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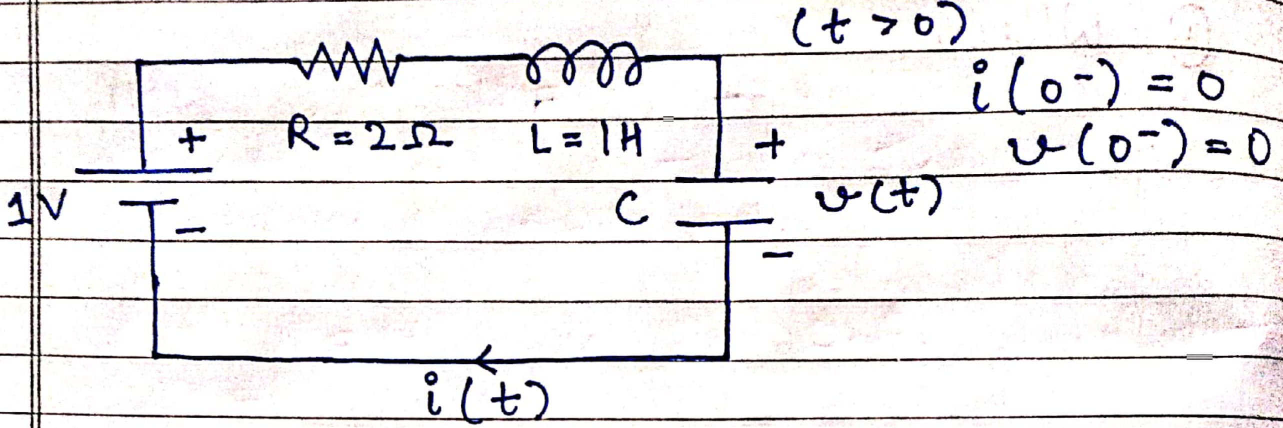
Questions and Answers :

Answer 1



① When $C = 25/9$ F

Apply Kirchhoff's Voltage Loop Rule.

$$1 - R\,i(t) - L\,\dot{i}(t) - \frac{q(t)}{C} = 0$$

Here $q(t)$ is the charge accumu- -lated on the capacitor at time t.

$$i(t) = \dot{q}(t)$$
$$\Rightarrow \dot{i}(t) = \ddot{q}(t)$$

Also for a capacitor $q = Cv$

$$\Rightarrow \dot{q}(t) = C\,\dot{v}(t)$$

$$\therefore\ 1 - RC\,\dot{v}(t) - LC\,\ddot{v}(t) - v(t) = 0$$

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$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = 1$$

Put the values of R, L, C.

$$\frac{25}{9}\frac{d^2v}{dt^2} + \frac{50}{9}\frac{dv}{dt} + v = 1$$

$$25\frac{d^2v}{dt^2} + 50\frac{dv}{dt} + 9v = 9$$

The characteristic eq^n of this differential eq^n will be -

$$25r^2 + 50r + 9 = 0$$

discriminant, $\sqrt{D} = \sqrt{b^2-4ac}$

$$\sqrt{D} = \sqrt{b^2-4ac} = \sqrt{50^2-4\cdot25\cdot9}$$

$$= \sqrt{1600} = 40$$

Because $\sqrt{D}$ is real (or D is positive), the nature of the roots is real and distinct.

Let $v_N(t)$ be the natural response.

roots x_1, x_2 of characteristic eq^n $= \frac{-b\pm\sqrt{D}}{2a} = -9/5, -1/5$

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$\therefore v_N(t) = a e^{-9t/5} + b e^{-t/5}$

$v_N(t)$ is the natural response where $a, b \in \mathbb{R}$.

Now let $v_F(t) = c$ be the particular solⁿ or the forced response of D.E.

Put $v_F(t)$ in the D.E. and get $c = 1$

$$v(t) = v_N(t) + v_F(t) \quad (t > 0)$$
$$= a e^{-9t/5} + b e^{-t/5} + 1$$

$v(0^+) = v(0^-) = 0$

$\dot{v}(0^+) = i(0^+)/C = i(0^-)/C = 0$

$\Rightarrow \quad a + b + 1 = 0$

$-\frac{9a}{5} - \frac{b}{5} = 0 \Rightarrow 9a + b = 0$

Solving the two we get,

$a = 1/8, \quad b = -9/8.$

$$\therefore v(t) = \frac{1}{8} e^{-9t/5} - \frac{9}{8} e^{-t/5} + 1.$$

$$\dot{v}(t) = \frac{9}{40}\left(e^{-t/5} - e^{-9t/5}\right) > 0 \ \forall \ t > 0$$

(monotonically increasing)

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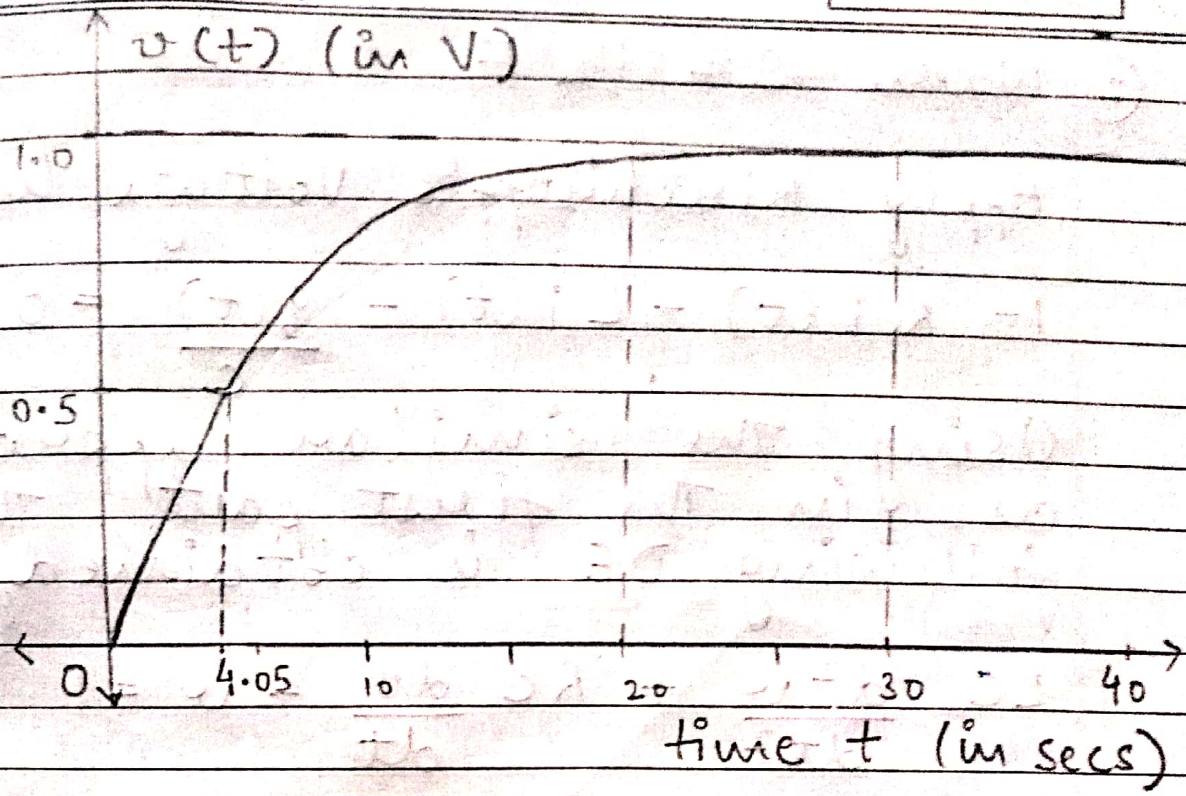

The circuit simulated on LTSpice and the corresponding graph obtained are shown on the next page.

