8

Testing Hypothesis for Two Population Parameters

SECTIONS

CHAPTER OBJECTIVES

In Chapter 7, we discussed how to test hypotheses about parameters in a single population.

In this chapter, we discuss testing hypothesis to compare population parameters of two populations.

Section 8.1 discusses a t-test for testing hypothesis of two population means when samples are independent and when samples are paired.

Section 8.2 discusses a F-test for testing hypothesis of two population variances.

Section 8.3 discusses a Z-test for testing hypothesis of two population proportions when samples are large enough.

8.1 Testing Hypothesis for Two Population Means

- There are many examples comparing means of two populations as follows:

  - Is there a difference between the starting salary of male graduates and of female graduates in this year's college graduates?
  - Is there a difference in the weight of the products produced in the two production lines?
  - Did the special training for typists to increase the speed of typing really bring about an increase in the speed of typing?

- As such, a comparison of the two population means (μ_1 and μ_2) is possible by testing hypothesis that the difference in the population means is greater than, or less than, or equal to zero. The comparison of two population means differs depending on whether samples are extracted independently from each population or not (referred to as paired samples).

8.1.1 Two Independent Samples

- Generally, testing hypothesis for two population means can be divided into three types, depending on the type of the alternative hypothesis as follows:

  1) $H_0 : \ \mu_1 - \mu_2 = D_0$ $\quad H_1 : \ \mu_1 - \mu_2 > D_0$

  2) $H_0 : \ \mu_1 - \mu_2 = D_0$ $\quad H_1 : \ \mu_1 - \mu_2 > D_0$

  3) $H_0 : \ \mu_1 - \mu_2 = D_0$ $\quad H_1 : \ \mu_1 - \mu_2 \neq D_0$

  Here D_0 is the value for the difference in population means to be tested.

- When samples are selected independently from each other in the population, the estimator of the difference of the population means $\mu_1 - \mu_2$ is the difference in sample means $\overline{X}_1 - \overline{X}_2$. The sampling distribution of all possible sample mean differences is approximately a normal distribution with the mean $\mu_1 - \mu_2$ and variance $\sigma_1^2/n_1 + \sigma_2^2/n_2$ if both sample sizes are large enough.
- Since the population variances σ_1^2 and σ_2^2 are usually unknown, estimates of these variances, s_1^2 and s_2^2, are used to test the hypothesis. The test statistic differs slightly depending on the assumption of two population variances. If two populations follow normal distributions and their variances can be assumed the same, the testing hypothesis for the difference of two population means uses the following statistic.

$$\frac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}} \qquad s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

  s_p^2 is an estimator of the population variance called as a **pooled variance** which is an weighted average of two sample variances s_1^2 and s_2^2 by using the sample sizes as weights when population variances are assumed to be the same.
- The above statistic follows a t-distribution with $n_1 + n_2 - 2$ degrees of freedom and it is used to test the difference of two population means as follows:

Table 8.1.1 Testing hypothesis of two population means
- independent samples, populations are normal distributions, two population variances are assumed to be equal

Type of Hypothesis	Decision Rule
1) $H_0:\ \mu_1 - \mu_2 = D_0$ $H_1:\ \mu_1 - \mu_2 > D_0$	If $\dfrac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}} > t_{n_1+n_2-2\,;\,\alpha}$, then reject H_0, else accept H_0
2) $H_0:\ \mu_1 - \mu_2 = D_0$ $H_1:\ \mu_1 - \mu_2 < D_0$	If $\dfrac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}} < -t_{n_1+n_2-2\,;\,\alpha}$, then reject H_0, else accept H_0
3) $H_0:\ \mu_1 - \mu_2 = D_0$ $H_1:\ \mu_1 - \mu_2 \neq D_0$	If $\left\lvert \dfrac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}} \right\rvert > t_{n_1+n_2-2\,;\,\alpha/2}$, then reject H_0, else accept H_0

※ If sample sizes are large enough ($n_1 > 30,\ \ n_2 > 30$), t distribution is approximately close to the standard normal distribution and the decision rule may use the standard normal distribution.

Example 8.1.1

Two machines produce cookies at a factory and the average weight of a cookie bag should be 270g. Cookie bags were sampled from each of two machines to examine the weight of the cookie bag. The average weight of 15 cookie bags extracted from the machine 1 was 275g and their standard deviation was 12g, and the average weight of 14 cookie bags extracted from the machine 2 was 269g and the standard deviation was 10g. Test whether weights of cookie bags produced by two machines are different at the 1% significance level. Check the test result using 『eStatU』.

Answer

- The hypothesis of this problem is $H_0:\ \mu_1 = \mu_2,\ \ H_1:\ \mu_1 \neq \mu_2$. Hence, the decision rule is as follows:

  ‘If $\left\lvert \dfrac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}} \right\rvert > t_{n_1+n_2-2\,;\,\alpha/2}$, then reject H_0’

  The information in this example can be summarized as follows:

  $$n_1 = 15,\ \overline{x}_1 = 275,\ s_1 = 12,$$
  $$n_2 = 14,\ \overline{x}_2 = 269,\ s_2 = 10$$

  Therefore,

  $$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} = \frac{(15-1)12^2 + (14-1)10^2}{15 + 14 - 2} = 122.815$$

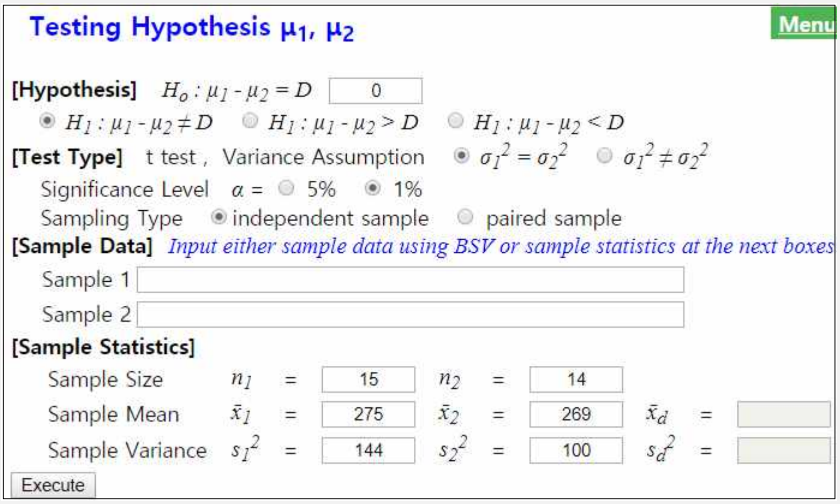
  $$\left\lvert \frac{275-269}{\sqrt{\dfrac{122.815}{15} + \dfrac{122.815}{14}}} \right\rvert = 1.457$$

  $$t_{15+14-2\,;\,0.01/2} = t_{27\,;\,0.005} = 2.7707$$

  Since 1.457 < 2.7707, H_0 can not be rejected.

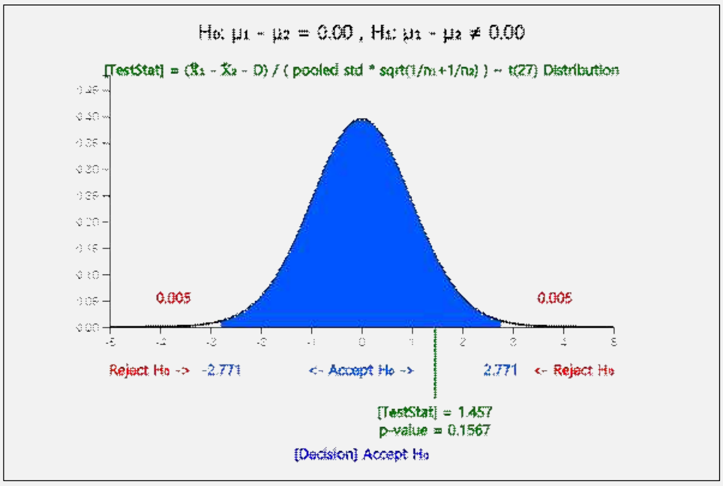
Example 8.1.1 Answer (continued)

- In『eStatU』menu, select 'Testing Hypothesis μ_1, μ_2', In the window shown in <Figure 8.1.1>, check the alternative hypothesis of not equal case at [Hypothesis], check the variance assumption of [Test Type] as the equal case, check the significance level of 1%, check the independent sample, and enter sample sizes n_1, n_2, sample means $\overline{x}_1$, $\overline{x}_2$, and sample variances as in <Figure 8.1.1>.



