Introduction to Statistics and Data Science using *eStat*

Chapter 8 Testing Hypothesis for Two Populations

8.2 Testing hypothesis for two population variances

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8.2 Testing hypothesis for two population variances

- **Examples of comparing two population variances.**

- **When comparing two population means, if the sample was small, decision rule for testing hypothesis were different depending on whether two population variances were the same or different.**

  How can we test if two unknown population variances are the same?

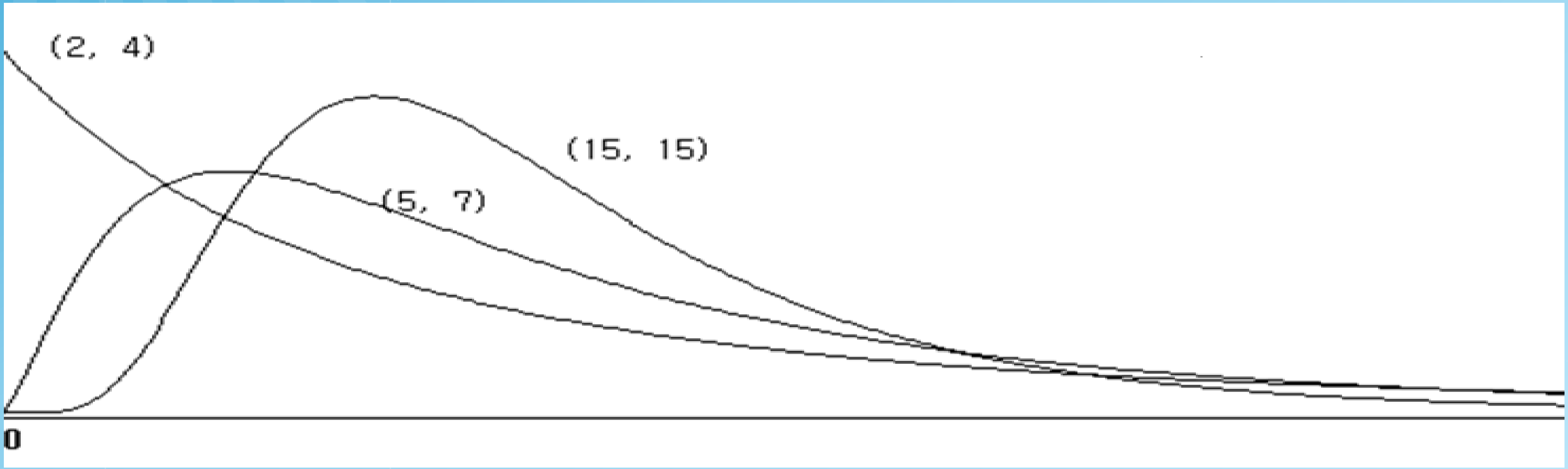
- **Quality of bolts used to assemble cars depends on strict specification for their diameter. The average diameter of these bolts is said to be the same for two companies.**

  How can you compare variances if smaller variances as superior?

8.2 Testing hypothesis for two population variances

- Sample statistic to test two population variances.

$$\frac{\left(\frac{S_1^2}{\sigma_1^2}\right)}{\left(\frac{S_2^2}{\sigma_2^2}\right)} \sim F_{n_1-1, n_2-1}$$



8.2 Testing hypothesis for two population variances

Table 8.2.1 Testing hypothesis for two population variances
- Two populations are normally distributed-

Type of Hypothesis	Decision Rule
1) $H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 > \sigma_2^2$	If $\frac{S_1^2}{S_2^2} > F_{n_1-1,n_2-1;\alpha}$, then reject H_0, else accept H_0
2) $H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 < \sigma_2^2$	If $\frac{S_1^2}{S_2^2} < F_{n_1-1,n_2-1;1-\alpha}$, then reject H_0, else accept H_0
3) $H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 \neq \sigma_2^2$	If $\frac{S_1^2}{S_2^2} < F_{n_1-1,n_2-1;1-\alpha/2}$ or $\frac{S_1^2}{S_2^2} > F_{n_1-1,n_2-1;\alpha/2}$, then reject H_0, else accept H_0

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[Example 8.2.1] A company that produces a bolt has two plants. One day, ten bolts produced in Plant 1 were sampled randomly and the variance of diameter was 0.11^2. 12 bolts produced in Plant 2 were sampled randomly and the variance of diameter was 0.13^2.

