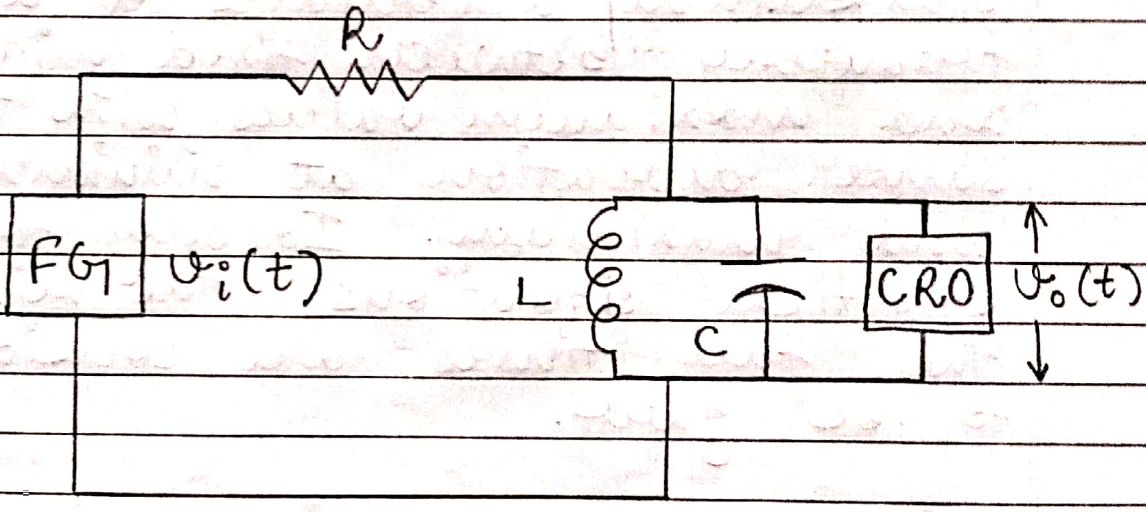
19CS10044 Nakul

PAGE NO. :
DATE : / /

Experiment 3B

Aim : To experimentally determine the first, third, and fifth harmonic fourier series coefficients for a square wave and verifying them with the theoretical values.

Circuit Diagram : (from Manual)



Theory and Formulae :

Fourier series is a periodic function composed of harmonically related sinusoids, combined by a weighted summation. For a periodic f^n it can be written as an infinite sum of sines and cosines functions each having some characteristic strength or amplitude and having frequency an integral multiple

19CS10044 Nahul

PAGE NO. :

DATE : / /

of the functions frequency.
Mathematically, a periodic function $f_p(t)$ can be written as-

$$f_p(t) = a_0 + \sum_{i=1}^{\infty} \left(a_i \cos(i\omega t) + b_i \sin(i\omega t) \right)$$

A square wave is a non-sinusoidal periodic waveform in which the amplitude alternates at a steady frequency between fixed minimum and maximum values with the same duration at minimum and maximum. Ideally for practical situations, the rise and the fall times are considered to be zero.

Using Fourier analysis, an ideal wave (square wave) with an amplitude of A can be represented as an infinite sum of sinusoidal waves.

$$s(t) = \frac{4A}{\pi} \sum_{k=1}^{\infty} \frac{\sin((2k-1)\omega t)}{(2k-1)}$$

$$s(t) = \frac{4A}{\pi} \left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{7}\sin(7\omega t) + \dots \right)$$

where $\omega = 2\pi f$ and f is the

19CS10044 Nalul

PAGE NO. :
DATE : / /

frequency of the square wave s(t). The ideal square wave contains only components of odd-integer harmonic frequencies. The coefficient or the amplitude of the nth sinusoidal term is called the nth harmonic coefficient (say h_n).