<Figure 8.1.1> Testing hypothesis for two population means using『eStatU』

- Click the [Execute] button will show the result of testing hypothesis as <Figure 8.1.2>.



<Figure 8.1.2> Testing hypothesis for two population means – case of the same population variances

- If variances of two populations are different, the test statistic

$$\frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

do not follow a t distribution even if populations are normally distributed. The testing hypothesis for two population means when their population variances are different is called a Behrens-Fisher problem and several methods to solve this problem have been studied. The Satterthwaite method approximates the degrees of freedom $n_1 + n_2 - 2$ of the t distribution in the decision rule in Table 8.1.1 with ϕ as follows:

$$\phi \;=\; \frac{\left[\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right]^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

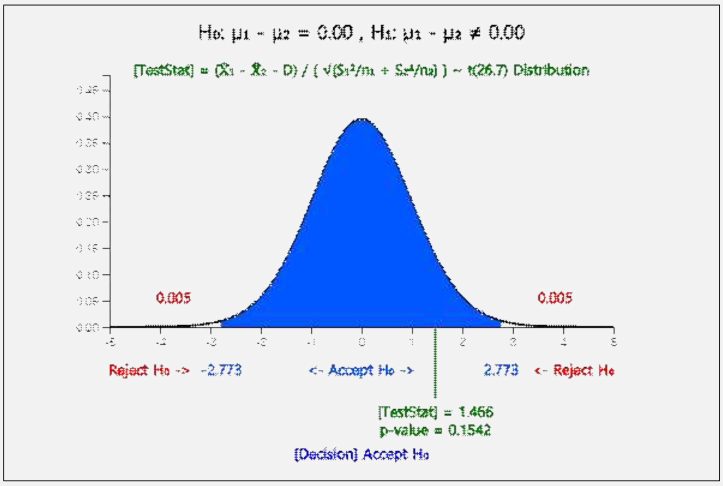
- Table 8.1.2 summarizes decision rule when two population variances are different.

Table 8.1.2 Testing hypothesis of two population means
- independent samples, populations are normal distributions,
two population variances are assumed to be different

Type of Hypothesis	Decision Rule
1) $H_0:\ \mu_1 - \mu_2 = D_0$ $H_1:\ \mu_1 - \mu_2 > D_0$	If $\dfrac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} > t_{\phi\,;\,\alpha}$, then reject H_0, else accept H_0
2) $H_0:\ \mu_1 - \mu_2 = D_0$ $H_1:\ \mu_1 - \mu_2 < D_0$	If $\dfrac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} < -t_{\phi\,;\,\alpha}$, then reject H_0, else accept H_0
3) $H_0:\ \mu_1 - \mu_2 = D_0$ $H_1:\ \mu_1 - \mu_2 \neq D_0$	If $\left\lvert \dfrac{(\overline{X}_1 - \overline{X}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} \right\rvert > t_{\phi\,;\,\alpha/2}$, then reject H_0, else accept H_0

Example 8.1.2	If two population variances are assumed to be different in [Example 8.1.1], test whether weights of cookie bags produced from two machines are equal or not at a 1% significance level. Check the test result using『eStatU』.
Answer	◆ Since the population variances are different, the degrees of freedom ϕ of t distribution is approximated as follows: $\phi \;=\; \dfrac{\left[\dfrac{12^2}{15} + \dfrac{10^2}{14}\right]^2}{\dfrac{\left(\dfrac{12^2}{15}\right)^2}{15-1} + \dfrac{\left(\dfrac{10^2}{14}\right)^2}{14-1}} \;=\; 26.67$ $t_{26.7\,;\,0.01/2} \;=\; 2.773$ Since 1.457 < 2.773, H_0 can not be rejected. ◆ In order to practice using『eStatU』, select the different population variances assumption $\sigma_1^2 \neq \sigma_2^2$ of [Test Type] in the window of <Figure 8.1.1> and click the [Execute] button to see the result as shown in <Figure 8.1.3>.

Example 8.1.2 Answer



<Figure 8.1.3> Testing hypothesis for two population means
– Case of two different population variances

Example 8.1.3

(Monthly wages by male and female)
Samples of 10 male and female college graduates this year were randomly taken and their monthly average wages were examined as follows: (Unit 10,000 KRW)

Male	272	255	278	282	296	312	356	296	302	312
Female	276	280	369	285	303	317	290	250	313	307

⇨ eBook ⇨ EX080103_WageByGender.csv.

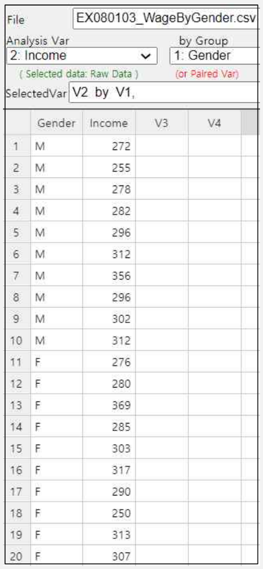
Using 『eStat』, answer the following questions.

1) If population variances are assumed to be the same, test the hypothesis at the 5% significance level whether the average monthly wage for male and female is the same.
2) If population variances are assumed to be different, test the hypothesis at the 5% significance level whether the average monthly wage for male and female is the same.

Answer

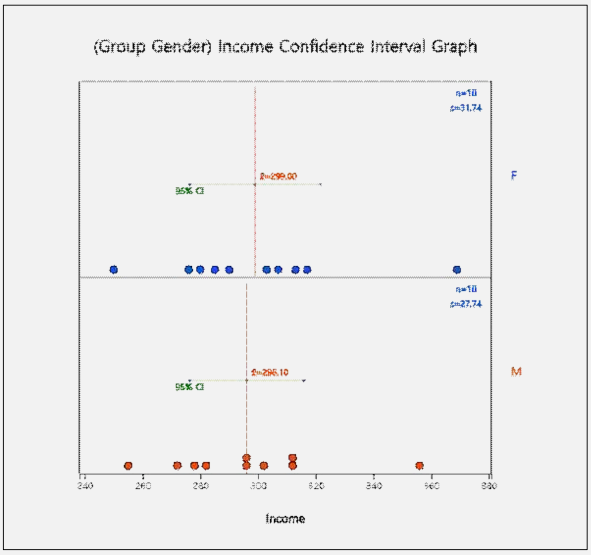
1) In 『eStat』, enter raw data of gender (M or F) and income as shown in <Figure 8.1.4> on the sheet. This type of data input is similar to all statistical packages. After entering the data, click the icon for testing two population means and select 'Analysis Var' as V2 and 'By Group' variable as V1. A 95% confidence interval graph that compares sample means of two populations will be displayed as <Figure 8.1.5>.

Example 8.1.3
Answer



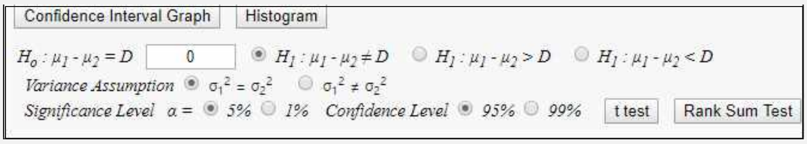
	Gender	Income	V3	V4
1	M	272		
2	M	255		
3	M	278		
4	M	282		
5	M	296		
6	M	312		
7	M	356		
8	M	296		
9	M	302		
10	M	312		
11	F	276		
12	F	280		
13	F	369		
14	F	285		
15	F	303		
16	F	317		
17	F	290		
18	F	250		
19	F	313		
20	F	307		

<Figure 8.1.4> Data input for testing two population means

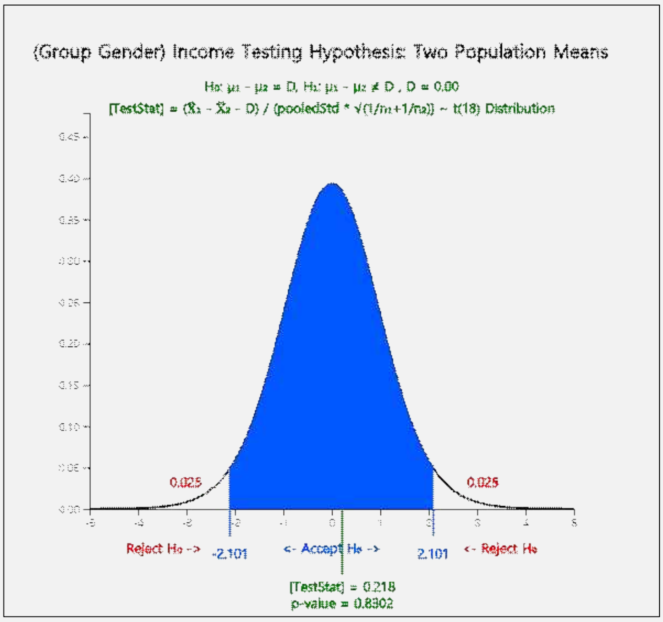


<Figure 8.1.5> Dot graph and confidence Intervals by gender for testing two population means

- In the options window as in <Figure 8.1.6> located at the below of the Graph Area, enter the average difference $D=0$ for the desired test, select the variance assumption $\sigma_1^2=\sigma_2^2$, select the 5% significance level and click the [t-test] button. Then the graphical result of testing hypothesis for two population means will be shown as in <Figure 8.1.7> and the test result as in <Figure 8.1.8>.



