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# Experiment 1-B
# Transient Response of RLC Circuit with Initial Conditions.

**Aim:** To determine the transient response of an RLC network in terms of the parameters $\tau$, $\xi$, $\omega$, $\omega_n$ and the initial conditions $i_L(0^-)$, $v_C(0^-)$.

**Theory:** $\xi$ is the normalised damping constant defined as -

$$\xi = \frac{\alpha}{\omega_0}$$

where $\alpha$ is $R/2L$ and $\omega_0$ is the natural frequency $1/\sqrt{LC}$.

Under damping conditions, the frequency of oscillation drops down to $\omega_0\sqrt{1-\xi^2}$ where $\xi$ is damping constant of that network.

Theoretically, RLC networks with non zero initial conditions can be solved by formulating a differential eq^n based on the I-V characteristics of the components R, L, C.

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$$V_R = I_R R \quad \text{(ohm's law)}$$

$$i_c(t) = C \frac{dv_c}{dt}$$

$$v_L(t) = L \frac{di_L}{dt}$$

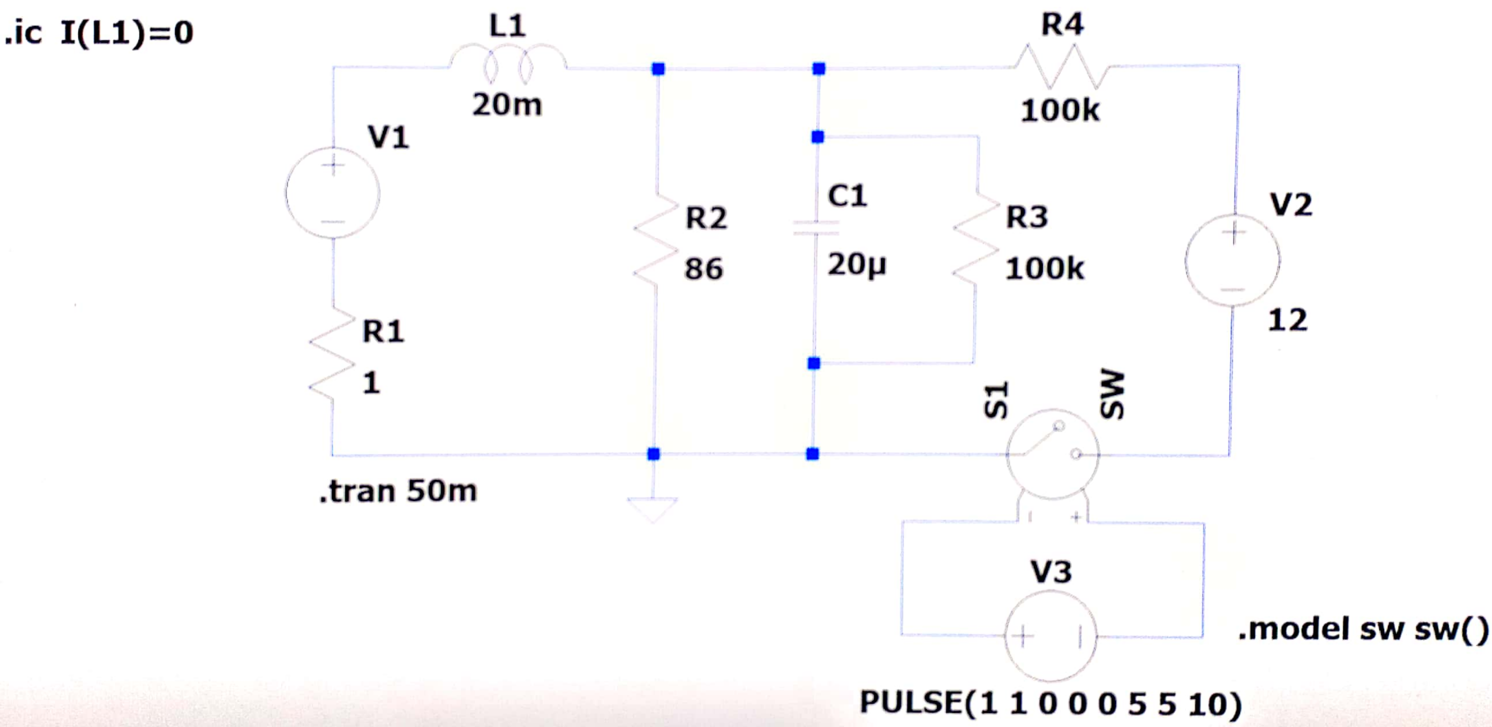
Besides, the RLC network can be transform to an s-domain network and all the components R, L, C can be treated as nothing but resistances which follow ohm's law but have their values in s-domain.
For a circuit with non-zero initial conditions however a constant voltage or current source might have to be imagined to write the resistances / impedances of L and C in s-domain as Ls and 1/Cs respectively.
These techniques will be studied more deeply in the calculations part.

## LTSpice Circuit Diagrams:

The circuit diagrams for this experiment follow from the next page.

