Chapter 7

Testing Hypothesis for Single Population Parameter

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Table of Contents

Chapter 7 Testing Hypothesis for Singe Population Parameter

7.1 Testing Hypothesis for a Population Mean

- Examples of testing hypothesis for a population mean are as follows.
  - The capacity of a cooky product is indicated as 200g. Will there be enough cookies in the indicated capacity?
  - At a light bulb factory, a newly developed light bulb advertises a longer bulb life than the past. Is this propaganda reliable?
  - Immediately after completing this year's academic test, students said that there will be an average English score of 5 points higher than last year. How can you investigate if this is true?
- The testing hypothesis test is to decide statistically which hypothesis is to use for the two hypotheses about the unknown population parameter using the sample.
- In this section, we examine the test of the population mean, population variance, and population proportion, which are most commonly used in testing hypothesis.

7.1 Testing Hypothesis for a Population Mean

[Ex 7.1.1] At a light bulb factory, the average life expectancy of a light bulb made by a conventional production method is known to be 1,500 hours and the standard deviation is 200 hours. Recently, the company is trying to introduce a new production method, with the average life expectancy of 1,600 hours for light bulbs. To confirm this argument, 30 samples were taken by simple random sampling and the sample mean was 1555 hours. Can you tell me that the new type of light bulb has a life of 1600 hours?

<Answer>

- The statistical approach to the question of this issue is first to make two assumptions about the different arguments for the population mean μ. Namely

$$H_0:\ \mu\ =\ 1500 \qquad H_1:\ \mu\ =\ 1600$$

7.1 Testing Hypothesis for a Population Mean

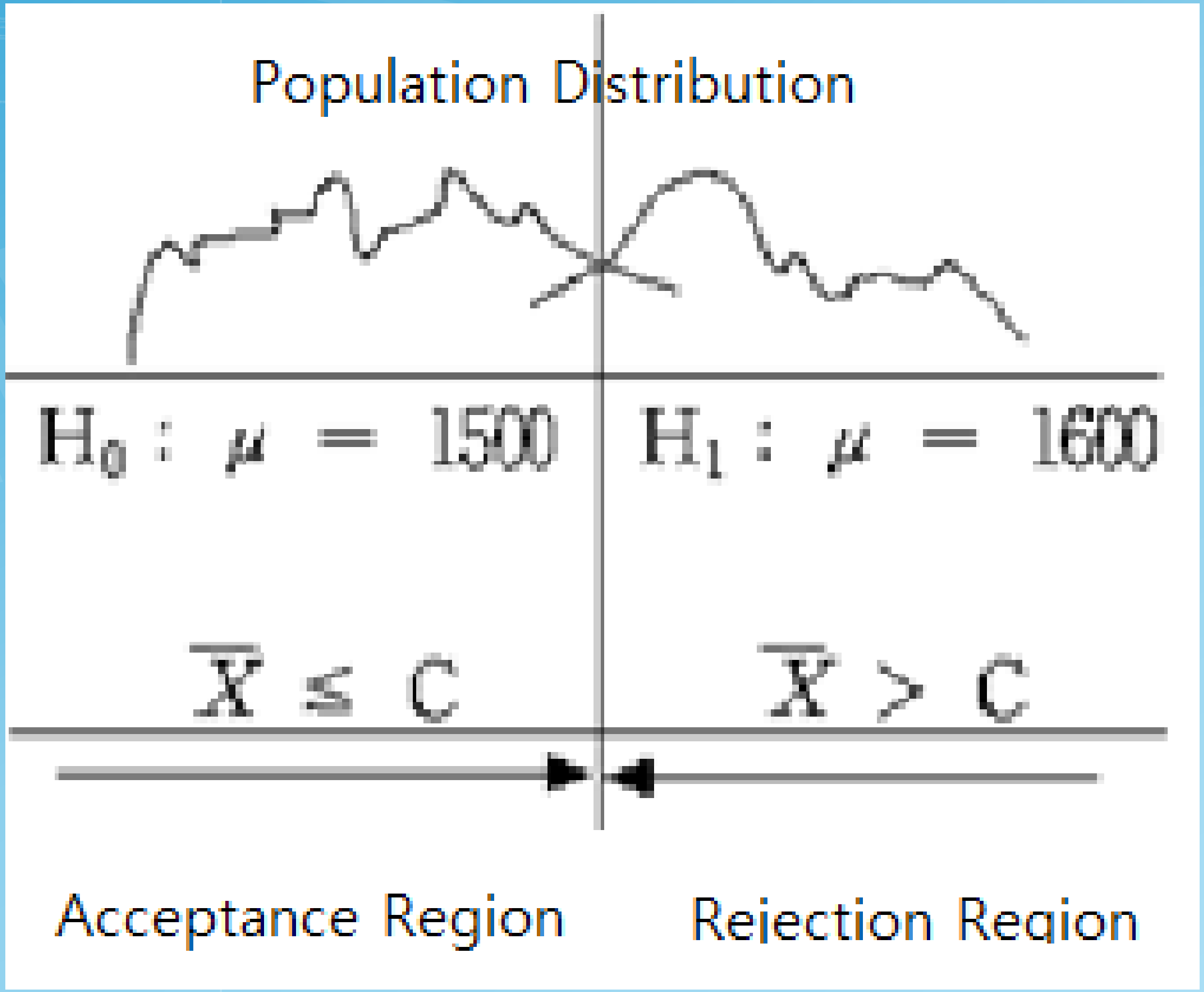
〈Ex 7.1.1 Answer〉

- H_0 is called null hypothesis and H_1 is alternative hypothesis. In most cases, the null hypothesis is defined as an existing known fact, and the alternative hypothesis is defined as new facts or changes in current beliefs. So when choosing between two hypotheses, the basic idea of testing hypothesis is 'unless there is a definite reason, we accept the null hypothesis (current fact) without choosing the alternative theory (the fact of the matter). This idea of testing hypothesis is referred to as a conservative decision making.
- A common sense criterion for choosing between two hypotheses would be 'which population mean of two hypothesis is closer to the sample mean'. Based on this common sense criteria which uses the concept of distance, the sample mean of 1555 is closer to H_1: μ = 1600 so the alternative hypothesis will be chosen. Statistical testing hypothesis is based on not only this common sense criteria, but also on the sampling distribution of $\overline{X}$. In other words, statistical testing hypothesis is to select a critical value C based on the sampling distribution theory and to make a decision rule as follows.

  If $\overline{X}$ is smaller than C, then the null hypothesis H_0 will be chosen, else reject H_0'

7.1 Testing Hypothesis for a Population Mean

〈Ex 7.1.1 Answer〉



7.1 Testing Hypothesis for a Population Mean

〈Ex 7.1.1 Answer〉

Table 7.1.1 Two types of errors in testing hypothesis

		Actual	
		H_0 is true	H_1 is true
Decision:	H_0 is true	Correct	Type 2 Error
	H_1 is true	Type 1 Error	Correct

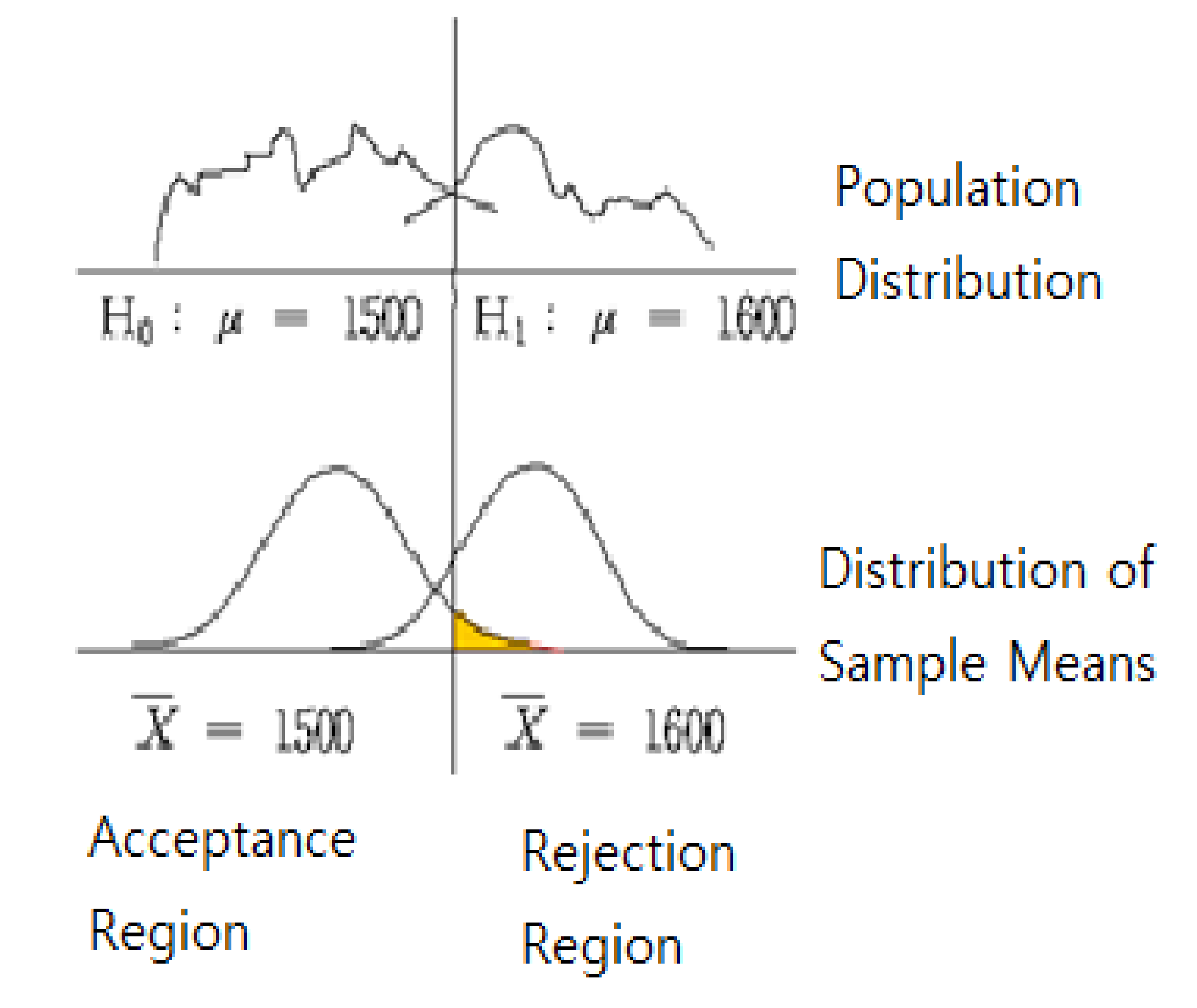
7.1 Testing Hypothesis for a Population Mean

〈Ex 7.1.1 Answer〉

- If you try to reduce one type of error when the sample size is a constant, then the other type of error is increasing. That is why we came up with a conservative decision-making method that defines the null hypothesis H_0 as 'past or present facts' and 'accept the null hypothesis unless there is a clear evidence for the alternative hypothesis.' In this conservative way, we try to reduce the type 1 error as much as possible that selects H_1 when H_0 is true, which would be more risky than type 2 error. Statistical testing hypothesis determines the tolerance for the probability of type 1 error, usually 5% or 1% for rigorous test, and use the selection criteria that satisfy this limitation. The tolerance for the probability that this type 1 error will occur is called a 'significance level' and is often expressed as α. The probability of the type 2 error is expressed as β .