<Figure 8.1.6> Options to test for two population means



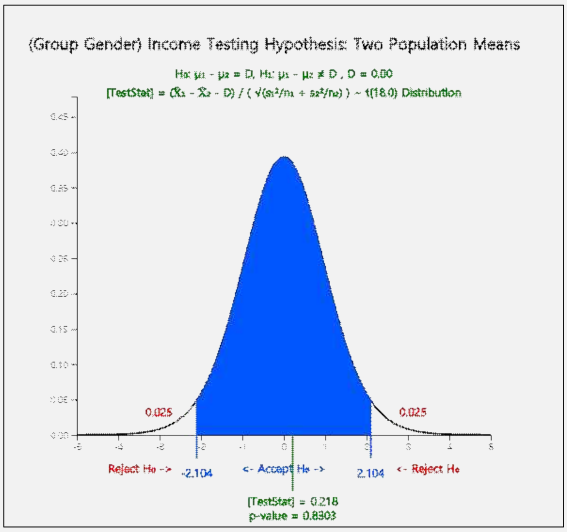
<Figure 8.1.7> Testing hypothesis for μ_1 and μ_2 – case of the same population variances

Example 8.1.3 Answer (continued)

Testing Hypothesis: Two Population Means	Analysis Var	Income	Group Name	Gender	
Statistics	Observation	Mean	Std Dev	std err	Population Mean 95% Confidence Interval
1 (F)	10	299.000	31.742	10.038	(276.293, 321.707)
2 (M)	10	296.100	27.739	8.772	(276.257, 315.943)
Total	20	297.550	29.051	6.496	(283.954, 311.146)
Missing Observations	0				
Hypothesis	Variance Assumption	$\sigma_1^2 = \sigma_2^2$			
$H_0 : \mu_1 - \mu_2 = D$	D	[TestStat]	t value	p-value	μ_1-μ_2 95% Confidence Interval
$H_1 : \mu_1 - \mu_2 \neq D$	0.00	Difference of Sample Means	0.218	0.8302	(-25.106, 30.906)

<Figure 8.1.8> result of testing hypothesis for two population means if population variances are the same

2) Select the variance assumption $\sigma_1^2 \neq \sigma_2^2$ at the option window and click [t-test] button under the graph to display the graph of the hypothesis test and the test result table as in <Figure 8.1.9> and <Figure 8.1.10>.



<Figure 8.1.9> Testing hypothesis for μ_1 and μ_2 – case of the different population variances

Testing Hypothesis: Two Population Means	Analysis Var	Income	Group Name	Gender	
Statistics	Observation	Mean	Std Dev	std err	Population Mean 95% Confidence Interval
1 (F)	10	299.000	31.742	10.038	(276.293, 321.707)
2 (M)	10	296.100	27.739	8.772	(276.257, 315.943)
Total	20	297.550	29.051	6.496	(283.954, 311.146)
Missing Observations	0				
Hypothesis	Variance Assumption	$\sigma_1^2 \neq \sigma_2^2$			
$H_0 : \mu_1 - \mu_2 = D$	D	[TestStat]	t value	p-value	μ_1-μ_2 95% Confidence Interval
$H_1 : \mu_1 - \mu_2 \neq D$	0.00	Difference of Sample Means	0.218	0.8303	(-25.142, 30.942)

<Figure 8.1.10> Result of testing hypothesis for two population means if population variances are different

[Practice 8.1.1]

(Oral Cleanliness by Brushing Methods)

Oral cleanliness scores were examined for 8 samples who are using the basic brushing method (coded 1) and 7 samples who are using the rotation method (coded 2). The data are saved at the following location of 『eStat』.

⇨ eBook ⇨ PR080101_ToothCleanByBrushMethod.csv.

1) If population variances are the same, test the hypothesis at the 5% significance level whether scores for both brushing methods are the same using 『eStat』.
2) If population variances are different, test the hypothesis at the 5% significance level whether scores for both brushing methods are the same using 『eStat』.

8.1.2 Paired Sample

- The testing hypothesis for two population means in the previous section is based on two samples extracted independently from each population. However, in some cases it is difficult to extract samples independently, or if samples are extracted independently, then the resulting analysis may be meaningless, because characteristics of each sample differ too much.
- For example, you want to give typists a special education to increase the speed of typing and want to see if this training has been effective in the speed of typing. In this case, if different samples are extracted before and after education, it is difficult to measure the effectiveness of education, because individual differences are severe. In order to overcome the individual difference for a typist who has sampled before training education, if you measure the typing speed before and after the training for the typist, the effect of special education can be well understood.
- A hypothesis test that uses same samples to perform similar experiments to compare means of two populations is called a paired comparison. In the paired comparison, we calculate the difference (d_i) between paired data x_{i1} and x_{i2} as shown in Table 8.1.3 and obtain the mean of differences ($\overline{d}$) and variance of differences (s_d^2).

Table 8.1.3 Data for a paired comparison

Sample of population 1 (x_{i1})	Sample of population 2 (x_{i2})	Difference of pair $d_i = x_{i1} - x_{i2}$
x_{11} x_{21} ... x_{n1}	x_{12} x_{22} ... x_{n2}	$d_1 = x_{11} - x_{12}$ $d_2 = x_{21} - x_{22}$ ... $d_n = x_{n1} - x_{n2}$
	Mean of d_i Variance d_i	$\overline{d} = \sum d_i / n$ $s_d^2 = \sum (d_i - \overline{d})^2/(n-1)$

- When two populations of normal distributions have the same mean, the sample statistic $\overline{d}/(s_d/\sqrt{n})$ follows a t distribution with the n-1 degrees of freedom. It allows the testing of the difference between two population means in case of the paired comparison as follows:

Table 8.1.4 Testing hypothesis of two population means (paired comparison)
- two populations are normal distributions, and paired sample case

Type of Hypothesis	Decision Rule
1) $H_0:\ \mu_1-\mu_2=D_0$ $H_1:\ \mu_1-\mu_2>D_0$	If $\dfrac{\overline{d}-D_o}{\dfrac{s_d}{\sqrt{n}}} > t_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0
2) $H_0:\ \mu_1-\mu_2=D_0$ $H_1:\ \mu_1-\mu_2<D_0$	If $\dfrac{\overline{d}-D_o}{\dfrac{s_d}{\sqrt{n}}} < -t_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0
3) $H_0:\ \mu_1-\mu_2=D_0$ $H_1:\ \mu_1-\mu_2\neq D_0$	$\left\lvert\dfrac{\overline{d}-D_o}{\dfrac{s_d}{\sqrt{n}}}\right\rvert > t_{n-1\,;\,\alpha/2}$, then reject H_0, else accept H_0

Example 8.1.4

The following is the result of a special training to improve the typing speed of eight typists before and after the training. Test whether or not the typing speed has increased at the 5% significance level. Assume that the speed of typing follows a normal distribution. Check the test result using『eStat』and『eStatU』.

id	Typing speed before training (unit: words/min)	Typing speed after training (unit: words/min)
1	52	58
2	60	62
3	63	62
4	43	48
5	46	50
6	56	55
7	62	68
8	50	57

Answer

- This problem is for testing the null hypothesis $H_0:\ \mu_1-\mu_2 = 0$ to the alternative hypothesis $H_1:\ \mu_1-\mu_2 < 0$ to compare the typing speed of typists before training (population 1) and after training (population 2) using paired samples. Therefore, the decision rule is as follows:

  ‘If $\dfrac{\overline{d}-D_o}{\dfrac{s_d}{\sqrt{n}}} < -t_{n-1\,;\,\alpha}$, then reject H_0.’