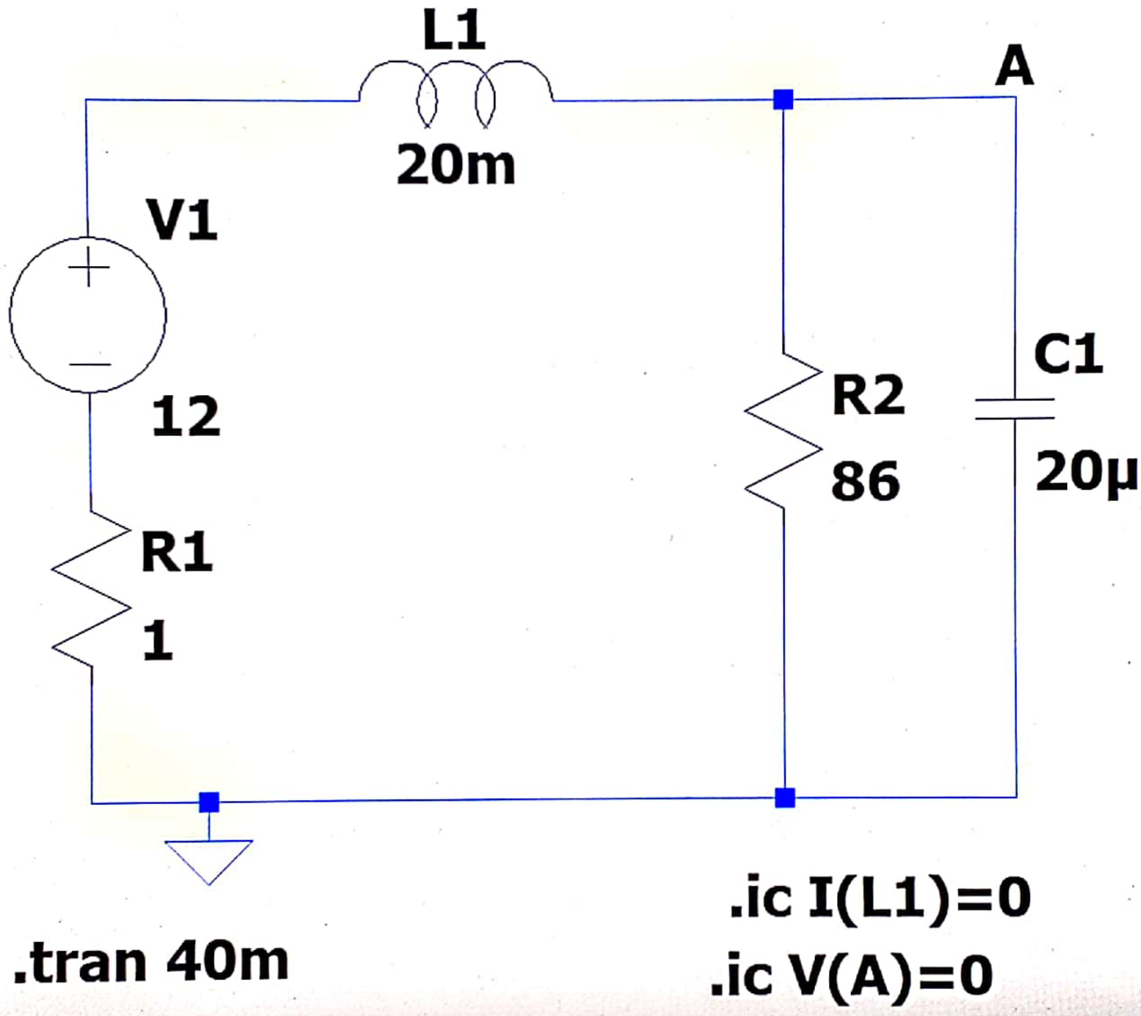
## EXPERIMENT 02 - PART B

## CIRCUIT 01



# EXPERIMENT 02 - PART B

## CIRCUIT 02



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## Observations :

**Part 01** - The voltage source $V_1$ was switched on and the transient was captured across the 20μF capacitor.
The waveform is shown on the next page.

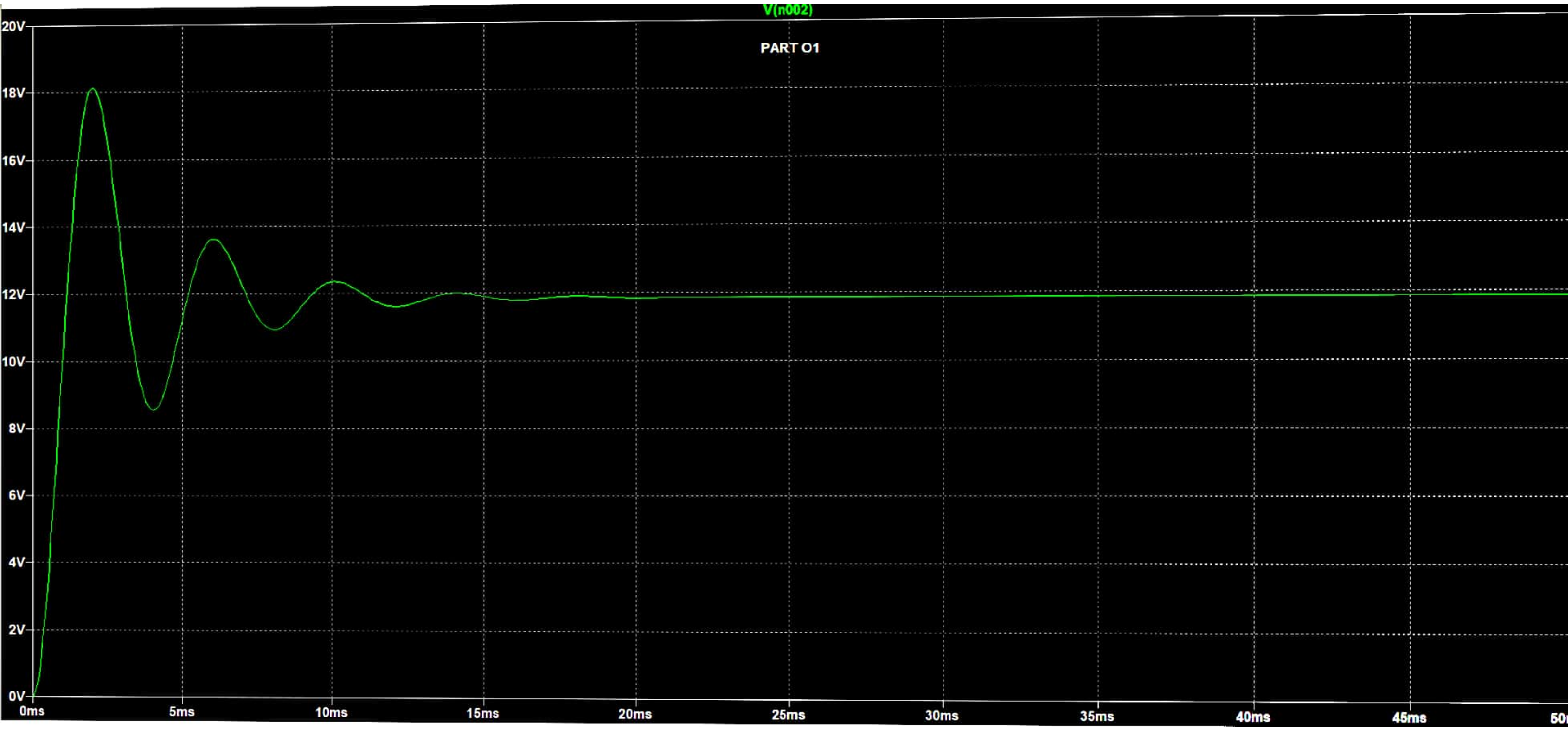
$\tau$ (time constant) = 6.12ms.