7.1 Testing Hypothesis for a Population Mean

⟨Ex 7.1.1 Answer⟩



7.1 Testing Hypothesis for a Population Mean

〈Ex 7.1.1 Answer〉

- If the population corresponds to the distribution of H_0 : μ = 1500, the sampling distribution of sample means is approximated by the central limit theorem as $N(1500, 200^2/30)$. If the population corresponds to the distribution of H_1 : μ = 1600, the sampling distribution of sample means is approximated as $N(1600, 200^2/30)$. The standard deviation for each population is assumed to be 200 from a historical data. Then the decision rule becomes as follows.

  ' If $\overline{X}$ < C, then accept H_0, else accept H_1 (i.e. reject H_0)'

  In Figure 7.1.2, the shaded area represents the probability of type 1 error. If we set the significance level, which is the tolerance level of type 1 error, is 5%, I.e. $P(\overline{X} < C) = 0.95$, C can be calculated by the normal distribution percentile as follows.

$$1500 + 1.645 \frac{200}{\sqrt{30}} = 1560.06$$

  Therefore the decision rule can be written as follows. .

  'If $\overline{X} < 1500 + 1.645 \frac{200}{\sqrt{30}} = 1560.06$, then accept H_0, else reject H_0 (accept H_1).'

7.1 Testing Hypothesis for a Population Mean

⟨Ex 7.1.1 Answer⟩

- In this problem, the observed sample mean of random variable $\overline{X}$ is $\overline{x}$= 1555 and H_0 is adopted. In other words, the hypothesis of H_0 : μ = 1500 is judged to be correct, which contradicts the results of common sense criteria than $\overline{x}$ = 1555 is closer to H_1 : μ = 1600 than H_0 : μ = 1500. This result can be interpreted that the sample mean of 1555 is not sufficient grounds to reject the null hypothesis by a conservative decision-making method.
- The above selection criteria are often written as follows, emphasizing that they are results from a conservative decision-making method.

  'If $\overline{X}$ < 1560.06, then do not reject H_0, else reject H_0.'

  In addition, the selection criteria above is written for calculation purposes as follows.

  'If $\dfrac{\overline{X}-1500}{\dfrac{200}{\sqrt{30}}} < 1.645$, then accept H_0, else reject H_0.'

  In this case, if $\overline{x}$ = 1555, then $\dfrac{1555-1500}{\dfrac{200}{\sqrt{30}}} = 1.506$ which is less than 1.645. Therefore accept H_0.

7.1 Testing Hypothesis for a Population Mean

Table 7.1.2 Testing hypothesis for population mean - known σ case

Type of Hypothesis	Decision Rule
1) $H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$	If $\dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > z_\alpha$, then reject H_0
2) $H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	If $\dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} < -z_\alpha$, then reject H_0
3) $H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	If $\left\lvert \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \right\rvert > z_{\alpha/2}$, then reject H_0

Note: The H_0 of 1) can be written as $H_0 : \mu \leq \mu_0$, 2) as $H_0 : \mu \geq \mu_0$

7.1 Testing Hypothesis for a Population Mean

☞ Decision rule using p-value

If p-value is less than the significance level rejech H_0, else accept H_0

Table 7.1.3 Calculation of p-value

Type of Hypothesis	p-value
1) $H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$	$P(\overline{X} > \overline{x}_{obs})$
2) $H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$P(\overline{X} < \overline{x}_{obs})$
3) $H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$If\ \overline{X}_{obs} > \mu_o\ ,\ 2P(\overline{X} > \overline{x}_{obs})\quad else\ 2P(\overline{X} < \overline{X}_{obs})$

Note : $\overline{x}_{obs}$ is the observed sample mean.

7.1 Testing Hypothesis for a Population Mean

Table 7.1.4 Testing hypothesis for population mean - unknown σ case (population is a normal distribution)

Type of Hypothesis	Decision Rule
1) $H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$	If $\dfrac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} > t_{n-1;\alpha}$, reject H_0
2) $H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	If $\dfrac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} < -t_{n-1;\alpha}$, reject H_0
3) $H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	If $\left\| \dfrac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \right\| > t_{n-1;\alpha/2}$, reject H_0

Note: The H_0 of 1) can be written as $H_0 : \mu \leq \mu_0$, 2) as $H_0 : \mu \geq \mu_0$

7.1 Testing Hypothesis for a Population Mean

[Example 7.1.2] The weight of a bag of cookies is 250 grams. Suppose the weight of the bags is normal. In the survey of 100 samples of bags, the average was 253 grams and the standard deviation was 10 grams.

1) Test hypothesis that the weight of the bag of cookies is 250g or larger and find a p-value. α= 1%
2) Test hypothesis whether or not the weight of the bag of cookies is 250g and find a p-value. α= 1%
3) Use 「eStatU」 to test the hypothesis above.

<Answer>

1) The hypothesis is a right tail test as H_0: μ = 250, H_1: μ > 250. Since the sample size is large (n =100), we can use Z distribution instead of t distribution. Decision rule is as follows.

   'If $(\overline{X} - \mu_0)/(\frac{S}{\sqrt{n}}) > z_\alpha$, then reject H_0, else accept H_0'

   'If $(253 - 250)/(\frac{10}{\sqrt{100}}) > z_{0.01}$, then reject H_0, else accept H_0'

   Since (253-250) / (10/10) = 3, $z_{0.01}$ = 2.326, H_0 is rejected. We can write the decision rule as follows.

   'If $\overline{X} > 250+2.326(\frac{10}{\sqrt{100}})$, then reject H_0, else accept H_0'

   'If $\overline{X} > 252.326$, then reject H_0, else accept H_0'

7.1 Testing Hypothesis for a Population Mean

- Since p-value is the probability of Type 1 error when the sample mean is the critical value. it can be calculated by the probability of $P(\overline{X} > 253)$. Since the distribution of $\overline{X}$ is approximately $N(250, \frac{100}{100})$ when H_0: μ = 250 is true, p-value is as follows.

  p-값 = $P(\overline{X} > 253)$ = $P(Z > (253-250)/(10/10))$ = $P(Z > 3)$ = 0.0013

2) The hypothesis is a two-sided test as H_0: μ = 250, H_1: $\mu \neq 250$. Since the sample size is large (n =100), we can use Z distribution instead of t distribution. Decision rule is as follows.

  'If $\left| \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \right| > z_{\alpha/2}$, then reject H_0, else accept H_0'

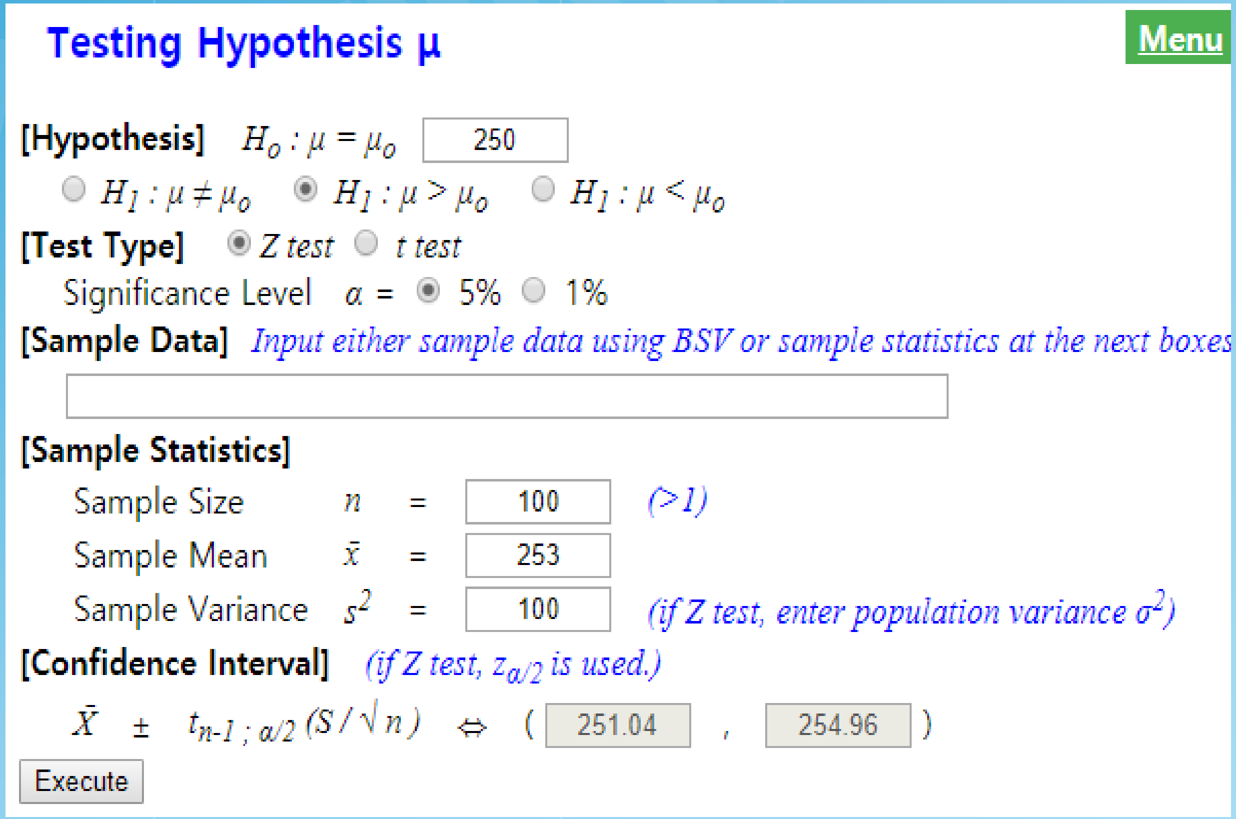
  'If $\left| \frac{253 - 250}{\frac{10}{\sqrt{100}}} \right| > z_{0.005}$, then reject H_0, else accept H_0'

