter) of 10 x 10 cm is 5000 W/(sr m2). Calculate the radiosity (radiant exitance). How much energy is emitted from the light source?

In this case, the radiosity is:

$$B = L \cdot \pi = 5000 \cdot \pi = 15700 \text{ W/m}^2$$

The energy emitted from the light source is:

$$E = B \cdot A = 15700 \cdot 0.1 \cdot 0.1 = 157 \text{ J}$$

Part 7

The radiance L = 6000 cos θ W/(m2 sr) for a non-diffuse emitter of area 10 by 10 cm. Find the radiant exitance. Also, find the power of the entire light source.

Power can be calculated as follows:

$$P = L \int_A dA_0 \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos(\theta) \sin(\theta) d\theta$$

Given the L, this becomes:

$$P = 6000 \int_A dA_0 \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^2(\theta) \sin(\theta) d\theta$$

which can be simplified to:

$$P = 6000 \cdot 2\pi \cdot \frac{1}{3} \cdot A = 4000\pi \cdot A$$

this results in:

$$P = 4000\pi \cdot 0.1 \cdot 0.1 = 12.57\text{W}$$

The radiant exitance is:

$$M = P / A = 12.57 / 0.1 \cdot 0.1 = 1257 \text{ W/m}^2$$