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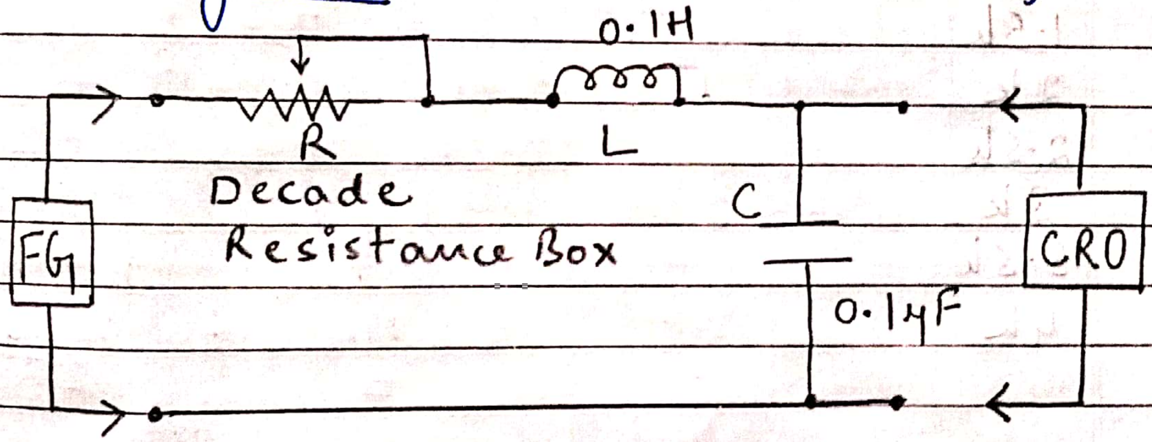
Experiment: 3A

Aim: To study the frequency response of an RLC circuit on Simulink, for various values of ξ (normalised damping constant) and comparing the practical results with the theoretical values.

Circuit Diagram: (from Manual)



Theory And Equations:

For the above RLC network, when the output is taken across the capacitor, the transfer function will be as follows. (apply voltage divider rule)

$$T(\omega) = \frac{1/j\omega C}{1/j\omega C + j\omega L + R}$$

$$T(\omega) = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

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$T(\omega)$ is a complex valued function whose magnitude gives the gain and arguement gives the phase difference b/w the input and output wave form.

$$|T(\omega)| = \frac{1}{[(1-\omega^2LC)^2 + (\omega RC)^2]^{1/2}}$$
$$= M(\omega) \quad (\text{gain})$$

$\angle T(\omega)$ ($\Delta\phi$ in radians) $= -\tan^{-1}\left(\frac{\omega RC}{1-\omega^2LC}\right)$ if $\omega < \omega_n$

$-\pi/2$ if $\omega = \omega_n$

$-\tan^{-1}\left(\frac{\omega RC}{1-\omega^2LC}\right) - \pi$ if $\omega > \omega_n$

(where $\omega_n = 1/\sqrt{LC}$).

For a series RLC network as in the diagram, ξ is defined as the normalised damping constant whose value is: $\xi = \frac{R}{2L\omega_n} = \frac{R}{2}\sqrt{\frac{C}{L}}$

The above network acts like a band pass filter if $\xi < 1/\sqrt{2}$. For this BPF the maximum gain M_m is given by $M_m = \frac{1}{2\xi\sqrt{1-\xi^2}}$. This maximum value of gain for this network occurs

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at $\omega_m = \omega_n \sqrt{1-2\xi^2}$. ($\xi < 0.707$)

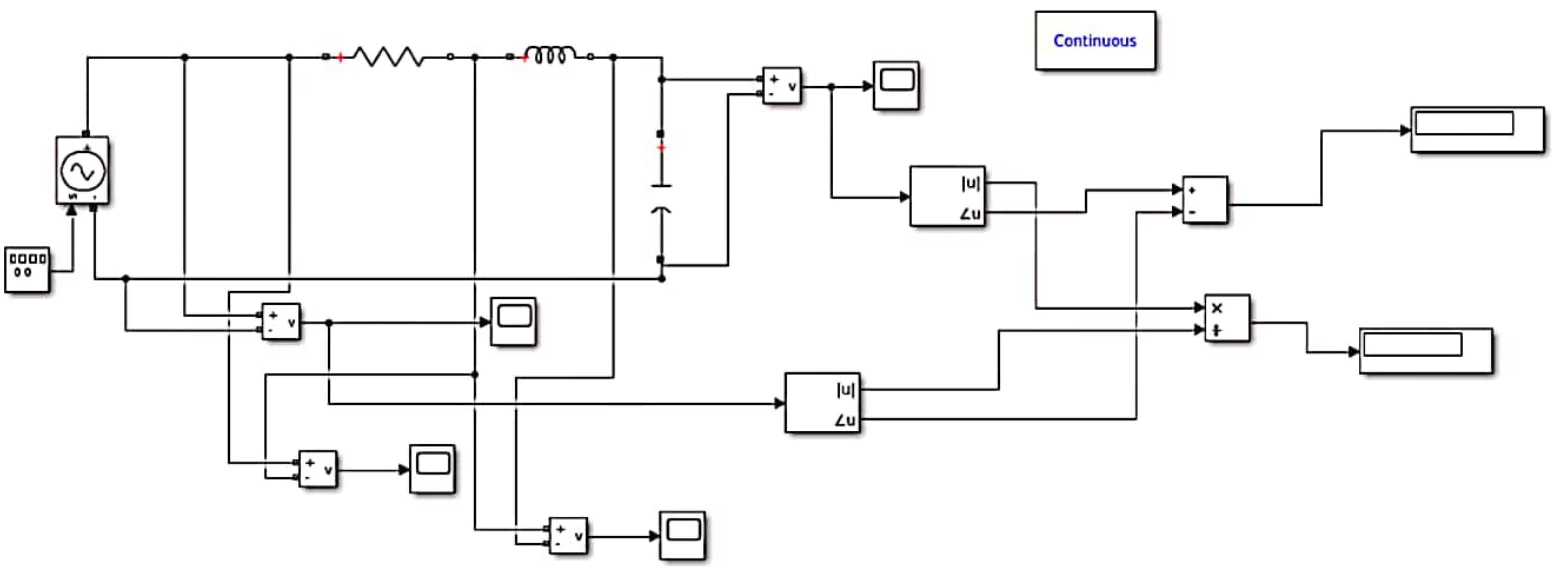
The peak value of the gain can be obtained by plugging in $\omega = \omega_m$ in the formula of M also.

Simulink Circuit :

The screenshot of the circuit simulated on simulink is pasted on the next page.

Equipment : (Simulink Blocks)

1. Signal Generator
2. Controlled Voltage Source
3. Voltage Measurement Block
4. Scope
5. Fourier Block
6. Display Block
7. Add Block Parameter
8. Product / Divide Block Parameter
9. Resistor
10. Inductor
11. Capacitor
12. powergui Block.


Continuous
|u|
∠u
|u|
∠u

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Part A :

Simulink Procedure :

1. Drag all the blocks that are mentioned in the "Equipments" from the library Browser to the main screen.
2. Connect all the blocks as shown in the Simulink circuit diagram.
3. Double-click on Signal Generator block parameter. Ensure that the waveform is sinusoidal.
4. Set frequency equal to 100 Hz and set the same frequency as the fundamental frequency for both the Fourier blocks.
5. Set values of inductance and capacitance as 0.1 H and 0.1 μF respectively.
6. Choose $\zeta = 0.2$ and calculated the corresponding value of resistance and set it.
7. Run the simulation. Record values of V_R, V_L, V_C from the scope. Record the values of gain and phase difference from display blocks.
8. Increase frequency in steps and repeat for each one of them (100 Hz to about 5 kHz). Do not forget to also change fundamental frequency of fourier blocks.