The theoretical plot sketched above and the plot obtained on LTSpice are very similar.

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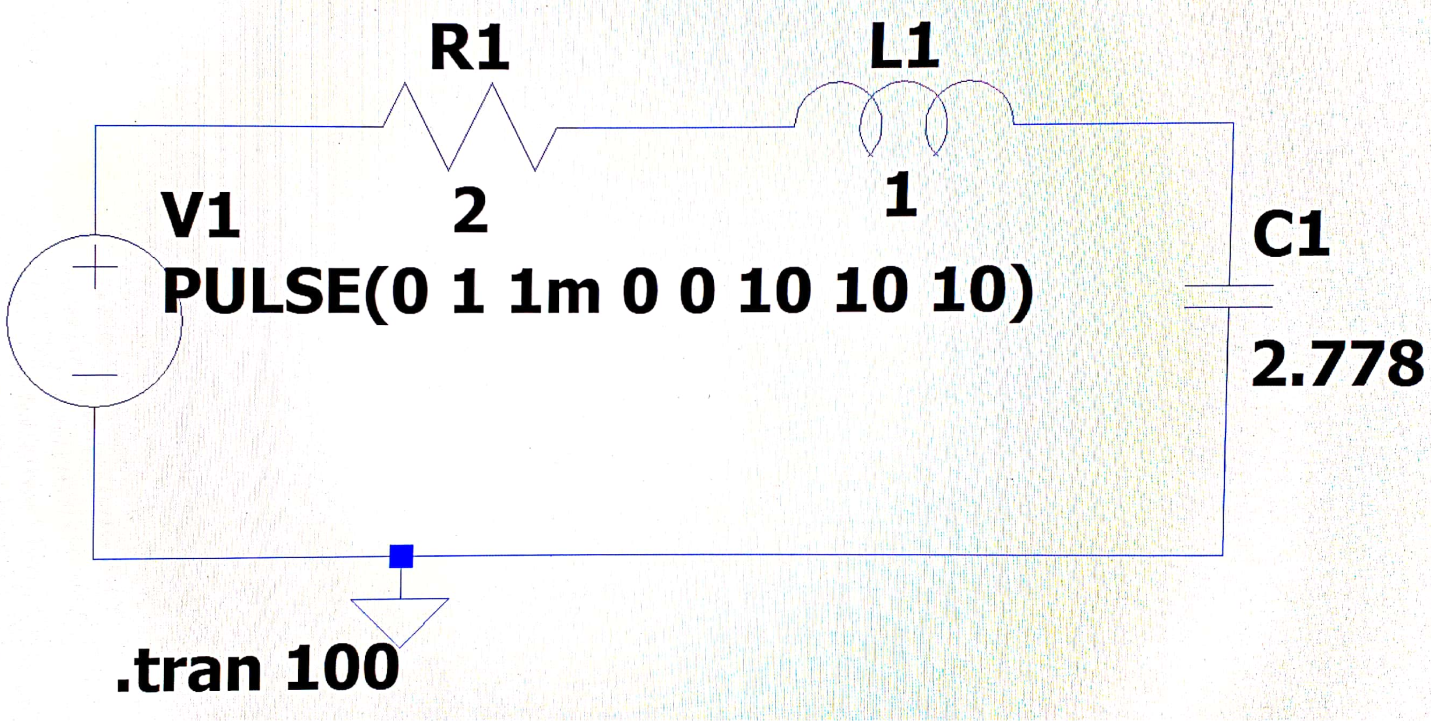
ANSWER 01 - PART 01



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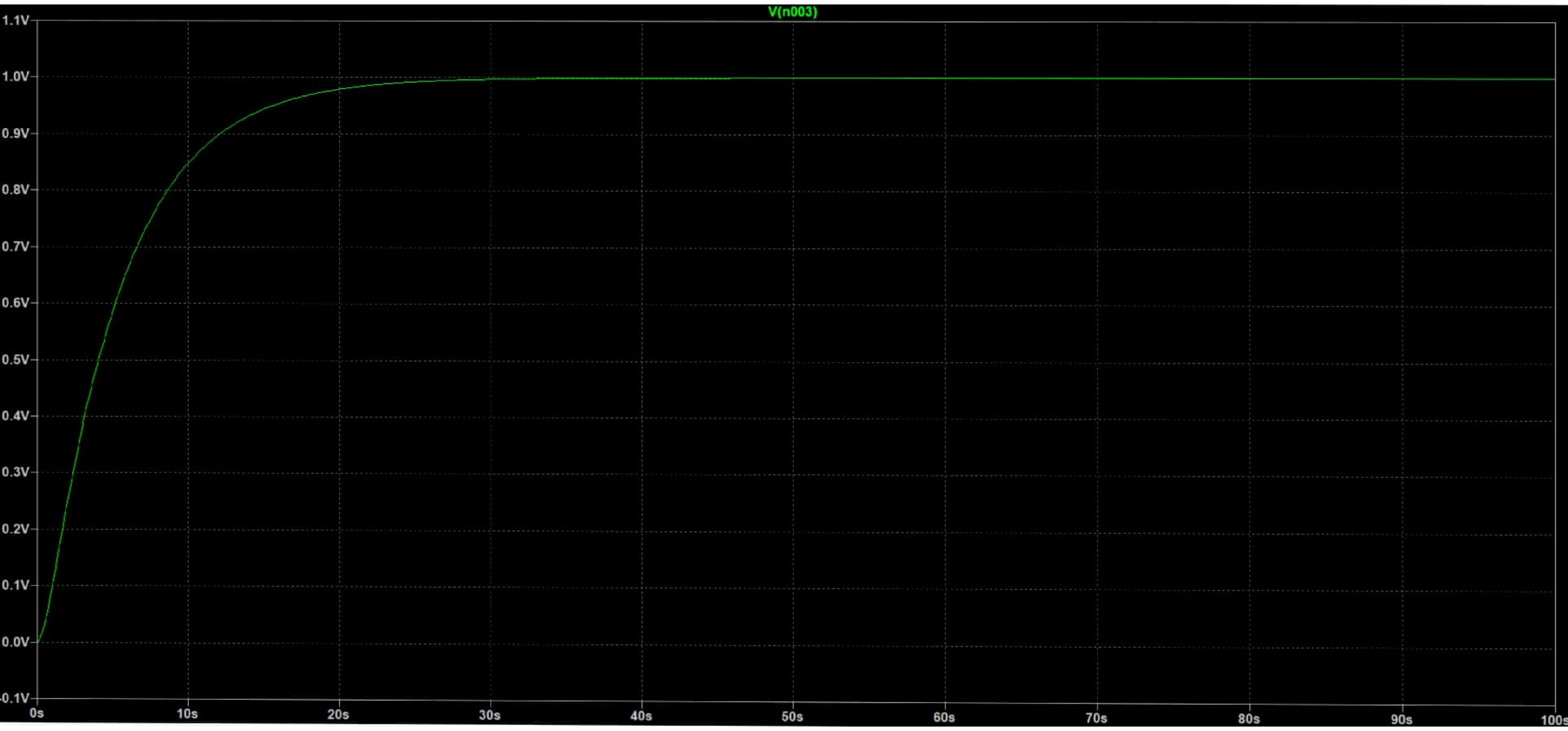