- Calculated differences (d_i) of paired samples before and after training, the mean ($\overline{d}$) and standard deviation (s_d) of differences are as follows:

Example 8.1.4 Answer

id	Typing speed before training (unit: words/min)	Typing speed after training (unit: words/min)	Difference d_i
1	52	58	−6
2	60	62	−2
3	63	62	1
4	43	48	−5
5	46	50	−4
6	56	55	1
7	62	68	−6
8	50	57	−7
			Mean $\overline{d} = -3.5$ Standard deviation $s_d = 3.16$

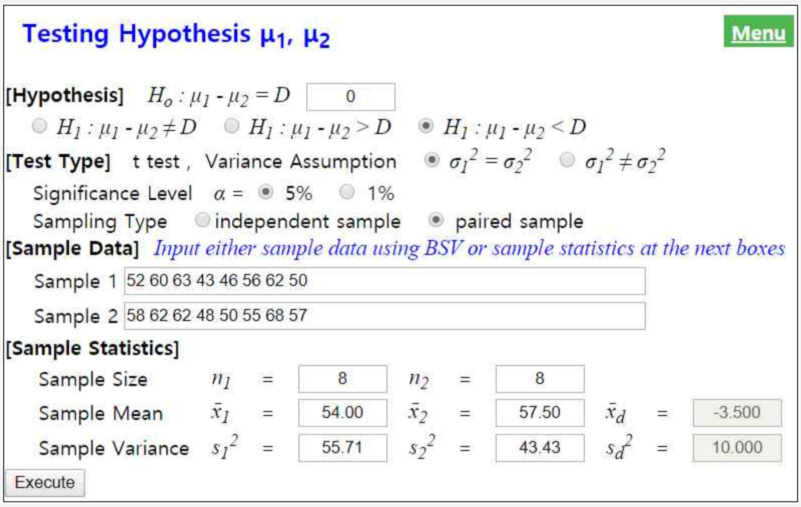
- The test statistic is as follows:

$$\frac{\overline{d} - D_o}{\frac{s_d}{\sqrt{n}}} = \frac{-3.5}{\frac{3.16}{\sqrt{8}}} = -3.13$$

$$-t_{n-1\,;\,\alpha} = -t_{8-1\,;\,0.05} = -t_{7\,;\,0.05} = -1.8946,$$

Therefore, H_0 is rejected and concludes that the training increased the typing speed.

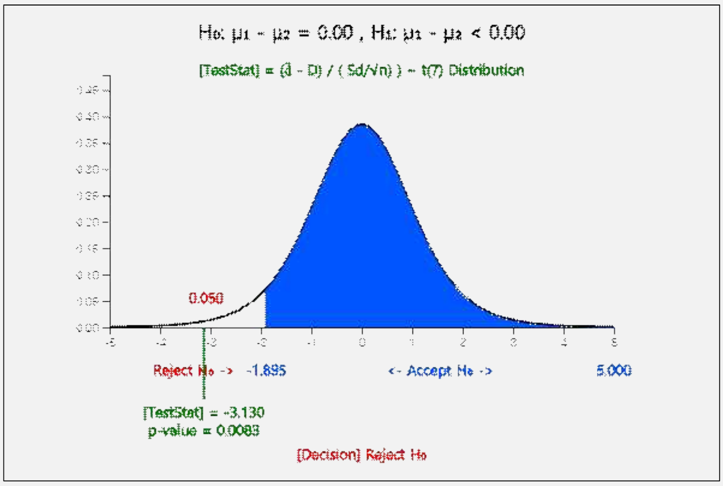
- In 『eStatU』 menu, select 'Testing Hypothesis: μ_1, μ_2', select the alternative hypothesis ◉ $H_1 : \mu_1 - \mu_2 < D$ at [Hypothesis], check the 5% significance level, check 'paired sample' at [Test Type], and enter data of sample 1 and sample 2 of paired samples at [Sample Data] as in <Figure 8.1.11>.



<Figure 8.1.11> Testing hypothesis for two population means using 『eStatU』 - paired sample

- Click the [Execute] button to calculate the sample mean and sample standard deviation of differences ($\overline{d}$ and s_d^2) and to show the result of the hypothesis test as <Figure 8.1.12>.

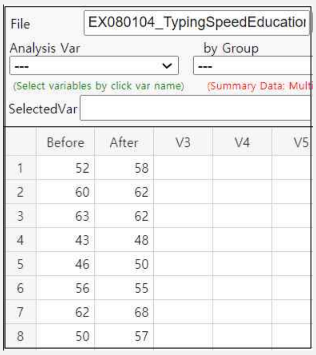
Example 8.1.4 Answer (continued)



<Figure 8.1.12> Result of testing hypothesis for two population means using 『eStatU』 - paired sample

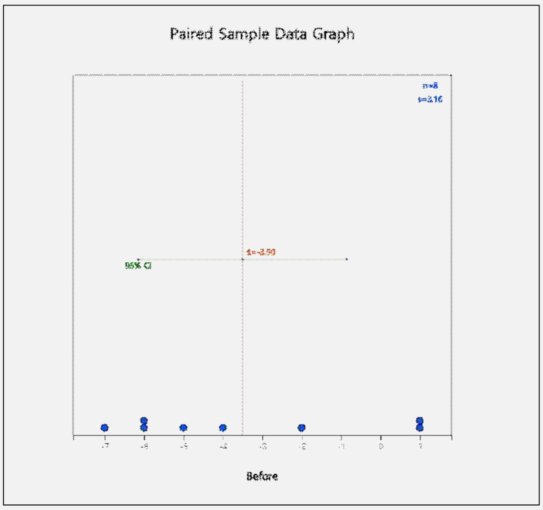
- In『eStat』, the paired data is entered in two columns as shown in <Figure 8.1.13>. Click the icon for testing two population means and select 'Analysis Var' as V1 and 'by Group' as V2 to show the dot graph and the confidence interval for differences of paired data as in <Figure 8.1.14>.

⇨ eBook ⇨ EX080104_TypingSpeedEducation.csv.



	Before	After	V3	V4	V5
1	52	58			
2	60	62			
3	63	62			
4	43	48			
5	46	50			
6	56	55			
7	62	68			
8	50	57			

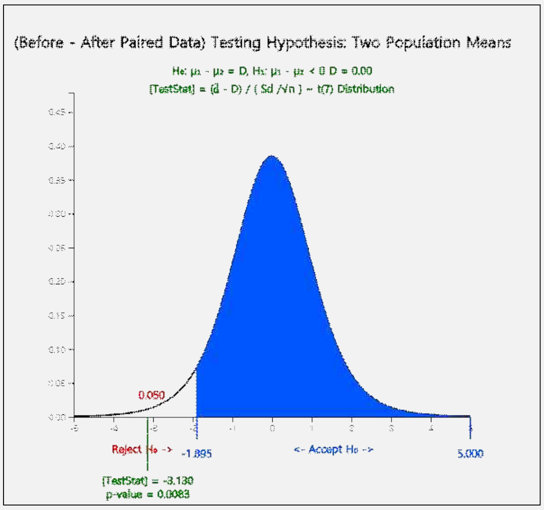
<Figure 8.1.13> Data input of paired sample



<Figure 8.1.14> Dot graph of difference data of paired sample

Example 8.1.4 Answer (continued)

- Enter the mean difference D = 0 for the desired test in the options window below the graph, select the 5% significance level, and press the [t-test] button to display the result of the hypothesis test for paired samples such as <Figure 8.1.15> and <Figure 8.1.16>.



<Figure 8.1.15> Testing hypothesis for two population means using 『eStat』 - paired sample

Testing Hypothesis: Two Population Means	Analysis Var	(Before - After)			
Statistics	Observation	Mean	Std Dev	std err	Population Mean 95% Confidence Interval
	8	-3.500	3.162	1.118	(-6.144, -0.856)
Missing Observations	0				
Hypothesis	Variance Assumption	$\sigma_1^2 = \sigma_2^2$			
$H_0 : \mu_1 - \mu_2 = D$	D	[TestStat]	t value	p-value	μ_1-μ_2 95% Confidence Interval
$H_1 : \mu_1 - \mu_2 < D$	0.00	Difference of Sample Means	-3.130	0.0083	(-6.144, -0.856)

<Figure 8.1.16> Result of testing hypothesis for two population means using 『eStat』 - paired sample

[Practice 8.1.2]

Randomly sampled data of (wife age, husband age) for 8 couples are as follows:

(28, 28) (29, 30) (18, 21) (29, 33) (22, 22) (18, 21) (40, 35) (24, 29)
⇨ eBook ⇨ PR080102_CoupleAge.csv.

