7

Testing Hypothesis for Single Population Parameter

μ σ p α, β

SECTIONS

7.1 Testing Hypothesis for a Population Mean

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- 7.4.1 Type 2 Error and Power of a Test
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CHAPTER OBJECTIVES

Estimation of population parameters using sample distributions was discussed in Chapter 6.

However, one might be interested in which one of two hypothesis about the population parameter is reasonable to accept.

This problem is called a testing hypothesis and we take samples and calculate the sample statistics to decide by using the sampling distributions discussed in Chapter 6.

In this chapter we discuss the testing hypothesis of the population mean, population variance and population proportion.

A method of testing hypothesis which considers both type 1 and type 2 error is also introduced.

7.1 Testing Hypothesis for a Population Mean

- Examples of testing hypothesis for a population mean are as follows:

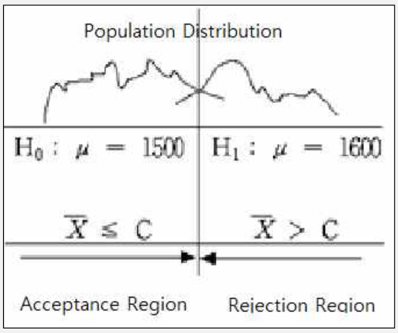
  - The weight of a cookie bag is indicated as 200g. Would there be enough cookies as the indicated weight?
  - At a light bulb factory, a newly developed light bulb advertises a longer bulb life than the past one. Is this propaganda reliable?
  - Immediately after completing this year's academic test, students said that there will be 5 points increase in the average English score higher than last year. How can you investigate if this is true?

- Testing hypothesis is the answer to the above questions (hypothesis). That is, the testing hypothesis is to decide statistically which hypothesis is to use for the two hypotheses about the unknown population parameter using samples. In this section, we examine the test of the population mean, population variance, and population proportion which are most commonly used in testing hypothesis.
- The following example explains the theory of testing hypothesis of the population mean in single population.

Example 7.1.1	At a light bulb factory, the average life expectancy of a light bulb made by a conventional production method is known to be 1500 hours and the standard deviation is 200 hours. Recently, the company is trying to introduce a new production method, with the average life expectancy of 1600 hours for light bulbs. To confirm this argument, 30 samples were taken from the new type of light bulbs by simple random sampling and the sample mean was $\overline{x}$ = 1555 hours. Can you tell me that the new type of light bulb has the average life of 1600 hours?
Answer	◆ A statistical approach to the question of this issue is first to make two assumptions about the different arguments for the population mean μ. Namely, H_0: μ = 1500 H_1: μ = 1600 ◆ H_0 is called a **null hypothesis** and H_1 is an **alternative hypothesis**. In most cases, the null hypothesis is defined as an ‘existing known fact’, and the alternative hypothesis is defined as ‘new facts or changes in current beliefs’. So when choosing between two hypotheses, the basic idea of testing hypothesis is 'unless there is a significant reason, we accept the null hypothesis (current fact) without choosing the alternative hypothesis (the fact of the matter). This idea of testing hypothesis is referred to as a ‘**conservative decision making**’. ◆ A common sense criterion for choosing between two hypotheses would be 'which population mean of two hypothesis is closer in distance to the sample mean'. Based on this common sense criteria which uses the concept of distance, the sample mean of 1555 is closer to H_1: μ = 1600 so the alternative hypothesis will be chosen. A statistical testing hypothesis is based not only on this common sense criteria, but also on the sampling distribution of $\overline{X}$. In other words, the statistical testing hypothesis is to select a critical value C based on the sampling distribution theory and to make a **decision rule** as follows: ‘If $\overline{X}$ is smaller than C, then the null hypothesis H_0 will be chosen, else reject H_0’

Example 7.1.1 Answer (continued)

- The area of $\{\overline{X} < C\}$ is called an **acceptance region** of H_0 and the area $\{\overline{X} \geq C\}$ is called a **rejection region** of H_0 (<Figure 7.1.1>).



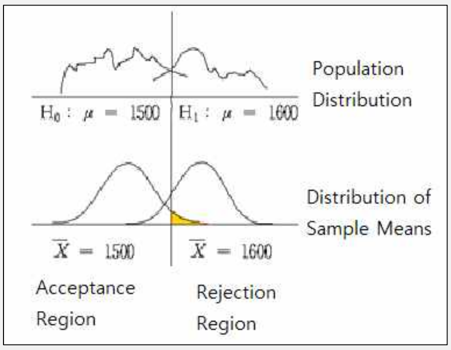
<Figure 7.1.1> Acceptance and rejection region of H_0

- If a hypothesis is chosen by this decision rule, there are always two possible errors in the decision. One is a **Type 1 Error** which accepts H_1 when H_0 is true, the other is a **Type 2 Error** which accept H_0 when H_1 is true. These errors can be summarized as Table 7.1.1.

Table 7.1.1 Two types of errors in testing hypothesis

	Actual: H_0 is true	Actual: H_1 is true
Decision: H_0 is true	Correct	Type 2 Error
H_1 is true	Type 1 Error	Correct

- If you try to reduce one type of error when the sample size is fixed, then the other type of error is increasing. That is why we came up with a conservative decision making method that defines the null hypothesis H_0 as 'past or present facts' and 'accept the null hypothesis unless there is a significance evidence for the alternative hypothesis.' In this conservative way, we try to reduce the type 1 error as much as possible that selects H_1 when H_0 is true, which would be more risky than the type 2 error. Testing hypothesis determines the tolerance for the probability of the type 1 error, usually 5% or 1% for rigorous test, and use the selection criteria that satisfy this limitation. The tolerance for the probability that this type 1 error will occur is called th**e significance level** and is often expressed as α. The probability of the type 2 error is expressed as β .
- If the significance level is established, the decision rule for the two hypotheses can be tested using the sampling distribution of all possible sample means in Chapter 6. <Figure 7.1.2> shows the distribution of populations for two hypotheses, and the distribution of all possible sample means for each population.



<Figure 7.1.2> Testing Hypothesis

Example 7.1.1 Answer (continued)	
	◆ If the population corresponds to the distribution of H_0 : μ = 1500, the sampling distribution of all possible sample means is approximated as $N(1500,\, 200^2/30)$ by the central limit theorem. If the population corresponds to the distribution of H_1 : μ = 1600, the sampling distribution of all possible sample means is approximated as $N(1600,\, 200^2/30)$. The standard deviation for each population is assumed to be 200 from a historical data. Then the decision rule becomes as follows: 'If $\overline{X}$ < C, then accept H_0, else accept H_1 (i.e. reject H_0)' In Figure 7.1.2, the shaded area represents the probability of the type 1 error. If we set the significance level, which is the tolerance level of the type 1 error, is 5%, i.e. $P(\overline{X}$ < $C)$ = 0.95, C can be calculated by finding the percentile of the normal distribution $N(1500,\, 200^2/30)$ as follows: $1500 + 1.645 \frac{200}{\sqrt{30}}$ = 1560.06 Therefore, the decision rule can be written as follows: 'If $\overline{X}$ $<$ 1560.06, then accept H_0, else reject H_0 (accept H_1).' ◆ In this problem, the observed sample mean of the random variable $\overline{X}$ is $\overline{x}$= 1555 and H_0 is accepted. In other words, the hypothesis of H_0 : μ = 1500 is judged to be correct, which contradicts the result of common sense criteria that $\overline{x}$ = 1555 is closer to H_1 : μ = 1600 than H_0 : μ = 1500. This result can be interpreted that the sample mean of 1555 is not a sufficient evidence to reject the null hypothesis by a conservative decision making method. ◆ The above decision rule is often written as follows, emphasizing that it is the result from a conservative decision making method. 'If $\overline{X}$ < 1560.06, then do not reject H_0, else reject H_0.' In addition, this decision rule can be written for calculation purpose as follows: 'If $\frac{\overline{X} - 1500}{\frac{200}{\sqrt{30}}}$ $<$ 1.645, then accept H_0, else reject H_0.' In this case, since $\overline{x}$ = 1555, $\frac{1555-1500}{\frac{200}{\sqrt{30}}}$ $= 1.506$ and it is less than 1.645. Therefore, we accept H_0.

- Since the testing hypothesis by the conservative decision making is only based on the probability of the type 1 error as seen in [Example 7.1.1], even if the alternative hypothesis is H_1 : μ > 1500, we will have the same decision rule.
- Generally, there are three types of alternative hypothesis in the testing hypothesis for the population mean as follows:

  1) H_1 : μ > μ_0 2) H_1 : μ < μ_0 3) H_1 : μ $\neq \mu_0$

  Since 1) has the rejection region on the right side of the sampling distribution of all possible sample means under the null hypothesis, it is called a **right-sided test.** Since 2) has the rejection region on the left side of the sampling distribution, it is called a **left-sided test**. Since 3) has rejection regions on both sides of the sampling distribution, it is called a **two-sided test**. The decision rule for each type of three alternative hypothesis are summarized in Table 7.1.2 when the population standard deviation is known and α is the significance level.