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observation Tables:

when $\xi = 0.2$ ($R = 400\,\Omega$)

f (Hz)	V_R (V)	V_L (V)	V_C (V)	gain M	$\Delta\phi$ (radians)
100	0.025	0.016	1.004	1.004	−0.025
300	0.079	0.049	1.034	1.034	−0.078
500	0.135	0.087	1.100	1.100	−0.138
700	0.213	0.134	1.212	1.212	−0.215
1k	0.383	0.241	1.526	1.526	−0.393
1.5k	0.959	0.602	2.543	2.543	−1.282
2k	0.656	0.412	1.305	1.305	−2.426
2.5k	0.394	0.247	0.627	0.627	−2.737
3k	0.283	0.178	0.376	0.376	−2.854
3.5k	0.224	0.140	0.254	0.254	−2.916
4k	0.186	0.117	0.185	0.185	−2.955

M_m at 1.5 kHz (from table)

Theoretically $M_m = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

occuring at $\omega_m = \omega_n\sqrt{1-2\xi^2}$

$$f_m = \frac{\omega_m}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-2\xi^2}$$

put $L = 0.1\,H$, $C = 0.1\,\mu F$, $\xi = 0.2$

$f_m = 1.526$ kHz (theoretical)
(close to practical value)

$$\frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times0.2\sqrt{1-0.2^2}} = 2.551$$ (close to 2.543)

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Calculations :

From the observation table,

$$\omega_m\text{-practical} = f_m\text{-practical}\cdot 2\pi$$
$$= 1.5 \times 2\pi \times 10^3 \text{ rad/s}$$
$$= 9424.78 \text{ rad/s}$$

$$\omega_m\text{-theoretical} = \omega_n\sqrt{1-2\xi^2}$$
$$= \frac{1}{\sqrt{LC}}\sqrt{1-2\xi^2}$$
$$= \frac{1}{\sqrt{0.1 \times 0.1 \times 10^{-6}}} \times \sqrt{1-2\times 0.2^2}$$
$$= 9591.66 \text{ rad/s.}$$

$$M_m\text{-practical} = 2.543.$$

$$M_m\text{-theoretical} = \left[(1-\omega_m^2 LC)^2 + (\omega_m RC)^2\right]^{-1/2}$$
$$= \left[(1-9591.66^2 \times 0.1^2 \times 10^{-6})^2 + (9591.66 \times 400 \times 0.1 \times 10^{-6})^2\right]^{-1/2}$$
$$= 2.551$$

$$(\Delta\phi)_m\text{-practical} = -1.282 \text{ rads}$$

$$(\Delta\phi)_m\text{-theoretical} = -\tan^{-1}\left(\frac{\omega_m RC}{1-\omega_m^2 LC}\right)$$
$$= -\tan^{-1}\left(\frac{9591.66 \times 400 \times 0.1 \times 10^{-6}}{1-9591.66^2 \times 0.1 \times 10^{-7}}\right)$$
$$= -1.365 \text{ rads}$$

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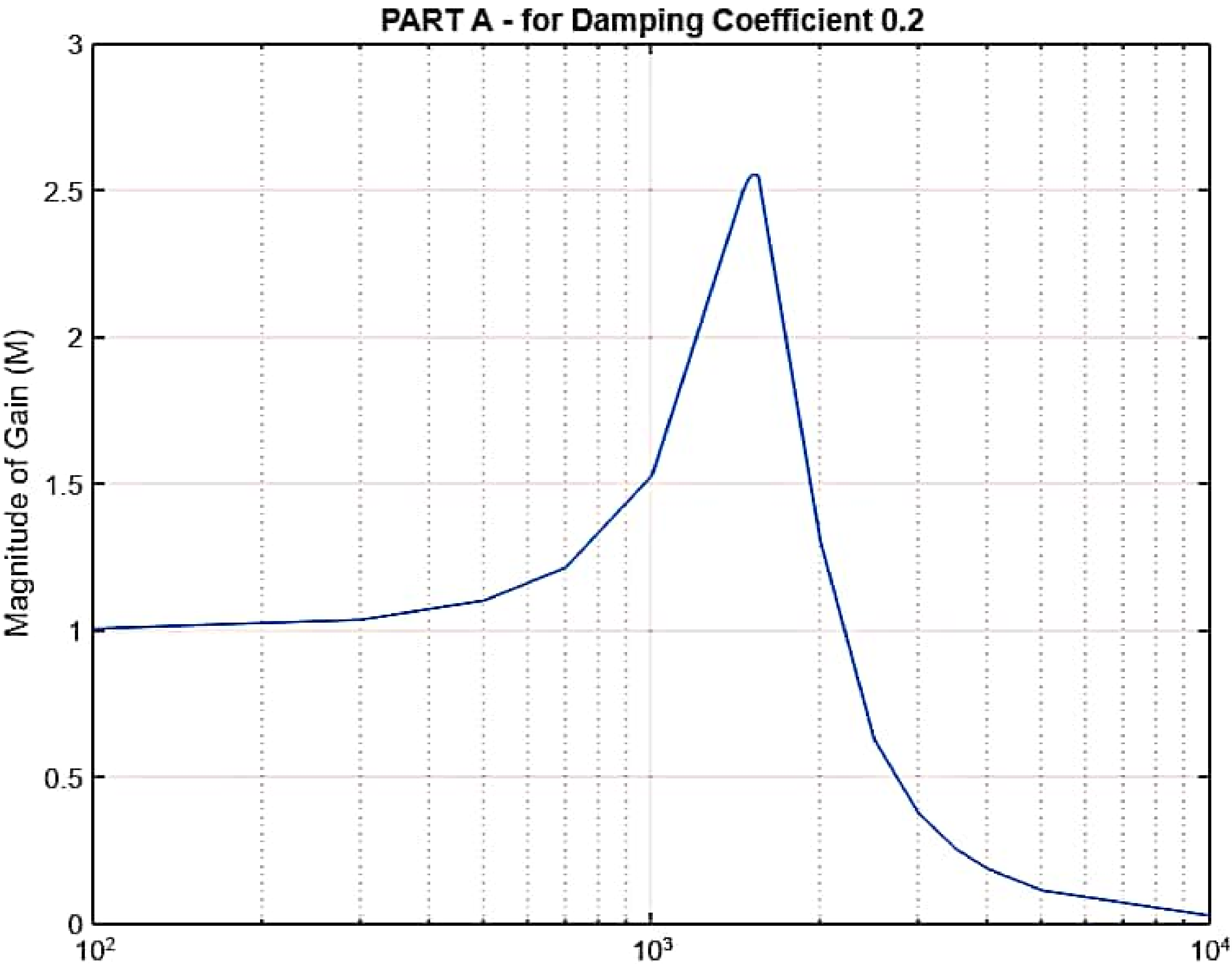
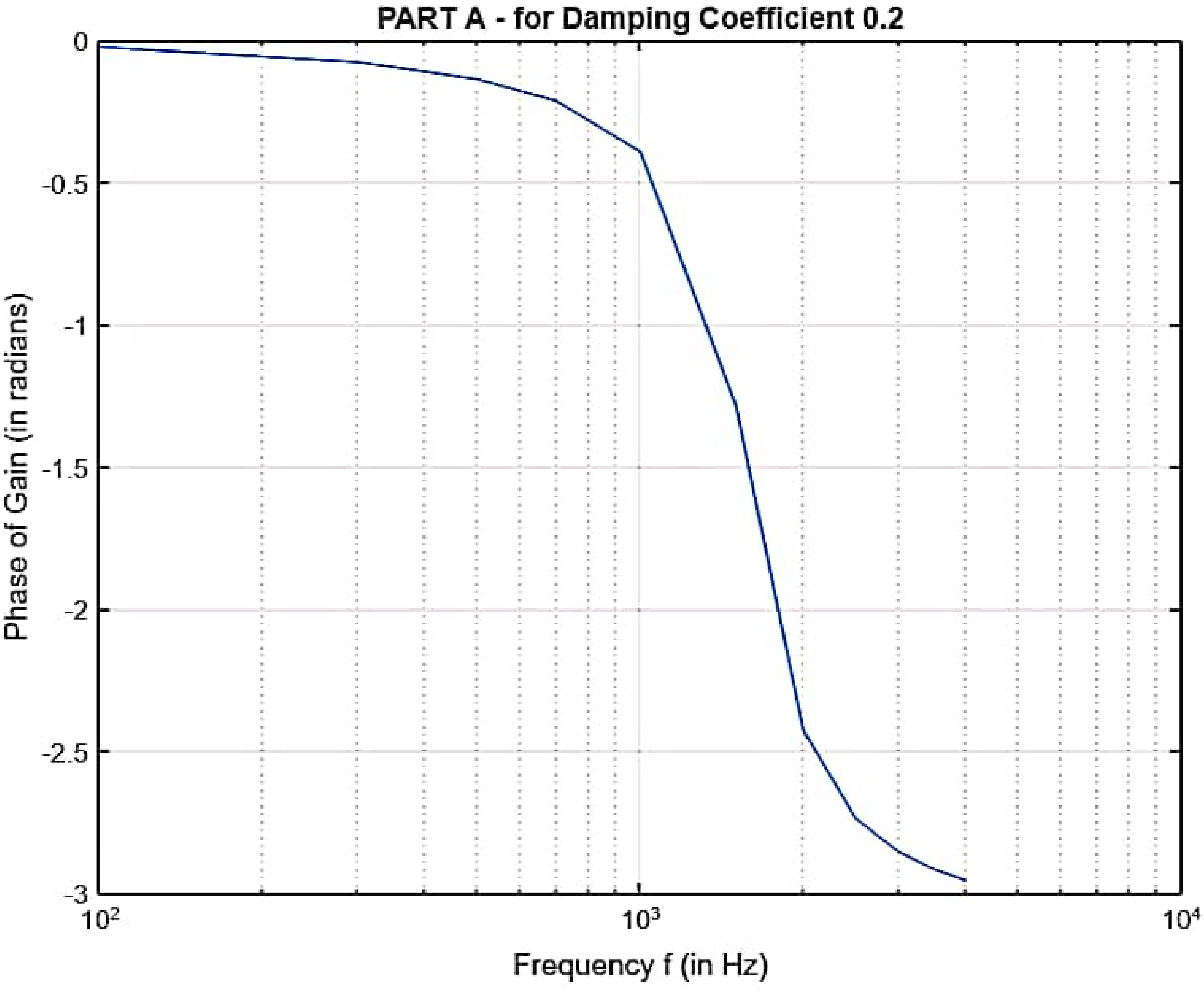
	Practical	Theoretical
ω_m	9424.78 rad/s	9591.66 rad/s
M_m	2.543	2.551
$(\Delta\phi)_m$	−1.282 rads	−1.365 rads.