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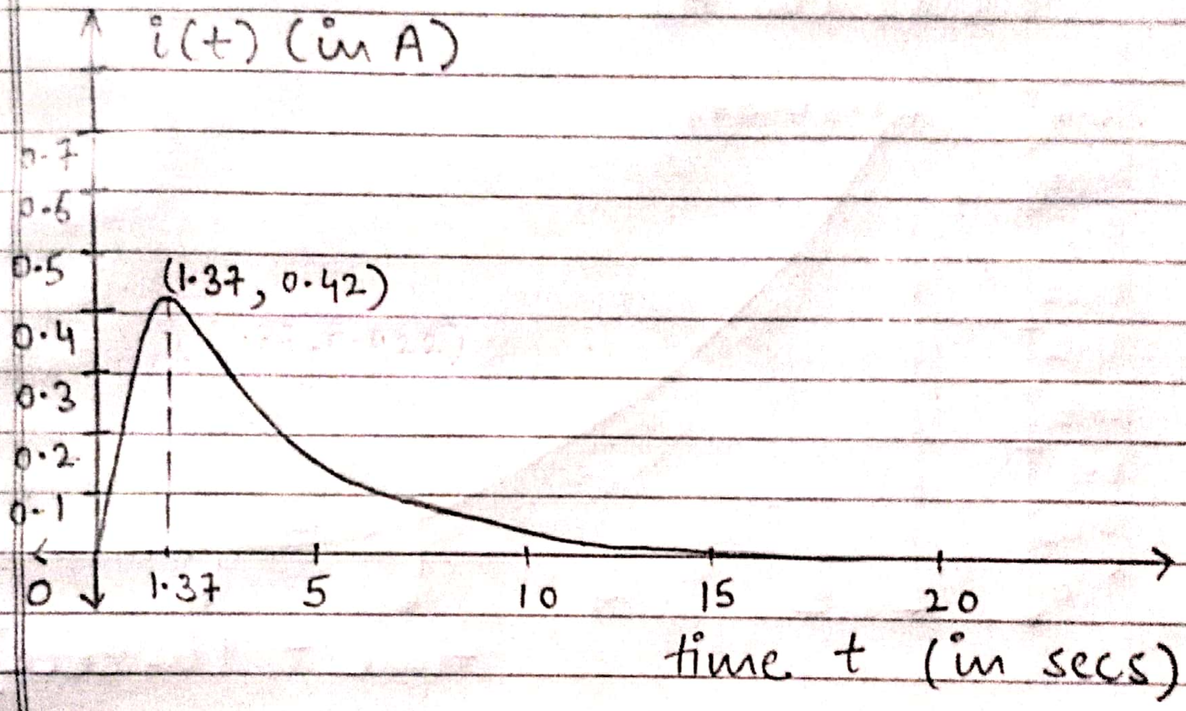
$$i(t) = C\,\dot{v}(t)$$

$$= \frac{5}{8}\left(e^{-t/5} - e^{-9t/5}\right).$$



The plot obtained from LTSpice simulation is shown on the next page.
Both plots give maxima at nearly the same time and the maximum current is also the same. The shape is similar too.

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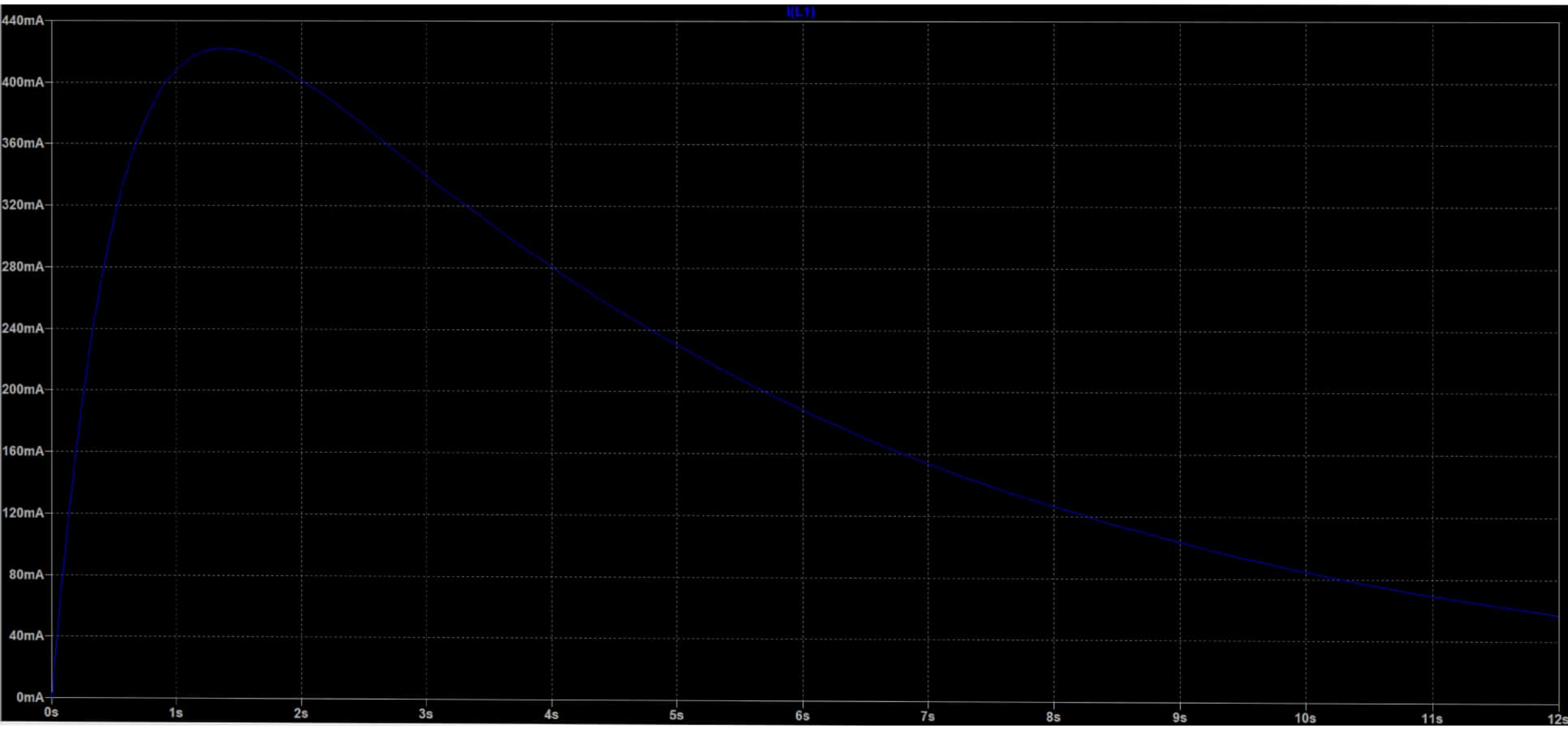

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(2) When $C = 1/2 \ F$

Apply Kirchhoff's Voltage loop Rule

$$1 - Ri(t) - L\dot{i}(t) - \frac{q(t)}{C} = 0$$

Using the similar approach as in the first part the following DE is obtained.

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = 1$$

Put values of R, L, C.

$$\frac{1}{2}\frac{d^2v}{dt^2} + 1.\frac{dv}{dt} + v = 1$$

$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + 2v = 2$$

The characteristic eq^n of this differential eq^n will be.-

$$r^2 + 2r + 2 = 0$$

discriminant, $D = b^2 - 4ac$
$= 4 - 4.1.2 = -4$

Because $D < 0$, the nature of the roots is complex.

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Let $v_N(t)$ be the natural response

roots x_1, x_2 of characteristic eq^n $= \frac{-b \pm \sqrt{D}}{2a} = -1 \pm i$

$v_N(t) = ae^{-t}\sin t + be^{-t}\cos t$
$(a, b \in \mathbb{R})$

Now let $v_F(t) = c$ be the forced response or the particular sol^n of DE.

Put $v_F(t)$ in the DE and get $c = 1$.