Table 7.1.2 Testing hypothesis for the population mean - known σ case

Type of Hypothesis	Decision Rule
1) $H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$	$If \ \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > z_\alpha$, $then\ reject\ H_0$
2) $H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$If \ \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} < -z_\alpha$, $then\ reject\ H_0$
3) $H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$If \ \left\lvert \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \right\rvert > z_{\alpha/2}$, $then\ reject\ H_0$

Note: The H_0 of 1) can be written as $H_0 : \mu \le \mu_0$, 2) as $H_0 : \mu \ge \mu_0$

- The following expression used for the decision rule is referred to as a **test statistic** for testing hypothesis of the population mean.

$$\frac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

- The population standard deviation σ of the test statistic is usually unknown. However, if the sample is large enough (approximately 30 or more), the hypothesis test can be performed using the sample standard deviation S instead of the population standard deviation σ.
- In [Example 7.1.1], if the sample mean is either 1555 or 1540, the null hypothesis can not be rejected, but degrees of evidence that the null hypothesis is not rejected are different. The degree of evidence that the null hypothesis is not rejected is measured by calculating the probability of the type 1 error when the observed sample mean value is considered as the critical value for decision, which is called th**e p-value**. That is, the p-value indicates where the observed sample mean is located among all possible sample means by considering the location of the alternative hypothesis. In [Example 7.1.1], the p-value for $\overline{X}$ = 1540 is the probability of sample means which is greater than $\overline{X}$ = 1540 by using $N(1500, 200^2/30)$. The higher the p-value, the stronger the reason for not being rejected. If H_0 is rejected, the smaller the p-value, the stronger the grounds for being rejected. Therefore, if the p-value is less than the significance level considered by the analyst, then H_0 is rejected, because it means that the sample mean is in the rejection region. Statistical packages provide this p-value.

Decision rule using p-value

Decision rule using p-value

If the p-value is less than the significance level, then H_0 is rejected, else H_0 is accepted.

- The calculation of the p-value depending on the type of the alternative hypothesis is summarized as in Table 7.1.3.

Table 7.1.3 Calculation of p-value

Type of Hypothesis	p-value
1) H_0 : $\mu = \mu_0$ H_1 : $\mu > \mu_0$	$\mathrm{P}(\overline{X} > \overline{x}_{obs})$
2) H_0 : $\mu = \mu_0$ H_1 : $\mu < \mu_0$	$\mathrm{P}(\overline{X} < \overline{x}_{obs})$
3) H_0 : $\mu = \mu_0$ H_1 : $\mu \neq \mu_0$	$If\ \overline{x}_{obs} > \mu_o$, $2P(\overline{X} > \overline{x}_{obs})$ $else\ 2P(\overline{X} < \overline{x}_{obs})$

Note : $\overline{x}_{obs}$ is the observed sample mean.

- If the population standard deviation σ is unknown and the population is a normal distribution, the test statistic

$$\frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}}$$

is a t distribution with (n-1) degrees of freedom. The testing hypothesis for the population mean can be done as Table 7.1.4 which replace the Z distribution with the t distribution in Table 7.1.2 and σ with S.

Table 7.1.4 Testing hypothesis for a population mean - unknown σ case (Assume that the population is a normal distribution)

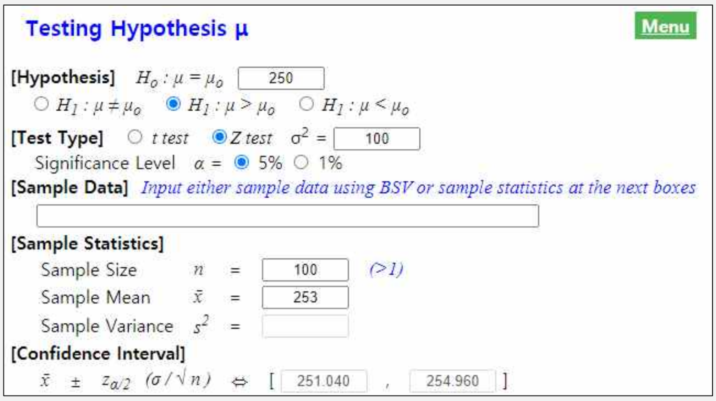
Type of Hypothesis	Decision Rule
1) H_0 : $\mu = \mu_0$ H_1 : $\mu > \mu_0$	$If\ \dfrac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} > t_{n-1\,;\,\alpha},\ reject\ H_0$
2) H_0 : $\mu = \mu_0$ H_1 : $\mu < \mu_0$	$If\ \dfrac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} < -t_{n-1\,;\,\alpha},\ reject\ H_0$
3) H_0 : $\mu = \mu_0$ H_1 : $\mu \neq \mu_0$	$If\ \left\lvert \dfrac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \right\rvert > t_{n-1\,;\,\alpha/2},\ reject\ H_0$

Note: The H_0 of 1) can be written as $H_0 :\ \mu \leq \mu_0$, 2) as $H_0 :\ \mu \geq \mu_0$

- If the sample size is large enough (approximately 30 or more), the t distribution is approximated to the standard normal distribution, so the testing hypothesis in Table 7.1.4 can be performed using the standard normal distribution, Z, instead of t distribution.

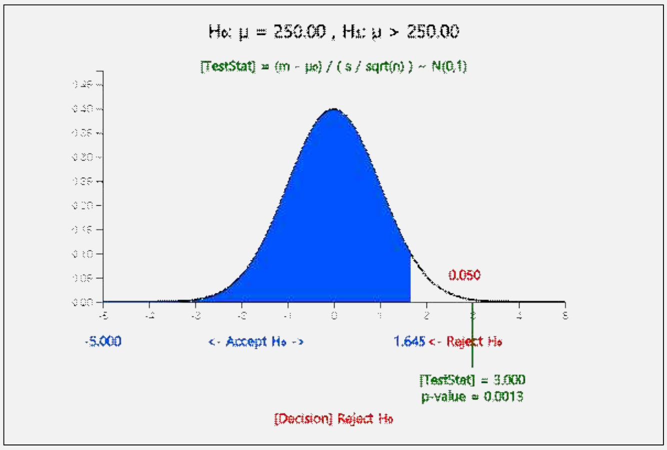
Example 7.1.2	The weight of a bag of cookies is supposed to be 250 grams. Suppose the weight of all bags of cookies is a normal distribution. In the survey of 100 samples of bags which were randomly selected, the sample mean was 253 grams and the standard deviation was 10 grams. 1) Test hypothesis whether the weight of the bag of cookies is 250g or larger and find the p-value. α = 1% 2) Test hypothesis whether or not the weight of the bag of cookies is 250g and find the p-value. α = 1% 3) Use 『eStatU』 to test the hypothesis above.
Answer	1) The hypothesis is a right tail test as H_0: μ = 250, H_1: μ > 250. Since the sample size is large (n=100), we can use Z distribution instead of t distribution. Decision rule is as follows: 'If $(\overline{X} - \mu_0)/(\frac{S}{\sqrt{n}}) > z_\alpha$, then reject H_0, else accept H_0' 'If $(253 - 250)/(\frac{10}{\sqrt{100}}) > z_{0.01}$, then reject H_0, else accept H_0' Since (253-250) / (10/10) = 3 and $z_{0.01}$ = 2.326, H_0 is rejected. We can write the above decision rule as follows: 'If $\overline{X} > 250 + 2.326(\frac{10}{\sqrt{100}})$, then reject H_0, else accept H_0' 'If $\overline{X} > 252.326$, then reject H_0, else accept H_0' ◆ Since the p-value is the probability of Type 1 error when the sample mean is the critical value. it can be calculated by the probability of $P(\overline{X}$ > 253). Since the distribution of $\overline{X}$ is approximately $N(250, \frac{100}{100})$ when H_0: μ = 250 is true, the p-value is as follows: p-value = $P(\overline{X}$ > 253) = $P(Z$ > (253-250)/(10/10)) = $P(Z$ > 3) = 0.0013 2) The hypothesis is a two-sided test as H_0: μ = 250, H_1: $\mu \neq$ 250. Since the sample size is large (n=100), we can use the Z distribution instead of the t distribution. Decision rule is as follows: 'If $\left\| \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \right\| > z_{\alpha/2}$, then reject H_0, else accept H_0' 'If $\left\| \frac{253 - 250}{\frac{10}{\sqrt{100}}} \right\| > z_{0.005}$, then reject H_0, else accept H_0' Since (253-250) / (10/10) = 3 and $z_{0.005}$ = 2.575, H_0 is rejected. The p-value can be calculated as follows: p-value = 2$P(\overline{X}$ > 253) = 2$P(Z$ > (253-50)/(10/10)) = 2$P(Z$ > 3) = 0.0026 3) In『eStatU』menu, select 'Testing Hypothesis μ', enter 250 at the box of μ_0 on [Hypothesis] and select the alternative hypothesis as the right test in the window shown in <Figure 7.1.3>. Check [Test Type] as Z test and check the significance level at 5%. At the [Sample Statistics], enter sample size 100, sample mean 253, and sample variance $10^2 = 100$. For the Z test, you must enter the population variance, but you may enter the sample variance, because the sample size is large enough.