$h_1 = 4A/\pi$ (fundamental/first harmonic)

$h_3 = 4A/3\pi$ (third harmonic)

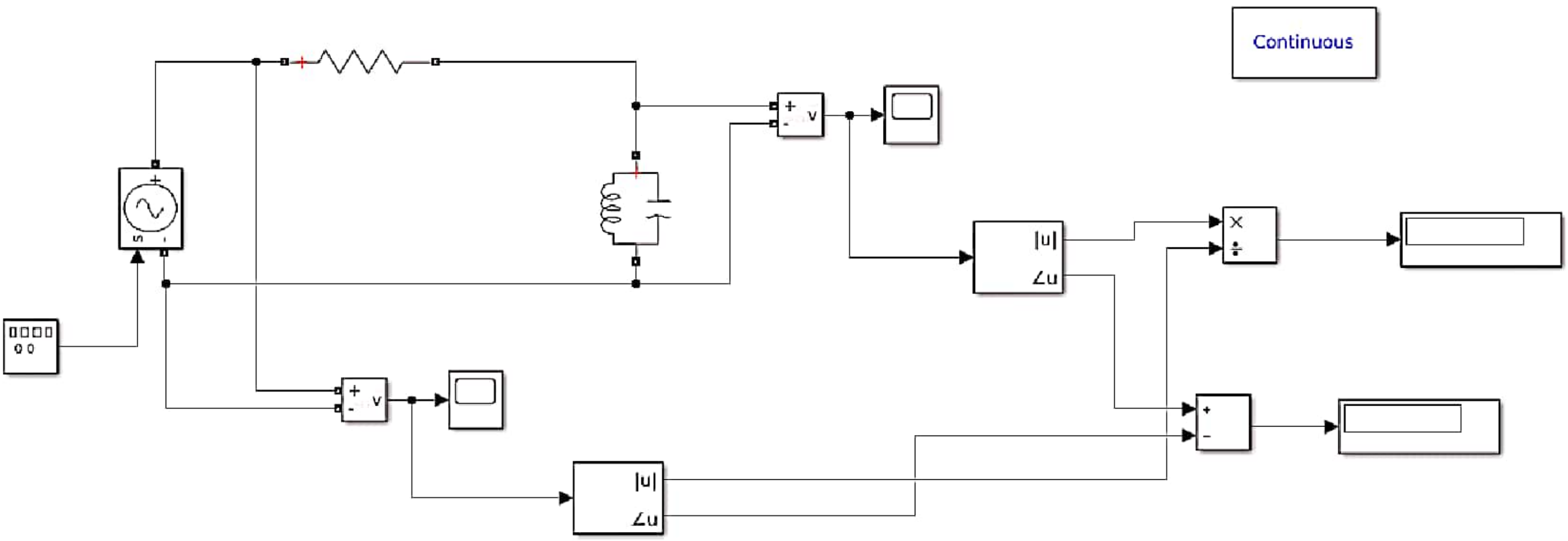
$h_5 = 4A/5\pi$ (fifth harmonic)

Simulink Circuit Diagram :

The screenshot of the circuit simulated on simulink is pasted on the next page.

Equipment : (Simulink Blocks).

1. Signal Generator
2. Controlled Voltage Source
3. Voltage Measurement Block
4. Scope
5. Fourier Block
6. Display Block
7. Product and Add Block Parameters
8. Resistor
9. Parallel LC branch
10. powergui block.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PAGE NO. :
DATE : / /

19CS10044 Nahul

Part A :

Simulink Procedure :

1. Drag all the blocks as mentioned in "Equipments" from the Library Browser to the main screen.
2. Connect all the blocks / components as shown in the Simulink circuit diagram.
3. Select the values of network components as R = 100kΩ, L = 10mH, C = 100nF and set a sinusoidal output from the signal generator with an amplitude of 3 V and initial frequency of 100 Hz.
4. Measure the corresponding output amplitude from the scope and then calculate the gain. Or directly record the gain from the display block.
5. Vary output frequency of the signal generator (and also simultaneously the fundamental frequency of the fourier blocks) gradually from 100 Hz to 100 kHz.
6. Draw plots on a semi-log scale of frequency vs gain (in dB) on Matlab.
7. Using Laplace Transform, derive the transfer function of the network with these parameters and plot its frequency response on Matlab.
8. Compare the two plots.