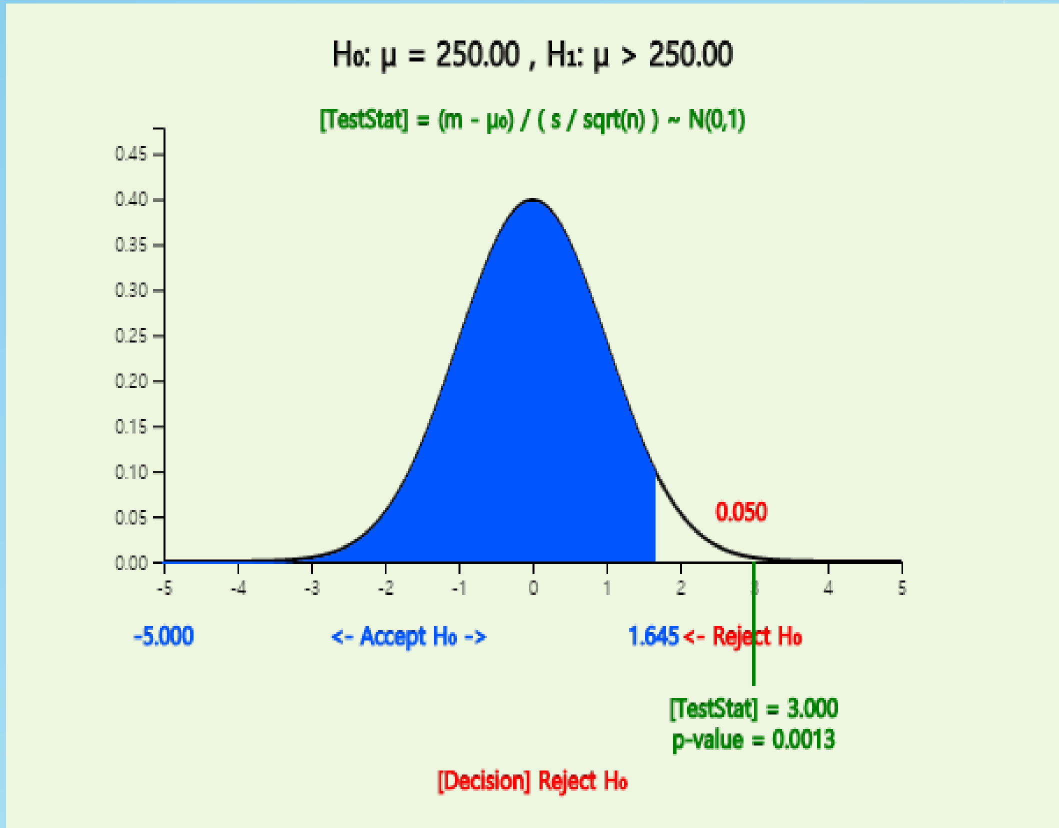
  Since (253-250) / (10/10) = 3, $z_{0.005}$ = 2.575, H_0 is rejected. p-value can be calculated as follows.

  p-값 = $2P(\overline{X} > 53)$ = $2P(Z > (53-50)/(10/10))$ = $2P(Z > 3)$ = 0.0026

7.1 Testing Hypothesis for a Population Mean

3) Select the alternative hypothesis as the right test in the window shown in <Figure 7.1.3>, which appears by selecting 「eStatU」 and 'Testing Hypothesis μ' and check [Test Type] as Z test. Then, check the significance level at 5%, and enter sample size 100, sample mean 253, and sample variance $10^2 = 100$. For the Z test, you must enter the population variance, but you may enter the sample variance because the sample size is large enough.





7.1 Testing Hypothesis for a Population Mean

[Example 7.1.3] Test whether the weight of the cookies is 250g or greater when the sample size is 16 in question [Example 7.1.2] and obtain a p-value. Check the results using 「eStat U」

<Answer>

- Since the population stadard deviation is unknown and the sample size is small, the decision rule is as follows.

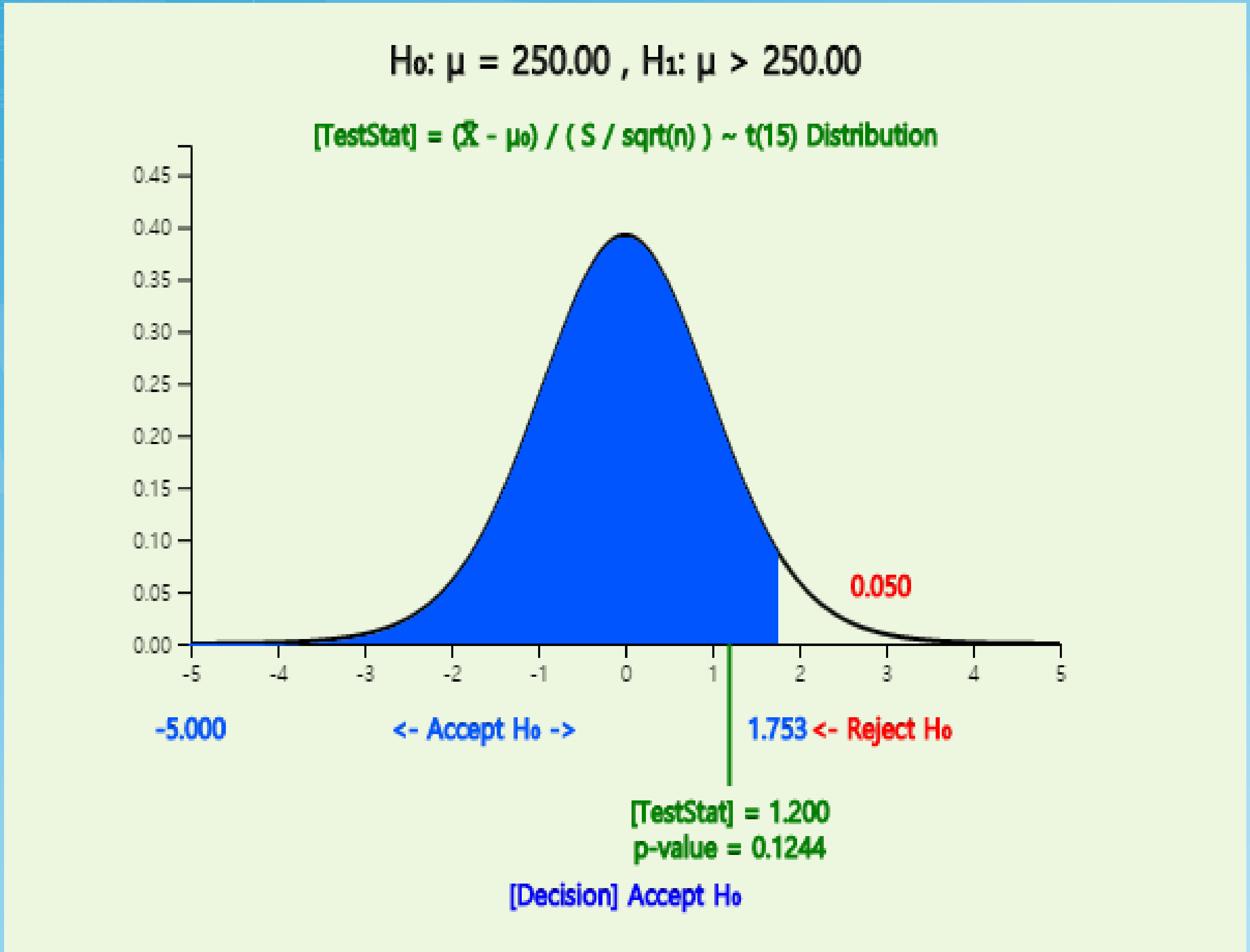
‘If $(\overline{X} - \mu_o)/(\frac{S}{\sqrt{n}}) > t_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0’

‘If $(253 - 250)/(\frac{10}{\sqrt{16}}) > t_{16-1\,;\,0.01}$, then reject H_0, else accept H_0’

Since the value of test statistic is $(253 - 250)/(\frac{10}{\sqrt{16}}) = 1.2$, $t_{15,0.01} = 2.602$, we accept H_0. Note that the decision rule can be written as follows.