- Test whether variances of the bolt from two plants are the same or not with the 5% significance level.
- Check the test result using『eStatU』.

<Answer>

- Hypothesis: $H_0 : \sigma_1^2 = \sigma_2^2$, $H_1 : \sigma_1^2 \neq \sigma_2^2$
- Decision rule is as follows:

  'If $\frac{S_1^2}{S_2^2} < F_{n_1-1,n_2-1;1-\alpha/2}$ or $\frac{S_1^2}{S_2^2} > F_{n_1-1,n_2-1;\,\alpha/2}$, then reject H_0'
- $\frac{S_1^2}{S_2^2} = \frac{0.11^2}{0.13^2} = 0.716$, $F_{n_1-1,n_2-1;1-\alpha/2} = F_{11,9;0.975} = 0.279$, $F_{11,9;0.025} = 3.912$
- Hence, H_0 can not be rejected that two variances are equal.

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<Answer of Ex 8.2.1>

Testing Hypothesis σ_1^2, σ_2^2 Menu

[Hypothesis] $H_o : \sigma_1^2 = \sigma_2^2$

◉ $H_1 : \sigma_1^2 \neq \sigma_2^2$ ○ $H_1 : \sigma_1^2 > \sigma_2^2$ ○ $H_1 : \sigma_1^2 < \sigma_2^2$

[Test Type] *F test*

Significance Level α = ◉ 5% ○ 1%

[Sample Data] *Input either sample data using BSV or sample statistics at the next boxes*