19CS10044 Nakul

PAGE NO.:
DATE: / /

observation Table :

Frequency (Hz)	gain (M)	20 log (gain) dB
100	0.0001	-80.00
500	0.0003	-70.46
700	0.0004	-67.96
1000	0.0006	-64.44
1500	0.0012	-58.42
2000	0.0015	-56.48
2500	0.0028	-51.06
3000	0.0029	-50.75
3500	0.0043	-47.33
4000	0.0068	-43.35
4500	0.0141	-37.02
4700	0.0231	-32.73
5000	0.2441	-12.25
5010	0.3214	-9.86
5020	0.5192	-5.69
5030	0.9333	-0.60
5040	0.7593	-2.39
5050	0.4266	-7.40
5060	0.2841	-10.93
5070	0.2104	-13.54
5100	0.1188	-18.50
5500	0.0173	-35.24
6000	0.0090	-40.92
7000	0.0047	-46.56
8000	0.0033	-49.63
9000	0.0026	-51.70
10000	0.0021	-53.56
15000	0.0012	-58.42

19CS10044 Nahul

PAGE NO. :
DATE : / /

20000	0.0008	-61.94
25000	0.0007	-63.10
30000	0.0005	-66.02
40000	0.0004	-67.96
50000	0.0003	-70.46
60000	0.0003	-70.46
70000	0.0002	-73.80
80000	0.0002	-73.80
90000	0.0002	-73.80
100000	0.0002	-73.80

Matlab Plots :

Both theoretical and experimental plots of this part are pasted on the next page.

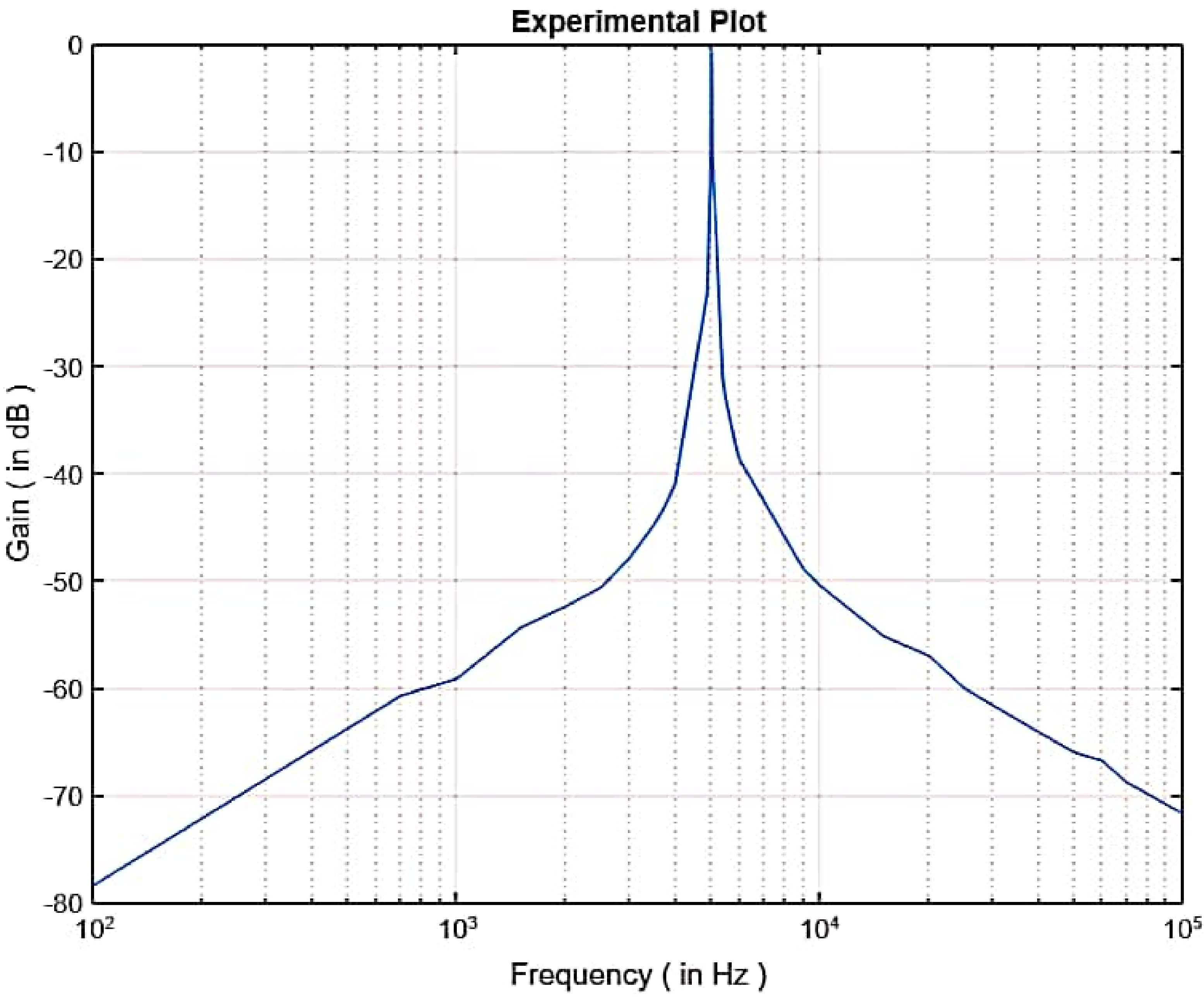
Calculations :