‘If $\overline{X} > 250 + 2.602(\frac{10}{\sqrt{16}})$, then reject H_0, else accept H_0’

7.1 Testing Hypothesis for a Population Mean

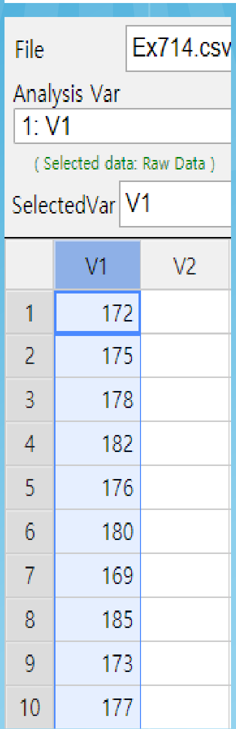


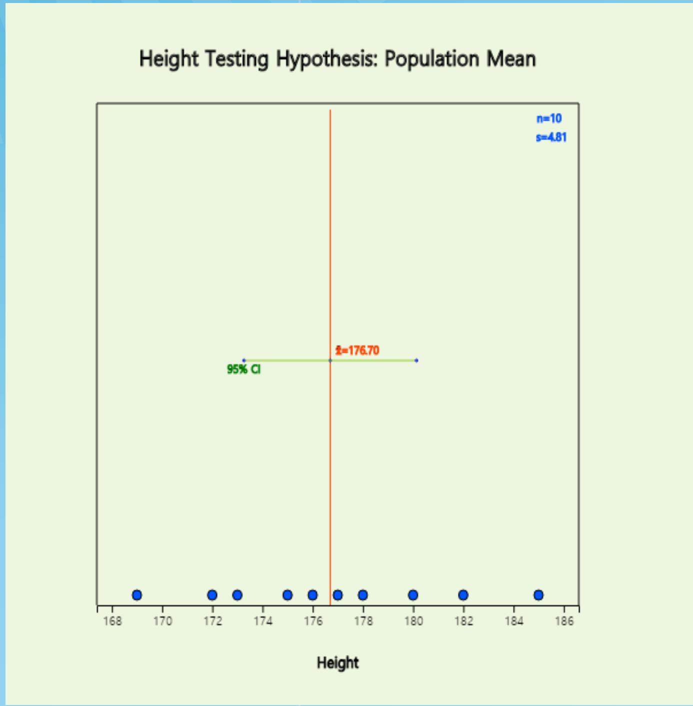
7.1 Testing Hypothesis for a Population Mean

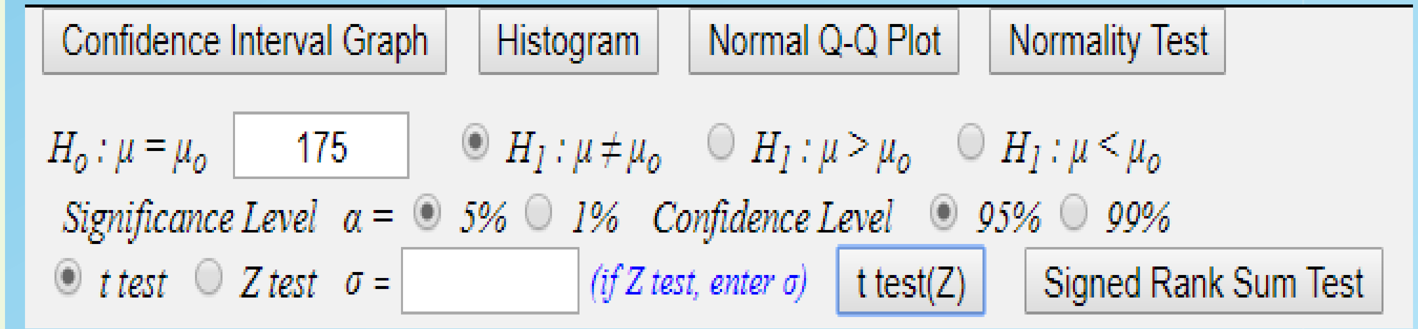
[Example 7.1.4] (「eStat」 Practice)
We sampled 10 male college students and examined their heights as follows.

172 175 178 182 176 180 169 185 173 177 (Unit cm)

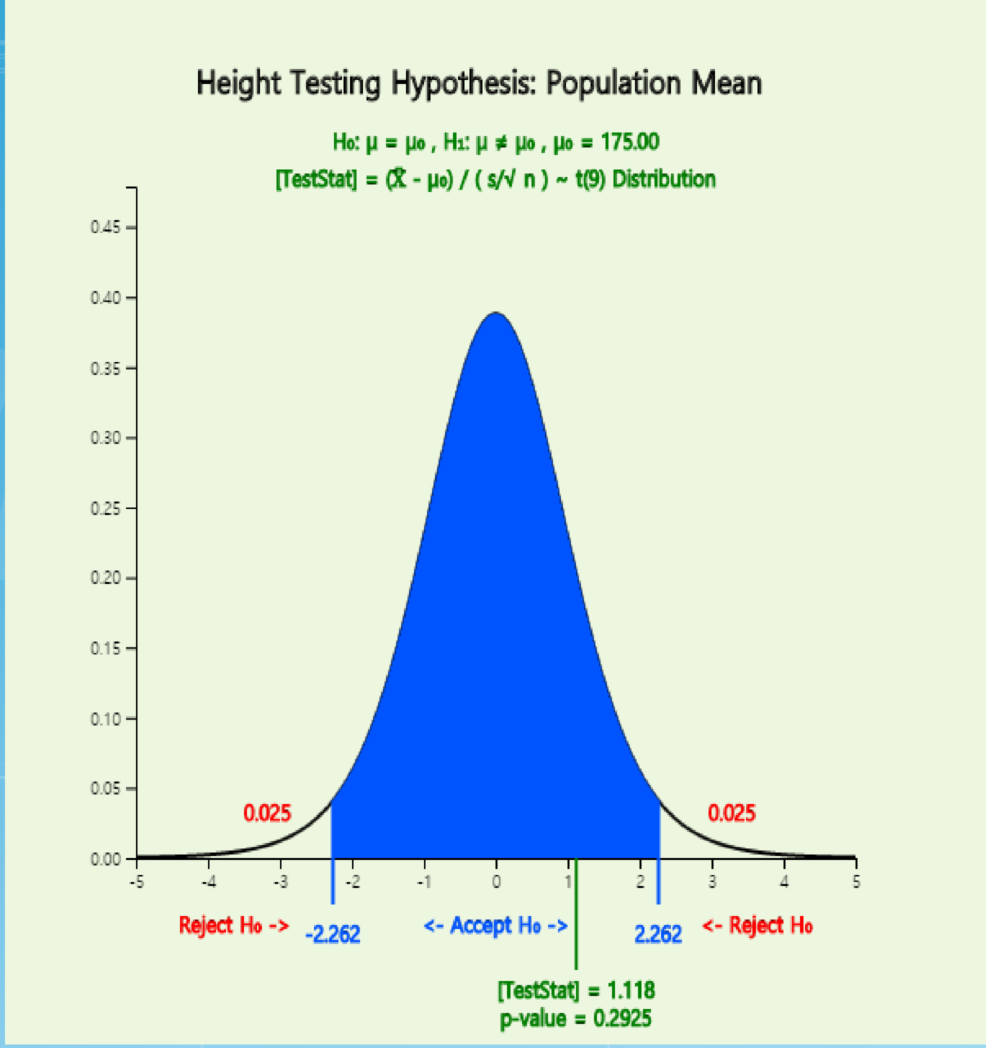
Test the hypothesis whether the population mean is 175cm or greater with a significant level of 5%.







7.1 Testing Hypothesis for a Population Mean



Testing Hypothesis: Population Mean	Analysis Var	Height			
Statistics	Observation	Mean	Std Dev	std err	Population Mean 95% Confidence Interval
	10	176.700	4.809	1.521	(173.260, 180.140)
Missing Observations	0				
Hypothesis					
$H_0 : \mu = \mu_0$	μ_0	[TestStat]	t value	p-value	
$H_1 : \mu \neq \mu_0$	175.00	sample mean	1.118	0.2925	

7.2 Testing Hypothesis for a Population Variance

- Examples for testing hypothesis of population variances are as follows.

  - The bolts of a company that currently supplies bolts to an automaker have an average diameter of 7mm and a variance of 0.25mm. Recently, rival companies have been applying for the supply, claiming that their companies' bolts have an average diameter of 7 millimeters and a variance of 0.16mm. How can I find out if this claim is true?

  - The variance of math score of the last year's college scholastic aptitude test was 100. This year's math problem is said to be much easier than last year's. How can I find out if the variance of math score of this year test is smaller than last year?

7.2 Testing Hypothesis for a Population Variance

Table 7.2.1 Testing hypothesis of population variance
- population is normally distributed -

Type of Hypothesis	Decision Rule
1) $H_0 : \sigma^2 = \sigma_0^2$: $H_1 : \sigma^2 > \sigma_o^2$:	If $\frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0
2) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 < \sigma_o^2$	If $\frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0
3) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 \neq \sigma_o^2$	If $\frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1\,;\,\alpha/2}$ or $\frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1\,;\,1-\alpha/2}$, then reject H_0, else accept H_0

Note: In 1) the null hypothesis can be written as $H_0 : \sigma^2 \leq \sigma_o^2$, in 2) $H_0 : \sigma^2 \geq \sigma_o^2$.

7.2 Testing Hypothesis for a Population Variance

[Example 7.2.1] One company produces a bolt of automotive. If the average bolt diameter is 15mm and its variance is less than 0.10^2, it can be delivered to the automotive company. Twenty-five of the most recent products were randomly sampled and their variance is 0.15^2. Assuming that the diameter of the bolt follows normal distribution,

1) Conduct testing yypothesis at a 5% significant level to determine if the product can be delivered to the automotive company.
2) Check the results using 「eStatU」.