Example 7.1.2 Answer (continued)



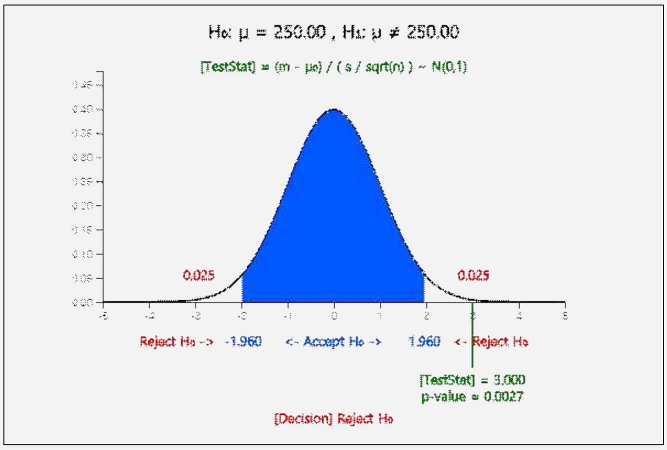
<Figure 7.1.3> 『eStatU』 Testing Hypothes for μ

- If you click the [Execute] button, the confidence Interval for μ is calculated and the testing result using 『eStatU』 will appear as in <Figure 7.1.4>.



<Figure 7.1.4> 『eStatU』 Testing Hypothes for μ
- Right Tail Test -

- If you select the two-tail test at [Hypothesis] of <Figure 7.1.3>, testing result using 『eStatU』 is as <Figure 7.1.5>.



<Figure 7.1.5> 『eStatU』 Testing Hypothes for μ
- Two Tails Test -

Example 7.1.3

When the sample size is 16 and the sample variance is 100 in [Example 7.1.2], test whether the average weight of the cookie bags is 250g or greater and obtain the p-value. Check the result using 『eStatU』

Answer

- Since the population standard deviation is unknown and the sample size is small, the decision rule is as follows:

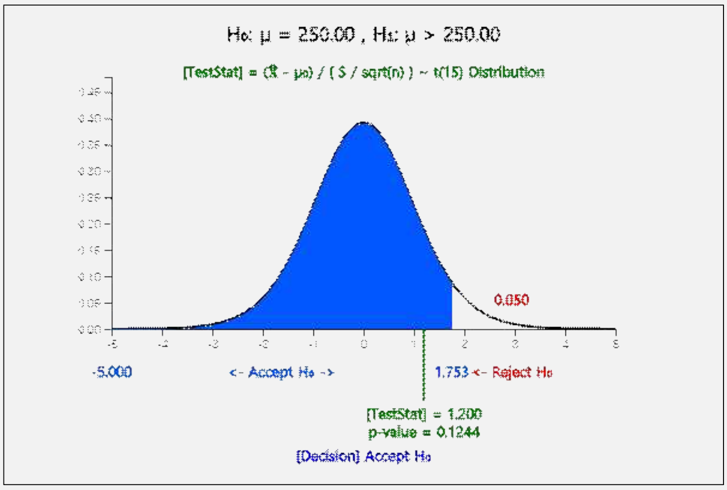
  'If $(\overline{X} - \mu_o)/(\frac{S}{\sqrt{n}}) > t_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0'

  'If $(253 - 250)/(\frac{10}{\sqrt{16}}) > t_{16-1\,;\,0.01}$, then reject H_0, else accept H_0'

  Since the value of test statistic is $(253 - 250)/(\frac{10}{\sqrt{16}}) = 1.2$, and $t_{15,0.01} = 2.602$, we accept H_0. Note that the decision rule can be written as follows:

  'If $\overline{X} > 250 + 2.602(\frac{10}{\sqrt{16}})$, then reject H_0, else accept H_0'

- In <Figure 7.1.3> of 『eStatU』, select the right-sided test of [Hypothesis], select the t-test on [Test Type] and enter sample size n = 16, then the test result is as <Figure 7.1.6> if you click the [Execute] button.



<Figure 7.1.6> Testing hypothesis for μ with t distribution using 『eStatU』

- Since the p-value is the probability that t_{15} is greater than the test statistics 1.200, the p-value is 0.124 by using the module of t-distribution in 『eStatU』.

Example 7.1.4

(Heights of college students)

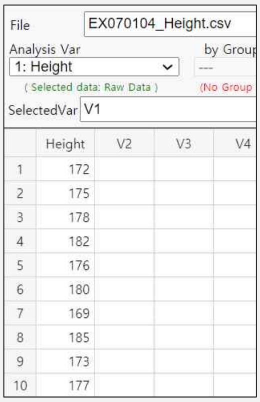
10 male students are sampled in a university and examined their heights as follows:

172 175 178 182 176 180 169 185 173 177 (Unit cm)
Ex ⇨ eBook ⇨ EX070104_Height.csv.

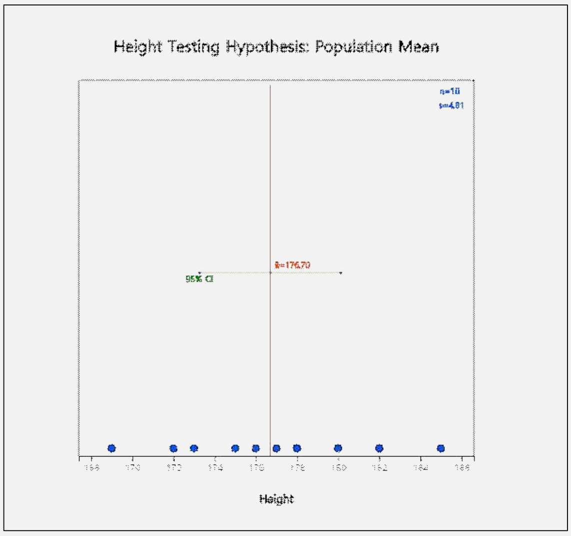
Test the hypothesis whether the population mean is 175cm or greater with the significance level of 5%.

Example 7.1.4 Answer

- After entering data on a sheet as shown in <Figure 7.1.7> in 『eStat』, clicking the testing hypothesis for the population mean $\boxed{\mu}$ and then clicking variable name V1 for 'Analysis Var' in variable selection box will result in a dot graph with 95% confidence interval as in <Figure 7.1.8>.

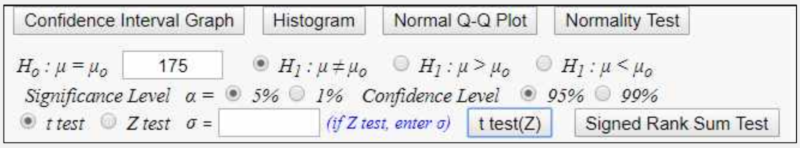


<Figure 7.1.7> Data input

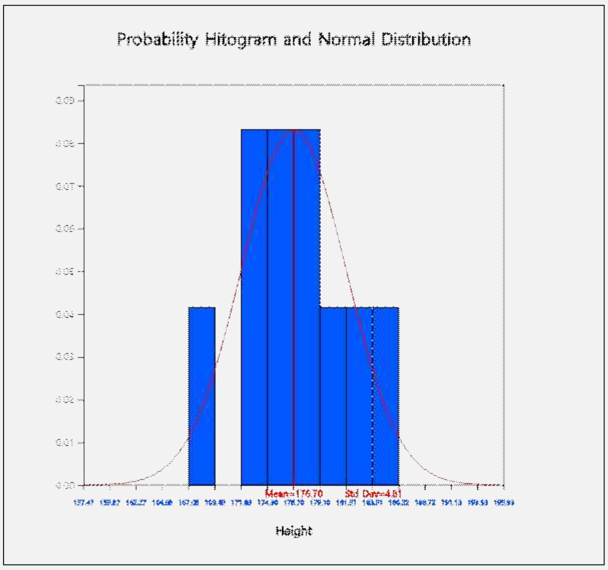


<Figure 7.1.8> Dot graph and confidence interval

- If you click [Histogram] button from option menu below the graph as in <Figure 7.1.9>, the corresponding histogram is appeared as in <Figure 7.1.10>. The histogram together with the normal distribution graph can be used to check whether the sample data comes from a normal distribution. The options such as [Normal Q-Q Plot] and [Normality Test] will be explained in chapter 11.