$v(t) = v_N(t) + v_F(t)$

$= e^{-t}(a\sin t + b\cos t) + 1$

$v(0^+) = v(0^-) = 0$
$\dot{v}(0^+) = i(0^+)/C = i(0^-)/C = 0$

$\Rightarrow v(0^+) = b + 1 = 0 \Rightarrow b = -1$
$\dot{v}(t) = e^{-t}(a\cos t - b\sin t)$
$- e^{-t}(a\sin t + b\cos t)$

$\dot{v}(0^+) = a - b = 0 \Rightarrow a = -1$

$\therefore v(t) = 1 - e^{-t}(\sin t + \cos t)$

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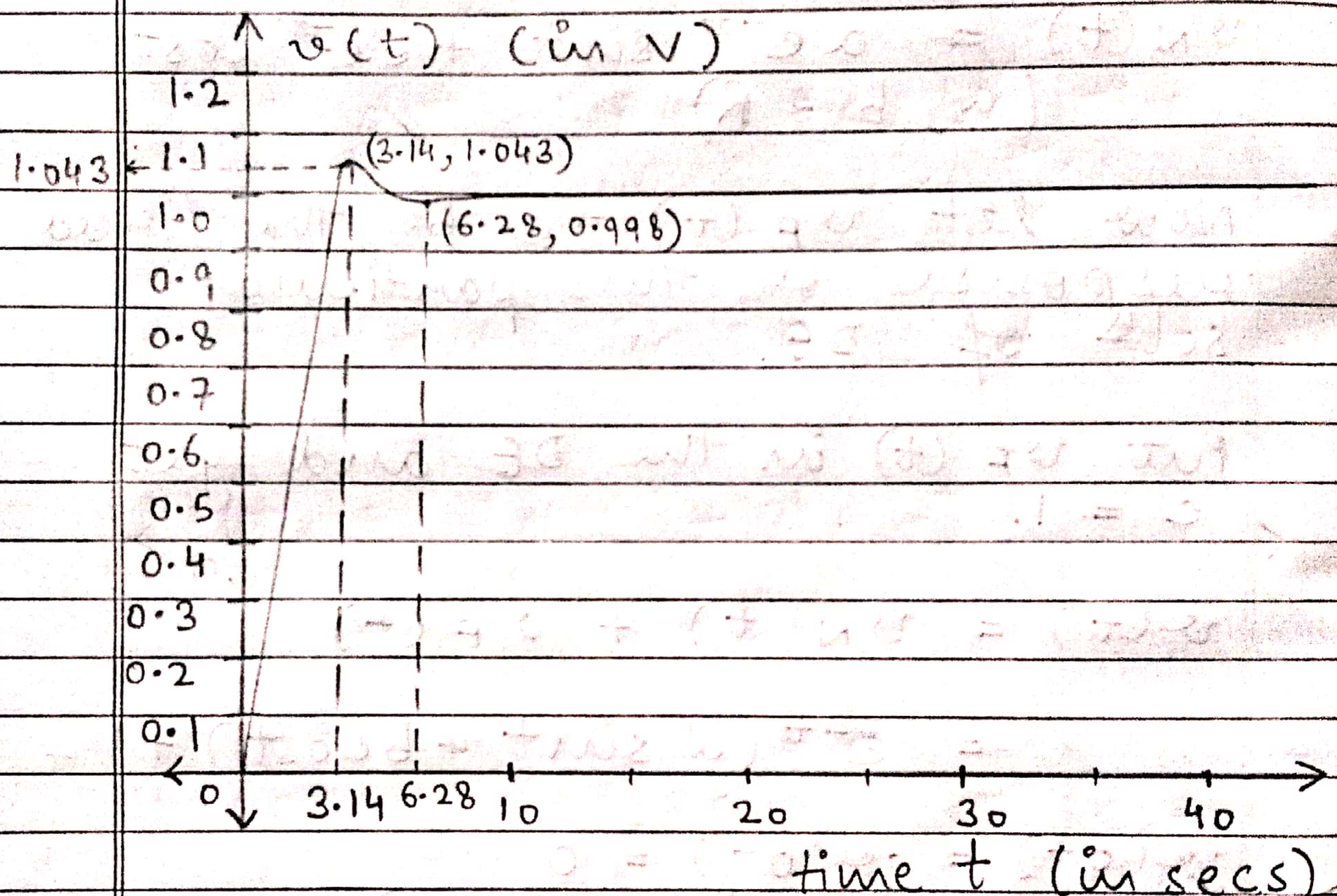
Now $i(t) = 2e^{-t} \sin t$

for maxima, $i(t) = 0$
$\Rightarrow t = \pi = 3.14.$

$v(t=\pi) = 1 + e^{-\pi} = 1.043$ V.



The circuit simulated on LTSpice and the corresponding graph obtained are shown on the next page.

The theoretical plot sketched above and the plot obtained on LTSpice are very similar.

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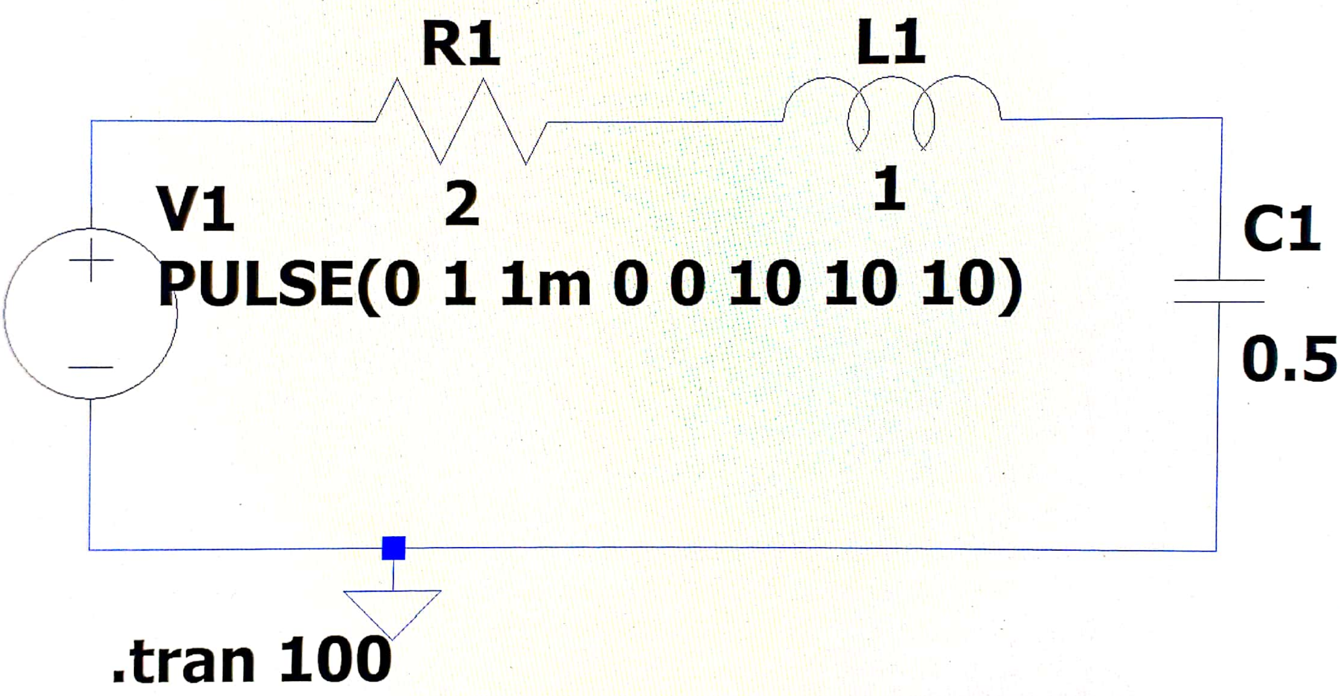
ANSWER 01 - PART 02