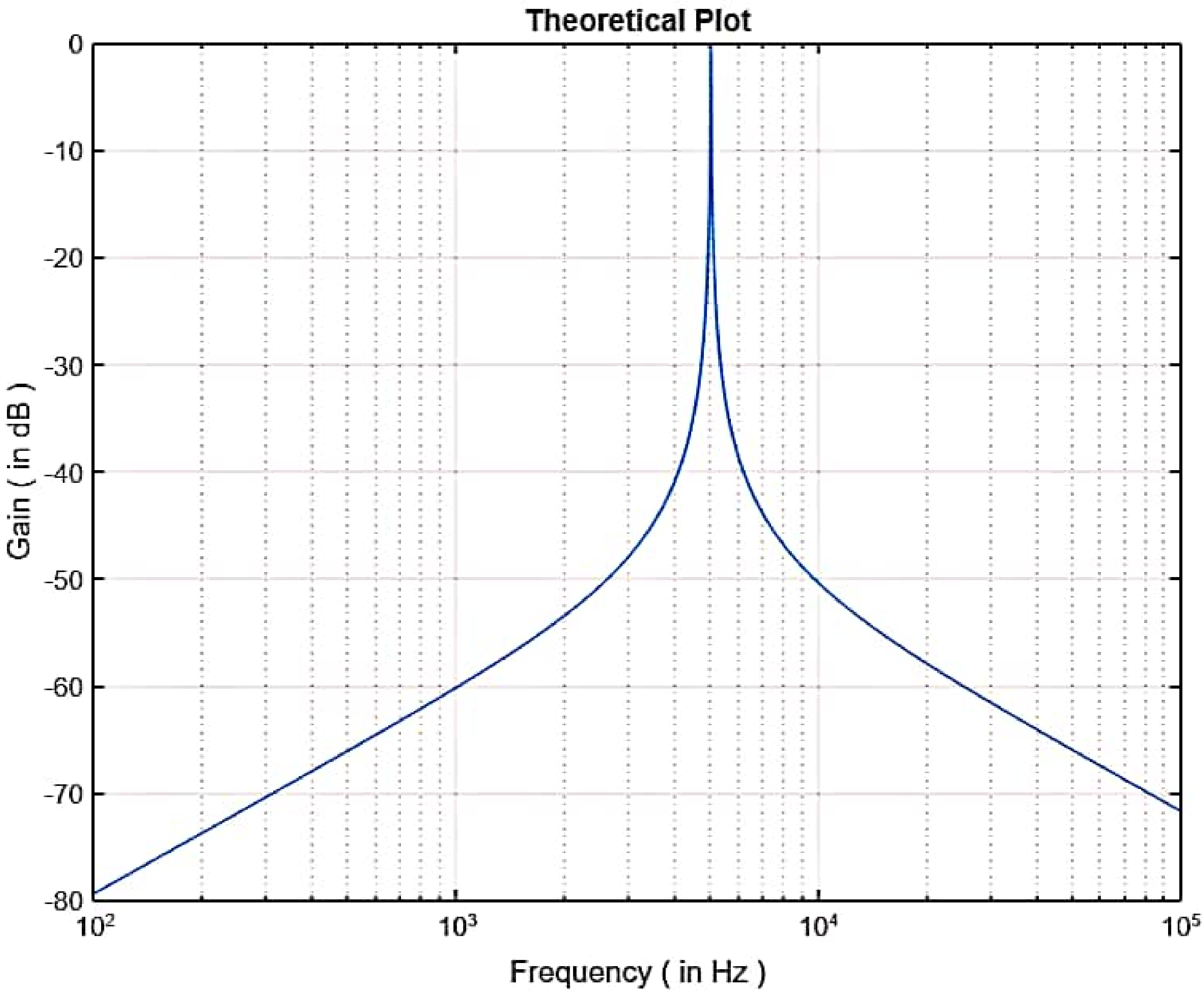
In s-domain write the inductance and capacitance as Ls and $1/Cs$ respectively.