〈Answer of Ex 7.2.1〉

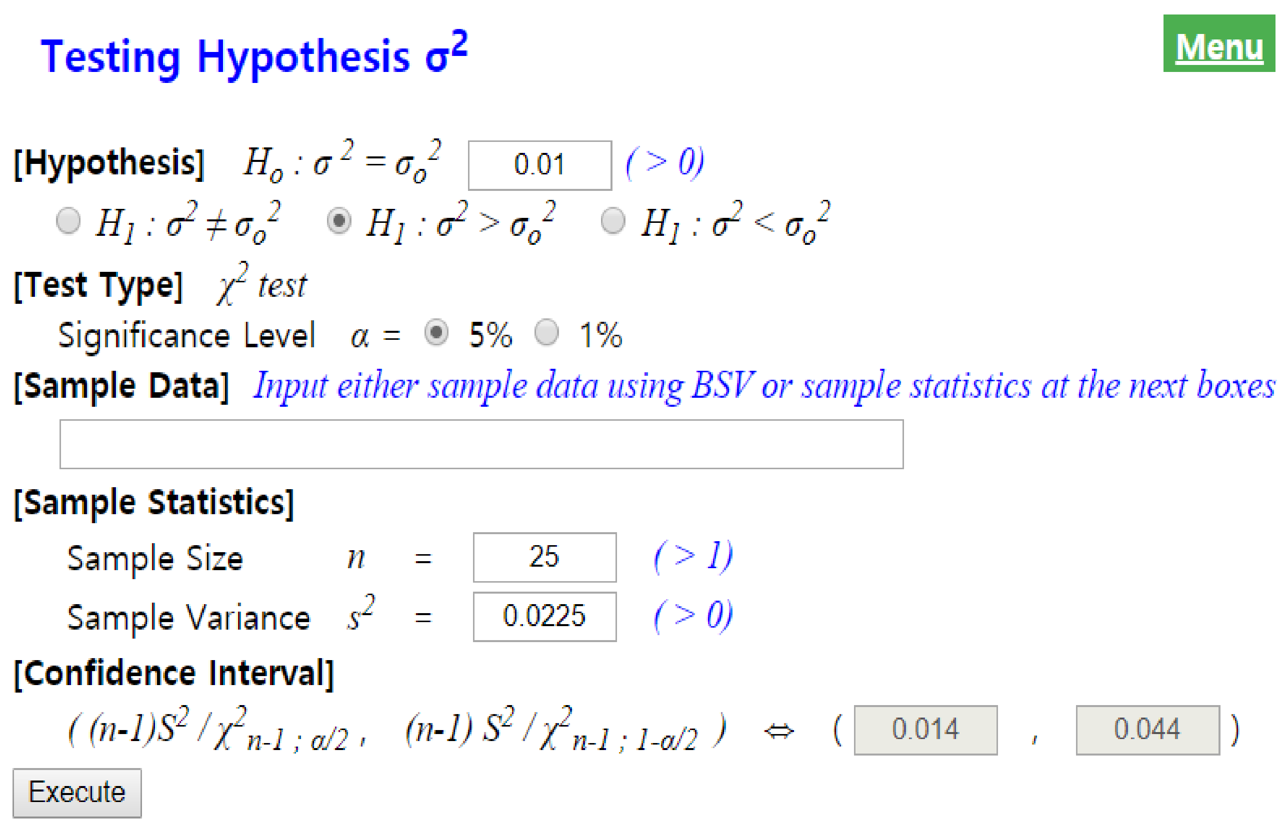
1) The hypothesis of this problem is H_0: $\sigma^2 \le 0.1^2$ H_1: $\sigma^2 > 0.1^2$ and its decision rule is as follows.

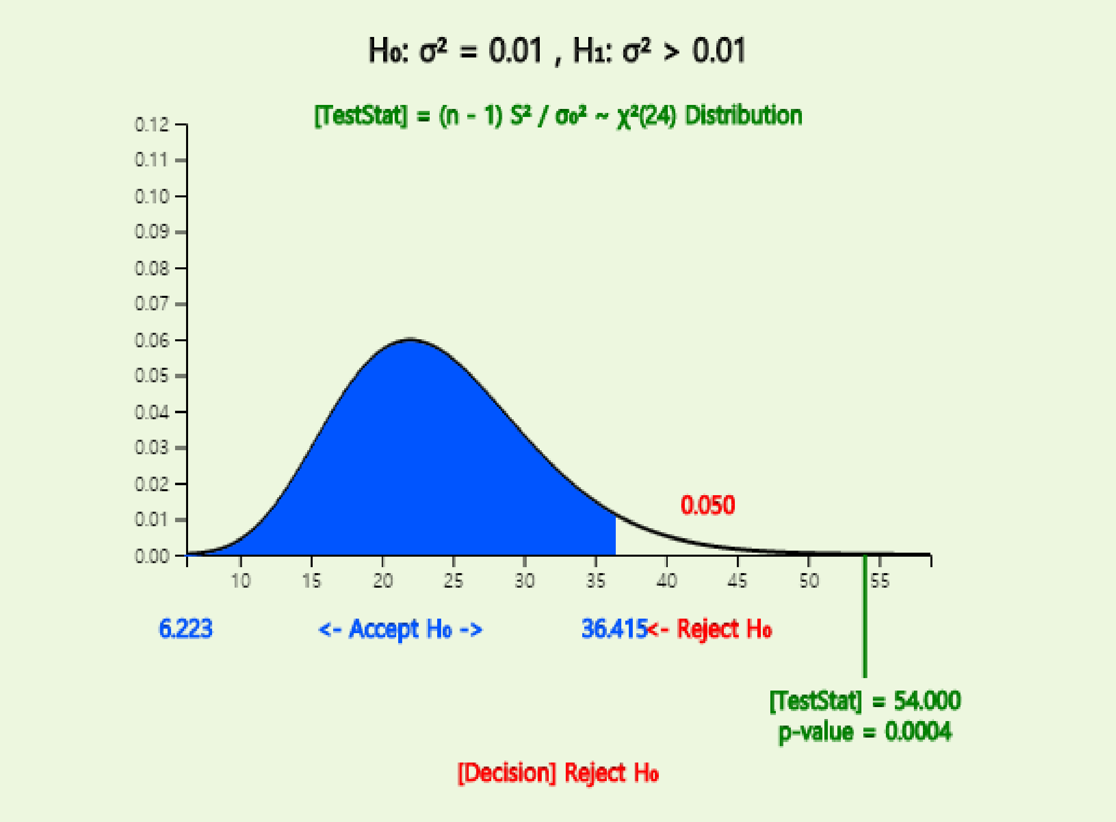
'If $\dfrac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0'

Note that $s^2 = 0.15^2 = 0.0225$, $(25-1) \times 0.15^2/0.1^2 = 54$ and $\chi^2_{25-1\,;\,0.05} = \chi^2_{24\,;\,0.05} = 36.42$. Therefore H_0 is rejected.

7.2 Testing Hypothesis for a Population Variance

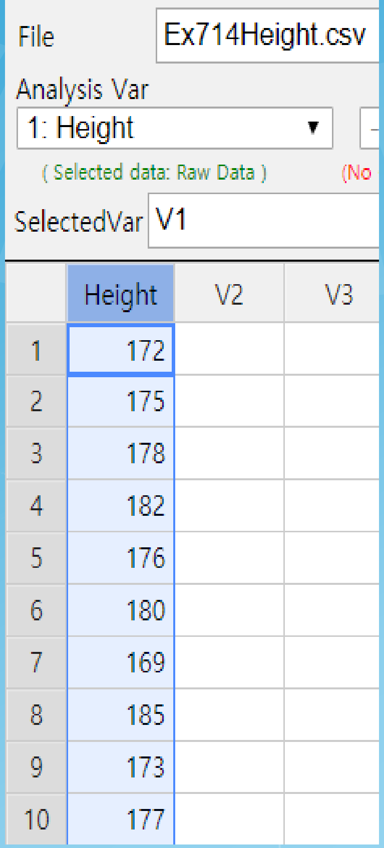
〈Answer of Ex 7.2.1〉

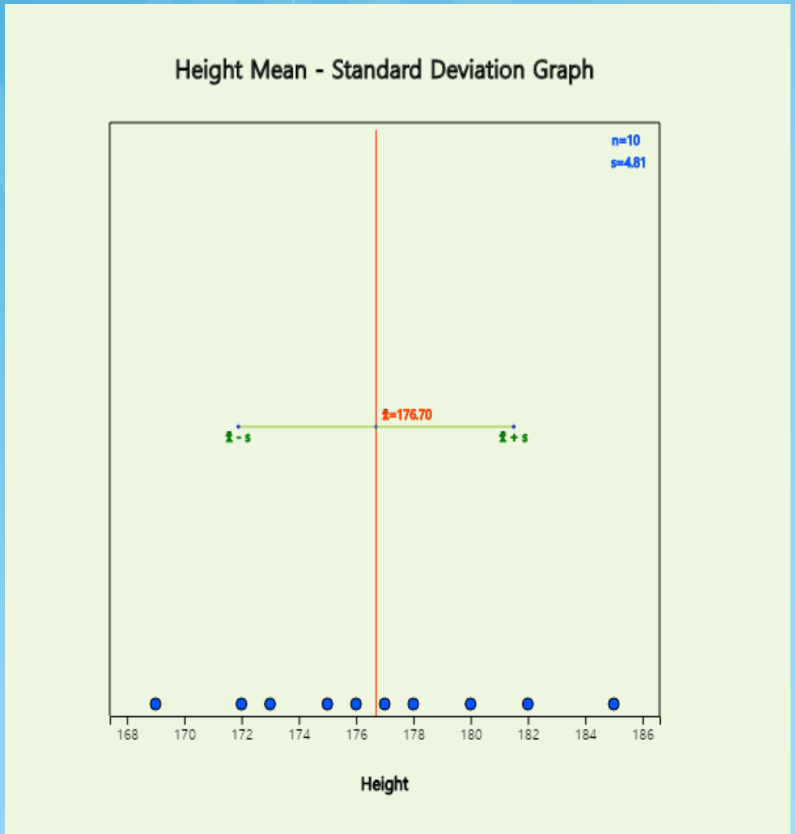


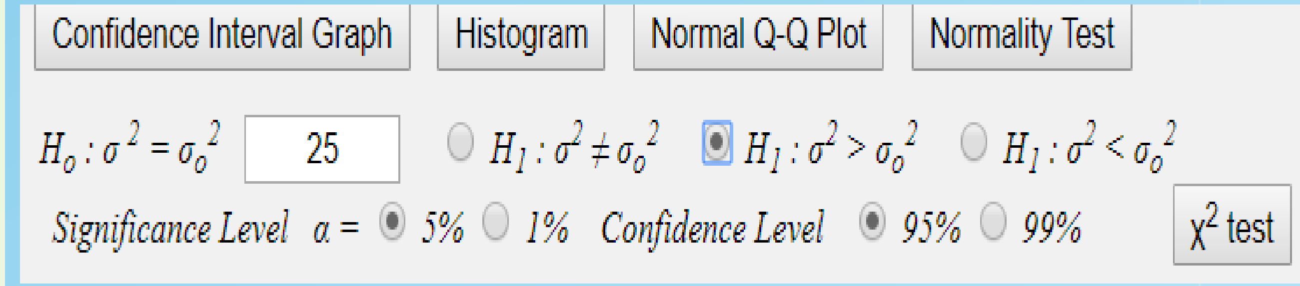


7.2 Testing Hypothesis for a Population Variance

[Ex 7.2.2] By using the height data of 10 male college students in [Ex 7.1.4], 172 175 178 182 176 180 169 185 173 177, test the hypothesis whether the population variance is greater than 25 at a significant level of 5%.