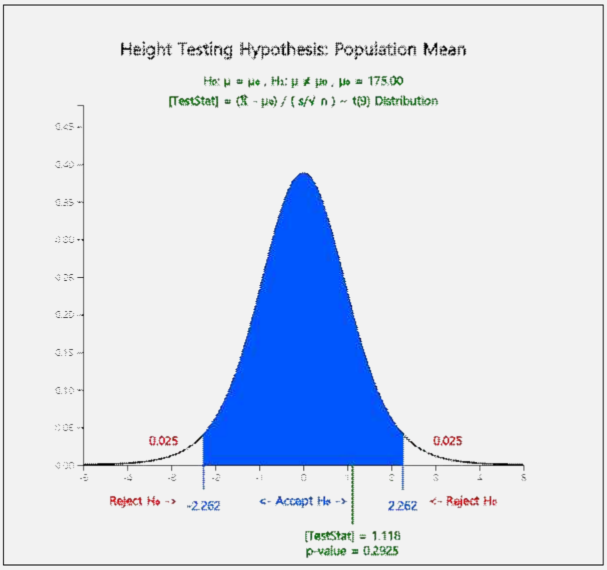
<Figure 7.1.9> Options for testing hypothesis of population mean



<Figure 7.1.10> Histogram and Normal Distribution

Exmaple 7.1.4 Answer (continued)

- Enter μ_0 = 175 at the box of option menu, select the right sided test and the significance level of 5%. Then press the [t-Test] button to display the hypothesis test graph as shown in <Figure 7.1.11> and a test result in the Log Area as in <Figure 7.1.12>.



<Figure 7.1.11> Testing hypothesis for population mean

Testing Hypothesis: Population Mean	Analysis Var	Height			
Statistics	Observation	Mean	Std Dev	std err	Population Mean 95% Confidence Interval
	10	176.700	4.809	1.521	(173.260, 180.140)
Missing Observations	0				
Hypothesis					
$H_0 : \mu = \mu_0$	μ_0	[TestStat]	t value	p-value	
$H_1 : \mu \neq \mu_0$	175.00	sample mean	1.118	0.2925	

<Figure 7.1.12> Testing hypothesis for population mean

- You can select Z-test in the option menu, but you have to enter the population standard deviation σ in this case.

[Practice 7.1.1]

The following data are weights of the 7 employees randomly selected who are working in the shipping department of a wholesale food company.

154, 186, 159, 174, 183, 163, 181 (unit pound)
Ex ⇨ eBook ⇨ PR070101_Height.csv.

Based on this data, can you say that the average weight of employees working in the shipping department is 160 or greater than 160? Use the significance level of 5%.

- The testing hypothesis of the population mean using the sample variance requires the assumption that the population is normally distributed. Testing whether sample data come from a normal population is called a goodness of fit test which will be explained in Chapter 11.
- In this section, we looked at the testing hypothesis for the population mean when the sample size is already given and only the significance level of the type 1 error is considered. In this case, if we try to reduce the probability of the type 1 error, then the probability of the type 2 error will be increased and we can not reduce both types of errors at the same time. Therefore, if the sample size was predetermined, or if data were given, only the type 1 error was considered as a conservative decision making to test the hypothesis. However, if the sample size can be selected by a researcher, there is a testing hypothesis that considers both types of errors together which will be explained in detail in section 7.4.

7.2 Testing Hypothesis for a Population Variance

- Examples for testing hypothesis of a population variance are as follows:

  - Bolts produced by a company are currently supplied to an automaker and have an average diameter of 7mm and a variance of 0.25mm. Recently, a rival company has applied for the supply, claiming that their company's bolts have the same average diameter of 7 mm but a variance of 0.16mm. How can I find out if this claim is true?
  - The variance of scores in mathematics on the last year's college scholastic aptitude test was 100. This year’s questions in mathematics test are said to be much easier than last year's. How can I test whether the variance of the mathematics score in this year is smaller than the last year?

- If you understand the testing hypothesis for the population mean, the testing hypothesis for the population variance differs only from the sampling distribution and the test statistic, but the basic concept is the same. In Chapter 6, we studied that the distribution of all possible sample variances multiplied by a constant, $(n-1)S^2/\sigma^2$, follows a chi-square distribution with n-1 degrees of freedom when the population is a normal distribution with variance σ^2. Using this theory, testing hypothesis for the population variance can be done as follows:

Table 7.2.1 Testing hypothesis of the population variance
- the population is normally distributed -

Type of Hypothesis	Decision Rule
1) $H_0 : \sigma^2 = \sigma_0^2$: $H_1 : \sigma^2 > \sigma_o^2$:	If $\frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0
2) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 < \sigma_o^2$	If $\frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1\,;\,1-\alpha}$, then reject H_0, else accept H_0
3) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 \neq \sigma_o^2$	If $\frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1\,;\,\alpha/2}$ or $\frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1\,;\,1-\alpha/2}$, then reject H_0, else accept H_0

Note: In 1) the null hypothesis can be written as $H_0 : \sigma^2 \le \sigma_o^2$, in 2) $H_0 : \sigma^2 \ge \sigma_o^2$.

Example 7.2.1

One company produces bolts for an automobile. If the average diameter of bolts is 15mm and its variance is less than or equal to 0.10^2, it can be delivered to the automobile company. Twenty-five of the most recent products were randomly sampled and their variance was 0.15^2. Assuming that the diameter of a bolt follows a normal distribution,

1) Conduct testing hypothesis at the 5% significance level to determine if the product can be delivered to the automotive company.
2) Check the result using 『eStatU』

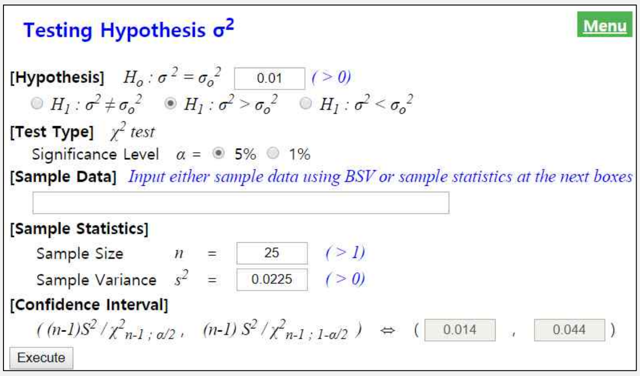
Example 7.2.1 Answer

1) The hypothesis of this problem is H_0: $\sigma^2 \le 0.1^2$ H_1: $\sigma^2 > 0.1^2$ and its decision rule is as follows:

‘If $\dfrac{(n-1)S^2}{\sigma_0^2}$ > $\chi^2_{n-1\,;\,\alpha}$, then reject H_0, else accept H_0’

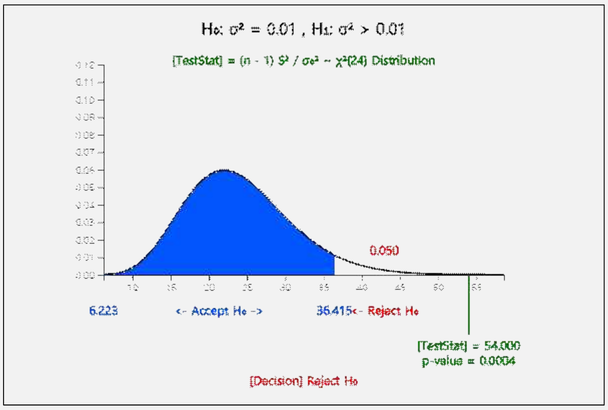
Note that s^2 = $0.15^2 = 0.0225$, (25-1)×$0.15^2/0.1^2$ = 54 and $\chi^2_{25-1\,;\,0.05}$ = $\chi^2_{24\,;\,0.05}$= 36.42. Therefore, H_0 is rejected.

2) Select ‘Testing Hypothesis σ^2’ in 『eStatU』. Enter σ_0^2 = $0.1^2 = 0.01$, select the right sided test and the 5% significance level as <Figure 7.2.1> in the input box. Then enter the sample size n = 25 and sample variance $s^2 = 0.15^2 = 0.0225$.



<Figure 7.2.1> Testing hypothesis for σ^2 using 『eStatU』

- If you click the [Execute] button, the confidence interval of σ^2 is calculated and testing result will be shown as <Figure 7.2.2>.



<Figure 7.2.2> Testing hypothesis for σ^2

Example 7.2.2

(Heights of college students)

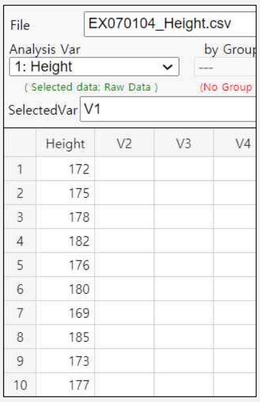
Using 『eStat』 and the height data of 10 male college students in [Example 7.1.4],

172 175 178 182 176 180 169 185 173 177,
⇨ eBook ⇨ EX070104_Height.csv.