$$Ls \parallel 1/Cs = \frac{L/C}{Ls + 1/Cs}$$

$$= \frac{Ls}{1 + LCs^2}$$

Apply simply voltage division rule in the s-domain circuit


Experimental Plot
0
-10
-20
-30
-40
-50
-60
-70
-80
Gain (in dB)
10^2
10^3
10^4
10^5
Frequency (in Hz)


Theoretical Plot
0
-10
-20
-30
-40
-50
-60
-70
-80
Gain (in dB)
10^2
10^3
10^4
10^5
Frequency (in Hz)

19CS10044 Nahul

PAGE NO.:
DATE: / /

and get the following transfer function G(s).

$$G(s) = \frac{(Ls \parallel 1/Cs)}{R + (Ls \parallel 1/Cs)}$$

$$G(s) = \frac{Ls}{R(1+LCs^2) + Ls}$$

For plotting the frequency response, put $s = j\omega$ in the transfer function G(s) and consider its magnitude for gain.

$$G(j\omega) = \frac{L\omega j}{L\omega j + R(1-LC\omega^2)}$$

$$|G(j\omega)| = M(\omega) = \frac{L\omega}{[(L\omega)^2 + R^2(1-LC\omega^2)^2]^{1/2}}$$

$$M(2\pi f) = \frac{2\pi f L}{[(2\pi f L)^2 + R^2(1-4\pi^2 f^2 LC)^2]^{1/2}}$$

$M(2\pi f)$ function gives gain as a f^n of frequency. Put values of R, L, C and get the following form

$$M(2\pi f) = \frac{6.283\times10^{-2} f}{[3.948\times10^{-3} f^2 + 10^{10}(1-3.948\times10^{-8} f^2)^2]^{1/2}}$$

19CS10044 Nakul

PAGE NO. :
DATE : / /

The maximum value of the gain $M(2\pi f)$ occurs at $f = \dfrac{1}{2\pi\sqrt{LC}}$.

Let fm be the frequency at which the maximum gain occurs for this network.

f_m (practical) = 5030 Hz (from table)

$$f_m \text{ (theoretical)} = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{10\cdot 10^{-3}\cdot 100\cdot 10^{-9}}}$$

$$= 5032.92 \text{ Hz}.$$

$$\% \text{ error} = \frac{5032.92 - 5030}{5032.92} = \underline{0.06\%}$$

Let the maximum attainable gain by the network be Mm.

