Functional completeness

You need to be alert to (usually minor) changes that may be made to the assignment statement or to the guidelines after the assignment is first put up. Refresh this frame and re-read the assignment carefully before you make your final submission.

Background

In Post's treatment of functional completeness, five classes of Boolean functions are defined

- T-preserving (T)
- F-preserving (F)
- Counting (L)
- Monotonic (M)
- Self-dual (S)

Post's functional completeness theorem is stated below

> A set $\mathbb{F}$ of Boolean connectives is functionally complete if and only if for each of the five defined classes, there is a member of $\mathbb{F}$ which does not belong to that class.

Examples of such functions **not** belonging to various classes are given below.

Not T-preserving

x_2	x_1	x_0	T'$_0$	T'$_1$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	1
1	1	1	0	0

Not F-preserving

x_2	x_1	x_0	F'$_0$	F'$_1$	F'$_2$
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	1	0
1	1	1	1	1	1

Not Counting

x_2	x_1	x_0	L'$_0$	L'$_1$	L'$_2$	L'$_3$
0	0	0	1	0	1	0
0	0	1	1	1	0	0
0	1	0	0	0	1	1
0	1	1	0	0	0	0
1	0	0	0	1	1	0
1	0	1	0	1	1	0
1	1	0	1	0	0	0
1	1	1	0	1	0	1

Not Monotonic

x_2	x_1	x_0	M'$_0$	M'$_1$	M'$_2$	M'$_3$	M'$_4$
0	0	0	1	1	1	1	1
0	0	1	1	0	0	0	0
0	1	0	0	0	0	1	0
0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	1
1	1	0	0	1	0	0	0
1	1	1	0	0	0	0	0

Not Self-dual

x_2	x_1	x_0	S'$_0$	S'$_1$	S'$_2$	S'$_3$	S'$_4$	S'$_5$
0	0	0	1	0	1	1	1	1
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	1	0	0
0	1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1
1	0	1	0	1	1	1	0	0
1	1	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1	1

Choice of functions

- Let there be n given functions of a particular class in the table
- Let your group number be m
- You should use the function with index $(n \mod m)$

Realisation of T', F', L', M' and S' functions

- Post's constructions assumes the availability of such functions
- For this experiment, a *cheating* strategy will be used for the realisation of these functions
- The required functions, as defined via these truth tables, should be realised using the available gates and encapsulated as components for use in the Post's construction steps

Define complementation using T', F' and L' functions

- Choose appropriate function T', F' and L' functions from the given tables of functions
- Construct the complementation function using Post's construction methods (with suitable encapsulations as components)

Define the constant functions T and F

- Choose appropriate function T', F', L' and S' functions from the given tables of functions
- Construct the complementation function using Post's construction methods (with suitable encapsulations as components)

Define two of the eight $g(p,q)$ functions with odd number of Ts in a row

- Let your group number be m
- You should implement one $g_\imath$ function with index $\imath = (n \mod 8)$
- You should implement another $g_\jmath$ function with index $(\imath + 3 \mod 8)$
- Choose appropriate function T', F', L', M' and S' functions from the given tables of functions