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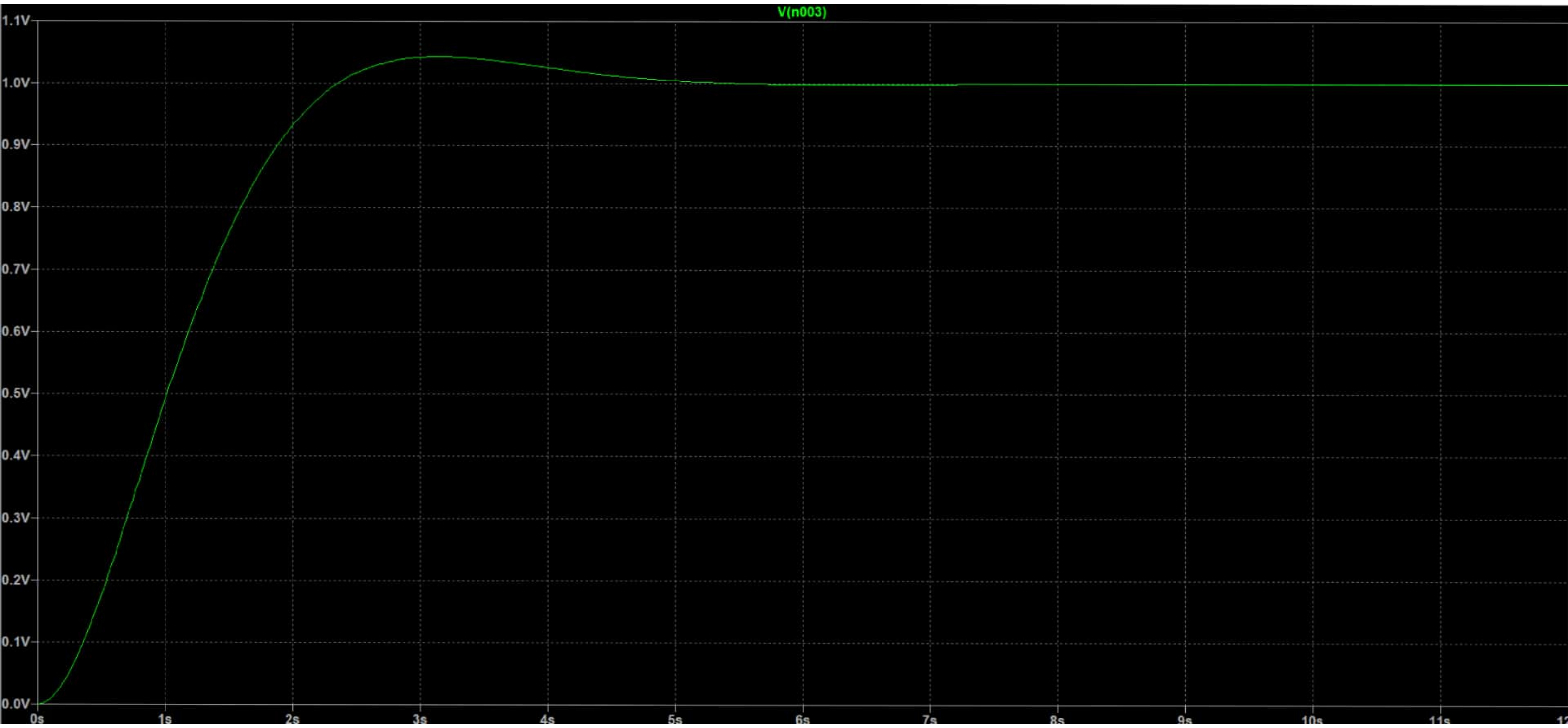

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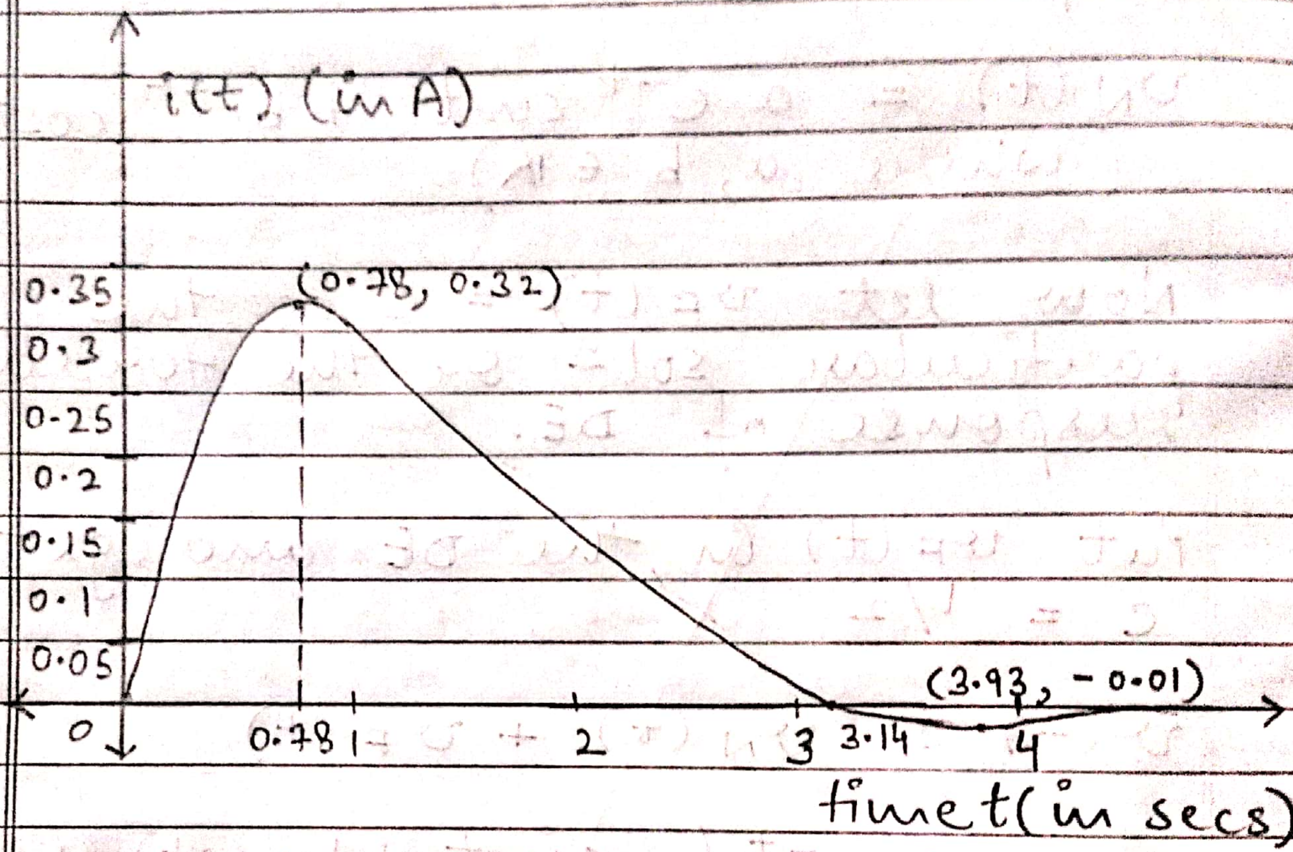
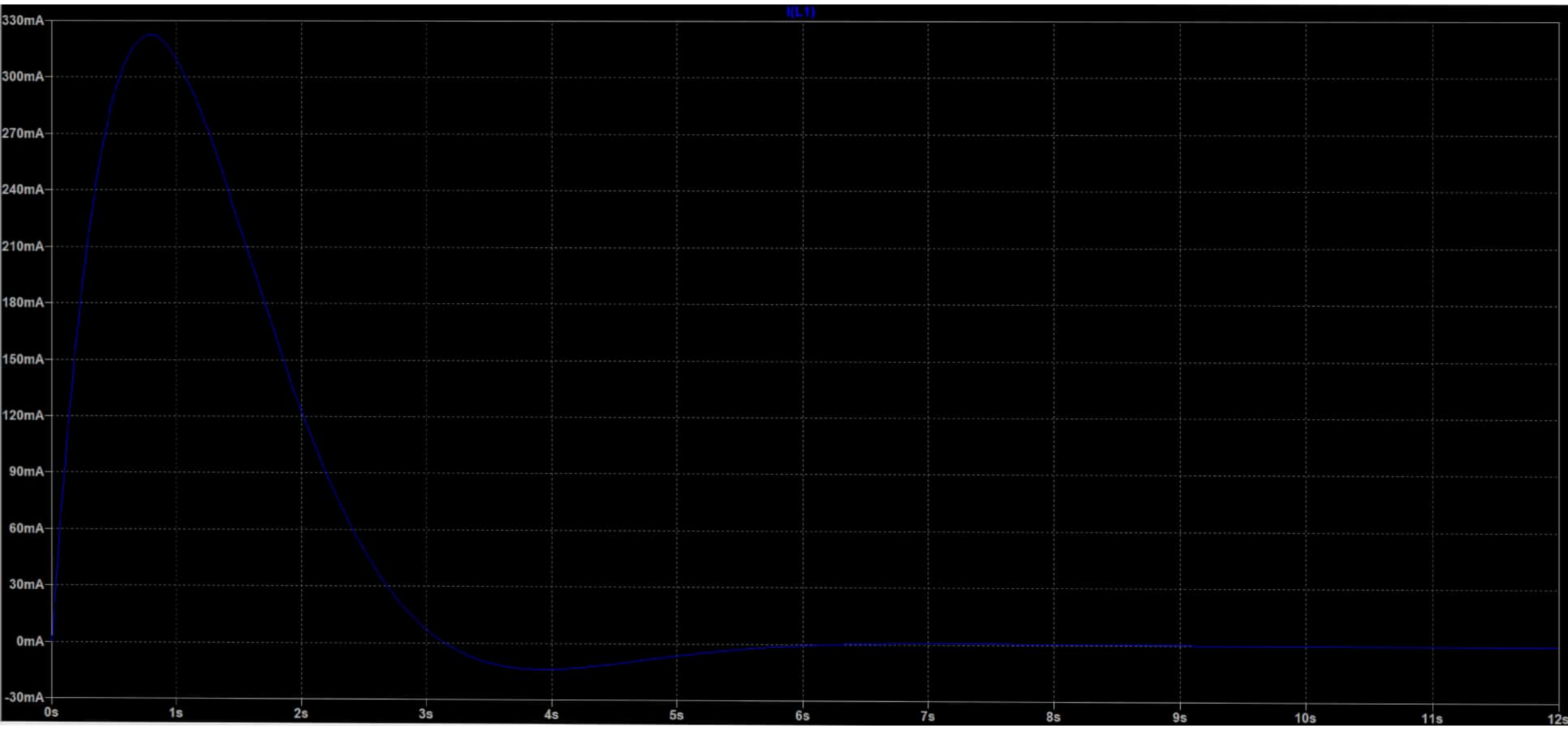
$$i(t) = C\,\dot{v}(t)$$