Discussion : The practical and theoretical values are quite close. According to the theory, the frequency response should be similar to that of a band pass filter (BPF). According to the observations, the gain first increases, reaches maximum and then decreases. This behaviour indeed is similar to a BPF.

Matlab Plots : The relavant plots for this part follow from the next page.


PART A - for Damping Coefficient 0.2
Magnitude of Gain (M)
Frequency f (in Hz)
3
2.5
2
1.5
1
0.5
0
10^2
10^3
10^4



PART A - for Damping Coefficient 0.2
0
-0.5
-1
-1.5
-2
-2.5
-3
Phase of Gain (in radians)
10^2
10^3
10^4
Frequency f (in Hz)

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Part B :

Simulink Procedure :

① follow steps 1 to 5 as given in part A.
② Choose $\xi = 0.5$ and calculate the corresponding value of resistance and set it.
③ follows steps 7 and 8 as given in part A.

Observation Table :

The observations for this part are given on the next page.

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Observation Table:

when $\xi = 0.5$ $(R = 1000\,\Omega)$

f (Hz)	V_R (V)	V_L (V)	V_C (V)	gain M	$\Delta\phi$ (radians)
100	0.063	0.039	1.002	1.002	−0.063
300	0.192	0.120	1.018	1.018	−0.193
500	0.329	0.207	1.048	1.048	−0.335
700	0.479	0.301	1.088	1.088	−0.500
1k	0.720	0.452	1.146	1.146	−0.804
1.5k	0.993	0.624	1.054	1.054	−1.452
2k	0.908	0.571	0.723	0.723	−2.002
2.5k	0.731	0.459	0.465	0.465	−2.321
3k	0.594	0.373	0.315	0.315	−2.506
1.2k	0.868	0.545	1.151	1.151	−1.051

M_m at 1.2kHz (from table)

Theoretically, $\omega_m = \omega_n\sqrt{1-2\xi^2}$

$$f_m = \frac{\omega_m}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-2\xi^2}$$

put the values and get $f_m = 1.125$ kHz (theoretical)

(close to the practical value)

$$M_{m\text{-theoretical}} = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times0.5\sqrt{1-0.5^2}}$$

$= 1.155$ (close to 1.151)

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Calculations:

From the observation table,

ω_m-practical $= f_m$-practical 2π

$= 1.2 \times 2\pi \times 10^3$ rad/s

$= 7539.82$ rad/s.

ω_m-theoretical $= \omega_n \sqrt{1-2\xi^2}$

$= \frac{1}{\sqrt{LC}} \sqrt{1-2\xi^2}$

$= \frac{1}{\sqrt{0.1 \times 0.1 \times 10^{-6}}} \times \sqrt{1-2\times 0.5^2}$

$= 7071.07$ rad/s.

M_m-practical $= 1.151$

M_m-theoretical $= \left[(1-\omega_m^2 LC)^2 + (\omega_m RC)^2\right]^{-1/2}$

$= \left[(1 - 7071.07^2 \times 0.1^2 \times 10^{-6})^2 + (7071.07 \times 1000 \times 10^{-7})^2\right]^{-1/2}$

$= 1.155$

$(\Delta\phi)_m$ (practical) $= -1.051$ rads.

$(\Delta\phi)_m$-theoretical $= -\tan^{-1}\left(\frac{\omega_m RC}{1-\omega_m^2 LC}\right)$

$= -\tan^{-1}\left(\frac{7071.07 \times 1000 \times 10^{-7}}{1 - 7071.07^2 \times 0.1 \times 10^{-7}}\right)$

$= -1.500$ rads.