M_m (practical) = 0.9333

M_m (theoretical) = $M(2\pi f_m)$ = 1.000

$$\% \text{ error} = \frac{1 - 0.933}{1} = \underline{6.70\%}$$

Discussion : The theoretical and practical values of fm and Mm are incredibly close. The two plots - theoretical and practical show similar behaviour. In both

19CS10044 Nahul

PAGE NO. :
DATE : / /

the plots, the dB value of the gain increases very sharply around the frequency f_m. By taking even finer steps during the experiment, the gain value of 1 could have been achieved. Anyhow the number of readings taken was sufficient to compare the theoretical plot and the experimental plot. It was because of sharp increase at the frequency f_m that the experimental value of M_m varied from the theoretical by 6.7%. The gain becomes incredibly small for frequencies less than 4.5kHz and those more than 5.5kHz. Because of the sharp increase around f_m, the band-width of the response in both the plots seems to be very less.

19CS10044 Nahul

PAGE NO. :
DATE : / /

Part B :

Simulink Procedure :

1. Repeat steps 1 and 2 from part A.
2. Select the values of network components as $R = 100k\Omega$, $L = 10mH$, $C = 100nF$. Set the square wave output from the signal generator with amplitude 3V and frequency 5kHz. (Do not forget to set the fundamental frequency of the fourier blocks every time the frequency of generator is reset).
3. Open the scope corresponding to the output voltage. Fine tune the frequency of the signal generator in steps of 10Hz. Reduce the step size to 1Hz where the change of the output waveform becomes more significant.
4. Take larger number of observations around the frequency where the output waveform resembles a sinusoid and the gain on the display block becomes maximum.
5. Record all observations. Choose the reading where the output waveform becomes a sinusoid with the maximum amplitude/gain.

19CS10044 Nakul

PAGE NO. :
DATE : / /

(6) Deduce the fundamental Fourier series coefficient from the value of this amplitude by dividing it by the input amplitude (3 Volts).

(7) Compare the practical value with the theoretical value.

* Observation Table and the relavant calculations are shown on the next page.

19CS10044 Nahul

PAGE NO. :
DATE : / /

Observation Table :

Frequency (Hz)	Output Amplitude (Volts)
3500	0.038
4000	0.053
5000	0.878
5010	1.629
5020	1.878
5030	3.416
5032.9	3.724
5032.92	3.727
5032.95	3.728
5033	3.729
5033.5	3.745
5033.75	3.742
5034	3.739
5035	3.697
5100	0.492
5500	0.072
6000	0.108

Calculations : Maximum amplitude M from the observation table is 3.745 at 5.0335 kHz.

a_1 (practical) $= \frac{M}{|V_{in}|} = \frac{3.745}{3} = 1.248$

a_1 (theoretical) $= 4/\pi = 1.273$

% error $= (1.273 - 1.248)/1.273 = 1.96\%$

The practical and theoretical values of the fundamental coefficient are close.

19CS10044 Nakul

PAGE NO.:
DATE: / /

Discussion: The theoretical and practical values of a_1 (the fundament fourier series coefficient) were very close; only an error of 1.96% was found. It is clear from the observation table that the steps were fine enough to obtain as close as possible value to the theoretical value. Even finer steps could have reduced the error.

19CS10044 Nahul

PAGE NO. :
DATE : / /

Part C :

Simulink Procedure :

1. Open the same circuit that was used in part B. Keep the values of all network components and all the connections intact.
2. Reset the frequency of signal generator (and fundamental frequency of fourier block) to 1680 Hz.
3. Repeat the steps 3, 4 and 5 from part B. Frequency of the signal generator must be tuned in fine steps.
4. Deduce the third harmonic fourier series coefficient from the value of the local peak amplitude by dividing it by the input amplitude (3 Volts).
5. Compare the practical value with the theoretical value.
6. Now reset the frequency of signal generator to 1kHz. Repeat the same experiment. Deduce the fifth harmonic fourier series coefficient from the value of the local peak around 1kHz and compare it with the theoretical value.