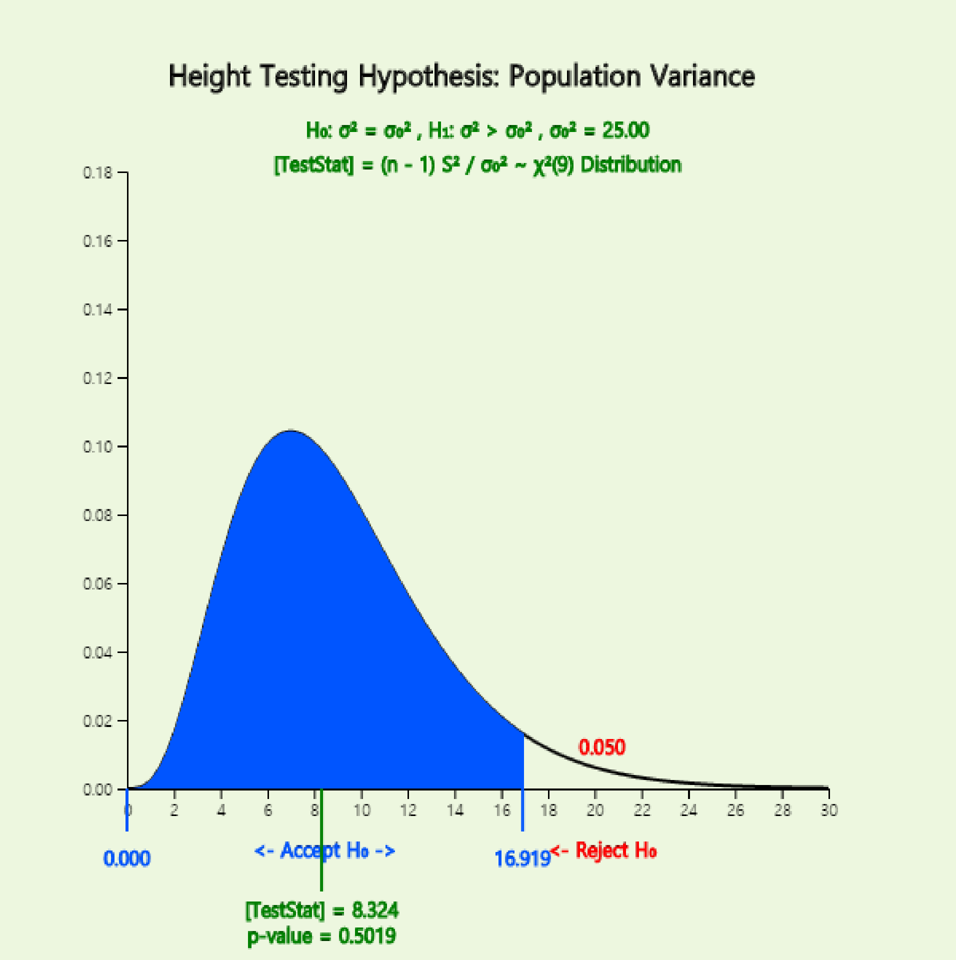






7.2 Testing Hypothesis for a Population Variance

⟨Answer of Ex 7.2.2⟩



Testing Hypothesis: Population Variance	Analysis Var	Height			
Statistics	Observation	Mean	Std Dev	std err	Population Variance 95% Confidence Interval
	10	176.700	4.809	1.521	(10.940, 77.063)
Missing Observations	0				
Hypothesis					
$H_0 : \sigma^2 = \sigma_0^2$	σ_0^2	[TestStat]	ChiSq value	p-value	
$H_1 : \sigma^2 > \sigma_0^2$	25.00	$(n-1)\ S^2 / \sigma_0^2$	8.324	0.5019	

7.3 Testing Hypothesis for a Population Proportion

- Let's take some examples of the need for testing hypothesis of unknown proportions of the population.

− Will the approval rating of a particular candidate exceed 50 percent in this year's presidential election?

− The unemployment rate was 7 percent last year. Has this year's unemployment rate increased?

− 10,000 car accessories are imported by ship, of which 2 percent were defective according to past experience. Is the defective product 2% this time again?

7.3 Testing Hypothesis for a Population Proportion

Table 7.3.1 Testing hypothesis for population proportion
- large sample case such as np_0 > 5, $n(1-p_0)$ > 5

Type of Hypothesis	Decision Rule
1) $H_0 : p = p_0$ $H_1 : p > p_0$	If $\dfrac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} > z_\alpha$, then reject H_0, else accept H_0
2) $H_0 : p = p_0$ $H_1 : p < p_0$	If $\dfrac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} < -z_\alpha$, then reject H_0, else accept H_0
3) $H_0 : p = p_0$ $H_1 : p \neq p_0$	If $\left\lvert \dfrac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} \right\rvert > z_{\alpha/2}$, then reject H_0, else accept H_0

Note: The null hypothesis in 1) can be written as $H_0 : p \leq p_0$ and in 2) as $H_0 : p \geq p_0$

7.3 Testing Hypothesis for a Population Proportion

[Example 7.3.1] A survey was conducted last month for the election of national assembly member. According to the survey of the last month, the approval rating of a particular candidate was 60 percent. In order to see if there is a change in the approval rating, a sample survey of 100 people has been conducted and 55 people supported it.

1) Test whether the current approval rating for a particular candidate is changed comparing with the one of last month of 60%. Use 5% the significance level.
2) Check the results using 「eStatU」.

<Answer>

1) The hypothesis of this problem is H_0 : $p = 0.6$, H_1 : $p \neq 0.6$. Since np_0 = 60, $n(1-p_0)$ = 40, it can be considered as a large sample and the decision rule is as follows.

$$\text{'If } \left| \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} \right| > z_{\alpha/2} \text{, reject } H_0\text{, else accept } H_0\text{'}$$

Since $\hat{p}$ = 55/100 = 0.55,

$$\left| \frac{0.55 - 0.6}{\sqrt{0.6(1-0.6)/100}} \right| = |-1.005| = 1.005, \quad z_{0.05/2} = z_{0.025} = 1.96$$

Hence H_0 is accepted.

⟨Answer of Ex 7.3.1⟩

Testing Hypothesis p

Menu

[Hypothesis] $H_o : p = p_o$ 0.6 $0 < p_o < 1$

◉ $H_1 : p \neq p_o$ ○ $H_1 : p > p_o$ ○ $H_1 : p < p_o$

[Test Type] Z test

Significance Level α = ◉ 5% ○ 1%

[Sample Data]

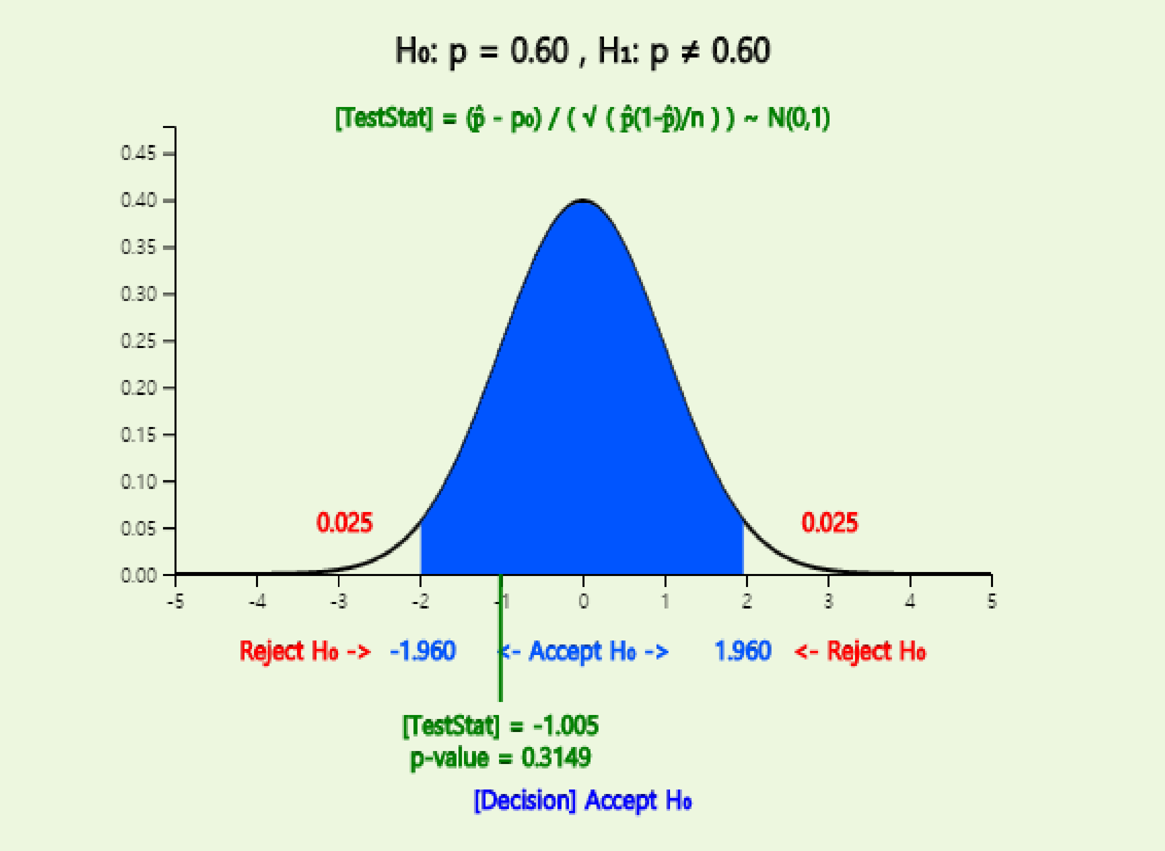
Sample Size n = 100

Sample Proportion $\hat{p}$ = 0.55 $0 < \hat{p} < 1$

Execute

[Confidence Interval]

$\hat{p} \pm z_{\alpha/2} \sqrt{(\hat{p}(1-\hat{p})/n)}$ ⇔ (0.452 , 0.648)



7.4 Testing Hypothesis with α and β simultaneously

- Since the testing hypothesis we have learned so far is a conservative decision-making method, we have created a critical value that reduces the probability of type one error α (error that rejects the null hypothesis even though it is true).