Test whether the population mean of wife's age is the same as the population mean husband's age or not. Use the significance level of 0.05.

8.2 Testing Hypothesis for Two Population Variances

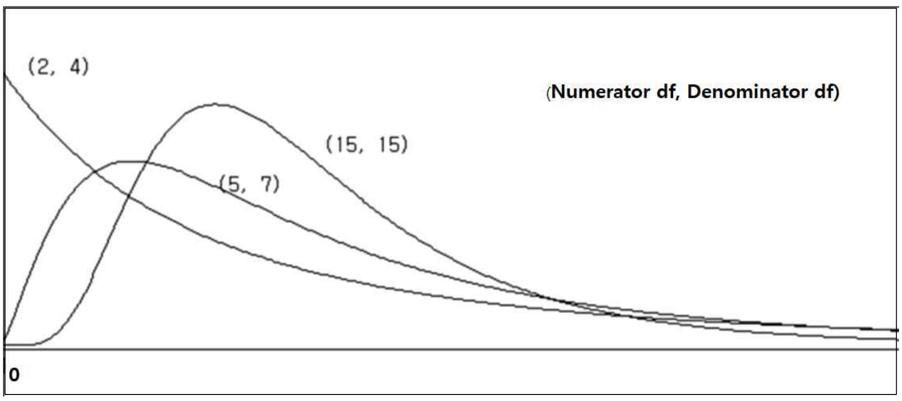
- Consider following examples to compare two population variances.

  - When comparing two population means in the previous section, we studied that if the sample size was small, the decision rule for testing hypothesis were different depending on whether two population variances were the same or different. So how can we test if two population variances are the same?
  - The quality of bolts used to assemble cars depends on the strict specification for their diameters. Average diameters of bolts produced by two factories were said to be the same and if the variance of diameters is smaller, it is considered as superior production. How can you compare variances of the diameter?

- When comparing variances (σ_1^2 and σ_2^2) of two populations, the ratio (σ_1^2/σ_2^2) of variances is calculated instead of comparing the difference in variances. If the ratio of variances is greater, smaller, or equal to 1, you can see that σ_1^2 is greater, smaller, or equal to σ_2^2. The reason for using the ratio of variances instead of the difference of variances is that it is easy to find the sampling distribution of the ratio of variances mathematically. If two populations follow normal distributions, and if n_1 and n_2 samples are collected randomly from each population, the ratio of two sample variances S_1^2 and S_2^2 such as

$$\frac{\left(\frac{S_1^2}{\sigma_1^2}\right)}{\left(\frac{S_2^2}{\sigma_2^2}\right)}$$

  follows a F distribution with the numerator degrees of freedom $n_1 - 1$ and the denominator degrees of freedom $n_2 - 1$. Using this fact, we can perform testing hypothesis on the ratio of population variances.
- F distribution is an asymmetrical distribution group with two parameters, the numerator degrees of freedom and denominator degrees of freedom. <Figure 8.2.1> shows F distributions for different parameters.



<Figure 8.2.1> F distribution of different degrees of freedom.

- Testing hypothesis for two population variances can be performed using the F distribution as following Table 8.2.1.

Table 8.2.1 Testing hypothesis for two population variances
- Two populations are normally distributed-

Type of Hypothesis	Decision Rule
1) $H_0:\ \sigma_1^2 = \sigma_2^2$ $H_1:\ \sigma_1^2 > \sigma_2^2$	If $\frac{S_1^2}{S_2^2} > F_{n_1-1,n_2-1\,;\,\alpha}$, then reject H_0, else accept H_0
2) $H_0:\ \sigma_1^2 = \sigma_2^2$ $H_1:\ \sigma_1^2 < \sigma_2^2$	If $\frac{S_1^2}{S_2^2} < F_{n_1-1,n_2-1\,;\,1-\alpha}$, then reject H_0, else accept H_0
3) $H_0:\ \sigma_1^2 = \sigma_2^2$ $H_1:\ \sigma_1^2 \neq \sigma_2^2$	If $\frac{S_1^2}{S_2^2} < F_{n_1-1,n_2-1\,;\,1-\alpha/2}$ or $\frac{S_1^2}{S_2^2} > F_{n_1-1,n_2-1\,;\,\alpha/2}$, then reject H_0, else accept H_0

Example 8.2.1

A company that produces a bolt has two plants. One day, 12 bolts produced in Plant 1 were sampled randomly and the variance of diameter was 0.11^2. 10 bolts produced in Plant 2 were sampled randomly and the variance of diameter was 0.13^2. Test whether variances of the bolt from two plants are the same or not with the 5% significance level. Check the test result using『eStatU』.

Answer

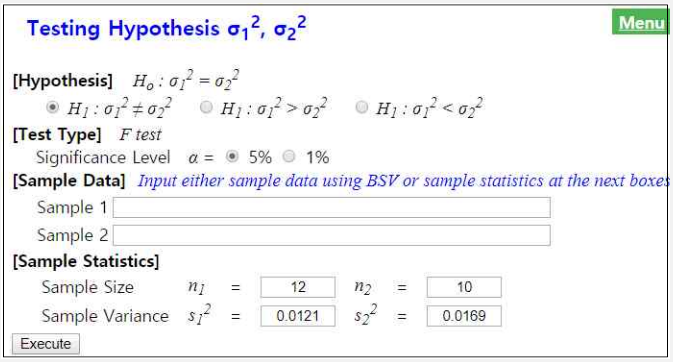
- The hypothesis of this problem is $H_0:\ \sigma_1^2 = \sigma_2^2,\ H_1:\ \sigma_1^2 \neq \sigma_2^2$ and its decision rule is as follows:

  ‘If $\frac{S_1^2}{S_2^2} < F_{n_1-1,n_2-1\,;\,1-\alpha/2}$ or $\frac{S_1^2}{S_2^2} > F_{n_1-1,n_2-1\,;\,\alpha/2}$, then reject H_0, else accept H_0’

  The test statistic using two sample variances $s_1^2,\ s_2^2$ and the percentile of F distribution is as follows:

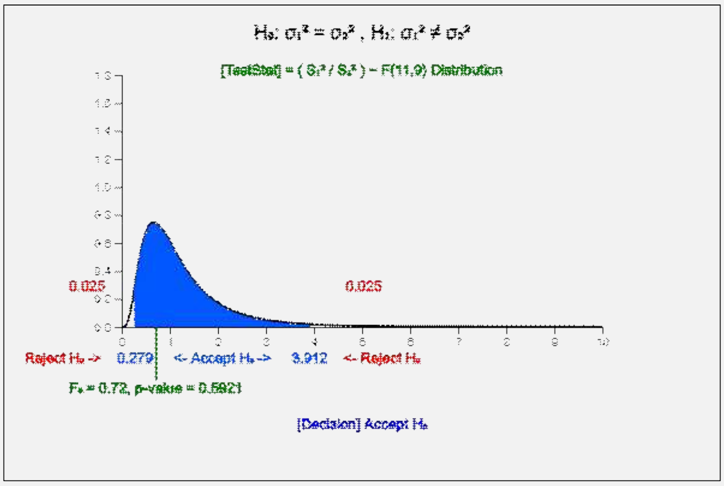
  $$\frac{s_1^2}{s_2^2} = \frac{0.0121}{0.0169} = 0.716$$
  $$F_{n_1-1,n_2-1\,;\,1-\alpha/2} = F_{11,9\,;\,0.975} = 0.279$$
  $$F_{n_1-1,n_2-1\,;\,\alpha/2} = F_{11,9\,;\,0.025} = 3.912$$

  Hence, the hypothesis H_0 can not be rejected and conclude that two variances are equal.
- In『eStatU』menu, select ‘Testing Hypothesis $\sigma_1^2,\ \sigma_2^2$. At the window shown in <Figure 8.2.2>, enter $n_1 = 12,\ n_2 = 10,\ s_1^2 = 0.0121,\ s_2^2 = 0.0169$. Click the [Execute] button to reveal the hypothesis test result shown in <Figure 8.2.3>.