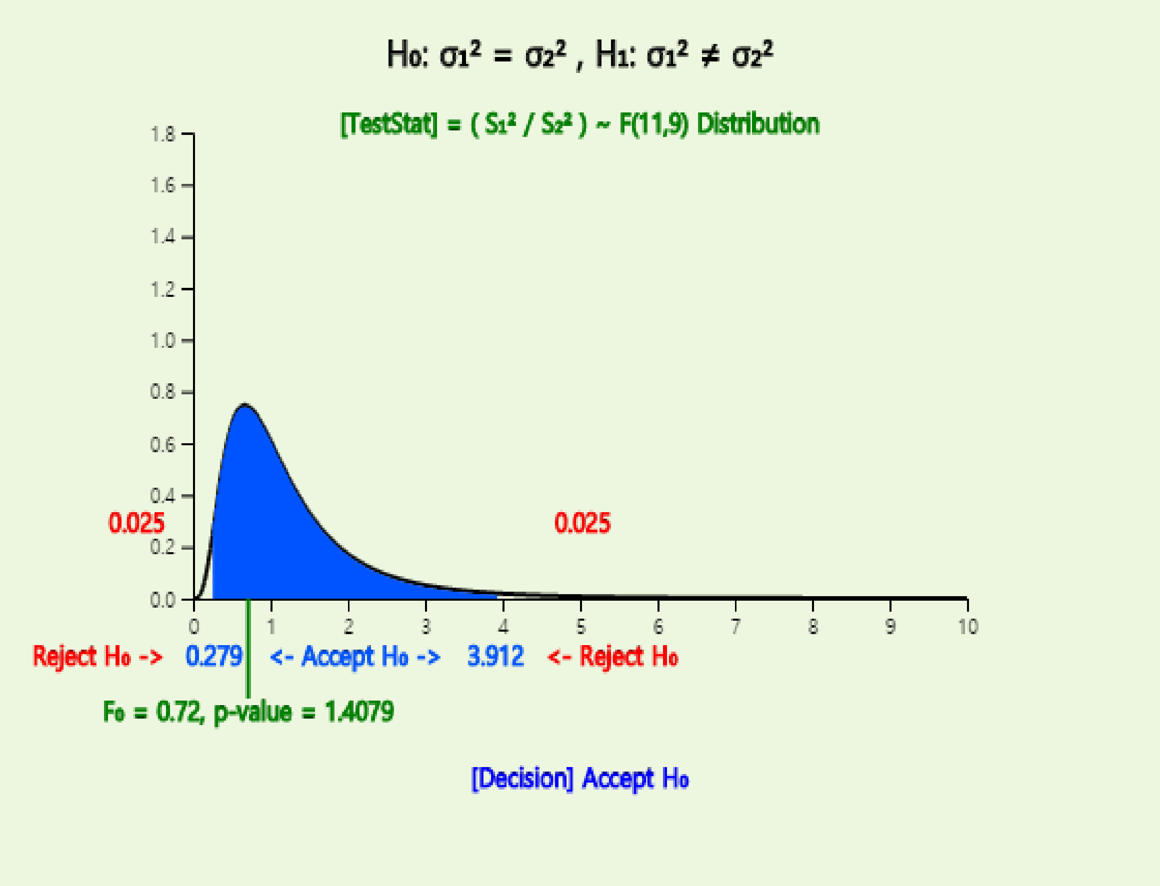
Sample 1

Sample 2

[Sample Statistics]

Sample Size	n_1 =	12	n_2 =	10
Sample Variance	s_1^2 =	0.0121	s_2^2 =	0.0169

Execute



8.2 Testing hypothesis for two population variances

[Example 8.2.2] In Example 8.1.3, A sample of 10 male and 10 female of college graduates this year was taken and the monthly average income was examined as follows. (Unit 10000 KRW)

- Test if the variances of the two populations are equal or not Using eStat.

Male	272 255 278 282 296 312 356 296 302 312
Female	276 280 369 285 303 317 290 250 313 307

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<Answer of Ex 8.2.2>

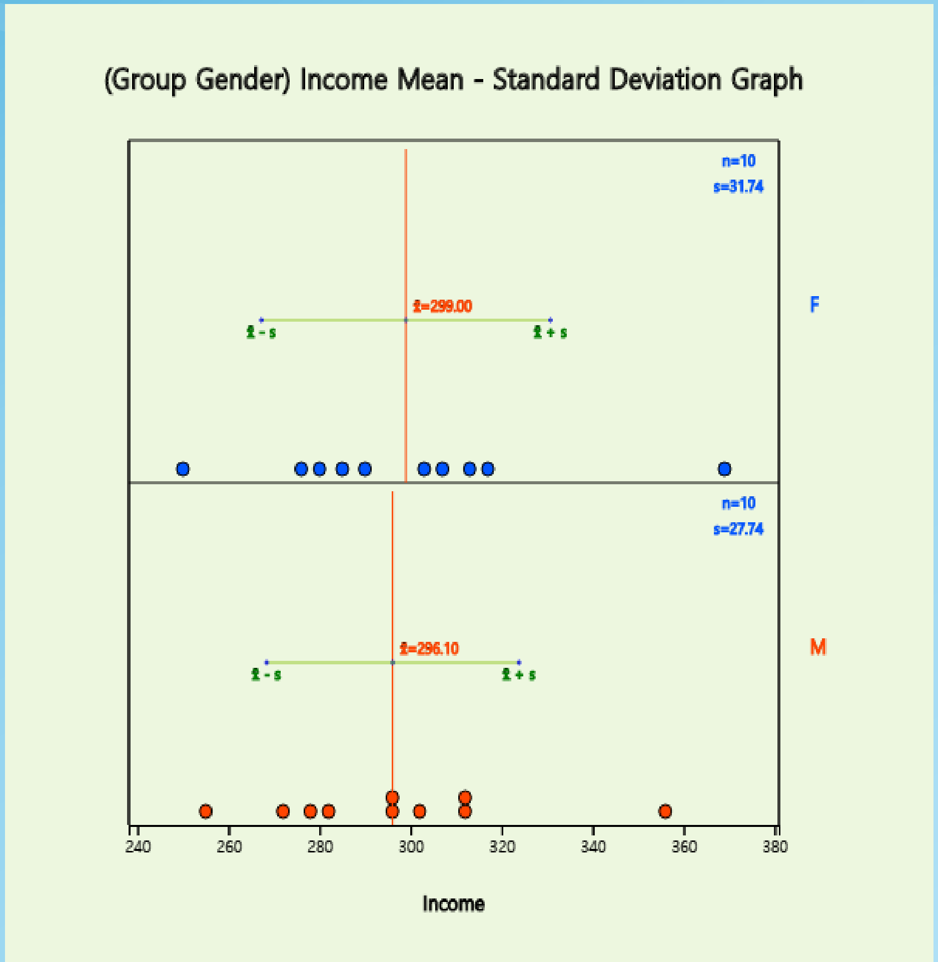
File: Ex813IncomByGender.csv

Analysis Var: 2: Income   by Group: 1: Gender

(Selected data: Raw Data)   (or Paired Var)