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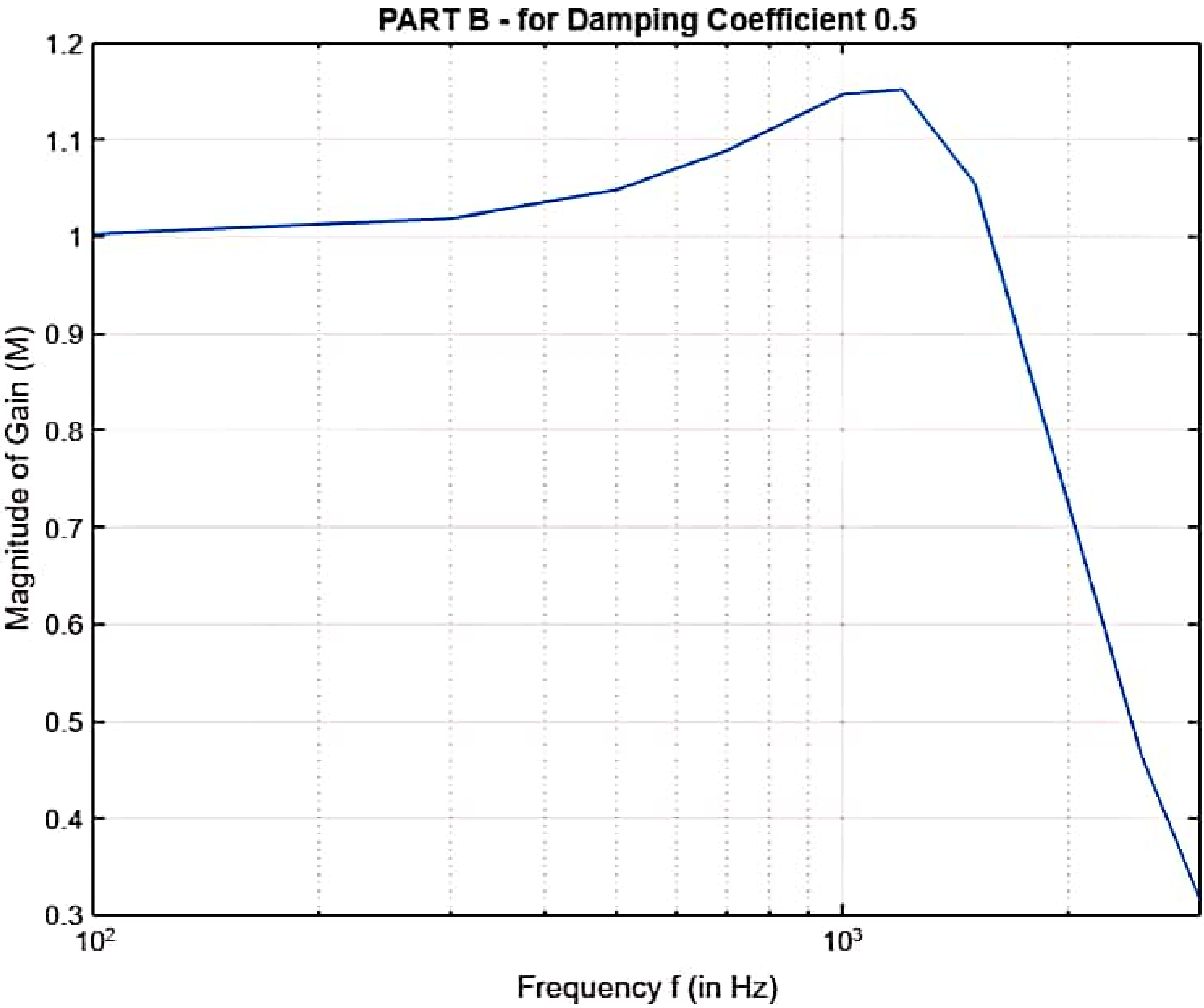
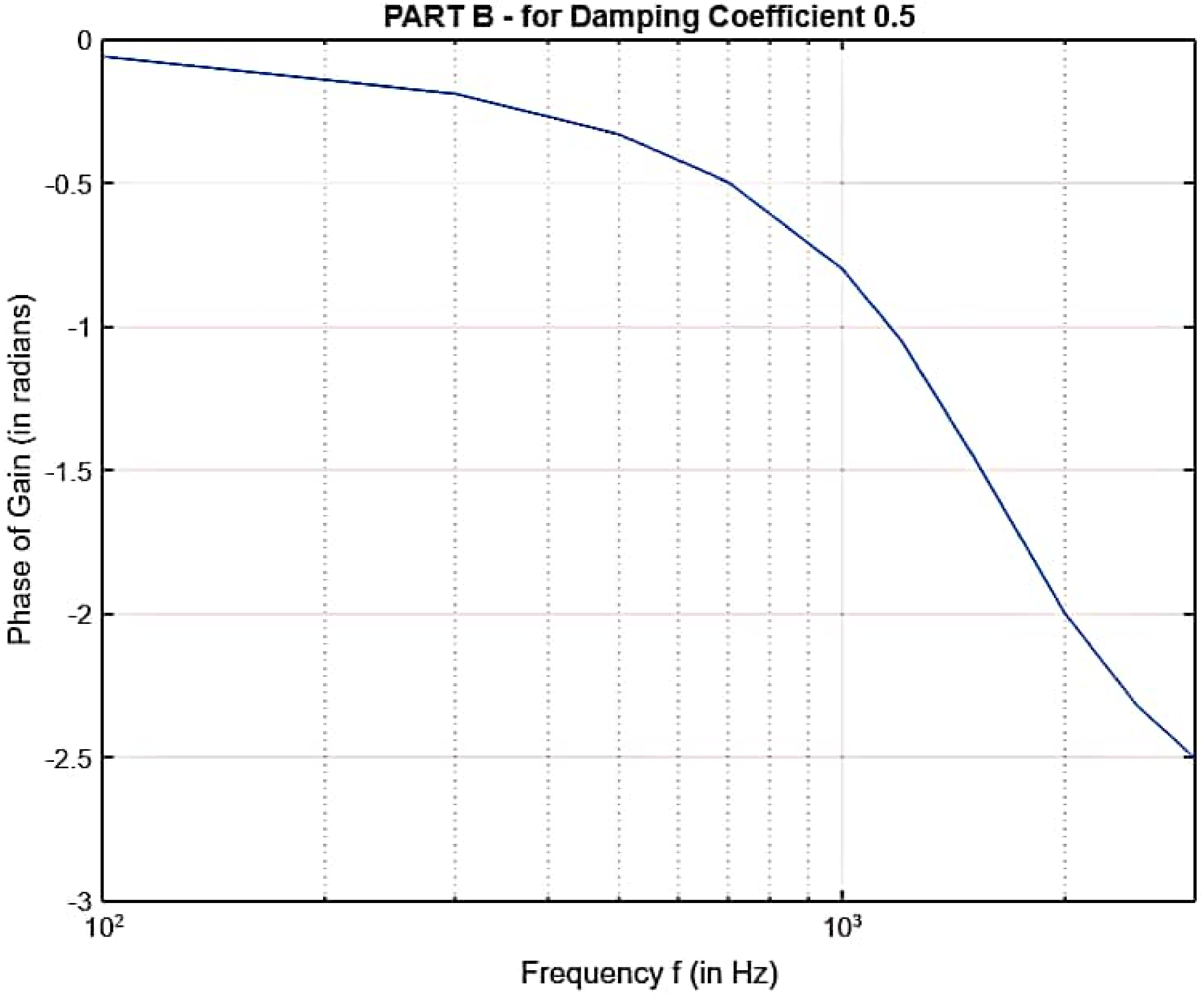
	Practical	Theoretical
ω_m	7071.07 rad/s	7539.82 rad/s
M_m	1.155	1.151
$(\Delta\phi)_m$	-1.051 rads	-1.500 rads

Discussions: The theoretical and practical values are quite close. The error in the values of $(\Delta\phi)_m$ and ω_m is though quite larger than in M_m. The observations recorded again show behaviour similar to that of a BPF in terms of gain. The error can be reduced by taking a large number of points/observations around the global maxima.

Matlab Plots: The relavant plots for this part follow from the next page.


PART B - for Damping Coefficient 0.5
Magnitude of Gain (M)
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
10^2
10^3
Frequency f (in Hz)


PART B - for Damping Coefficient 0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
Phase of Gain (in radians)
10^2
10^3
Frequency f (in Hz)

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Part C :

Simulink Procedure :

1. Follow steps 1 to 5 as given in part A.
2. Choose $\xi = 0.7$ and calculate the corresponding value of resistance and set it.
3. Follows steps 7 and 8 as given in part A.

Observation Table :

The observations for this part are given on the next page.

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Observation Table:

when $\xi = 0.7$ ($R = 1400\Omega$)

f (Hz)	V_R (V)	V_L (V)	V_C (V)	gain M	$\Delta\phi$ (radians)
100	0.088	0.055	1.000	1.000	−0.088
300	0.264	0.166	1.000	1.000	−0.267
500	0.441	0.276	0.997	0.997	−0.454
700	0.607	0.381	0.985	0.985	−0.652
1k	0.824	0.517	0.937	0.937	−0.968
1.5k	0.996	0.626	0.755	0.755	−1.486
2k	0.950	0.597	0.540	0.540	−1.890
2.5k	0.832	0.523	0.378	0.378	−2.158
3k	0.719	0.452	0.272	0.272	−2.339
3.5k	0.626	0.393	0.203	0.203	−2.465
5k	0.444	0.279	0.101	0.101	−2.681

M_m at 200Hz (from table)
(M remains nearly constant from 100 Hz to 300 Hz hence the average was taken).