<Figure 8.2.2> Data input for testing hypothesis of two population variances using『eStatU』

Example 8.2.1 Answer (continued)



<Figure 8.2.3> Testing hypothesis for two population variances using『eStatU』

Example 8.2.2

(Income of college graduates, data of [Example 8.1.3])

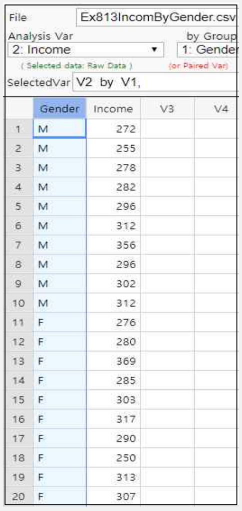
Samples of 10 male and 10 female graduates of the college this year were taken and the average monthly income were examined as follows. Test whether variances of two populations are equal.

Male	272	255	278	282	296	312	356	296	302	312	
Female	276	280	369	285	303	317	290	250	313	307	(Unit 10000 KRW)

⇨ eBook ⇨ EX080103_WageByGender.csv.

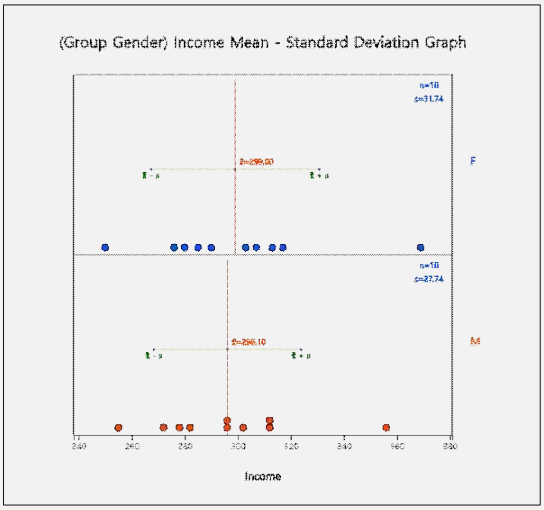
Answer

- In『eStat』, enter the gender and income in two columns on the sheet as shown in <Figure 8.2.4>. This type of data input is similar to all statistical packages. Once you entered the data, click on the icon for testing two population variances and select 'Analysis Var' as V2 and 'By Groups' as V1. Then a mean-standard deviation graph for each group will be appeared as in <Figure 8.2.5>.



	Gender	Income	V3	V4
1	M	272		
2	M	255		
3	M	278		
4	M	282		
5	M	296		
6	M	312		
7	M	356		
8	M	296		
9	M	302		
10	M	312		
11	F	276		
12	F	280		
13	F	369		
14	F	285		
15	F	303		
16	F	317		
17	F	290		
18	F	250		
19	F	313		
20	F	307		

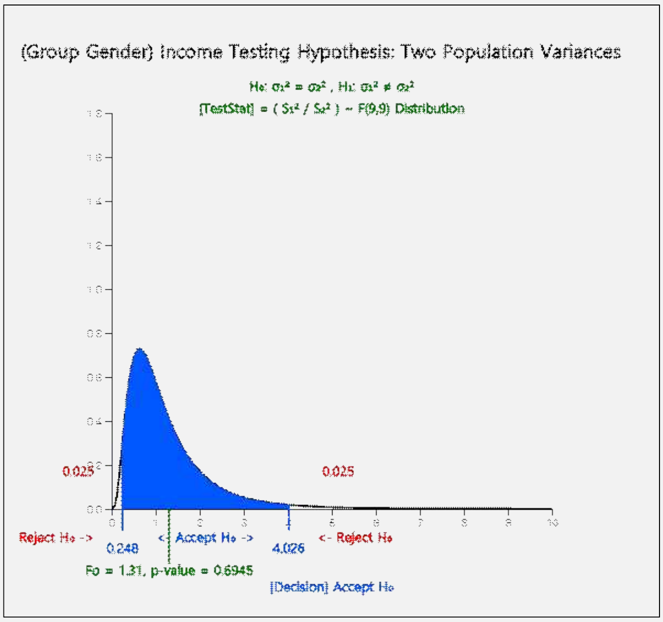
<Figure 8.2.4> Data input for testing two population variances



<Figure 8.2.5> Dot graph and mean-standard deviation interval of each group

Example 8.2.2 Answer (continued)

- If you click the [F-Test] button int the options window below the graph, a test result graph using F distribution such as <Figure 8.2.6> is appeared in the Graph Area and the result table is appeared as in <Figure 8.2.7> appears in the Log Area.



<Figure 8.2.6> Testing hypothesis for two population variances

Testing Hypothesis: Two Population Variances	Analysis Var	Income	Group Name	Gender	
Statistics	Observation	Mean	Std Dev	std err	Population Variance 95% Confidence Interval
1 (F)	10	299.000	31.742	10.038	(476.692, 3358.034)
2 (M)	10	296.100	27.739	8.772	(364.032, 2564.408)
Total	20	297.550	29.051	6.496	(488.092, 1800.362)
Missing Observations	0				
Hypothesis					
$H_0 : \sigma_1^2 = \sigma_2^2$		[TestStat]	F-value	p-value	σ_1^2 / σ_2^2 95% Confidence Interval
$H_1 : \sigma_1^2 \neq \sigma_2^2$		S_1^2 / S_2^2	1.309	0.6945	(0.325, 5.272)

<Figure 8.2.7> Result table of testing two population variances

[Practice 8.2.1]

Tire products from two companies are known to have the same average life span of 80,000km. However, there seems to be a difference in the variance. Sixteen tires from each of the two companies were randomly selected and run under similar conditions to measure their life span. The sample variance was 4,500 and 2,500, respectively.
Using『eStatU』, test the null hypothesis that the variances of the tire life of two products are the same at the 5% significance level.

8.3 Testing Hypothesis for Two Population Proportions

- Consider the following examples which compare two population proportions.
  - Is there a gender gap in the approval rating for a particular candidate in this year's presidential election?
  - A factory has two machines that make products. Do two machines have different defect rates?
- Comparing proportions p_1 and p_2 of two populations is possible by testing the difference between two proportions $p_1 - p_2$ as the comparison of two population means. The difference in sample proportions $\hat{p}_1 - \hat{p}_2$ from two populations follows a normal distribution with the mean $p_1 - p_2$ and variance $p_1(1-p_1)/n_1 + p_2(1-p_2)/n_2$ when two sample sizes are large enough. Since we do not know population proportions p_1 and p_2 to estimate the variance, weighted average value $\overline{p}$ for two sample proportions $\hat{p}_1$ and $\hat{p}_2$ by using sample sizes as weights is used as follows:

$$\overline{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

- The testing hypothesis for two population proportions uses the following test statistic.

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}}$$

Table 8.3.1 Testing hypothesis for two population proportions
- two independent large samples -

Type of Hypothesis	Decision Rule
1) $H_0 : p_1 = p_2$ $H_1 : p_1 > p_2$	If $\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} > z_\alpha$, then reject H_0, else accept H_0
2) $H_0 : p_1 = p_2$ $H_1 : p_1 < p_2$	If $\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} < -z_\alpha$, then reject H_0, else accept H_0
3) $H_0 : p_1 = p_2$ $H_1 : p_1 \neq p_2$	If $\left\lvert \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} \right\rvert > z_{\alpha/2}$, then reject H_0, else accept H_0

Example 8.3.1

A survey was conducted for a presidential election and samples were selected independently from both male and female populations. 54 out of 225 samples from the male population supported the candidate A and 52 out of 175 samples from the female population supported the candidate A. Test whether there is a difference in approval ratings of the male and female populations with the 5% significance level. Check the result using 『eStatU』.

Answer

- The hypothesis of this problem is H_0 : $p_1 = p_2$, H_1 : $p_1 \neq p_2$ and its decision rule is as follows:

$$\text{'If } \left| \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} \right| > z_{\alpha/2}, \text{ then reject } H_0, \text{ else accept } H_0\text{'}$$

- Since $\hat{p}_1$= 54/225 = 0.240, $\hat{p}_2$= 52/175 = 0.297, $\overline{p}$ and the test statistic can be calculated as follows:

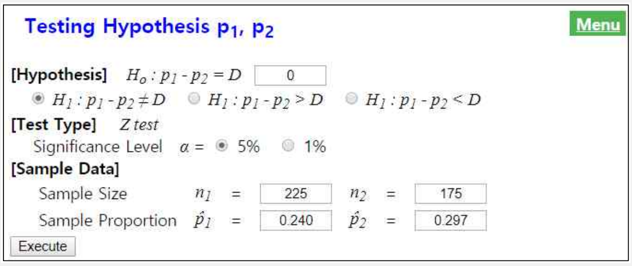
$$\overline{p} = (54 + 52) / (225 + 175) = 106 / 400 = 0.265$$

$$\left| \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} \right| = \left| \frac{0.240 - 0.297}{\sqrt{\frac{0.265(1-0.265)}{225} + \frac{0.265(1-0.265)}{175}}} \right| = 1.28$$

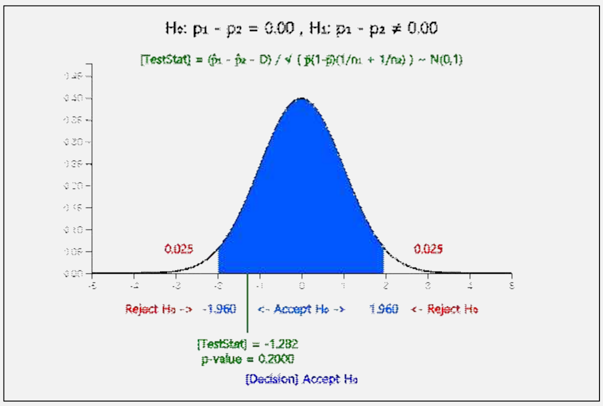
$$z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$$

Therefore, the hypothesis H_0 can not be rejected and we conclude that there is not enough evidence that the approval ratings of male and female are different.