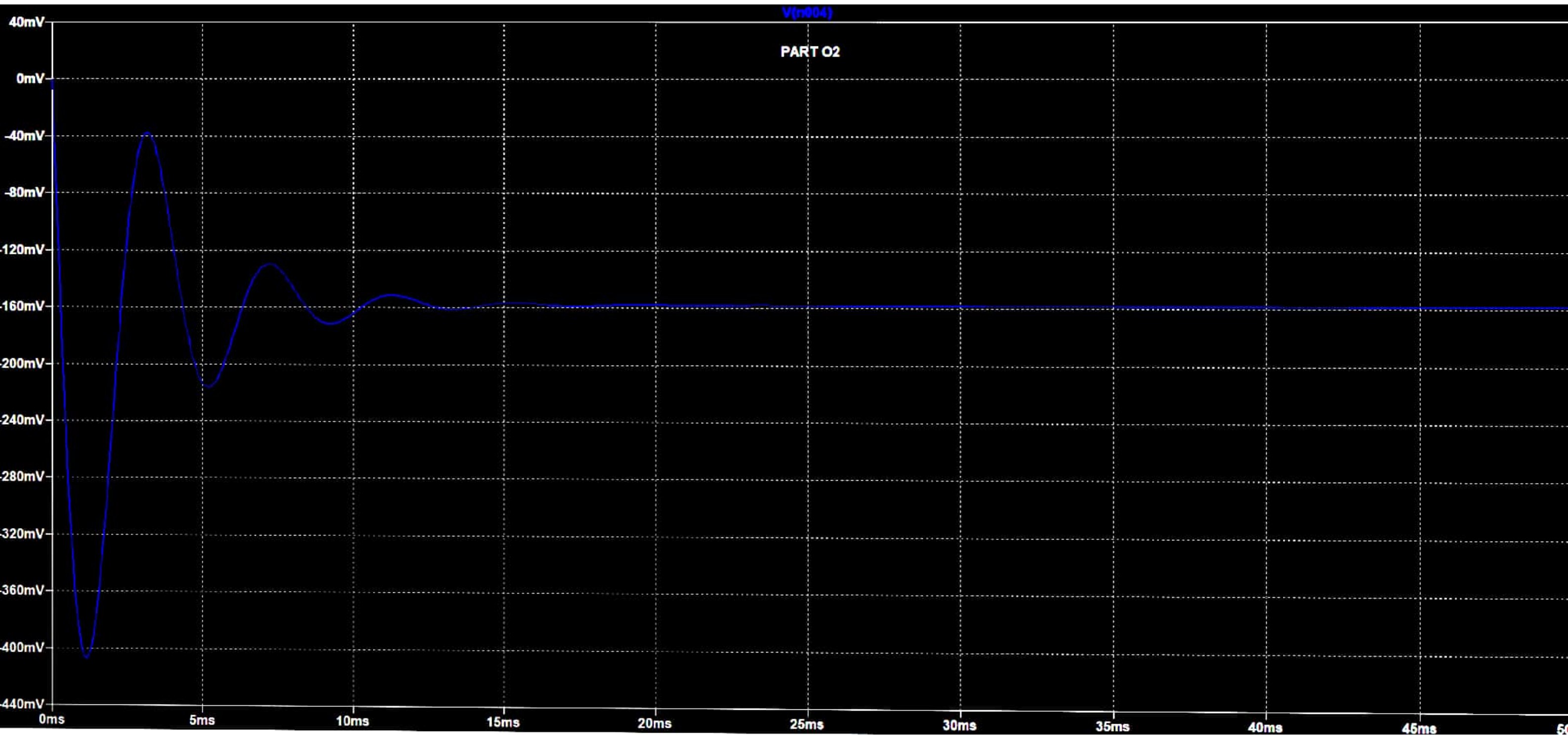
**Part 02** - The value of $R_2$ was changed to 75 Ω to adjust the current through 20mH inductor to less than 0.75A.
The transient of the inductor current was recorded via capturing the output across 1Ω shunt resistance.
The waveform is shown on the next page.

$\tau$ (time constant) = 3.31ms

**Part 03** - The voltage across the capacitor was captured for different values of $v_c(0^-)$. (C = 20 μF).

| $v_c(0^-)$ | Max overshoot | f(Hz) | Time Constant | $v_c(\infty)$ |
|---|---|---|---|---|
| 12V | +3.28V | 251.26 | 2.59ms | 11.86V |
| 5V | +163.06V | 248.14 | 2.91ms | 11.86V |
| 2.5V | +222.45V | 245.70 | 3.94ms | 11.86V |


V(n002)
PART O1
20V
18V
16V
14V
12V
10V
8V
6V
4V
2V
0V
0ms
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms


V(n004)
PART O2
40mV
0mV
-40mV
-80mV
-120mV
-160mV
-200mV
-240mV
-280mV
-320mV
-360mV
-400mV
-440mV
0ms
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms

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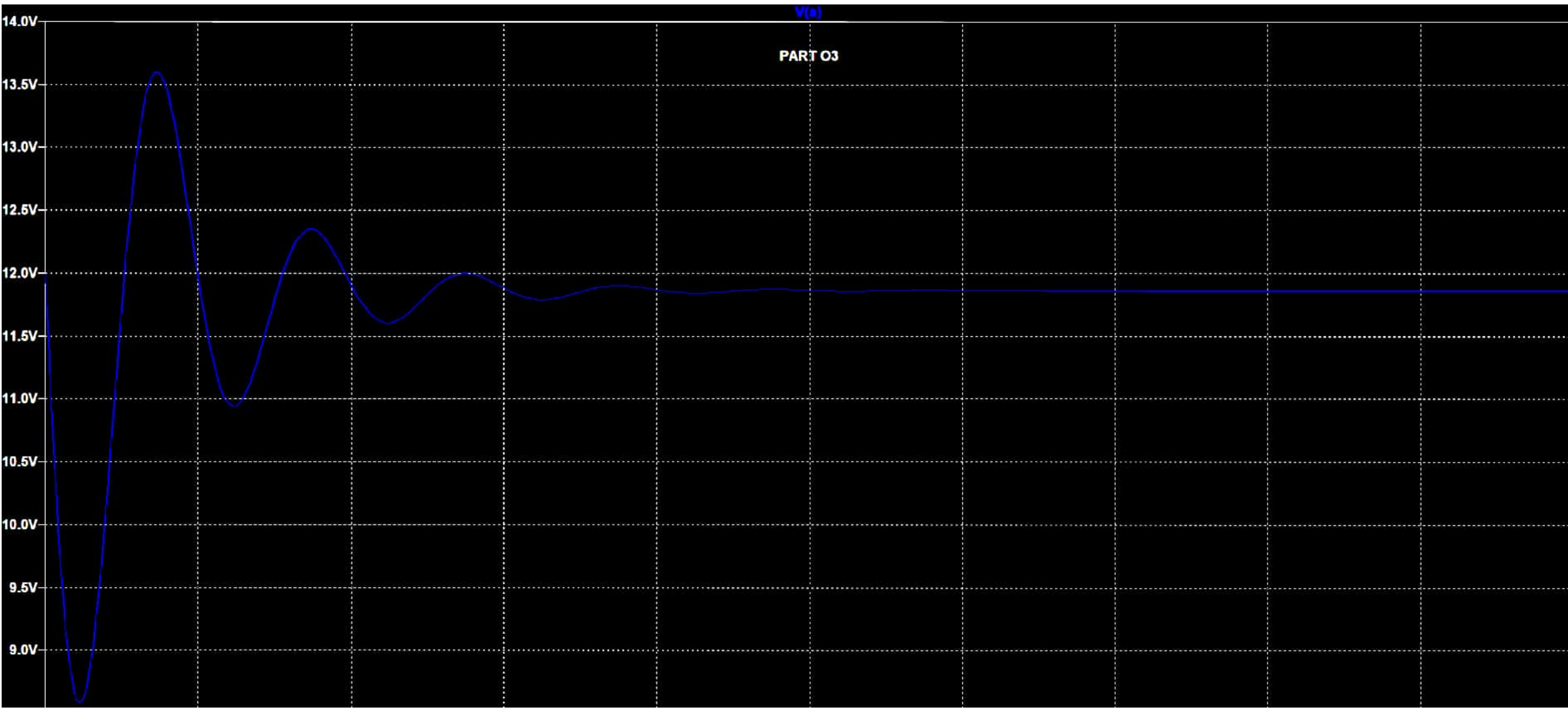
Part 04 The voltage across the capacitor was captured for different capacitances ( $v_c(0^-) = 5V$, $R_2 = 86\Omega$).