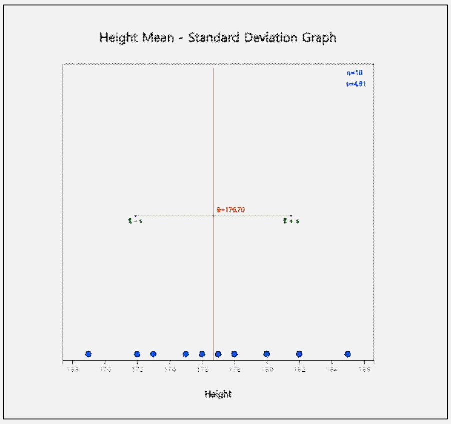
test the hypothesis whether the population variance is greater than 25 at the significance level of 5%.

Answer

- After entering data as shown in <Figure 7.2.3> on the sheet in 『eStat』 , click the icon of testing hypothesis for variance, $\boxed{\sigma}$, and select ‘Height’ as the Analysis Var to display a dot graph of data with (average) ± (standard deviation) interval as in<Figure 7.2.4>.

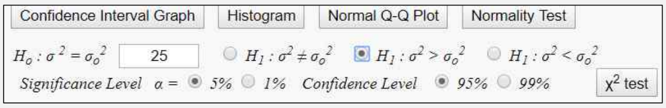


<Figure 7.2.3>
『eStat』 data input

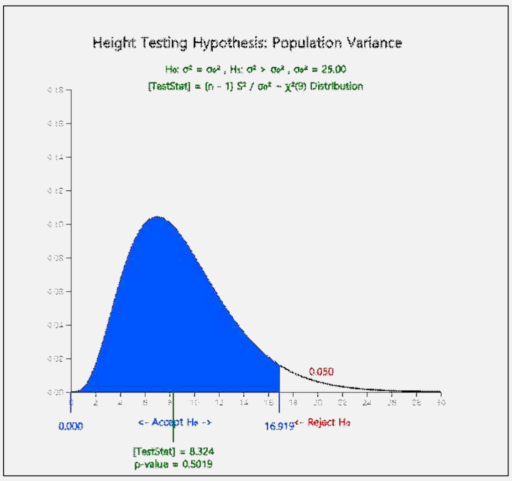


<Figure 7.2.4> Dot graph and (Mean) ± (Std Dev) for Testing hypothesis of σ^2

- In the option box under the Graph Area (<Figure 7.2.5>), enter σ_0^2 = 25, and select the alternative hypothesis as right-sided test, significance level as 5%. By clicking [χ^2 test] button, the result of the testing hypothesis will be shown as <Figure 7.2.6> and the result table as <Figure 7.2.7>.



<Figure 7.2.5> Option menu for testing hypothesis of σ^2



<Figure 7.2.6> Testing hypothesis for σ^2

Example 7.2.2 Answer (continued)

Testing Hypothesis: Population Variance	Analysis Var	Height			
Statistics	Observation	Mean	Std Dev	std err	Population Variance 95% Confidence Interval
	10	176.700	4.809	1.521	(10.940, 77.063)
Missing Observations	0				
Hypothesis					
$H_0 : \sigma^2 = \sigma_0^2$	σ_0^2	[TestStat]	ChiSq value	p-value	
$H_1 : \sigma^2 > \sigma_0^2$	25.00	$(n-1)\ S^2 / \sigma_0^2$	8.324	0.5019	

<Figure 7.2.7> Testing hypothesis for σ^2

- It is necessary to assume that the population is normally distributed to test the hypothesis of the population variance. Testing whether the population is normally distributed using sample data is called a goodness of fit test which will be discussed in Chapter 11. You may test the normality approximately by using a histogram with a normal distribution which can be drawn from the option box in <Figure 7.2.5>. In addition, [Normal Q-Q Plot] can be used to test the normality.

[Practice 7.2.1]

If the variance of the diameter of a metal washer product is less than 0.05^2, then the production process is under control. 21 samples were randomly selected from the assembly line and its variance is 0.06^2. According to this data, is the assembly process out of control with the significance level of 0.05?

7.3 Testing Hypothesis for a Population Proportion

- Consider the following examples for testing hypothesis of the population proportion.

  - Will the approval rating of a particular candidate exceed 50 percent in this year's presidential election?
  - The unemployment rate was 7 percent last year. Has this year's unemployment rate increased?
  - 10,000 car accessories are imported by ship, of which 2 percent was defective according to the past experience. Is the defective product 2% this time again?

- When the sample size is large enough, the sampling distribution of all possible sample proportions ($\hat{p}$) is approximated to a normal distribution with the mean of population proportion (p) and the variance of $p(1-p)/n$. Therefore, the testing hypothesis for the population proportion is similar to the testing hypothesis for the population mean as Table 7.3.1. If np > 5 and $n(1-p)$ > 5, it is usually considered as a large sample.

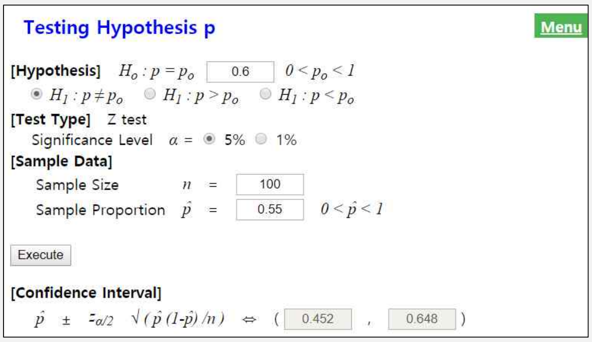
Table 7.3.1 Testing hypothesis for population proportion
– large sample case such as np_0 > 5, $n(1-p_0)$ > 5

Type of Hypothesis	Decision Rule
1) H_0 : $p = p_0$ H_1 : $p > p_0$	If $\frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$ > z_α , then reject H_0, else accept H_0
2) H_0 : $p = p_0$ H_1 : $p < p_0$	If $\frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$ < -z_α , then reject H_0, else accept H_0
3) H_0 : $p = p_0$ H_1 : $p \neq p_0$	If $\left\lvert \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} \right\rvert$ > $z_{\alpha/2}$, then reject H_0, else accept H_0

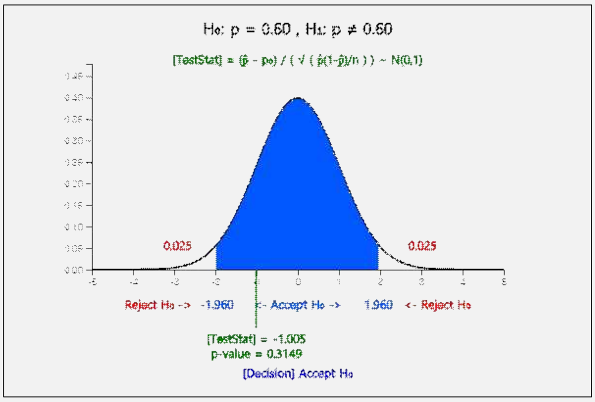
Note: The null hypothesis in 1) can be written as H_0 : $p \leq p_0$ and in 2) as H_0 : $p \geq p_0$

Example 7.3.1	A survey was conducted last month for the election of a national assembly member. According to the survey of the last month, the approval rating of a particular candidate was 60 percent. In order to see if there is a change in the approval rating, a sample survey of 100 people has been conducted and 55 people supported it. 1) Test whether the current approval rating for a particular candidate is changed comparing with the one of last month of 60%. Use 5% significance level. 2) Check the result using 『eStatU』.
Answer	1) The hypothesis of this problem is $H_0:\ p = 0.6,\ H_1:\ p \neq 0.6$. Since np_0 = 60, $n(1-p_0)$ = 40, it can be considered as a large sample and the decision rule is as follows: 'If $\left\lvert \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} \right\rvert$ > $z_{\alpha/2}$, reject H_0, else accept H_0' Since $\hat{p}$ = 55/100 = 0.55, $\left\lvert \frac{0.55-0.6}{\sqrt{0.6(1-0.6)/100}} \right\rvert = \lvert -1.005 \rvert = 1.005,\quad z_{0.05/2}$ = $z_{0.025}$ = 1.96 Hence, H_0 is accepted. 2) Select 'Testing Hypothesis p' at 『eStatU』 menu. Enter $p_0 = 0.6$, select the two sided test and the 5% significance level as <Figure 7.3.1> in the input box window. Then enter the sample size n = 100, and the sample proportion $\hat{p} = 0.55$. If you click the [Execute] button, the confidence interval of p is calculated and testing result will be shown as in <Figure 7.3.2>.