- In 『eStatU』 menu, select 'Testing Hypothesis p_1, p_2' and enter $n_1 = 225$, $\hat{p}_1 = 0.240$, $n_2 = 175$, $\hat{p}_2 = 0.297$ as shown in <Figure 8.3.1>. Clicking the [Execute] button will show the result of the hypothesis test as shown in <Figure 8.3.2>.



<Figure 8.3.1> Data input for testing two population proportions in 『eStatU』



<Figure 8.3.2> Result of testing hypothesis for two population proportions using 『eStatU』

Example 8.3.2

In 2000, a simple random sampling of 1,000 people aged 15 to 29 across the country examined the status of marriage, and 63.5 percent were single. In 2020, another 1,000 people were surveyed independently, with 69.8 percent of them being single. From this fact, can you say that there has been a tendency to get married late in recent years? In other words, test at the 5% significance level whether the population aged 15 to 29 in 2020 is more likely to be single than in 2000. What is the p-value of this test?

Answer

- The hypothesis of this problem is $H_0 : p_1 = p_2$, $H_1 : p_1 < p_2$ and its decision rule is as follows:

$$\text{If } \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} < -z_\alpha, \text{ then reject } H_0, \text{ else accept } H_0$$

- Since $\hat{p}_1$ = 0.635 and $\hat{p}_2$ = 0.698, $\bar{p}$ and the test statistic are as follows:

$$\bar{p} = \frac{1000 \times 0.635 + 1000 \times 0.698}{1000 + 1000} = \frac{0.635 + 0.698}{2} = 0.667$$

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} = \frac{0.635 - 0.698}{\sqrt{\frac{0.667(1-0.667)}{1000} + \frac{0.667(1-0.667)}{1000}}} = -2.989$$

$$-z_\alpha = -z_{0.05} = -1.645$$

Therefore, H_0 is rejected. and conclude that the proportion of unmarried people in 2020 has been increased. p-value can be calculated as follows:

$$p\text{-value} = P(Z < -2.989) = 0.0014$$

[Practice 8.3.1]

In a company, the labor union found that 63 percent of 200 salesmen who did not receive a college education wanted to take it back even now. The company did a similar study 10 years ago and it was only 58 percent of 100 salesmen wanted it. Test the null hypothesis that the desire for college education is not different from 10 years ago using the significance level of 0.05. Samples were selected independently.

- In the previous two examples of comparing two population proportions, two sample proportions were calculated from independent samples.
- Suppose two candidates ran in an election and one thousand samples were selected to test whether there was any difference on the candidate's approval rating. The approval ratings p_1 and p_2 of two candidates obtained from the sample are not independent, because unlike two previous examples they are calculated from one set of samples. So the test method should be different. The following statistic are used to test whether there is a difference in approval ratings of two candidates.

$$\frac{\hat{p}_1 - \hat{p}_2}{\sigma_{\hat{p}_1 - \hat{p}_2}} \;, \text{ where } \sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1) + p_2(1-p_2) + 2p_1p_2}{n}}$$

$\sigma_{\hat{p}_1 - \hat{p}_2}$ is the standard error of $\hat{p}_1 - \hat{p}_2$.

- Assuming that two population proportions are equal, the estimated value $\sigma_{\hat{p}_1 - \hat{p}_2}$ is as follows

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{2\overline{p}}{n}} \quad , \quad \overline{p} = (\widehat{p_1} + \widehat{p_2})/2$$

- If the sample size is large, the test statistic follows a normal distribution which allows proper testing hypothesis according to the form of the alternative hypothesis. As such, it is important to distinguish between sample proportions from independent samples and not independent samples when we compare two population proportions.

Exercise

8.1 An analyst studies two types of advertising methods (A and B) tried by retailers. The variable is the sum of the amount spent on advertising over the past year. The following is the sample statistics extracted independently from retailers of each type. (Unit million USD)

Type A:	n_1 = 60	$\overline{x}_1$ = 14.8	s_1^2 = 0.180
Type B:	n_2 = 70	$\overline{x}_2$ = 14.5	s_2^2 = 0.133

From these data, can you conclude that type A retailers have invested more in advertising than type B retailers? (Significance level = 0.05)

8.2 Paper making plants are looking to buy one of two forests. The followings are diameters of 50 trees sampled from each forest. From these data, test at the significance level of 0.05 whether the trees in area B are on average smaller than those in area A. What is the p-value of this test?

Area A:	$\overline{x}_1$ = 28.25	s_1^2 = 25
Area B:	$\overline{x}_2$ = 22.50	s_2^2 = 16

8.3 In order to check the period of residence at the current house in region A and B, the following statistics were examined from simple random samples of 100 households in A and 150 households in B. From this data, can households in A area live shorter on average than those in B? (Significance level = 0.05)

Region A	$\overline{x}_1$ = 33 months	s_1^2 = 900
Region B	$\overline{x}_2$ = 49 months	s_2^2 = 1,050

8.4 An advertising analyst surveyed how much working men and housewives were exposed to advertisements on radio, TV, newspaper and magazines. The survey item was the number of advertisements that each group encountered in a particular week and the sample mean and standard deviation of each group are as shown in the table below. From these data, can you say that housewives are exposed to more advertisements on average than working men? (Significance level = 0.05)

Group	n	Sample mean	Sample Standard Deviation
Working men	100	200	50
Housewives	144	225	60

8.5 One company wants to test whether a female employee uses the phone longer than a male employee. A sample survey of 10 males and 10 females for one-day call time measurement are as follows. Is there a difference in the average call time between male and female? Use the 5% significance level.

Male	8	6	4	6	2	2	4	8	10	10
Female	4	4	10	2	8	4	10	8	13	14

(unit minutes)

8.6 One factory tries to compare the adhesion of motor oil from two companies. Among the products of each company, 32 products were randomly selected and tested as follows. Based on these data, can you conclude that the adhesion means of the two company products are different? (Significance level = 0.05.)

Company A	13 21 60 35 38 10 36 24 35 35 45 19 42 11 35 39 25 17 51 25 52 25 11 11 55 44 25 41 16 47 50 18
Company B	46 52 66 65 71 67 47 48 58 42 66 69 60 80 45 47 69 75 43 46 74 73 43 70 51 72 65 45 76 48 56 64

8.7 An industrial psychologist thinks that the big factor that workers change jobs is self-esteem to workers' individual work. The scholar thinks that workers who change jobs frequently (group A) have lower self-esteem than those who do not (group B). The following data are used to measure the score of self-esteem by sampling each group independently.

Group A	60 45 42 62 68 54 52 55 44 41
Group B	70 72 74 74 76 91 71 78 78 83 50 52 66 65 53 52

Can this data support the psychologist's idea? Assume that scores of the population are normally distributed and that the population variance is not known but the same. (Significance level = 0.01)

8.8 In a business administration department of a university, a debate arose over claims that men have more knowledge of the stock market than women. To calm the dispute, the instructor sampled each of 15 men and women independently and tested them for knowledge of the stock market. The result is as follows:

Women	73 96 74 55 91 50 46 82 79 79 50 46 81 83
Men	57 78 42 44 91 65 63 60 97 85 92 42 86 81 64

According to the data, on average, can you say that men have more knowledge of the stock market than women? Use the significance level of 0.05. What assumptions do you need?