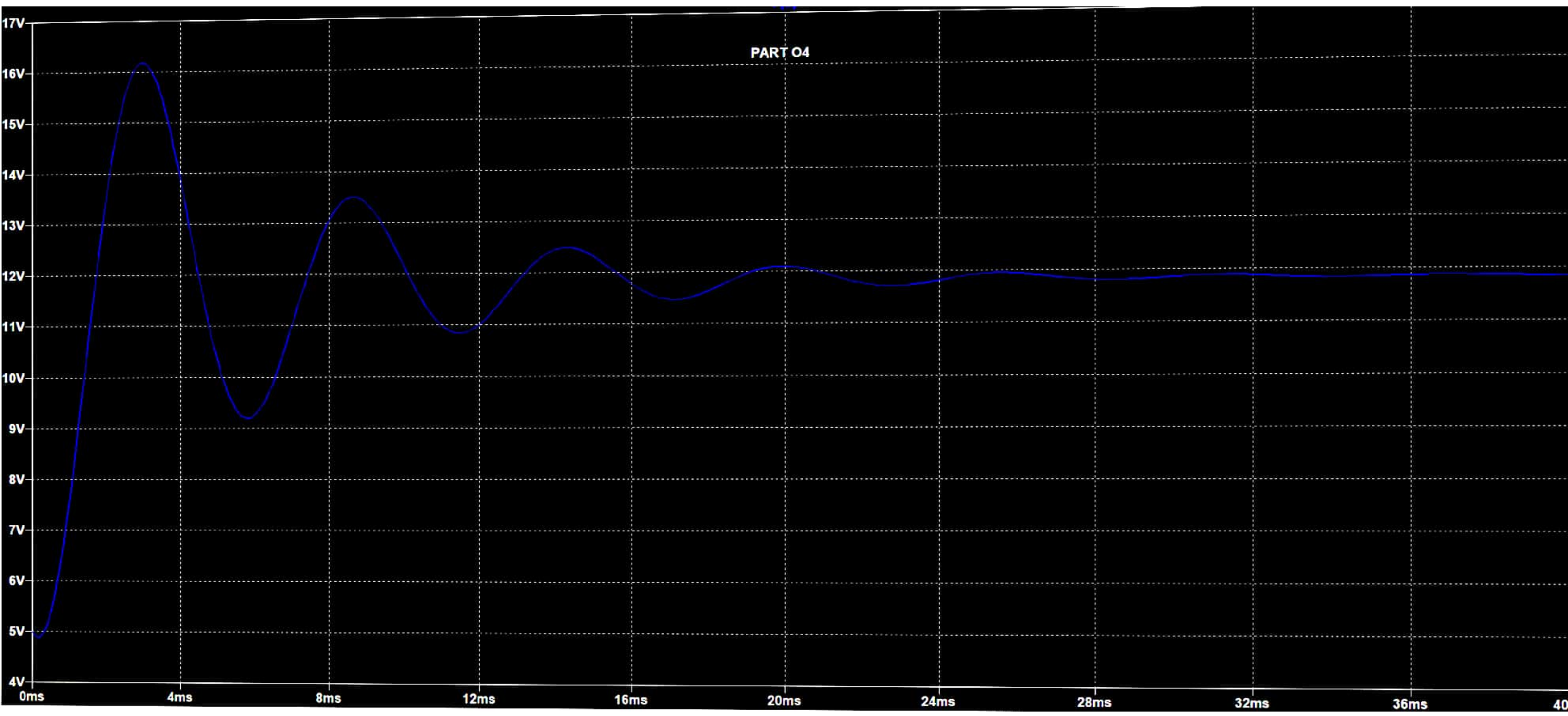
| C | Max Overshoot | f (Hz) | Time Constant | $v_c(\infty)$ |
|---|---|---|---|---|
| 40 μF | + 4.31 V | 173.61 | 8.62 ms | 11.86 V |
| 60 μF | + 4.56 V | 142.25 | 10.47 ms | 11.86 V |
| 100 μF | + 4.78 V | 111.23 | 17.91 ms | 11.86 V |