Example 7.3.1 Answer (continued)



<Figure 7.3.1> Data input for testing hypothesis of p using『eStatU』



<Figure 7.3.2> Testing hypothesis for p using『eStatU』

[Practice 7.3.1]

A university wants to build a parking lot for students. School authorities think more than 20 percent of students go to school by car. 100 students were randomly selected and 18 of them said that they go to school by car. Test at the significance level of 0.05 whether the school authorities' thinking is correct.

- If the sample size is small, the testing hypothesis for population proportion uses the binomial distribution and it will be explained in the Sign Test in Chapter 10.

7.4 Testing Hypothesis with α and β simultaneously

- Since the testing hypothesis we have learned so far is a conservative decision making method, we first decide a critical value that reduces the probability of the type 1 error α (the error that rejects the null hypothesis even though it is true). This decision rule is intended to keep the null hypothesis unless there is a sufficient evidence of the alternative hypothesis which is a new fact or risky. Thus, the probability of the type 2 error β was not considered at all in the decision rule. However, sometimes it is unclear which one should be the null hypothesis and which one should be the alternative hypothesis. Depending on the problem, both types of errors are important and should be considered simultaneously. If the analyst can determine the sample size, a testing hypothesis that takes into account both α and β can be performed.

7.4.1 Type 2 Error and Power of a Test

- Consider the following example to find out how to calculate the probability of the type 2 error β.

Example 7.4.1

For the testing hypothesis of [Example 7.1.1], calculate the probability of the type 2 error β if the significance level is 5%. Check this result using 『eStatU』.

Answer

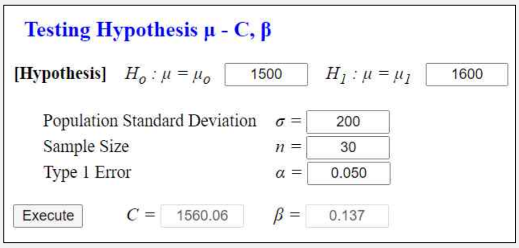
- The hypothesis in [Example 7.1.1] is H_0 : $\mu = 1500$, H_1 : $\mu = 1600$, the population standard deviation is assumed σ = 200, the sample size is n=30 and hence the decision rule is as follows if the significance level is 5%.

  ‘If $\overline{X}$ < 1500 + (1.645)$\frac{200}{\sqrt{30}}$ = 1560.06, reject H_0, else accept H_0’

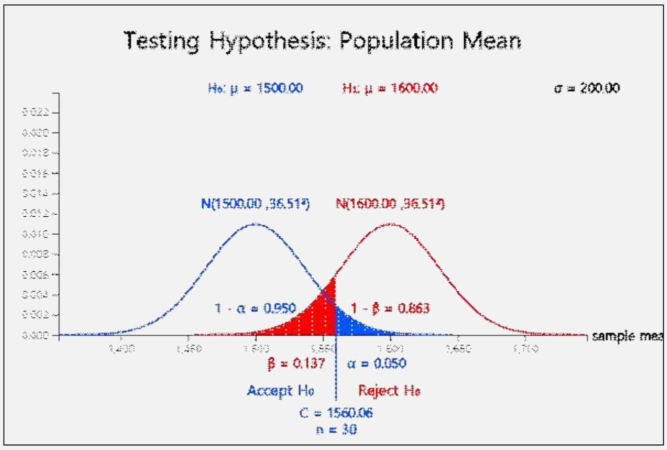
- Hence, the type 2 error which is the probability of ‘H_0 is true when H_1 is true’ can be calculated as follows:

  β = $\mathrm{P}(\overline{X}$ < 1560.06 | H_1 is true)
  = $\mathrm{P}((\overline{X}$-1600)/(200/ $\sqrt{30}$) < (1560.06-1600)/(200/ $\sqrt{30}$))
  = $\mathrm{P}(Z$ < -1.09) = 0.137

- Select ‘Testing μ - C, β’ at 『eStatU』 menu. Enter $\mu_0 = 1500$, $\mu_1 = 1600$, $\sigma = 200$, $\alpha = 0.05$, $n = 30$ in the input box window as <Figure 7.4.1> and click the [Execute] button. The result of the testing hypothesis, the critical value C and the probability of the type 2 error β, will be shown as in <Figure 7.4.2>.



<Figure 7.4.1> Testing hypothesis for μ with α using 『eStatU』



<Figure 7.4.2> Calculation of β and power using 『eStatU』

Example 7.4.2

In [Example 7.1.1], if the null hypothesis is not changed, but the alternative hypothesis is changed as follows:

$$H_0 \;:\; \mu = 1500, \quad H_1 \;:\; \mu = 1580$$

1) Calculate the probability of the type 2 error β if the significance level is 5%.
2) Check this result using 『eStatU』.

Answer

1) Although the alternative hypothesis has been changed to $H_1 : \mu = 1580$, the decision rule will not be changed in case of the conservative decision making, because the alternative hypothesis is the same type of $H_1 : \mu > 1500$.

‘If $\overline{X}$ < 1500 + (1.645)$\frac{200}{\sqrt{30}}$ = 1560.06, reject H_0, else accept H_0’

◆ Hence, the probability of type 2 error is as follows:

β = $\mathrm{P}(\overline{X}$ < 1560.06 | H_1 is true)
= $\mathrm{P}($ ($\overline{X}$-1580)/(200/ $\sqrt{30}$) < (1560.06-1580)/(200/ $\sqrt{30}$))
= $\mathrm{P}(Z$ < -0.546) = 0.293

2) In order to calculate β using 『eStatU』, enter $\mu_1 = 1580$ in <Figure 7.4.1> and click the [Execute] button.

[Practice 7.4.1]

Calculate the followings using 『eStatU』.

1) If $H_0 \;:\; \mu = 50$, $H_1 \;:\; \mu = 52$, n = 25, σ = 3, α = 0.05, find β.
2) If $H_0 \;:\; \mu = 50$, $H_1 \;:\; \mu = 54$, n = 25, σ = 3, α = 0.01, find β.
3) If $H_0 \;:\; \mu = 50$, $H_1 \;:\; \mu = 56$, n = 25, σ = 5, α = 0.05, find β.

- Comparing [Example 7.4.1] and [Example 7.4.2], the probability of the type 2 error occurring when $H_1 \;:\; \mu = 1600$ is less than that of $H_1 \;:\; \mu = 1580$, so the ability to judge is greater. In other words, the closer the population mean of H_1 is to the population mean of H_0, the less discriminating ability it becomes.
- Generally, the discriminating ability of two hypothesis is compared by using the following **power of a test**.

  Power = 1 - (Probability of the type 2 error) = 1 - β

  A large power increases the discriminating ability of the hypothesis test.
- The power of a test can be obtained for any μ_1 of the alternative hypothesis $H_1 : \mu = \mu_1$. It means that the power is a function over the value of μ_1 and it is called a **power function.**
- A function of the probability that the null hypothesis is correct when the null hypothesis is true is called an **operating characteristic function**.

  Operating characteristic function = 1 - (Probability of the type 1 error) = 1 - α

Example 7.4.3

In [Example 7.1.1], calculate the power of the following alternative hypothesis. Use α = 0.05. By using this, approximate the power function.

1) H_1 : μ = 1500	2) H_1 : μ = 1510	3) H_1 : μ = 1520
4) H_1 : μ = 1530	5) H_1 : μ = 1540	6) H_1 : μ = 1550
7) H_1 : μ = 1560	8) H_1 : μ = 1570	9) H_1 : μ = 1580
10) H_1 : μ = 1590	11) H_1 : μ = 1600	12) H_1 : μ = 1610

Answer

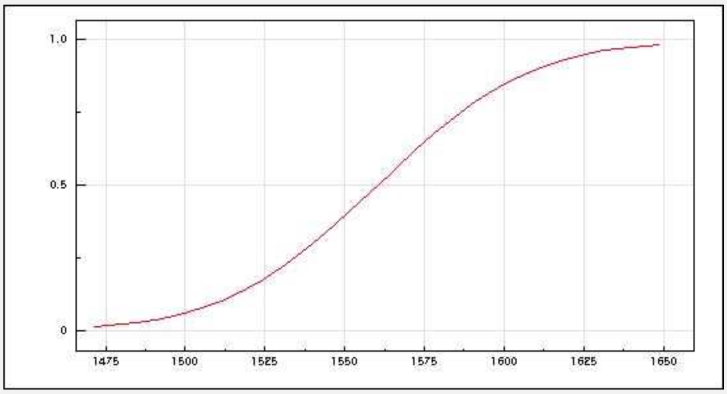
- Although the alternative hypotheses are different, the decision rule is the same as follows:

  'If $\overline{X}$ < 1500 + (1.645)$\frac{200}{\sqrt{30}}$ = 1560.06, then accept H_0, else reject H_0'

  Hence, if we calculate the probability of the type 2 error as [Example 7.4.2], the power of each test is as follows:

Alternative Hypothesis	β	Power = 1 - β
1) H_1 : μ = 1500	0.95	0.05
2) H_1 : μ = 1510	0.91	0.09
3) H_1 : μ = 1520	0.86	0.14
4) H_1 : μ = 1530	0.79	0.21
5) H_1 : μ = 1540	0.71	0.29
6) H_1 : μ = 1550	0.61	0.39
7) H_1 : μ = 1560	0.50	0.50
8) H_1 : μ = 1570	0.39	0.61
9) H_1 : μ = 1580	0.29	0.71
10) H_1 : μ = 1590	0.21	0.79
11) H_1 : μ = 1600	0.14	0.86
12) H_1 : μ = 1610	0.09	0.91

- The power function can be approximated by connecting points of (μ, 1 - β) in each test as <Figure 7.4.3>.