8.9 An oil company has developed a gasoline additive that will improve the fuel mileage of gasoline. We used 16 pairs of cars to compare the fuel mileage to see if it actually improved. Each pair of cars has the same details as its structure, model, engine size, and other relationship characteristics. When driving the test course using gasoline, one of the pair selected randomly and added additives, the other of the pair was driving the same course using gasoline without additives. The following table shows the km per liter for each of pairs. Is this data a basis for saying that additives increase fuel mileage? Assume that the fuel mileage is normally distributed. Use 5% significance level.

(unit: km / liter)

pair	Additive (X1)	No Additive (X2)	pair	Additive (X1)	No Additive (X2)
1	17.1	16.3	9	10.8	10.1
2	12.7	11.6	10	14.9	13.7
3	11.6	11.2	11	19.7	18.3
4	15.8	14.9	12	11.4	11.0
5	14.0	12.8	13	11.4	10.5
6	17.8	17.1	14	9.3	8.7
7	14.7	13.4	15	19.0	17.9
8	16.3	15.4	16	10.1	9.4

8.10 A study deals with a survey on whether car accidents in a village can be reduced effectively by increasing the number of street lamps. The following table shows the average number of accidents per night, one year before and one year after putting street lamps on 12 locations. Does this data provide evidence that street lamps have reduced nightly car accidents? Use the 5% significance level.

Location	A	B	C	D	E	F	G	H	I	J	K	L
Before	8	12	5	4	6	3	4	3	2	6	6	9
After	5	3	2	1	4	2	2	4	3	5	4	3

8.11 The survey result of (wife's age, husband's age) by sampling 16 couples are as follows

(28, 28) (29, 30) (18, 21) (29, 33) (22, 22) (18, 21) (40, 35) (24, 29)
(21, 31) (20, 24) (20, 34) (23, 25) (33, 39) (33, 35) (40, 29) (39, 40)

Test whether the wife's age is the same as the husband's age or not. Use the significance level of 0.05.

8.12 One person is considering the use of a test to compare between two population means. 16 samples are randomly taken from two populations and their sample variances are 28.5 and 9.5. Is this data shows evidence that two population variances are the same? (Significance level = 0.05)

8.13 Certain studies have been planned to compare the two relaxing drugs for office workers in stressful jobs. A medical team sampled eight workers for each of two drugs and collected data on the strain. Two sample variances are s_1^2 = 2916 and s_2^2= 4624. Using the significance level of 0.05, can this data be said to differ in two population variances of tension? Explain necessary assumptions.

8.14 Let X and Y be the number of days it takes for a plant to sprout its wide leaves and narrow leaves, respectively. The measured data are as follows:

$$n_x = 13,\ \overline{x} = 18.97,\ s_x^2 = 9.88,$$
$$n_y = 9,\ \overline{y} = 23.20,\ s_y^2 = 4.08.$$

If $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$, test the following hypothesis using the 5% significance level.

$$H_0 : \sigma_x^2/\sigma_y^2 = 1,\ H_1 : \sigma_x^2/\sigma_y^2 > 1$$

8.15 Both tire products are known to have an average life span of 80,000 km. However, there seems to be a difference in the variance. Sixteen tires from each of two companies were randomly selected and run under similar conditions to measure their life span. Sample variances were 4,500 and 2,200, respectively.

1) Test the null hypothesis that variances of the tire life of two products are the same at the significance levels of both 0.10 and 0.05.
2) Obtain 90% and 95% confidence intervals of the ratio σ_1^2/σ_2^2.

8.16 A carpet manufacturer is looking for materials that can withstand temperature above 250 degree Fahrenheit. One of two materials is a natural material and the other is a cheap artificial material, which both have the same properties except for heat-resistant levels. As a result of a heat-resistant experiment by independently selecting 250 samples from each of two materials, 36 samples from natural materials and 45 samples from man-made materials failed at temperatures above 250 degrees Fahrenheit. Is there a difference in the heat resistance of two materials from this data using the significance level is 0.05?

8.17 A labor union of a company found that 63 percent of 150 salespeople who did not receive a college education wanted to take it back even now. The company did a similar study 10 years ago, when only 58 percent of 160 people wanted it. Test the null hypothesis that the desire for college education is not different from 10 years ago using the significance level of 0.05. Samples were selected independently.

8.18 When we extracted 200 companies of the type A and examined them, we found that 12% of them spent more than 1% of their total sales on advertising. The other 200 companies of the type B independently selected and examined, we found 15% of them spent more than 1% of their total sales on advertising. Test the following hypotheses with the significance level of 0.05.

$$H_0 : p_B \le p_A, \quad H_1 : p_B > p_A$$

8.19 In a company, a study was conducted on the leisure activities of sales staffs and managing staffs. 400 persons were selected independently from each of sales and managing staffs. 288 sales and 260 managing staffs answered that they usually spend their leisure time on sports activities. From this data, can you say that the percentage of two groups for the leisure time spent on sports activities is the same? Use the significance level of 0.05.

8.20 In September 2013, a research institute surveyed 260 men and 263 women about a political issue and the response result is as follows. Do you think there are significant differences in their way of thinking on the political issue?' Specify the null hypothesis and the alternative hypothesis and test at the 5% significance level.

	Men	Women
Yes	57%	65%
No	43%	35%

8.21 In order to see whether the unemployment rate in two cities are different, samples of 500 people were randomly selected from two cities and found unemployment persons were 35 and 25 respectively. Can you say that the unemployment rate in two cities is different? Describe the necessary assumptions and calculate the p-value.

Multiple Choice Exercise

8.1 One professor claims that 'A student who studies in the morning will get better math score than a student who studies in the evening.' Assume that μ_1 is the average exam score of students who study in the morning and μ_2 is the average exam score of students who study in the evening. What is the null hypothesis of this test?

① $\mu_1 > \mu_2$ ② $\mu_1 \geqq \mu_2$
③ $\mu_1 \neq \mu_2$ ④ $\mu_1 = \mu_2$

8.2 What is the alternative hypothesis of the test of the above question 8.1?

① $\mu_1 > \mu_2$ ② $\mu_1 \geqq \mu_2$
③ $\mu_1 \neq \mu_2$ ④ $\mu_1 = \mu_2$

8.3 A researcher claims that “After age of 40 and over, there is no difference in weight between male and female.” Assume the average weight of males whose age is 40 and more is μ_1 and the average weight of females whose age is 40 and more is μ_2. What is the alternative hypothesis of the test?

① $\mu_1 = \mu_2$ ② $\mu_1 \neq \mu_2$
③ $\mu_1 > \mu_2$ ④ $\mu_1 < \mu_2$

8.4 We want to test whether two population means are equal or not using t-test. Which one of the following is not a required assumption?

① Populations are normal distributions.
② Two population variances are the same.
③ Samples are selected independently.
④ Samples are collected using cluster sampling method.

8.5 Which sampling distribution is used to test whether two population means are equal or not when sample sizes are small?

① Normal distribution
② t-distribution
③ Chi square distribution.
④ F-distribution

8.6 16 couples are randomly selected to compare their ages as follows. What is the name of this kind of data?
(woman age, man age)
(28, 28) (29, 30) (18, 21) (29, 33) (22, 22) (18, 21) (40, 35) (24, 29)
(21, 31) (20, 24) (20, 34) (23, 25) (33, 39) (33, 35) (40, 29) (39, 40)

① independent data ② paired data
③ random data ④ cluster data

8.7 Which sampling distribution is used to test whether two population variances are equal or not when populations are normally distributed?

① Normal distribution
② t-distribution
③ Chi square distribution.
④ F-distribution

8.8 Which sampling distribution is used to test whether two population proportions are equal or not when sample sizes are large enough?

① Normal distribution
② t-distribution
③ Chi square distribution.
④ F-distribution

8.9 In a company, a comparative study was conducted on leisure activities of sales staffs and managing staffs. 400 staffs selected independently from each of sales staffs and management staffs and surveyed. We found that 288 sales staffs and 260 managing staffs answered that they usually spend their leisure time on sports activities. Which of the following is the null hypothesis for comparing two groups?

① $p_1 = p_2$ ② $p_1 \neq p_2$
③ $p_1 > p_2$ ④ $p_1 < p_2$

8.10 Which of the following is the alternative hypothesis in question 8.9?

① $p_1 = p_2$ ② $p_1 \neq p_2$
③ $p_1 > p_2$ ④ $p_1 < p_2$

(Answers)
8.1 ④, 8.2 ①, 8.3 ②, 8.4 ④, 8.5 ②, 8.6 ②, 8.7 ④, 8.8 ①, 8.9 ①, 8.10 ②,