$$= e^{-t}\sin t$$



The plot for i(t) obtained on LTSpice is shown on the next page.
These two plots are very similar



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(3) When $C = 1F$

Apply Kirchhoff's Voltage Loop Rule.

$$1 - Ri(t) - L\dot{i}(t) - \frac{q(t)}{C} = 0$$

Using the similar approach as in the first part, the following DE is obtained.

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = 1$$

Put values of R, L, C.

$$1 \cdot \frac{d^2v}{dt^2} + 2\frac{dv}{dt} + v = 1$$

The characteristic eq^n of this differential eq^n will be -

$$r^2 + 2r + 1 = 0$$

discriminant, $D = b^2 - 4ac$
$= 2^2 - 4.1.1 = 0$

Because $D = 0$, the nature of roots is real and repeated.

Let $v_N(t)$ be the natural response.

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roots $x_1; x_2$ of characteristic eq$^{\underline{n}}$ $= \frac{-b \pm \sqrt{D}}{2a} = -1, -1$

$v_N(t) = ae^{-t} + bte^{-t}$
$(a, b \in \mathbb{R})$

Now let $v_F(t) = c$ be the forced response or the particular sol$^{\underline{n}}$ of DE.

Put $v_F(t)$ in DE and get $c = 1$.

$v(t) = v_N(t) + v_F(t)$

$= e^{-t}(a + bt) + 1$

$v(0^+) = v(0^-) = 0$
$\dot{v}(0^+) = i(0^+)/c = i(0^-)/c = 0$

$\Rightarrow v(0^+) = a + 1 = 0 \Rightarrow a = -1$

$\dot{v}(t) = e^{-t} b + (a + bt)(-e^{-t})$
$\dot{v}(0^+) = b - a = 0 \Rightarrow b = -1$

$\Rightarrow v(t) = 1 - e^{-t}(1 + t)$

$\dot{v}(t) = te^{-t} > 0 \quad \forall\, t > 0$
$\therefore$ the graph must be monotonically increasing

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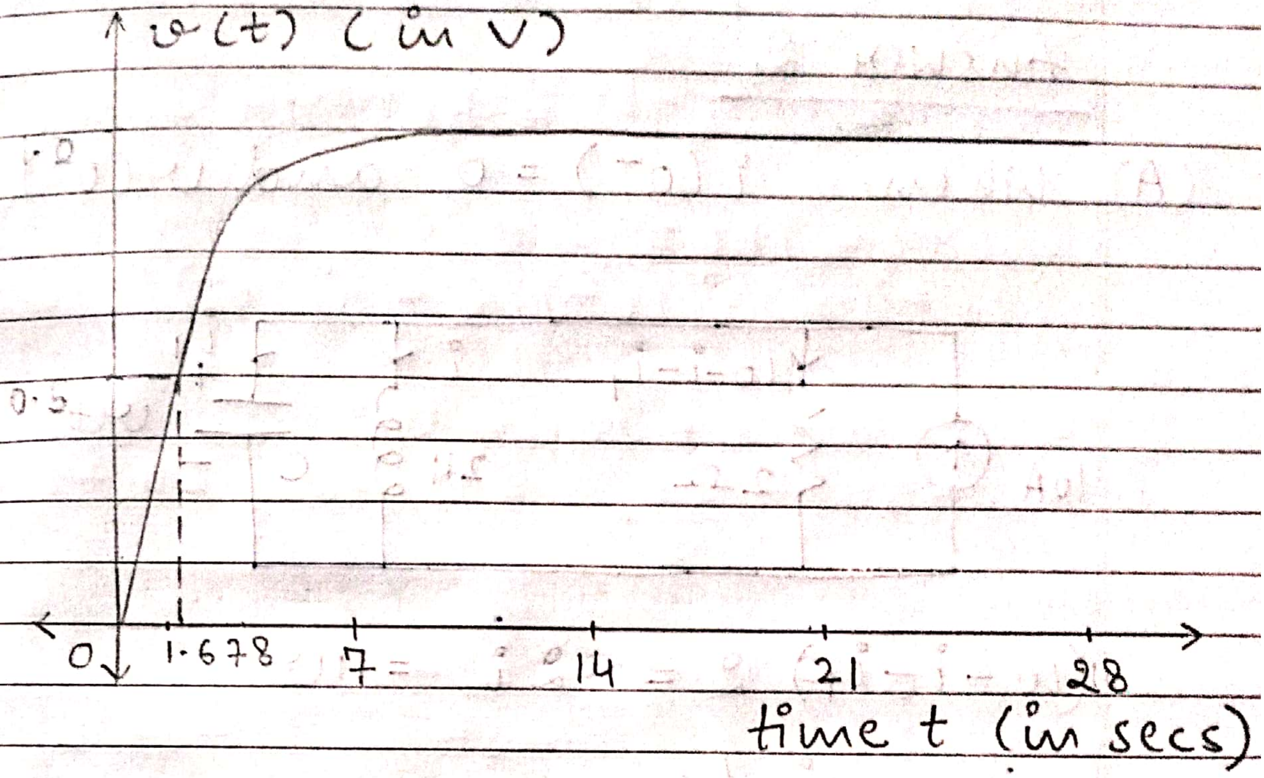

The circuit simulated on LTSpice and the corresponding graph obtained are shown on the next page.