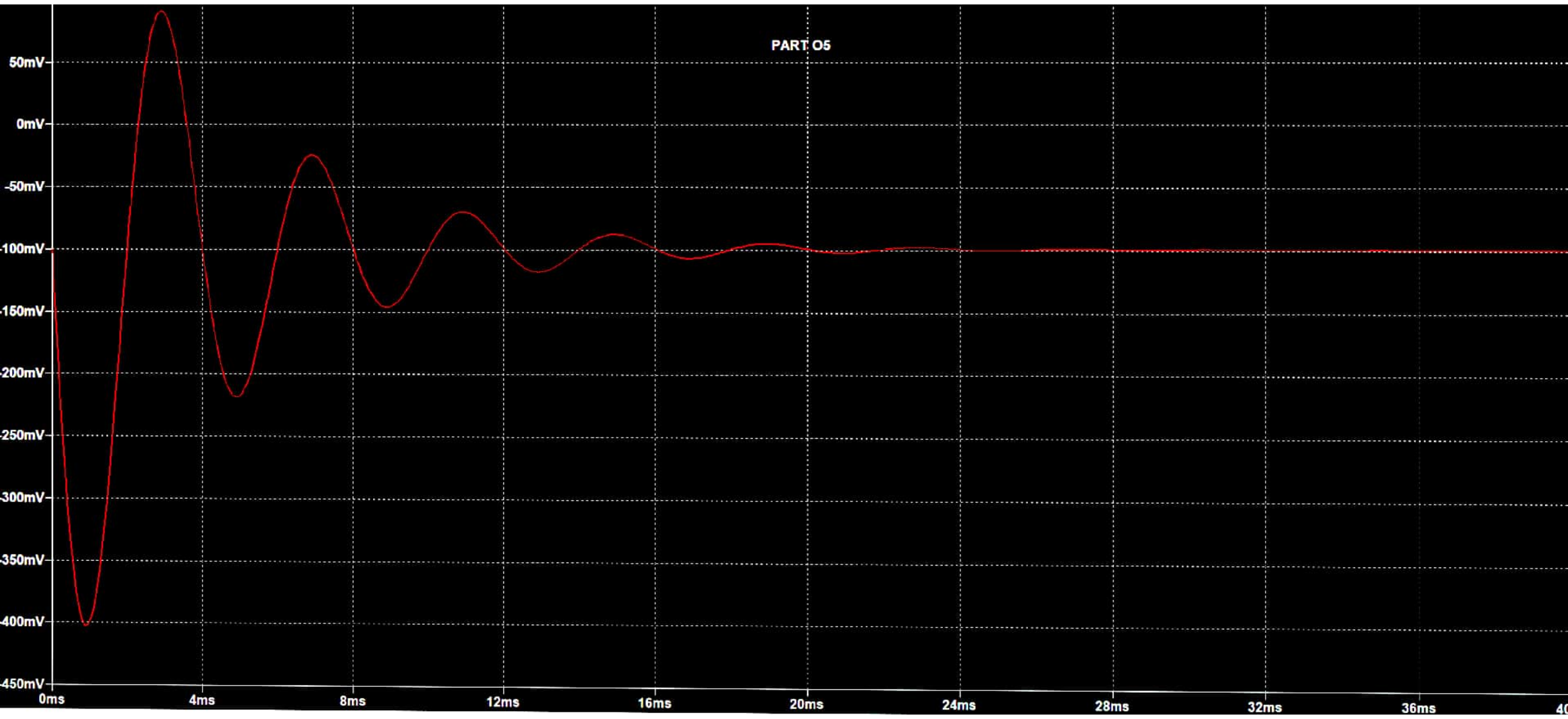
Part 05 The voltage across the shunt resistance $R_1 = 1\Omega$ was captured for different values of $i_L(0^-)$. ($C = 20\,\mu F$)

| $i_L(0^-)$ (A) | $R_2$ ($\Omega$) | M.O. (mV) | f (Hz) | Time Constant | $v_{R_1}(0)$ (mV) | $v_{R_1}(\infty)$ (mV) |
|---|---|---|---|---|---|---|
| 0.10 | 120 | 176.27 | 248.76 | 4.47 ms | 99.75 | 99.17 |
| 0.30 | 40 | 128.96 | 213.67 | 9.69 ms | 299.74 | 292.67 |
| 0.60 | 20 | 103.99 | 198.81 | 14.88 ms | 599.72 | 571.40 |

★ The plots obtained on LTSpice for the first entries in table 3, 4 and 5 follow from the next page.


V(a)
PART O3
14.0V
13.5V
13.0V
12.5V
12.0V
11.5V
11.0V
10.5V
10.0V
9.5V
9.0V


PART O4
17V
16V
15V
14V
13V
12V
11V
10V
9V
8V
7V
6V
5V
4V
0ms
4ms
8ms
12ms
16ms
20ms
24ms
28ms
32ms
36ms



PART O5
50mV
0mV
-50mV
-100mV
-150mV
-200mV
-250mV
-300mV
-350mV
-400mV
-450mV
0ms
4ms
8ms
12ms
16ms
20ms
24ms
28ms
32ms
36ms

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## Sample Calculations :

① Analytical determination of $v_c(t)$.

By KVL, $V_1 - L\frac{di}{dt} - v_c - R_1 i = 0$

By KCL, $i = C\frac{dv_c}{dt} + \frac{v_c}{R_2}$

$\therefore LC\frac{d^2v_c}{dt^2} + \left(\frac{L}{R_2} + R_1 C\right)\frac{dv_c}{dt} + v_c = V_1$

Now $v_c(0^-) = 5V$, $\dot{v}_c(0^-) = 0$ V/s.
$L = 20$ mH, $C = 20\,\mu F$,
$R_2 = 86\,\Omega$, $R_1 = 1\,\Omega$, $V_1 = 12V$

$\Rightarrow 4\times10^{-7}\frac{d^2v_c}{dt^2} + 2.52\times10^{-4}\frac{dv_c}{dt} + v_c = 12$

* Taking one sided Laplace transform on both sides.

$$4\times10^{-7}\left(s^2V_c(s) - 5s\right) + 2.52\times10^{-4}\left(sV_c(s) - 5\right) + V_c(s) = 12/s.$$

where $V_c(s)$ is $\mathcal{L}(v_c(t)\,u(t))$.

* upon rearranging, (and apply partial fractions)

$$V_c(s) = \frac{12}{s} - \frac{(7(s+315) + 2205)}{(s+315)^2 + 24\times10^5}$$

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$$\mathcal{L}^{-1}(V_c(s)) = \left(12 - 7e^{-315t}\cos(1549t) - 1.423\,e^{-315t}\sin(1549t)\right) \cdot u(t).$$

$$\Rightarrow v_c(t) = 12 - e^{-315t}\left(7\cos(1549t) + 1.423\sin(1549t)\right)$$

$$\lim_{t \to \infty} (v_c(t)) = 12\text{ V}. \quad (v_{c,max})$$

∴ The transient part of the output is the term added to 12V.

$$\underset{\text{(transient)}}{v_c^T(t)} = e^{-315t}\left(7\cos(1549t) + 1.423\sin(1549t)\right)$$

upon comparing $v_c^T(t)$ with

$$K_1 e^{-\sigma t}\cos\omega t + K_2 e^{-\sigma t}\sin\omega t.$$

$K_1 = 7V \qquad K_2 = 1.423V$

$\omega = 1549$ rad/s

$\sigma = 315\ \text{sec}^{-1}$

Now $\sigma = \xi\omega_n$

and $\omega = \omega_n\sqrt{1-\xi^2}$.