<Figure 7.4.3> Power function of [Example 7.4.1]

- In case of a two sided test, the power function is a V-shaped, because the type 2 error may appear on either side of the null hypothesis. If the V-shaped valley is deep, it is generally considered to have highly discriminating ability against the null hypotheses.

[Practice 7.4.2]

If H_0 : $\mu = 50$, n = 25, σ = 5, α = 0.05, calculate the power of the following alternative hypothesis. By using this, approximate the power function.

1) H_1 : μ = 51 2) H_1 : μ = 52 3) H_1 : μ = 53
4) H_1 : μ = 54 5) H_1 : μ = 55 6) H_1 : μ = 56

7.4.2 Testing Hypothesis with α and β

- If the sample size is not predetermined and the analyst can determine it, the testing hypothesis can be performed with the desired level of α and β as following example.

Example 7.4.4

Consider the testing hypothesis on the bulb life such as $H_0 :\ \mu = 1500$, $H_1 :\ \mu = 1570$. Find the sample size n and the decision rule which satisfies α of 5% and β of 10%. Assume that the population standard deviation σ is 200 hours. Check the result using 『eStatU』.

Answer

- Let n be the sample size and C be the critical value of a decision rule. The probability of the type 1 error α and the probability of the type 2 error β are defined as follows:

$$\alpha = P(\overline{X} > C \mid H_0 \text{ is true})$$
$$\beta = P(\overline{X} < C \mid H_1 \text{ is true})$$

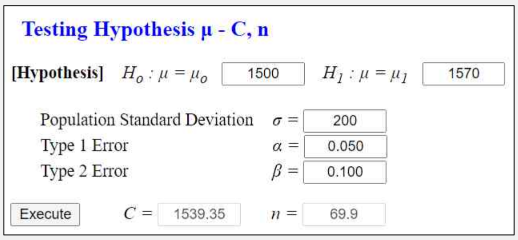
- If H_0 is true, the sampling distribution of $\overline{X}$ is $N(1500, \frac{200^2}{n})$ and if H_1 is true, the sampling distribution of $\overline{X}$ is $N(1570, \frac{200^2}{n})$. If α = 0.05 and β = 0.10, then $z_{0.05}$ = 1.645 and $z_{0.90}$ = -1.280. Hence, n and C should satisfy both of the following equations and they can be calculated by solving the two system of equations.

$$C = 1500 + 1.645 \times (200/\sqrt{n})$$
$$C = 1570 - 1.280 \times (200/\sqrt{n})$$

The solution is n = 69.8, C = 1539.4. i.e., the sample size is 70 approximately and the decision rule is as follows:

'If $\overline{X}$ > 1539.4, then reject H_0, else accept H_0'

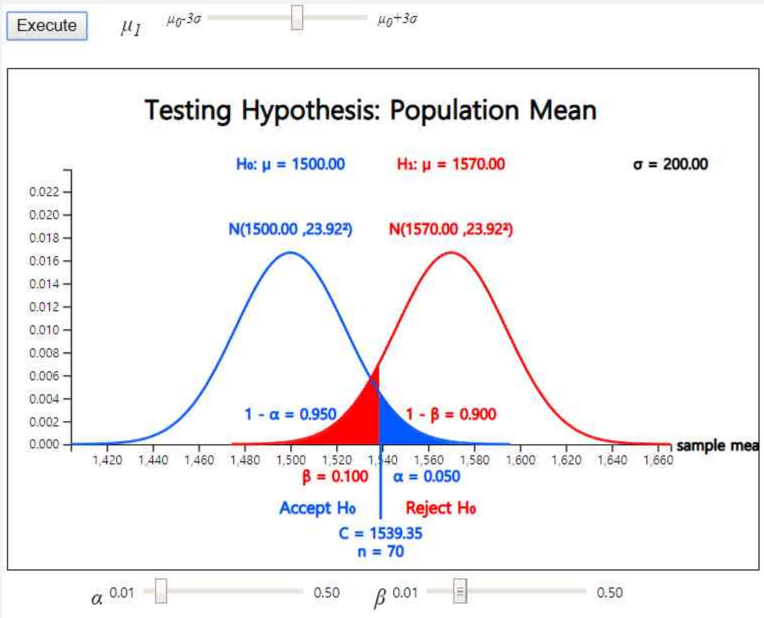
- Select 'Testing μ - C, n' at 『eStatU』 menu. Enter $\mu_0 = 1500$, $\mu_1 = 1570$, $\alpha = 0.05$, $\beta = 0.10$ in the input box window as <Figure 7.4.4>.



<Figure 7.4.4> Testing hypothesis for μ with α, β using 『eStatU』

Example 7.4.4 Answer (continued)

- If you click the [Execute] button, you will see the test result of 『eStatU』 as in <Figure 7.4.5>. The critical value C and the sample size n are calculated.



<Figure 7.4.5> Testing hypothesis for μ with α, β using 『eStatU』

[Practice 7.4.3]

If H_0 : $\mu = 50$, H_1 : $\mu = 52$, σ = 3, α = 0.05, β = 0.10, find n using 『eStatU』.

Exercise

7.1 Assume that the distribution of a population is a normal distribution with the standard deviation of 50. The sample mean of 25 samples randomly selected from the population is 70. Test the hypothesis $H_0 : \mu = 100 \;\; H_1 : \mu \neq 100$ with the significance level of 0.01 and obtain the p-value.

7.2 A bolt manufacturer claims that the average of bolt length is 4.5 cm, the standard deviation is 0.020 cm and it is normally distributed. When 16 samples were taken randomly, the sample mean was 4.512. Can you say that the actual average length of the bolt is different from that of the manufacturer's claim? Assume that the significance level is 0.01.

7.3 The amount of water needed in the process of producing a compound with distilled water added to any component in a fixed amount depends on the purity of the component. In the manufacturer's experience, the water requirement for a normal production is 6 liters and the standard deviation is 1 liter. A sample mean from samples of nine products was 7 liters. Can products be considered as a normal production at the significance level of 0.05?

7.4 A psychologist is working on physically disabled workers. Based on the past experience, the psychologist believed that the average social (relationship) score of these disabled workers was greater than 80. Twenty employees were sampled from the score population to obtain the following result:

99, 69, 91, 97, 70, 99, 72, 74, 74, 76,
96, 97, 68, 71, 99, 78, 76, 78, 83, 66.

The psychologist wants to know if the average social score of the population is right. Assume that the population follows a normal distribution and its standard deviation is 10. Test with the significance level of 0.05.

7.5 The following is the weights of the 10 employees randomly selected who are working in the shipping department of a wholesale food company.

154, 154, 186, 243, 159, 174, 183, 163, 192, 181 (unit pound)

Based on this data, can you say that the average weight of employees working in the shipping department is greater than 160 pound? Use the significance level of 5%.

7.6 A company that makes tar on a roof wants an average of less than 3 percent of impurities. By sampling 30 barrels of tar, can this data suggest that the population mean is less than 3% when the proportion of impurities is as follows? Use the significance level of 5%.

3 3 1 1 0.5 2 2 4 5 4 5 3 1 3 1
4 1 1 4 2 5 3 1 1 1 0.75 1.5 3 3 2

7.7 In a large manufacturer, the manager of the company claims that the average adaptation score of all unskilled workers is greater than 60. In order to check this claim, 40 unskilled workers were randomly selected and their test scores of adaptation scores were as follows:

73 57 96 78 74 42 55 44 91 91 50 65 46 63 82 60 97 79 85 79
92 50 42 46 86 81 81 83 64 76 40 57 78 66 84 96 94 70 70 81

If the population variance is 280, test the hypothesis at the significance level of 0.05 whether the manager's argument is correct. What is the p-value?

7.8 Existing tires of a company have an average life span of 54,000 km and a standard deviation of 10,000 km. Two researchers conducted a performance test for new tires independently. The first researcher tested 25 tires and obtained an average life span of 48,000 km, while the second researcher tested 100 to get 50,000 km. Which test result gives more reliable statistical evidence that the average life span of a new tire is less than the existing one? Assume that the standard deviation has not changed.