Theoretically, $\omega_m = \omega_n\sqrt{1-2\xi^2}$

$$f_m = \frac{\omega_m}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-2\xi^2}$$

put the values and get $f_m = 225$Hz (theoretical)
(close to practical value).

$$M_m\text{(theoretical)} = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.000$$
(exactly equal to practical value)

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Calculations :

From the observation table,

ω_m- practical $= f_m$- practical 2π

$= 200 \times 2\pi$

$= 1256.64$ rad/s.

ω_m- theoretical $= \omega_n \sqrt{1-2\xi^2}$

$= \frac{1}{\sqrt{LC}} \sqrt{1-2\xi^2}$

$= \frac{1}{\sqrt{0.1 \times 0.1 \times 10^{-6}}} \times \sqrt{1-2\times 0.7^2}$

$= 1414.21$ rad/s.

M_m- practical $= 1.000$

M_m- theoretical $= \left[(1-\omega_m^2 LC)^2 + (\omega_m RC)^2\right]^{-1/2}$

$= \left[(1-1414.21^2 \times 0.1 \times 10^{-7})^2 + (1414.21 \times 1400 \times 10^{-7})^2\right]^{-1/2}$

$= 1.020$

$(\Delta\phi)_m$- practical $= -0.267$ rads

$(\Delta\phi)_m$- theoretical $= -\tan^{-1}\left(\frac{\omega_m RC}{1-\omega_m^2 LC}\right)$

$= -\tan^{-1}\left(\frac{1414.21 \times 1400 \times 10^{-7}}{1-1414.21^2 \times 10^{-8}}\right)$

$= -0.199$ rads

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	Practical	Theoretical
ω_m	1256.64 rad/s	1414.21 rad/s
M_m	1.000	1.020
$(\Delta\phi)_m$	−0.267 rads	−0.199 rads

Discussions: The formulae discussed in the theory related to ω_m and M_m are only valid when ξ is less than $1/\sqrt{2}$ (0.707). In this part, the value of ξ was very close to this critical value and hence the frequency response was almost monotonic because of less value of ω_m. Experimentally, due to the constraint of the number of decimal places that can displayed on the display block of Simulink, the gain appeared to be constant in the range of 100 Hz to 300 Hz. In this case though it is confirmed that f_m lies b/w 100 Hz and 300 Hz, its exact (or even close) value cannot be experimentally determined and hence the average 200Hz was considered. It is perhaps due to this assumption and the closeness of ξ value to the critical value, the gap b/w theoretical and the practical

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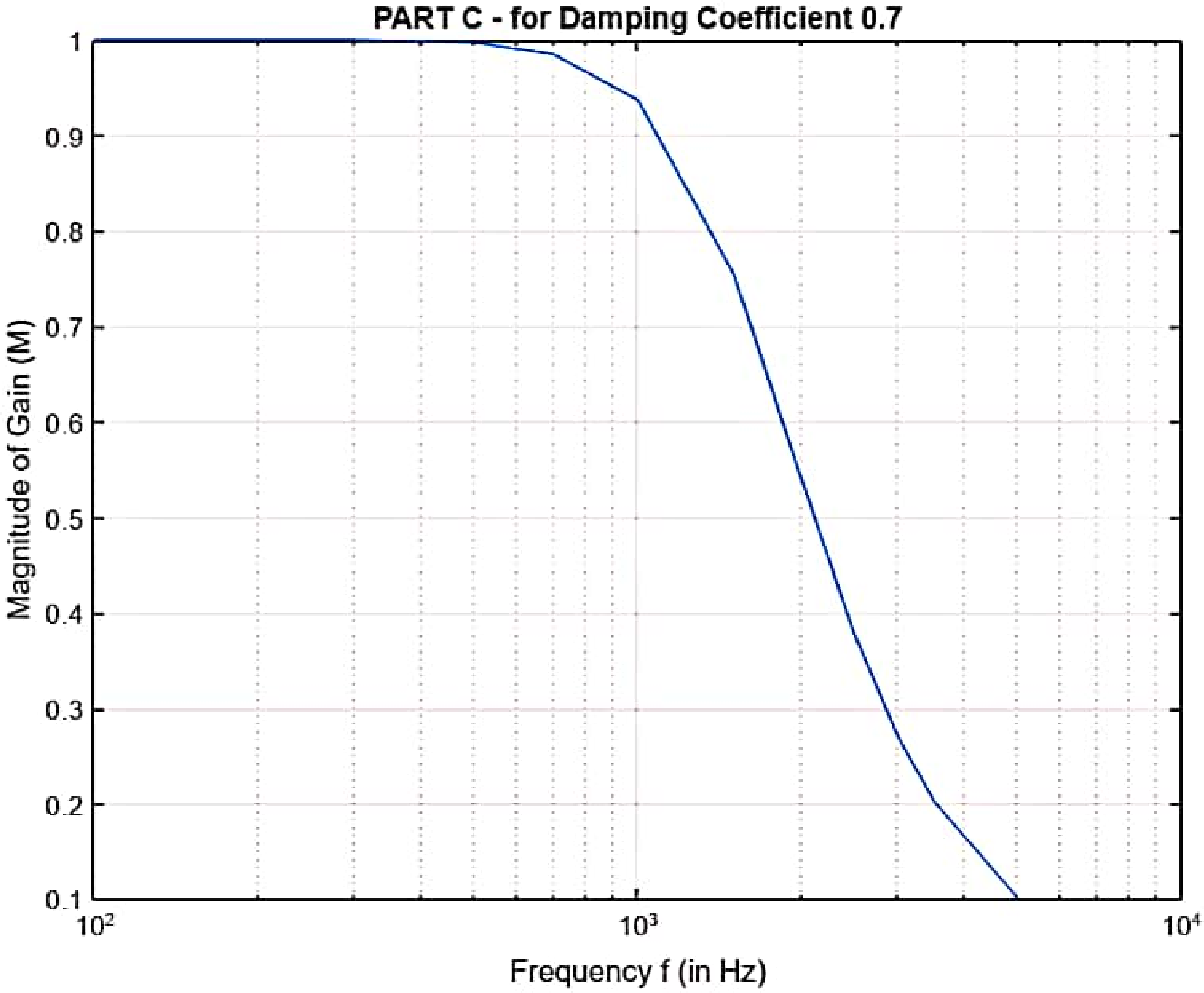
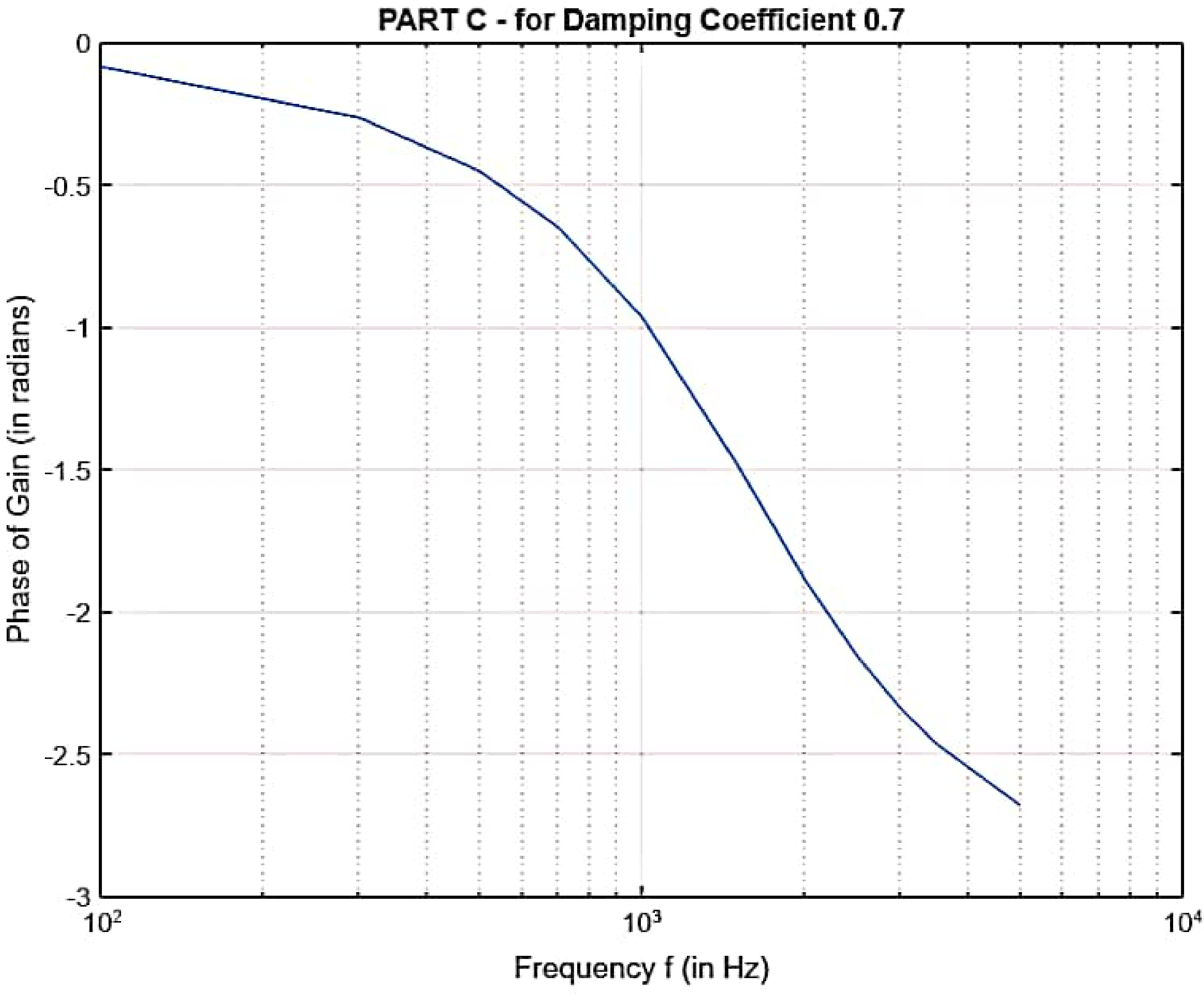
values is quite large, especially for $(\Delta\phi)_m$ (11.14% error in the value of ω_m).