19CS10044 Nakul

PAGE NO. :
DATE : / /

observation Tables :

Table 1

Frequency (Hz)	Output Amplitude (Volts)
1600	0.047
1650	0.113
1670	0.396
1677	1.177
1677.5	1.230
1677.6	1.241
1677.63	1.249
1677.64	1.242
1677.65	1.241
1677.67	1.250
1677.68	1.244
1677.7	1.244
1678	1.251
1678.25	1.242
1678.5	1.222
1679	1.150
1800	0.030
2000	0.024
2500	0.016
3000	0.046

19CS10044 Nahul

PAGE NO. :
DATE : / /

Table 2

Frequency (in Hz)	Output Amplitude (Volts)
100	0.016
300	0.027
500	0.013
700	0.034
1000	0.169
1003	0.287
1005	0.512
1006	0.686
1006.1	0.701
1006.3	0.719
1006.5	0.742
1006.6	0.749
1006.63	0.750
1006.65	0.751
1006.66	0.753
1006.67	0.752
1006.7	0.752
1006.75	0.748
1006.85	0.746
1007	0.744
1100	0.016
1300	0.010

19CS10044 Nahul

Calculations : Maximum amplitude M from table 1 is 1.251 that occurs at a frequency of 1.678 kHz.

$$a_3 \text{ (practical)} = \frac{M}{|V_{in}|} = \frac{1.251}{3} = 0.417$$

$$a_3 \text{ (theoretical)} = \frac{4}{3\pi} = 0.424$$

$$\% \text{ error} = \frac{0.424 - 0.417}{0.424} = 1.65\%$$

The practical and theoretical values of the third harmonic coefficient are close.

Maximum amplitude M' from table 2 is 0.753 that occurs at a frequency of 1.00666 kHz.

$$a_5 \text{ (practical)} = \frac{M'}{|V_{in}|} = \frac{0.753}{3} = 0.251$$

$$a_5 \text{ (theoretical)} = \frac{4}{5\pi} = 0.255$$

$$\% \text{ error} = \frac{0.255 - 0.251}{0.255} = 1.57\%$$

The practical and theoretical values of the fifth harmonic coefficient are also close.

19CS10044 Nahul

PAGE NO. :
DATE : / /

Discussion : The theoretical and practical values of a_3 (third harmonic fourier series coefficient) and a_5 (fifth harmonic fourier series coefficient) are very close, only having an error of 1.65% and 1.57% respectively. The experiment hence successfully determined the values of the three harmonics of fourier series with an at-most error of 2%. This error can perhaps be reduced by finer tuning.

Final Discussion & Applications :

The experimental was successful in determining the values of fourier series coefficients of a periodic square waveform to a fair degree of error. In all the three parts (A, B and C), the experimental results were in agreement with the theoretical results discussed in "Theory and formulae" and the "Calculations" sections. The experiment perhaps would have been much easier with real equipment than on Simulink. A frequency knob is present on a physical signal generator that can be conveniently rotated back and forth to get finely tuned frequency responses. On Simulink, it

19CS10044 Nakul

PAGE NO. :
DATE : / /

was very tedious to reset the frequency of the signal generator for each observation by opening the settings of the block every time. Moreover it was extremely time-consuming especially when a significant number of readings had to be taken.

Anyway, the results obtained from the Simulink simulation were same as the ones expected from a real experiment. As far as the applications of this experiment are concerned, they are numerous. It is really helpful to obtain the magnitude spectrum for arbitrary periodic signals, which cannot necessarily be analysed theoretically. This kind of experiment can be useful to isolate narrow bands of frequencies components from a compound waveform. Fourier transformation is used in a wide range of applications such as image analysis, image filtering, image reconstruction and compression. It is an important image processing tool used to decompose an image into sine-cosine components.