- The criteria is intended to keep the null hypothesis unless there is sufficient evidence of the alternative hypothesis which is a new fact or risky. Thus, the probability of type two error (β) was not considered at all in the selection criteria.

- However, sometimes it is unclear which fact should be the null hypothesis and which one should be the alternative hypothesis. Depending on the problem, both kinds of errors are important and should be considered simultaneously. At this time, if the analyst can determine the sample size, a testing hypothesis that takes into account both α and β can be performed.

7.4 Testing Hypothesis with α and β simultaneously

7.4. 1 β and the power of test

[Example 7.4.1] For testing hypothesis of [Example 7.1.1], calculate the probability of type two error β if the significant level is 5%. Check this result using 「eStatU」.

<Answer>

- The hypothesis in [Example 7.1.1] is H_0 : $\mu = 1500$, H_1 : $\mu = 1600$, the population standard deviation is assumed σ = 200, the sample size is n=30 and hence the decision rule is as follows if the significance level is 5%.

  'If $\overline{X}$ < $1500 + (1.645)\dfrac{200}{\sqrt{30}}$ = 1560.06, reject H_0, else accept H_0'

- Hence the type two error which is the probability of 'H_0 is true when H_1 is true' can be calculated as follows.

$$\beta = P(\overline{X} < 1560.06 \mid H_1 \text{ is true})$$
$$= P((\overline{X}\text{-}1600)/(200/\sqrt{30}) < (1560.06\text{-}1600)/(200/\sqrt{30}))$$
$$= P(Z < -1.09) = 0.137$$

7.4 Testing Hypothesis with α and β simultaneously

⟨Answer of Ex 7.4.1⟩

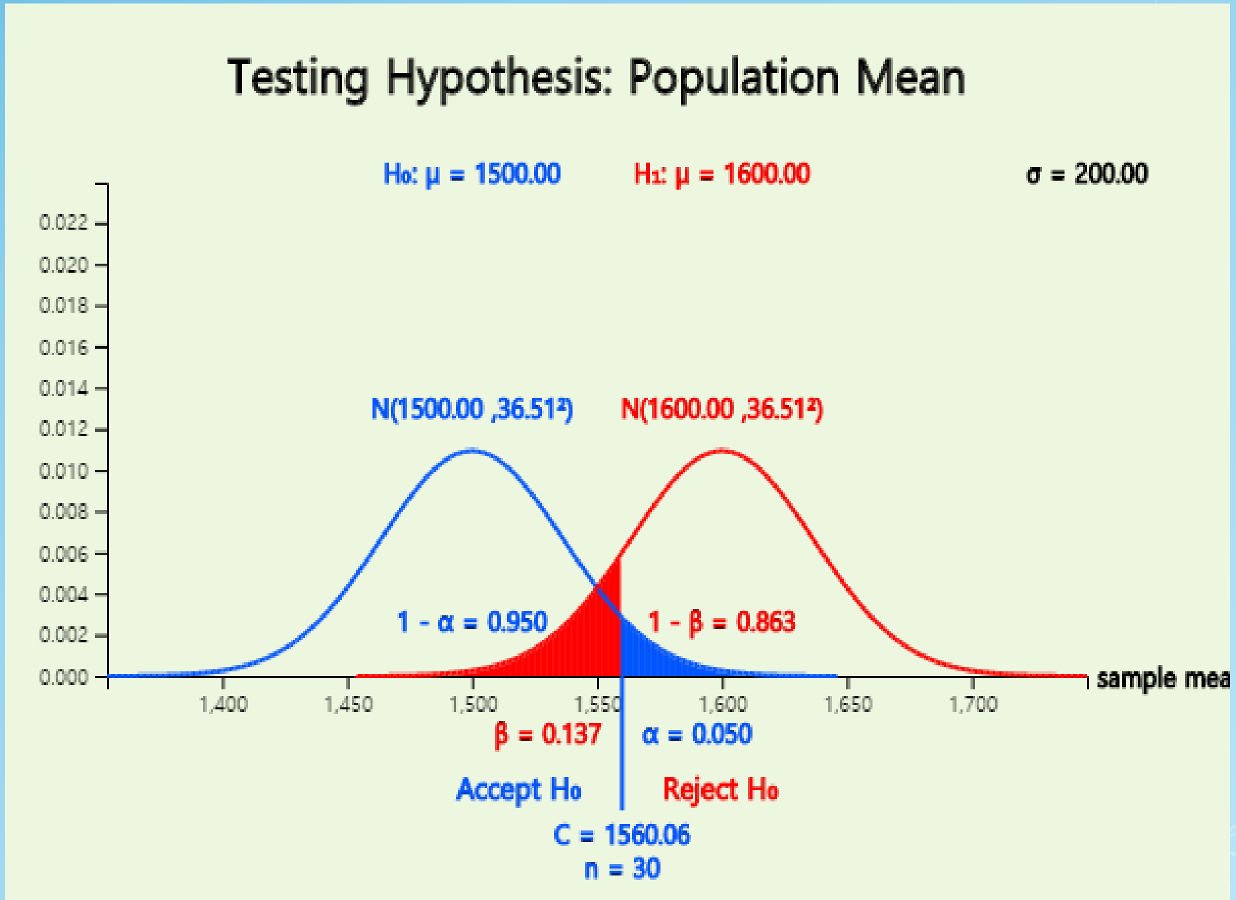
Testing μ - C, β Menu

[Hypothesis] $H_o : \mu = \mu_o$ 1500 $H_1 : \mu = \mu_1$ 1600

Population Standard Deviation $\sigma =$ 200

Type 1 Error $\alpha =$ 0.050 Sample Size $n =$ 30

Execute μ_1 μ_0-3σ μ_0+3σ



7.4 Testing Hypothesis with α and β simultaneously

[예 7.4.2] In [Example 7.1.1], if the null hypothesis is not changed but the alternative hypothesis is changed as follows. The significance level is 5%.

$$H_0 \; : \; \mu = 1500, \quad H_1 \; : \; \mu = 1580$$

1) calculate the probability of type two error β if the significant level is 5%.

2) Check this result using 「eStatU」.

<Answer>

1) Although the alternative hypothesis has been changed to $H_1 : \; \mu = 1580$, the decision rule will not be not changed in case of conservative decision making because the alternative hypothesis is the same type of $H_1 : \; \mu > 1500$.

'If $\overline{X}$ < 1500 + (1.645)$\frac{200}{\sqrt{30}}$ = 1560.06, reject H_0, else accept H_0'

Hence the probability of type two error is as follows.

β = P($\overline{X}$ < 1560.06 | H_1 이 참일 때)

= P(($\overline{X}$-1580)/(200/ $\sqrt{30}$) < (1560.06-1580)/(200/ $\sqrt{30}$))

= P(Z < -0.546) = 0.293

2) In order to calculate β using 「eStatU」, enter $\mu_1 = 1580$ in <Figure 7.4.1> and click [Execute] button.

7.4 Testing Hypothesis with α and β simultaneously

7.4. 1 β and the power of test

- Generally, the discriminating ability of two hypothesis tests is compared by using the following power of a test.

  Power = 1 - (Probability of type two error) = 1 - β

  A large power increases the discriminating ability of the hypothesis test.
- The power of a test can be obtained for any μ_1 of the alternative hypothesis $H_1 : \mu = \mu_1$. It means that the power is a function over the value of μ and it is called a **power function.**
- The function of probability that the null hypothesis is correct when the null hypothesis is true is called an **operating characteristic function.**

  Operational characteristic function = 1 - (Probability of type 1 error) = 1 - α

7.4 Testing Hypothesis with α and β simultaneously

7.4. 1 β and the power of test

[Example 7.4.3] In [Example 7.1.1], calculate the power of the following alternative hypothesis. Use α = 0.05. By using this power, approximate the power function.

1) $H_1 : \mu = 1500$ 2) $H_1 : \mu = 1510$ 3) $H_1 : \mu = 1520$

4) $H_1 : \mu = 1530$ 5) $H_1 : \mu = 1540$ 6) $H_1 : \mu = 1550$

7) $H_1 : \mu = 1560$ 8) $H_1 : \mu = 1570$ 9) $H_1 : \mu = 1580$

10) $H_1 : \mu = 1590$ 11) $H_1 : \mu = 1600$ 12) $H_1 : \mu = 1610$

<Answer>

- Although the alternative hypotheses are different, the decision rule is the same as follows.