Matlab Plots : The relavant plots for this part follow from the next page.


PART C - for Damping Coefficient 0.7
Magnitude of Gain (M)
Frequency f (in Hz)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
10^2
10^3
10^4


PART C - for Damping Coefficient 0.7
Phase of Gain (in radians)
0
-0.5
-1
-1.5
-2
-2.5
-3
10^2
10^3
10^4
Frequency f (in Hz)

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Part D :

Simulink Procedure :

1. Follow steps 1 to 5 as given in part A.
2. Choose $\xi = 0.95$ and calculate the corresponding value of resistance and set it.
3. Follows steps 7 and 8 as given in part A.

Observation Table :

The observations for this part are given on the next page.

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Observation Table:

when $\xi = 0.95$ $(R = 1900\Omega)$

f (Hz)	V_R (V)	V_L (V)	V_C (V)	gain M	$\Delta\phi$ (radians)
100	0.119	0.004	0.997	0.997	−0.119
300	0.348	0.035	0.972	0.972	−0.356
500	0.552	0.091	0.925	0.925	−0.585
700	0.719	0.167	0.861	0.861	−0.803
1k	0.892	0.295	0.747	0.747	−1.101
1.5k	0.998	0.495	0.558	0.558	−1.508
2k	0.972	0.643	0.407	0.407	−1.808
2.5k	0.897	0.742	0.301	0.301	−2.027
3k	0.814	0.808	0.227	0.227	−2.189
4k	0.668	0.884	0.142	0.142	−2.409
5k	0.558	0.923	0.094	0.094	−2.549

* Monotonic behaviour / increase of M as the frequency is decreased.
* No local maxima observed for non-zero frequencies.