The theoretical plot sketched above and the plot obtained on LTSpice are very similar.

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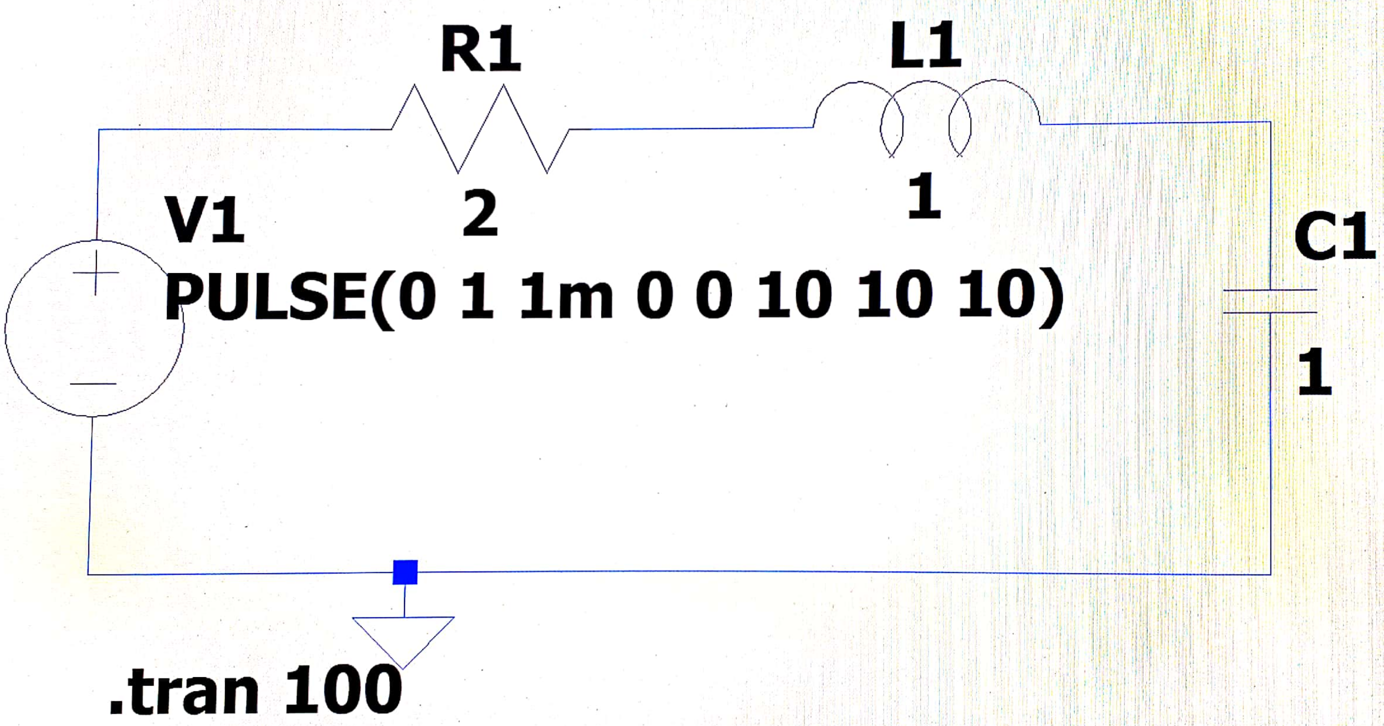
ANSWER 01 - PART 03



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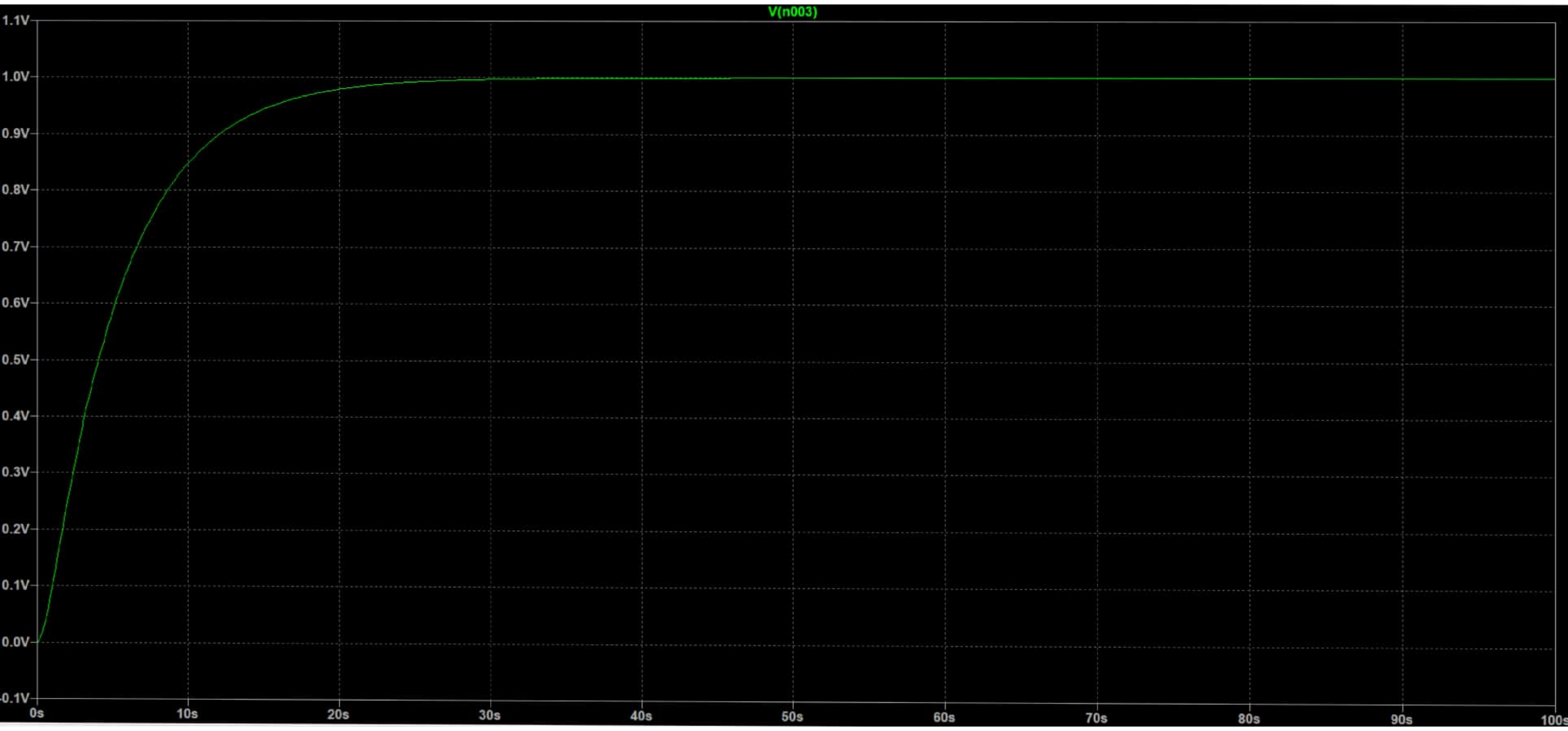