$\sqrt{1-\xi^2} = \omega\xi/\sigma$

Put the values and get

$\xi = 0.199.$

$\omega_n = \sigma/\xi = 1580.70$ rad/s.

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② Experimental verification of analytical sol$^{n}$ of $v_c(t)$.

$$v_c(t) = V_{max} + e^{-\sigma t}(K_1 \cos\omega t + K_2 \sin\omega t)$$

In the observation table, the following values deduced from the waveform are mentioned

$v_c(\infty) = 11.86\,V$

$\tau = 2.91\,ms$

$f = 248.14\,Hz$

$$V_{max} = \lim_{t\to\infty} v_c(t) = 11.86\,V$$

$\sigma = 1/\tau = 343.64$

$\omega = 2\pi f = 1559\ rad/s.$

$v_c(0) = V_{max} + K_1 = 5$

$\Rightarrow K_1 = 5 - 11.86 = -6.86\,V.$

$\dot{v}_c(0) = 0$

$\Rightarrow K_2\omega - \sigma K_1 = 0$

$\Rightarrow K_2 = -1.51\,V.$

$$\therefore v_c(t) = 11.86 - e^{-343.64t}(6.86\cos(1559t) + 1.51\sin(1559t))$$

(very similar to analytical sol$^{n}$)

On solving $\sqrt{1-\xi^2} = \omega\xi/\sigma$

we get $\xi = 0.215$

$\omega_n = \sigma/\xi = 1596.42\ rad/s.$

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③ Analytical determination of $i_L(t)$.

By KVL, $V_1 - i_L R_1 - L\frac{di_L}{dt} - v_c = 0$

By KCL, $C\frac{dv_c}{dt} + \frac{v_c}{R_2} = i_L$ — (1)

$$v_c = V_1 - i_L R_1 - L\frac{di_L}{dt} \quad -(2)$$

$$\frac{dv_c}{dt} = -R_1\frac{di_L}{dt} - L\frac{d^2 i_L}{dt^2} \quad -(3)$$

Put (2) and (3) in (1).

$$V_1 = R_2 LC\frac{d^2 i_L}{dt^2} + \frac{di_L}{dt}(L + R_1R_2C) + (R_1 + R_2)\, i_L$$

Now, $i_L(0^-) = 0.1\,A$, $\dot{i}_L(0^-) = 0\,A/s$.
$L = 20\,mH$, $C = 20\,\mu F$, $R_2 = 120\,\Omega$,
$R_1 = 1\,\Omega$, $V_1 = 12\,V$.

$$4.8\times10^{-5}\frac{d^2 i_L}{dt^2} + 2.24\times10^{-2}\frac{di_L}{dt} + 121\, i_L = 12.$$

Take one-sided Laplace transform on both sides.

$$4.8\times10^{-5}\left(s^2 I_L(s) - 0.1\, s\right) + 2.24\times10^{-2}\left(s\, I(s) - 0.1\right) + 121\, I_L(s) = 12/s.$$

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where $I_L(s) = \mathcal{L}(i_L(t)u(t))$

* Upon rearranging and forming partial fractions, we get.

$$I_L(s) = \frac{12}{121s} + \frac{1.23\times10^{-4} \times 1570}{(s+233.33)^2+(1570)^2}$$

$$+ \frac{8.26\times10^{-4}(s+233.33)}{(s+233.33)^2+(1570)^2}$$

* Taking laplace inverse both sides.

$$i_L(t) = \frac{12}{121} + 10^{-4}e^{-233.3t}\left(1.23\sin(1570t) + 8.26\cos(1570t)\right) A.$$

$$\lim_{t\to\infty} i_L(t) = 99.17\ mA = i_{L,max}.$$

$$\underset{\text{(transient)}}{i_L^T(t)} = 10^{-4}e^{-233.33t}\left(1.23\sin(1570t) + 8.26\cos(1570t)\right)$$

On comparing $i_L^T(t)$ with

$$K_1e^{-\sigma t}\cos(\omega t) + K_2e^{-\sigma t}\sin(\omega t).$$

$K_1 = 8.26\times10^{-4}$ A, $K_2 = 1.23\times10^{-4}$ A

$\sigma = 233.33\ sec^{-1}$,

$\omega = 1570$ rad/s.

On solving $\sqrt{1-\xi^2} = \omega\xi/\sigma$

$\xi = 0.147$ ; $\omega_n = \sigma/\xi = 1587$ rad/sec

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(4) Experimental verification of the analytically determined $i_L(t)$.

$$i_L(t) = I_{max} + e^{-\sigma t}(K_1 \cos\omega t + K_2 \sin\omega t)$$

The magnitude of current flowing through inductor in A is same as the magnitude of voltage drop across shunt resistor $R_1$ in V because its resistance is $1\Omega$.

* All the values used below are in obs. table

$i_L(0) = 0.1A$

$\lim_{t\to\infty} i_L(t) = 99.17 \times 10^{-3}$ A.

$\therefore \quad I_{max} = 9.917 \times 10^{-2}$ A.

$i_L(0) = I_{max} + K_1 = 0.1$

$\Rightarrow \quad K_1 = 8.30 \times 10^{-4}$ A

$\tau = 4.47 \times 10^{-3}$ sec (time constant)

$\sigma = 1/\tau = 223.71$ $sec^{-1}$

$f = 248.76$ Hz

$\omega = 2\pi f = 1563$ rad/sec

$\dot{i}_L(0) = 0$

$\Rightarrow \quad K_2\omega = \sigma K_1$

$K_2 = \sigma K_1/\omega = 1.18 \times 10^{-4}$ A.

Put all the values in the eq^n of $i_L(t)$.