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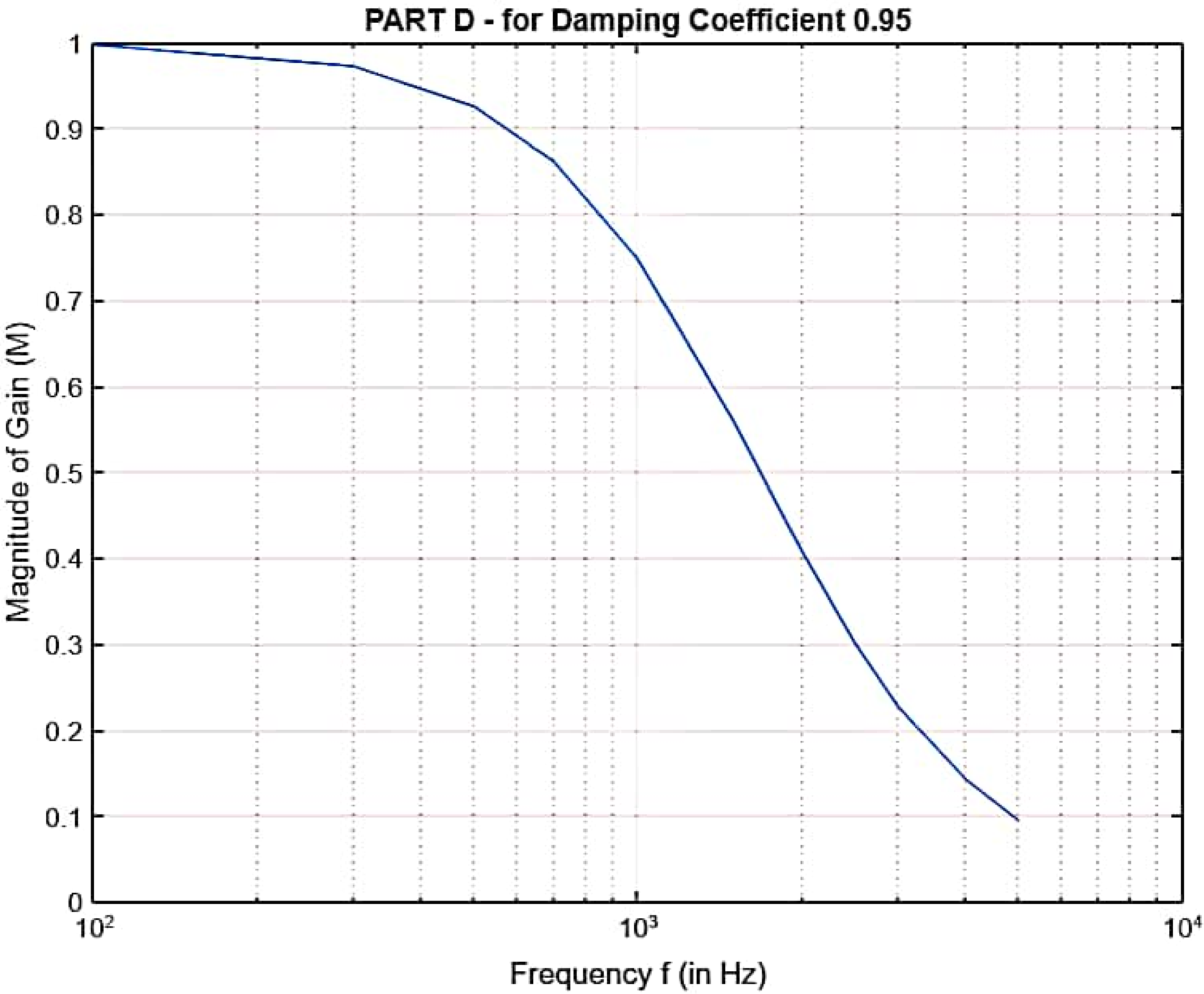
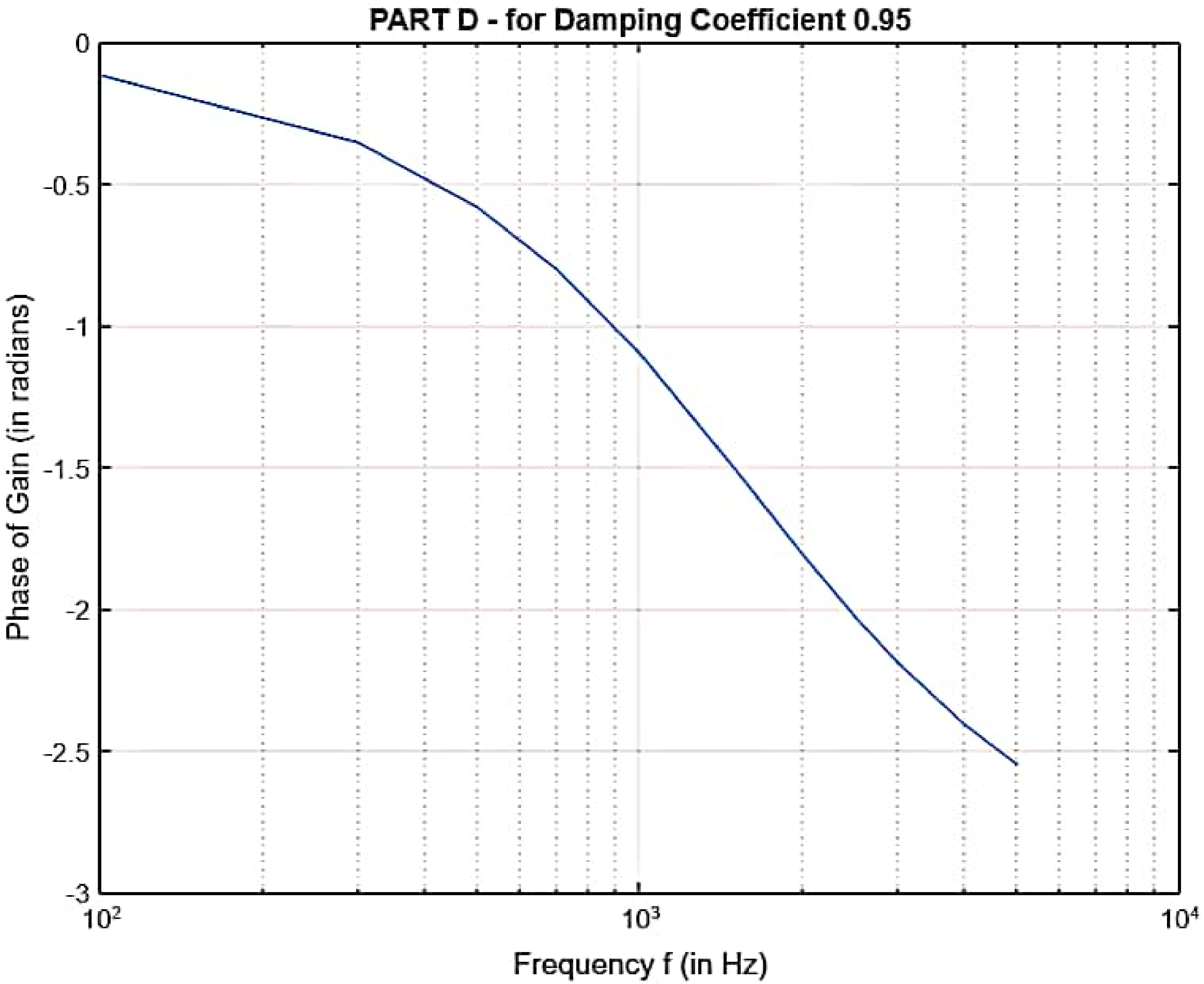
Discussion: Since in this case, ξ has a value of 0.95 (> 0.707); there occurs no maxima for a non zero frequency from the observat^{ly} table and from the graph it is clear that the gain of the network decreases monotonically as frequency is increased. The phase difference $\Delta\phi$ however shows the same behaviour as in other parts, i.e. its magnitude increases monotonically as frequency is increased from very small value to larger values.

Matlab Plots: The relavant plots for this part follow from the next page.



PART D - for Damping Coefficient 0.95
Magnitude of Gain (M)
Frequency f (in Hz)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^2
10^3
10^4


PART D - for Damping Coefficient 0.95
Phase of Gain (in radians)
0
-0.5
-1
-1.5
-2
-2.5
-3
10^2
10^3
10^4
Frequency f (in Hz)

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Part E :

Simulink Procedure :

① Follow steps 1 to 5 as given in part A.
② Choose E_g = 1.5 and calculate the corresponding value of resistance and set it.
③ Follows steps 7 and 8 as given in part A.

Observation Table :

The observations for this part are given on the next page.

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Observation Table :

when $E_g = 1.5$ (R = 3000Ω)

f (Hz)	V_R (V)	V_L (V)	V_C (V)	gain M	Δφ (radians)
100	0.186	0.004	0.986	0.986	−0.187
300	0.506	0.032	0.894	0.894	−0.530
500	0.723	0.076	0.767	0.767	−0.808
700	0.853	0.125	0.647	0.647	−1.022
1k	0.952	0.199	0.505	0.505	−1.260
1.5k	1.000	0.314	0.353	0.353	−1.531
2k	0.988	0.414	0.262	0.262	−1.723
2.5k	0.955	0.500	0.203	0.203	−1.872
3k	0.912	0.573	0.161	0.161	−1.994
4k	0.817	0.685	0.108	0.108	−2.184
5k	0.728	0.762	0.077	0.077	−2.325

* Monotonic decrease of gain as frequency is increased.
* No local maxima observed for non-zero frequencies.