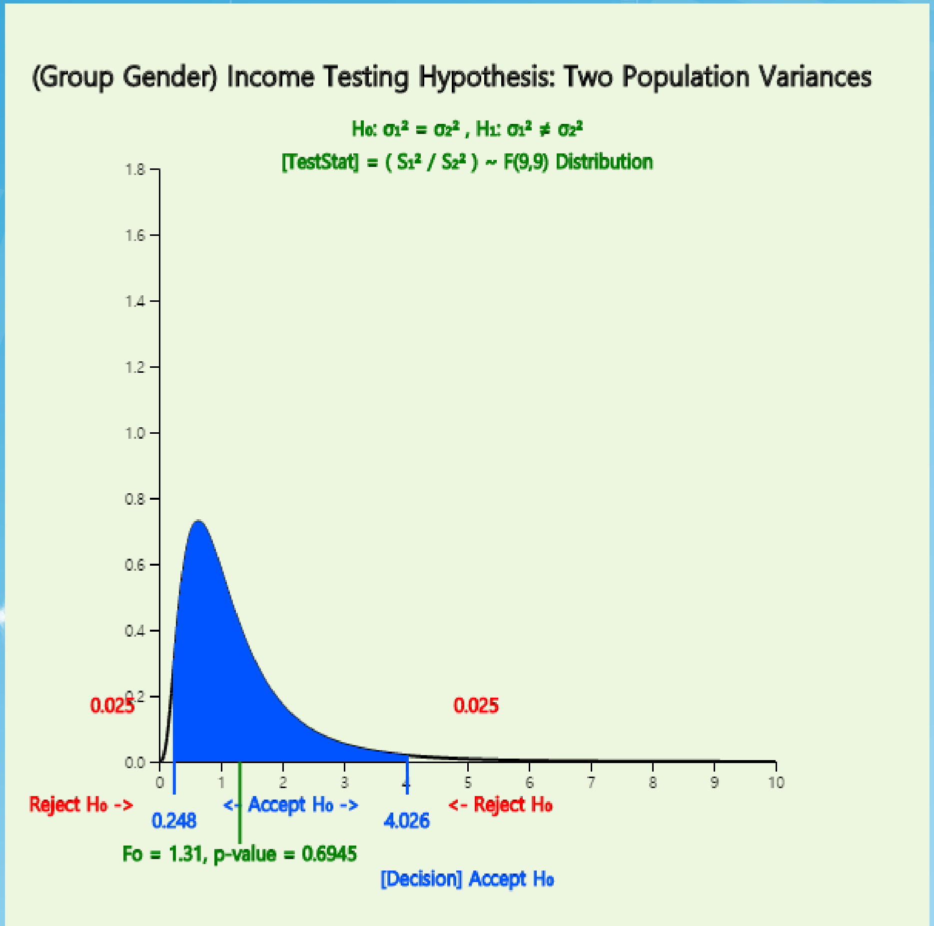
SelectedVar: V2 by V1,

	Gender	Income	V3	V4
1	M	272		
2	M	255		
3	M	278		
4	M	282		
5	M	296		
6	M	312		
7	M	356		
8	M	296		
9	M	302		
10	M	312		
11	F	276		
12	F	280		
13	F	369		
14	F	285		
15	F	303		
16	F	317		
17	F	290		
18	F	250		
19	F	313		
20	F	307		



8.2 Testing hypothesis for two population variances

<Answer of Ex 8.2.2>



Testing Hypothesis: Two Population Variances	Analysis Var	Income	Group Name	Gender	
Statistics	Observation	Mean	Std Dev	std err	Population Variance 95% Confidence Interval
1 (F)	10	299.000	31.742	10.038	(476.692, 3358.034)
2 (M)	10	296.100	27.739	8.772	(364.032, 2564.408)
Total	20	297.550	29.051	6.496	(488.092, 1800.362)
Missing Observations	0				
Hypothesis					
$H_0 : \sigma_1^2 = \sigma_2^2$		[TestStat]	F-value	p-value	σ_1^2 / σ_2^2 95% Confidence Interval
$H_1 : \sigma_1^2 \neq \sigma_2^2$		S_1^2 / S_2^2	1.309	0.6945	(0.325, 5.272)

Thank you