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$$i_L(t) = 9.917 \times 10^{-2} + e^{-223.7t}(8.30\cos\omega t + 1.18\sin\omega t) \times 10^{-4}$$

put $\omega = 1563$

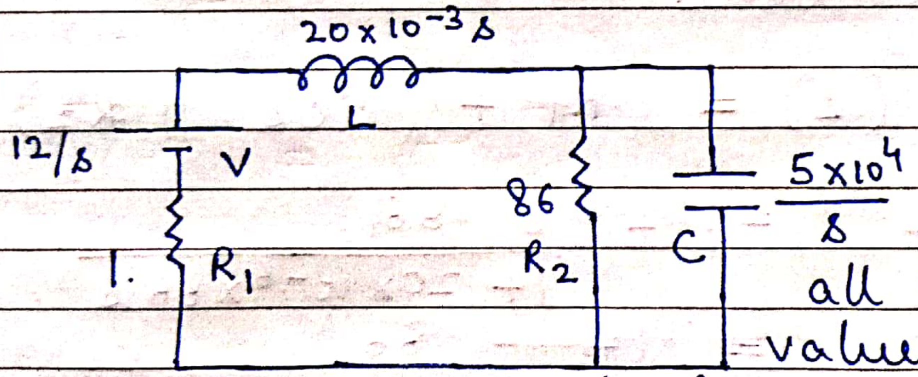
$$i_L(t) = 9.917 \times 10^{-2} + e^{-223.7t} \times 10^{-4}\left(8.30\cos(1563t) + 1.18\sin(1563t)\right)$$

(very similar to the analytical sol$^n$)

On solving, $\sqrt{1-\xi^2} = \omega\xi/\sigma$

$\xi = 0.142$, $\omega_n = \sigma/\xi = 1579$ rad/sec

(5) Finding transform-domain network.




all voltage values are in V and impedance values in $\Omega$.

$$Z_i(s) = R_1 \sim (R_2 \| C) \sim L$$

$$= R_1 + \frac{R_2/Cs}{R_2 + 1/Cs} + Ls$$

$$= 1 + \frac{86}{1 + 20 \times 86 \times 10^{-6} s} + 20 \times 10^{-3} s$$

$$Z_i(s) = \frac{3.44 \times 10^{-5} s^2 + 2.17 \times 10^{-2} s + 87}{1 + 1.72 \times 10^{-3} s}$$

$$Z_o(s) = (R_1 + Ls) \,||\, (R_2) \,||\, (1/Cs)$$

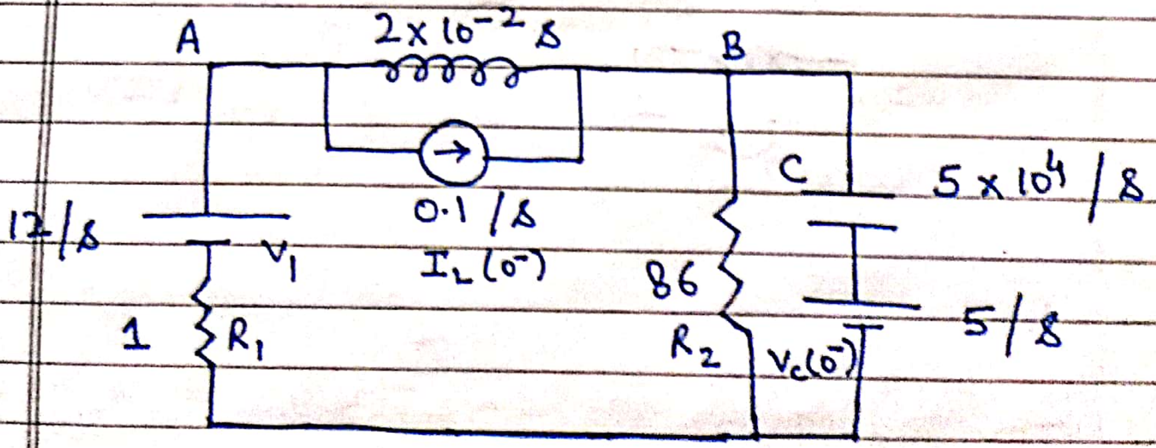
$$= \left[ \left( \frac{1}{R_1 + Ls} \right) + \frac{1}{R_2} + Cs \right]^{-1}$$

$$= \left( \frac{1}{1 + 20 \times 10^{-3} s} + \frac{1}{86} + 2 \times 10^{-5} s \right)^{-1}$$

$$Z_o(s) = \frac{86 + 1.72 \times 10^{-3} s}{3.44 \times 10^{-8} s^2 + 2.17 \times 10^{-2} s + 87}$$

(6) Determining $i_L(t)$ through transfer domain network and Thevenin's theorem

$I_L(0^-) = 0.1A$, $V_c(0^-) = 5V$




$Z_{Th}(s)$ (b/w A, B)

$= R_1 \sim (R_2 \,||\, C)$

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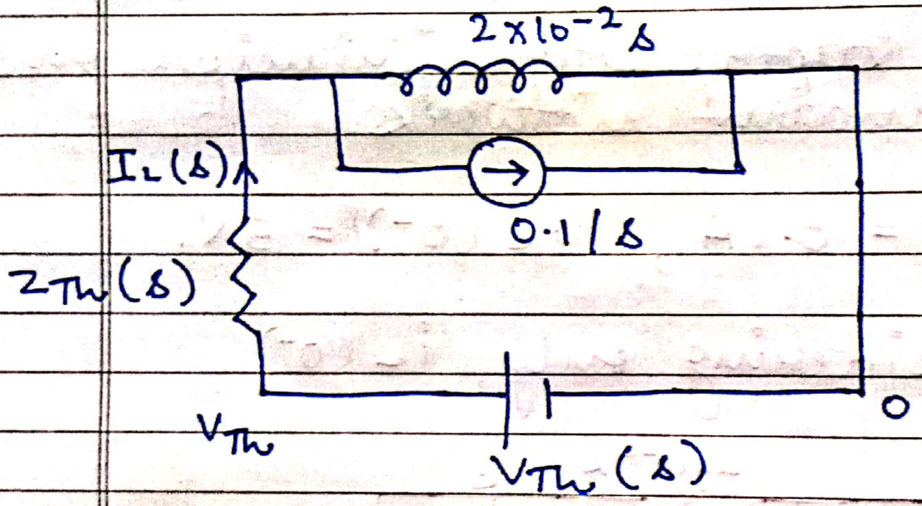
$$Z_{Th}(s) = 1 + \frac{86 \times 5\times10^4/s}{86 + 5\times10^4/s}$$

$$V_{Th}(s)\ (V_A - V_B) = \cancel{\frac{12}{s} - \frac{5}{s} = 7/s}$$

$$= \frac{12}{s} - \frac{5/s}{5\times10^4/s + 86} \times 86$$

$$= \frac{12}{s} - \frac{430}{86s + 5\times10^4}$$




$$I_L(s) = \frac{0.1}{s} + \frac{V_{Th} - I_L Z_{Th}}{2\times10^{-2}s}$$

$$I_L(s) = \frac{0.1 + 50V_{Th}}{s + 50Z_{Th}}$$

$$I_L(s) = \frac{0.1s^2 + 408.14s + 3.49\times10^5}{s(s^2 + 631.39s + 2.53\times10^6)}$$

(put the values of $V_{Th}$ and $Z_{Th}$)

Now expand $I_L(s)$ into partial fractions.