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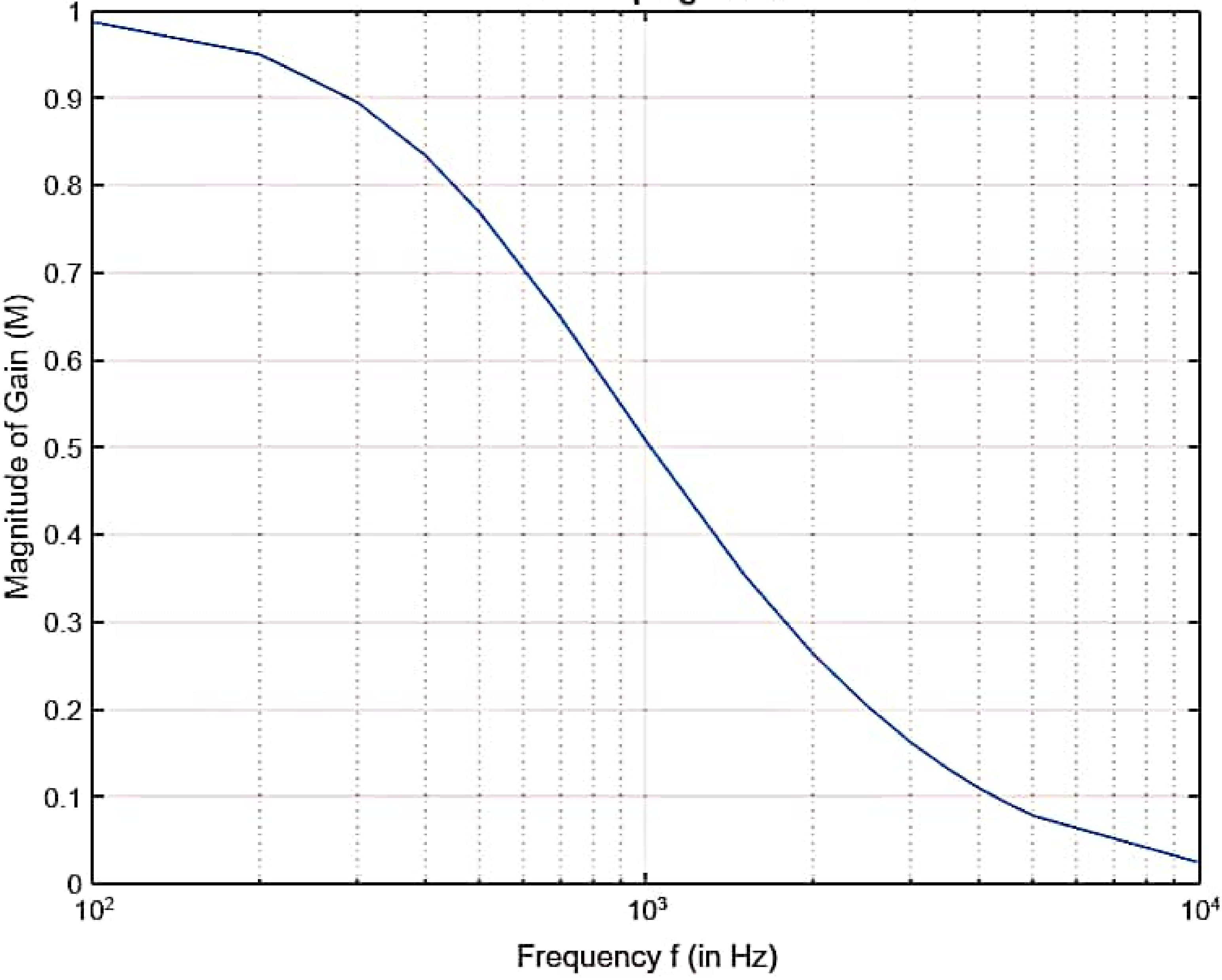
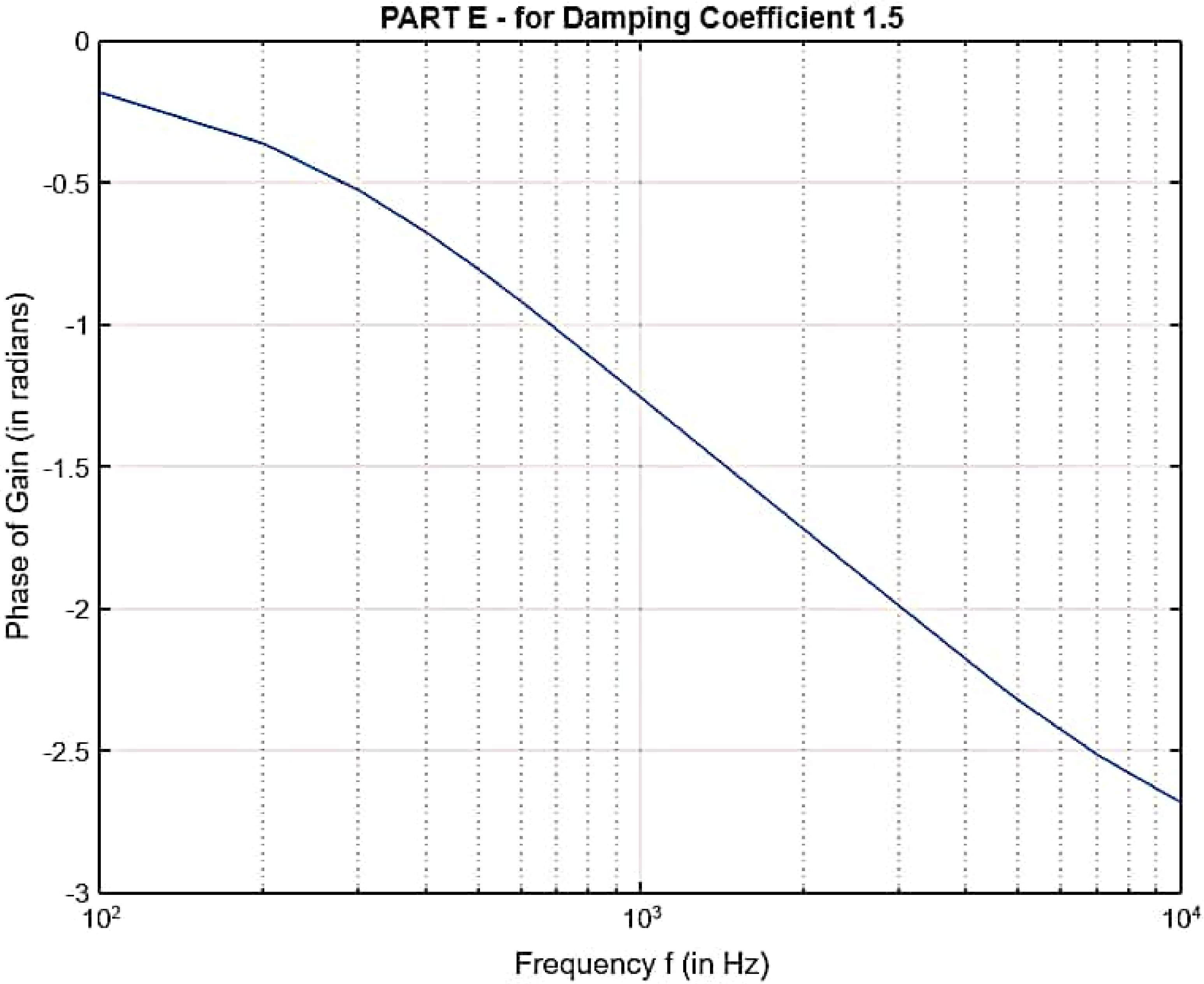
Discussion: Since in this case, ξ has a value of 1.50 (> 0.707), there occurs no maxima for a non zero frequency from the observat^y table and from the graph it is clear that the gain of the network decreases monotonically as frequency is increased. The phase difference $\Delta\phi$ however shows the same behaviour as in other parts, i.e. its magnitude increases monotonically as frequency is increased from very small value to larger values.

Matlab Plots: The relavant plots for this part follow from the next page.
One thing that should be noted from the Magnitude plots of part D and E is that though both of them are strictly decrease the initial rate of decrease in part E is more than in part D.

PART E - for Damping Coefficient 1.5




PART E - for Damping Coefficient 1.5
0
-0.5
-1
-1.5
-2
-2.5
-3
Phase of Gain (in radians)
10^2
10^3
10^4
Frequency f (in Hz)

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Final Discussion : The results shown by this experiment conducted on Simulink were very similar to the theoretical results and also the ones expected from a real physical experiment. In the "Calculations" section for all the parts it was found at that the theoretical and practical values of maximum gain (M_m) and angular frequencies corresponding to this gain (ω_m) are close. In some parts (like when $\xi = 0.7$) the error was more than in other parts (like for $\xi = 0.2$). Not only the gain but also the phase difference $\Delta\phi$ behaved as predicted in theory. For all the values of ξ, the magnitude / absolute value of $\Delta\phi$ increased monotonically. The rate of this increase however was observed to get reduced at high frequencies. This is also in agreement with the theory that $\Delta\phi$ should approach $-\pi$ (-3.141) radians as the frequency approaches infinity.

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As far as the applications of frequency response experiment are concerned not only for a series RLC circuit but in general, they are widespread. In amplifiers (BJT, MOSFET), the frequency response knowledge is important to obtain the maximum gain from it. outside the mid-frequency region in BJT, the gain reduces gradually, hence reducing the performance of the amplifier.

For LTI systems, once its frequency response has been measured, its characteristics can be approximated with arbitrary accuracy with a digital filter. Besides various figures related to frequency responses like 3dB frequency, low and high cut-off bandwidth and quality factor are very important for passive filters (low-pass, high-pass, band-pass, band-block) and other devices. Since the maximum power that can be obtained from an RLC circuit is directly related to the operating frequency, in radios and FM This functionality is very important to tune the channel.