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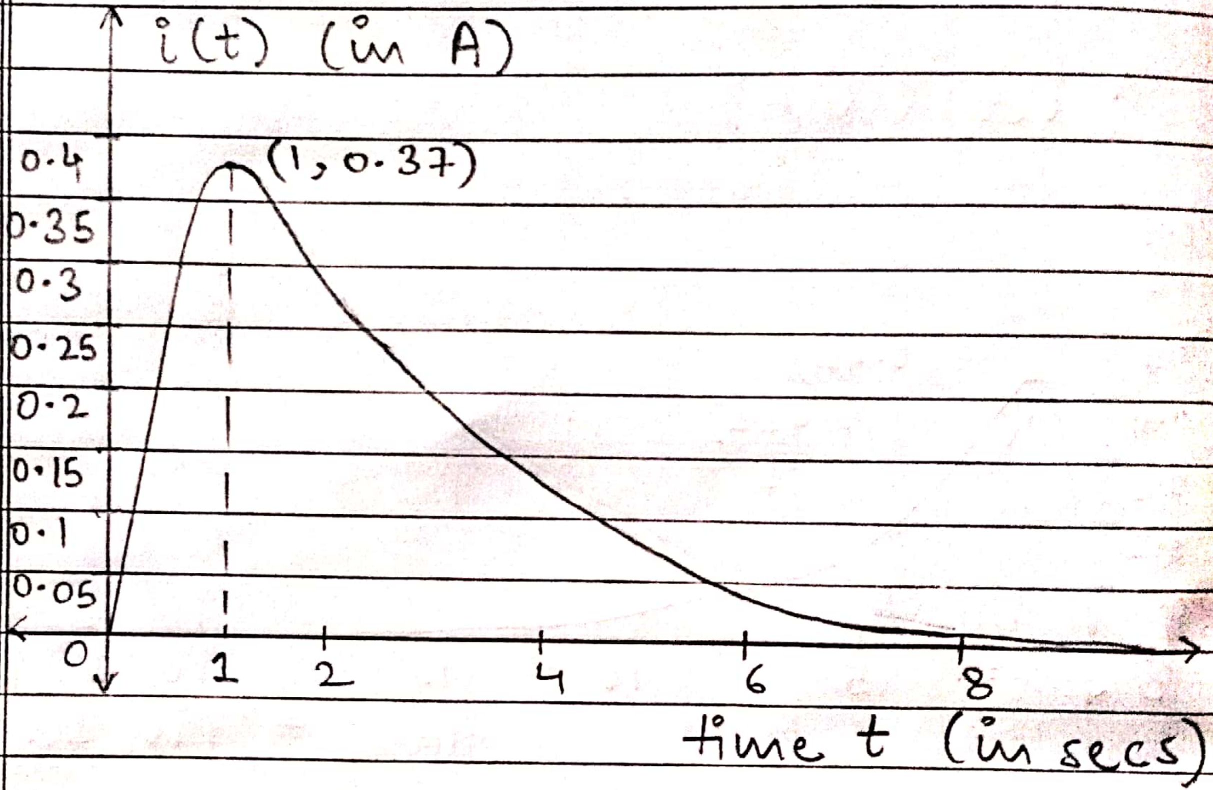
$$i(t) = C\, \dot{v}(t)$$
$$= t e^{-t}$$



The plot obtained from the LTSpice simulation is shown on the next page. It is very similar to the plot drawn above.

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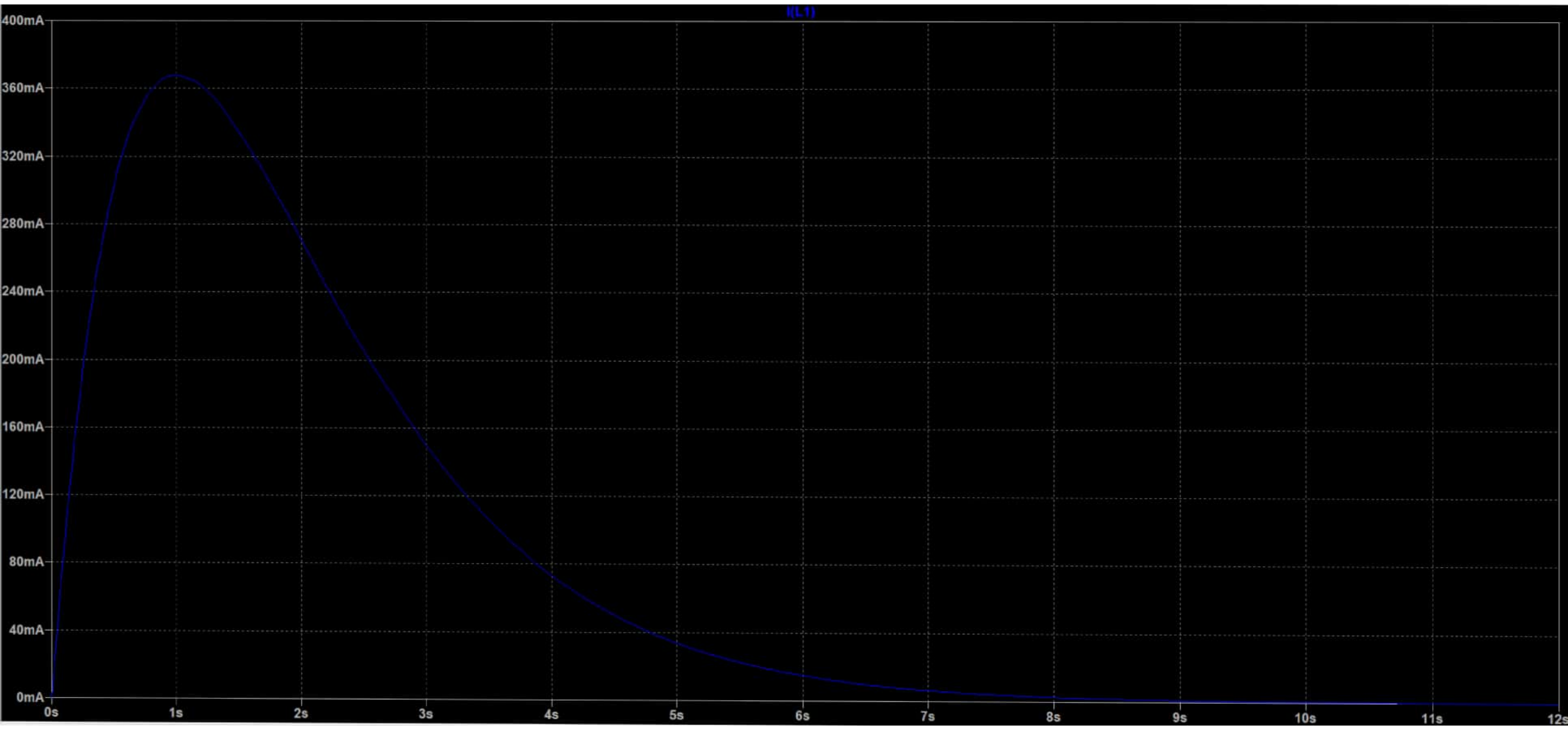

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