7.9 Suppose n = 100 and σ = 8.4 are given to test the hypothesis $H_0 : \mu = 75.0,\ H_1 : \mu < 75.0$.

1) If the null hypothesis is rejected when the sample mean is less than 73.0, what is the probability of the type 1 error?
2) If we test the alternative hypothesis $H_1 : \mu > 75.0$ and the decision rule is the same as above, what is the probability of the type 1 error?

7.10 A random variable follows a normal distribution N(μ, 4). Test the hypothesis H_0 : μ = 0 with the significance level of 0.10 if n = 25 and $\bar{x}$ = 0.28. Does the 90% confidence interval include μ = 0?

7.11 A sample of size 21 was randomly taken from a population that follows the normal distribution and its sample variance was 10. Test the null hypothesis H_0 : σ^2 = 15 and the alternative hypothesis H_1 : σ^2 ≠ 15 with the significance level of 0.05.

7.12 If the variance of diameters of metal washer products is less than 0.00005^2, then the production process is under control. 31 samples were randomly selected from the assembly line and its variance is 0.000061^2. According to this data, is the assembly process out of control with the significance level of 0.05? What assumptions do you need to get an answer?

7.13 For a manufacturer to make a product, the variance of tensile strength of the synthetic fiber must be less than or equal to five. When 25 samples are randomly selected from the new shipment and the variance is seven. Does this data provide sufficient evidence for the manufacturer to reject the shipment? Assume the significance level of 0.05 and that the tensile strength of the fiber follows approximately a normal distribution.

7.14 In a process of filling the container, the average weight is set to 8g and the variance of the weight shall be σ^2 = 4g to satisfy the given tolerances. 25 container samples were randomly selected and its standard deviation was 2.8g.

1) If the weight is assumed to follow a normal distribution, is the population variance greater than the prescribed value σ^2? Test at the significance level of 0.01.
2) What is the range of sample variances that can not be rejected? Are these values symmetrical to the prescribed value of 2g? Why is that?

7.15 A university wants to build a student parking lot. School authorities think more than 20 percent of students go to school by car. 250 students are randomly selected and 65 of them said that they go to school by car. Test at the significance level of 0.05 whether the school authorities' thinking is correct.

7.16 The accountant of a company thinks that more than 20% of the statement of expenses includes at least one mistake. 400 statements of expenses were randomly selected and found at least one mistake in 100 statements. Test whether the accountant's belief is correct with the significance level of 0.05.

7.17 A researcher met 200 office workers of a company who changed their job last year. Thirty of them stated that they changed their work, because they could not expect for promotion. Can you say that less than 20% of the employees were changed their job, because of their promotion? The significance level is 0.05.

7.18 A statistician threw coins 24,000 times, with 12012 front and 11988 back. Does this data support the null hypothesis that there is 0.5 chance that the front face will appear? Test at the significance level of 0.05.

7.19 A high school student made dice from wood. It doesn't look like a cube, so it's not one-sixth of a chance of six at the top. I rolled the dice 18,000 times and got 3,126 of six at the top.

1) Determine whether these 3126 occurrences are statistically significant by obtaining the p-value.
2) Since it is too much to demand that a hand-made dice have an exact one-sixth chance of producing six, we decided to allow the error of 0.01 chance to be a fair dice. Using the above sample experiment, obtain a confidence interval for the probability of a 6 and determine whether it is acceptable to be viewed as a fair dice (Note: It is difficult to distinguish between the statistical significance and the practical significance by relying solely on testing hypothesis.)

** Obtain a power function for each of the following situations and draw a graph:

7.20 H_0 : $\mu = 51$, H_1 : $\mu < 51$, n = 25, σ = 3, α = 0.05.

7.21 H_0 : $\mu = 516$, H_1 : $\mu > 516$, n = 16, σ = 32, α = 0.05.

7.22 H_0 : $\mu = 3$, H_1 : $\mu \neq 3$, n = 100, σ = 1, α = 0.05.

7.23 H_0 : $\mu = 4.25$, H_1 : $\mu > 4.25$, n = 81 , σ= 1.8 , α = 0.01.

7.24 On question 7.21, find n, C and the decision rule when β = 0.10 and μ_1 = 520.

7.25 On question 7.23, find n, C and the decision rule when β = 0.05 and μ_1 = 4.52.

7.26 On question 7.23, find n, C and the decision rule when β = 0.03 and μ_1 = 5.

7.27 The standard deviation of the resistance of a wire made by a factory is known to be 0.02 ohms. An electronics company has decided not to buy the wire if there is sufficient evidence that the average resistance of the wire is greater than 0.4 ohms. The company's management adheres to the policy of the significance level $\alpha = 0.05$ and the sample size $n = 100$ in statistical tests.

1) Describe the proper null and alternative hypothesis.
2) Describe the type 1 and type 2 errors.
3) Obtain the rejection region of the test.
4) Draw the power function and operating characteristic curve.

7.28 In order to test whether the coins were fair, the following decision criteria were established:

① If the number of fronts when a coin thrown 100 times is in between 40 and 60, the null hypothesis is adopted.
② In other cases, the null hypothesis is rejected.

1) What is the null and alternative hypothesis?
2) What is the probability of the type 1 error in this test?
3) Mark the critical value of this test using the standard normal distribution.

4) Calculate the probability of the type 2 error when p = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9.

Multiple Choice Exercise

7.1 What is the type 1 error?

① Error rejecting the null hypothesis when it is true
② Error accepting the null hypothesis when it is true
③ Error rejecting the null hypothesis when it is not true
④ Error accepting the null hypothesis when it is true

7.2 Which hypothesis is accepted when the null hypothesis is rejected?

① null hypothesis ② significant hypothesis
③ alternative hypothesis ④ basic hypothesis

7.3 Which of the following statements is true?

① The conclusion about a statistical test is always absolute.
② Rejecting null hypothesis implies we accept the alternative hypothesis.
③ In order to reduce the type 1 error, significance level of 0.05 is better than 0.01.
④ In order to reduce the type 1 error, significance level of 0.01 is better than 0.05.

7.4 What is the meaning of the significance level $\alpha = 0.05$?

① 5% of decisions to reject the null hypothesis when it is true if the same test method was repeated repeatedly
② 5% of decisions to reject the alternative hypothesis when it is true if the same test method was repeated repeatedly
③ The probability of accepting the null hypothesis is 0.05.
④ The probability of accepting the alternative hypothesis is 0.05.

7.5 What is the critical value to reject the null hypothesis if the alternative hypothesis is right sided when the sampling distribution to test the population mean is the standard normal distribution?

① 2.58 ② 1.64 ③ -2.33 ④ 1.96

7.6 If the null hypothesis is $H_0 : \mu = \mu_0$, what is the two-sided alternative hypothesis?

① $H_1 : \mu > \mu_0$ ② $H_1 : \mu < \mu_0$
③ $H_1 : \mu = \mu_0$ ④ $H_1 : \mu \neq \mu_0$

7.7 The following lists the sequence of a statistical hypothesis test. What's the right order?

a. Set a hypothesis.
b. Determine the significance level.
c. Determine whether or not to accept the hypothesis.
d. Calculate the test statistic.
e. Find the rejection region

① a→d→e→b→c
② a→d→b→e→c
③ b→e→d→a→c
④ a→c→b→d→e

7.8 Which of the following statements is incorrect?

① The rejection of the null hypothesis means accepting the alternative hypothesis.
② The maximum allowable probability of the type 1 error is the significance level.
③ Error accepting the null hypothesis when it is false is the type 1 error.
④ Error accepting the alternative hypothesis when it is false is the type 2 error.

7.9 If $H_0 : \sigma^2 = \sigma_0^2$, what is the decision rule of the two sided test with the significance level of α?

① If $\dfrac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1,\alpha}$, reject H_0.
② If $\dfrac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1,\alpha}$, reject H_0.
③ If $\dfrac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1,\alpha/2}$, reject H_0.
④ If $\dfrac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1,\alpha/2}$ or $\dfrac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1,1-\alpha/2}$, reject H_0.

7.10 When a coin was thrown 10,000 times to check whether it was normal, the front and back came out 5020 times and 4980 times. What is the hypothesis you want to test?

① $H_0 : \mu = 10000$ ② $H_0 : \mu > 5000$
③ $H_0 : p > 0.5$ ④ $H_0 : p \neq 0.5$

7.11 What is the power of a test?

① α ② 1 - α
③ β ④ 1 - β

7.12 What is the operating characteristic probability?

① α ② 1 - α
③ β ④ 1 - β

(Answers)
7.1 ①, 7.2 ③, 7.3 ②, 7.4 ①, 7.5 ②, 7.6 ④, 7.7 ②, 7.8 ③, 7.9 ④, 7.10 ③,
7.11 ④, 7.12 ②