  'If $\overline{X} < 1500 + (1.645)\dfrac{200}{\sqrt{30}} = 1560.06$, then accept H_0, else reject H_0'

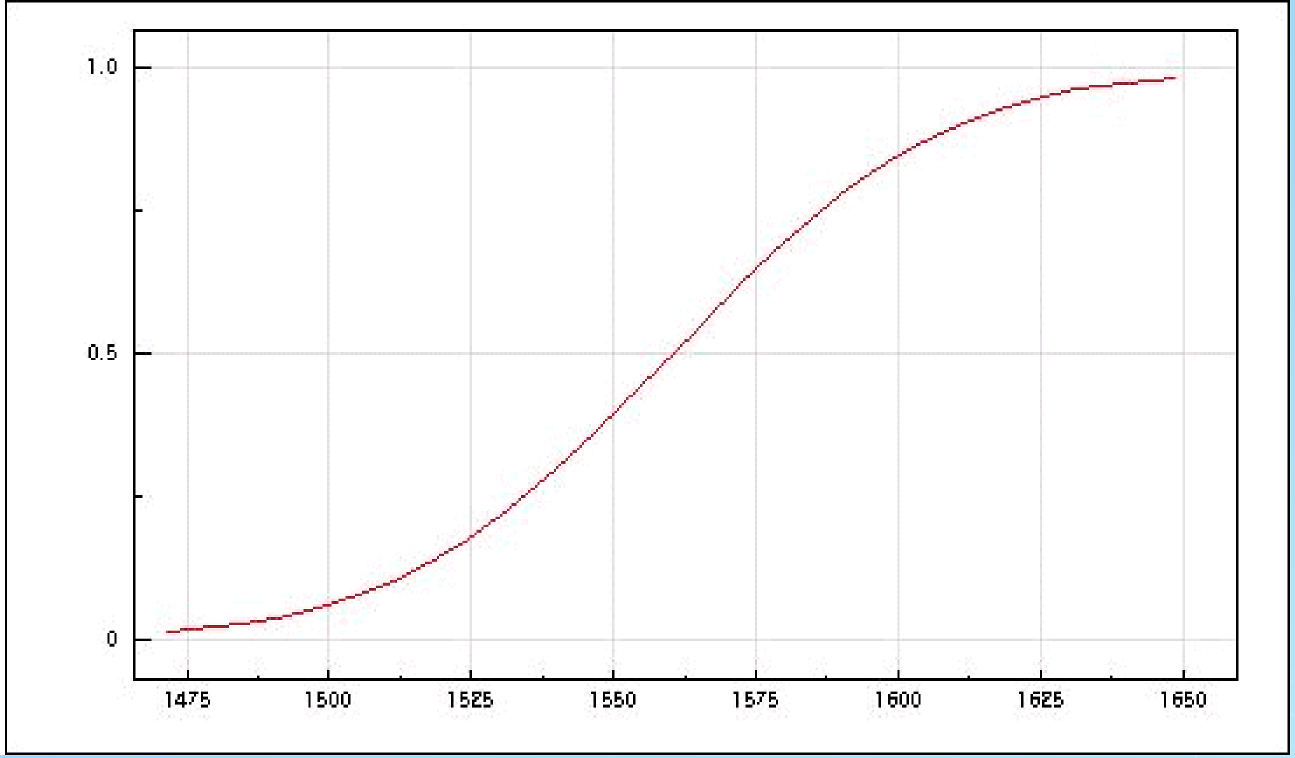
  Hence if we calculate the probability of type two error as [Example 7.4.2], the power of each test is as follows.

7.4 Testing Hypothesis with α and β simultaneously

⟨Answer of Ex 7.4.3⟩

Alternative Hypothesis	β	Power = 1 - β
1) H_1 : μ = 1500	0.95	0.05
2) H_1 : μ = 1510	0.91	0.09
3) H_1 : μ = 1520	0.86	0.14
4) H_1 : μ = 1530	0.79	0.21
5) H_1 : μ = 1540	0.71	0.29
6) H_1 : μ = 1550	0.61	0.39
7) H_1 : μ = 1560	0.50	0.50
8) H_1 : μ = 1570	0.39	0.61
9) H_1 : μ = 1580	0.29	0.71
10) H_1 : μ = 1590	0.21	0.79
11) H_1 : μ = 1600	0.14	0.86
12) H_1 : μ = 1610	0.09	0.91

- Power function



7.4 Testing Hypothesis with α and β simultaneously

7.4. 2 Testing Hypothesis with α and β

[Example 7.4.4] Consider a testing hypothesis on the bulb life such as $H_0: \mu = 1500$, $H_1: \mu = 1570$. Find the sample size n and the decision rule which satisfies α of 5% and β of 10%. Assume that the population standard deviation σ is 200 hours. Check the results using 「eStatU」.

<Answer>

- Let n be the sample size and C be the critical value of a decision rule. The probability of type one error α and the probability of type two error β are defined as follows.

  $\alpha = P(\overline{X} > C \mid H_0 \text{ is true})$
  $\beta = P(\overline{X} < C \mid H_1 \text{ is true})$

- If H_0 is true, the sampling distribution of $\overline{X}$ is $N(1500, \frac{200^2}{n})$ and if H_1 is true, the sampling distribution of $\overline{X}$ is $N(1570, \frac{200^2}{n})$. If α = 0.05 and β = 0.10, then $z_{0.05}$ = 1.645 and $z_{0.90}$ = -1.280. Hence n and C satisfy both of the following equations and they can be calculated by solving the two system of equations.

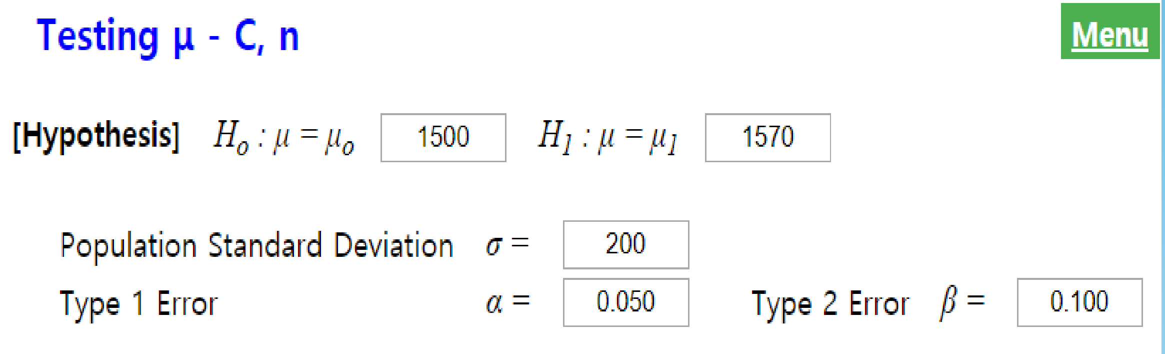
  $C = 1500 + 1.645 \times (200/\sqrt{n})$
  $C = 1570 - 1.280 \times (200/\sqrt{n})$

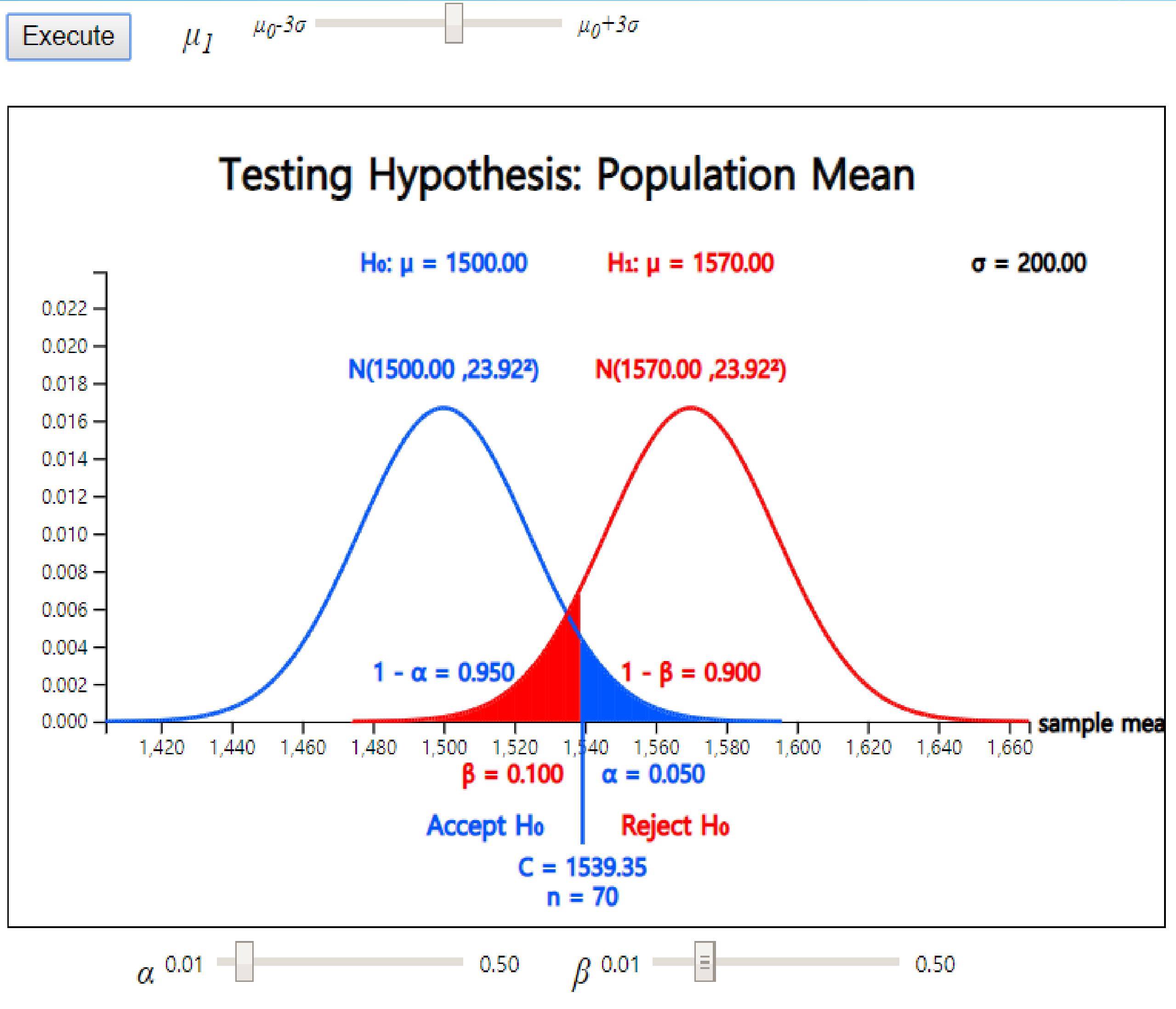
  The solution is n = 69.8, C = 1539.4. I.e., the saple size is 70 and the decision rule is as follows.

  'If $\overline{X}$ > 1539.4, then reject H_0, else accept H_0'

7.4 Testing Hypothesis with α and β simultaneously

〈Answer of Ex 7.4.4〉





7.5 Summary

- Testing Hypothesis for a Population Mean
- Testing Hypothesis for a Population Variance
- Testing Hypothesis for a Population Proportion
- Testing Hypothesis with α and β simultaneously
  - β and Power of a Test
  - Testing Hypothesis with α and β

Thank you