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$$I_L(s) = \frac{0.108}{s} - 0.0379\left(\frac{s+315.69}{(s+315.69)^2 + 2.43\times10^6}\right) + 333.02\left(\frac{1}{(s+315.69)^2 + 2.43\times10^6}\right)$$

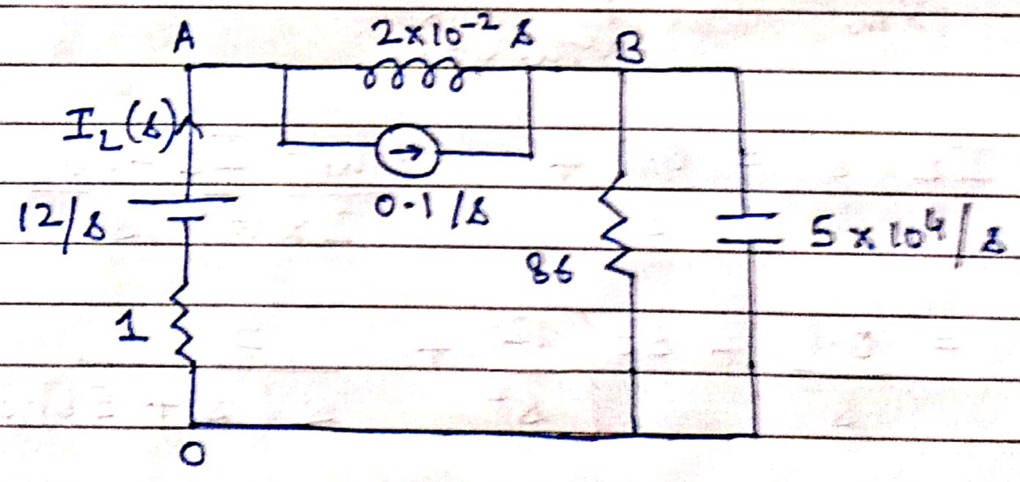
Take Laplace inverse on both sides.

$$i_L(t) = 0.1 - 0.0379\, e^{-315.69t} \cos(1559t) + 0.214\, e^{-315.69t} \sin(1559t).$$

(7) Applying superposition principle on the s-domain network.

$i_L(0^-) = 0.1A$, $v_c(0^-) = 5V$.

(i) considering only $i_L(0^-)$




$$V_A = \frac{12}{s} - I_L(s)$$

$$V_B = \frac{I_L \times 5\times10^4/s}{\frac{5\times10^4}{s} + 86} \times 86 = \frac{5\times10^4\, I_L}{s + 581.39}$$

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$$I_L(s) = \frac{V_A - V_B}{2\times10^{-2}s} + \frac{0.1}{s}$$

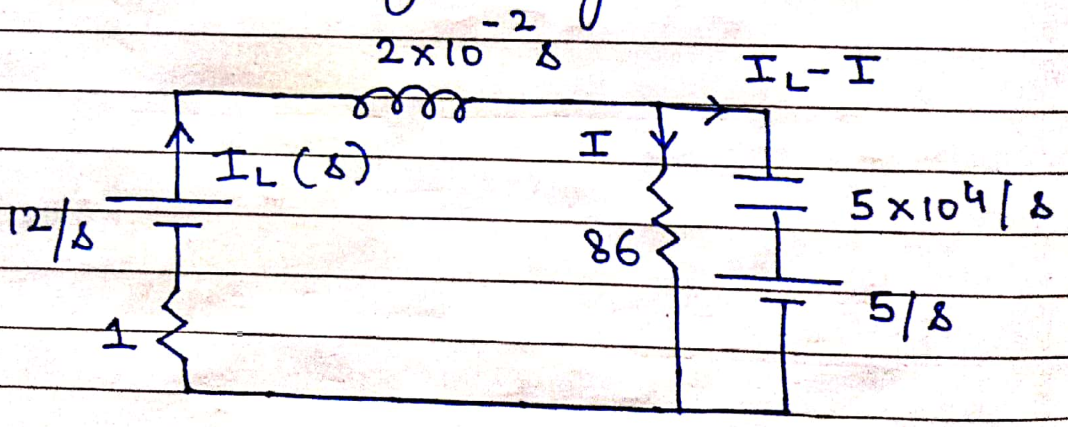
$$s I_L = 0.1 + 50\left(\frac{12}{s} - I_L - \frac{5\times10^4 I_L}{s+581.39}\right)$$

$$I_L = \frac{(0.1+600/s)(s+581.39)}{25\times10^5 + (s+50)(s+581.39)}$$

Break into partial fractions and take Laplace inverse on both sides.

$$i_{L_1}(t) = 0.138 - 0.038e^{-315.69t}\cos(1558t) + 0.374e^{-315.69t}\sin(1558t)$$

(ii) considering only $v_c(0^-)$




$$86I = \frac{5}{s} + \frac{5\times10^4}{s}(I_L - I) \quad -(1)$$

$$\frac{12}{s} - I_L - 2\times10^{-2}sI_L = 86I \quad -(2)$$

Put $I$ from (1) in (2)

$$I_L(s) = \frac{12(s+581.39) - 5s}{s(s+581.39)(2\times10^{-2}s+1) + 5\times10^4 s}$$

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Break into partial fractions and take Laplace inverse on both sides

$$i_{L_2}(t) = 0.138 - 0.138\, e^{-315.69t} \cos(1558t) + 0.197\, e^{-315.69t} \sin(1558t).$$

By superposition theorem, add the two $f^n$s together.

$$i_L(t) = i_{L_1}(t) + i_{L_2}(t)$$

$$i_L(t) = 0.276 - 0.176\, e^{-315.69t} \cos(1558t) + 0.571\, e^{-315.69t} \sin(1558t)$$

**Discussion:** The transient response of RLC network with initial conditions was successfully studied. Various parameters like $\sigma$, $\omega$, $\omega_n$, $\xi$, $v_c(\infty)$, $v_c(0^+)$ obtained from the practical were extremely close to the values determined theoretically. Differential $eq^n$s were formulated theoretically to confirm the plots obtained from LTspice. The plots followed the behaviour that was predicted by the theoretical $eq^n$s. The values of experimentally determined $T$, $f$, $\omega$, $\omega_n$ and $\sigma$ were used to find the closed form $eq^n$s of $i_L(t)$ and $v_c(t)$. These $eq^n$s

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were very similar to the eq^n s derived from scratch using the initial value conditions. Besides the eq^n s of $i_L(t)$ (or $v_c(t)$) obtained from superposition theorem were also obeying the practical observations.

The analytical and experimental results differ slightly and that might be because of the number of decimals places we take in our calculations. Moreover the values of time constant and frequency that were obtained from the practical LTSpice plots might not be 100% correct. An explicit eq^n gives a more correct estimate of these values than just a plot.