Material de Apoyo Docente

Universidad de Talca

Facultad de Psicología

MODELOS LONGITUDINALES CON VARIABLES LATENTES PARA LA INVESTIGACIÓN EN PSICOLOGÍA

Modelos de ecuaciones estructurales (SEM)

Fabiola Gómez

08 de octubre 2024

Paquetes a utilizar en esta sesión.

a. *lavaan*: Un paquete para la estimación de modelos de variables latentes como CFA, además permite realizar path analysis, modelación de ecuaciones estructurales, curvas de crecimiento latente, etc.
b. *psych*: Paquete que incluye funciones útiles para la investigación en psicología.
c. *MVN*: Paquete que usaremos para evaluar normalidad multivariada.

Durante esta sesión trabajaremos con una base de datos que contiene información de 543 jóvenes mayores de 18 años. Estos datos contienen información sobre el sexo de los participantes (hombre= 0 y mujer= 1), edad (años cumplidos) y nivel de religiosidad. Además contiene los resultados en las siguientes escalas: Justificación del Sistema (3 ítems) y Homonegatividad Moderna (7 ítems).Todas las escalas están medidas con una escala likert de cinco puntos.

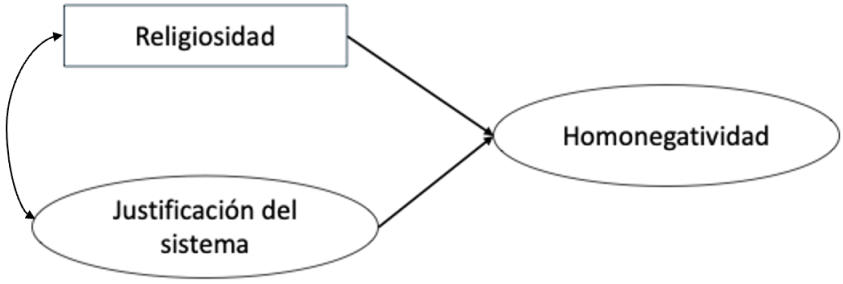
Los investigadores están interesados en evaluar el efecto que tiene la religiososdad y la justificación del sistema sobre los niveles de homonegatividad moderna. Adicionalmente, quieren evaluar si este modelo es invariante entre hombres y mujeres.
Ellos hipotetizan que estas dos variables tendrán un efecto positivo y significativo sobre los niveles de homonegatividad hacia los hombres gays. Además, se espera que las relaciones hipotetizadas (modelo estructural) sean invariantes entre hombres y mujeres.

Cargar la base de datos y exploración de las variables

```
setwd("/Users/fa/Dropbox/Docencia/2024/Doctorado/Optativo Doctorado UTalca/Clases/Clase

base<- read.csv("sem.csv", header = T, sep = ";", na= -9)

names(base) # nombre de las variables

##  [1] "CASE"         "sexo"         "edad"         "religiosidad" "sj_01"
##  [6] "sj_02"        "sj_03"        "homo1_gay"    "homo2_gay"    "homo3_gay"
```

```
## [11] "homo4_gay"    "homo5_gay"    "homo6_gay"    "homo7_gay"

str(base) # estructura de la base de datos y variables

## 'data.frame': 543 obs. of  14 variables:
##  $ CASE        : int  524 34 564 576 540 77 115 272 294 457 ...
##  $ sexo        : int  1 1 1 1 1 1 0 1 1 1 ...
##  $ edad        : int  24 24 24 23 23 22 22 22 22 22 ...
##  $ religiosidad: int  5 1 6 7 4 3 1 4 2 5 ...
##  $ sj_01       : int  2 1 2 3 3 3 3 1 2 2 ...
##  $ sj_02       : int  2 1 3 2 2 2 2 3 1 2 ...
##  $ sj_03       : int  1 1 3 4 4 2 2 2 1 3 ...
##  $ homo1_gay   : int  2 3 3 1 1 3 2 3 1 2 ...
##  $ homo2_gay   : int  3 2 2 1 1 2 1 1 1 2 ...
##  $ homo3_gay   : int  2 2 4 2 5 2 1 3 1 4 ...
##  $ homo4_gay   : int  3 2 4 4 3 2 1 1 1 3 ...
##  $ homo5_gay   : int  3 2 3 2 1 2 1 2 1 3 ...
##  $ homo6_gay   : int  4 2 3 1 1 3 2 2 1 2 ...
##  $ homo7_gay   : int  3 2 4 1 1 3 1 1 1 3 ...

library(psych)
describe(base[3:14], skew = F) # descriptivos sin sesgo y curtosis

##              vars   n  mean   sd median min max range   se
## edad            1 540 21.33 2.12     21  18  25     7 0.09
## religiosidad    2 541  3.17 1.82      3   1   7     6 0.08
## sj_01           3 543  2.15 0.87      2   1   5     4 0.04
## sj_02           4 543  2.23 0.97      2   1   5     4 0.04
## sj_03           5 541  1.90 0.94      2   1   5     4 0.04
## homo1_gay       6 541  1.99 1.07      2   1   5     4 0.05
## homo2_gay       7 541  1.53 0.99      1   1   5     4 0.04
## homo3_gay       8 540  2.45 1.30      2   1   5     4 0.06
## homo4_gay       9 541  2.09 1.19      2   1   5     4 0.05
## homo5_gay      10 540  2.24 1.27      2   1   5     4 0.05
## homo6_gay      11 540  2.15 1.23      2   1   5     4 0.05
## homo7_gay      12 541  2.14 1.26      2   1   5     4 0.05
```

Especificación del modelo



Estimación y evaluación de ajuste del modelo estructural

```
library(lavaan)

## This is lavaan 0.6-18
## lavaan is FREE software! Please report any bugs.

##
## Attaching package: 'lavaan'

## The following object is masked from 'package:psych':
##
##     cor2cov

#### Especificación del modelo

mod1<- '# Modelo de medición
        justif=~ sj_01 + sj_02 + sj_03
        homoneg=~ homo1_gay + homo2_gay + homo3_gay +
                  homo4_gay + homo5_gay + homo6_gay +
                  homo7_gay
        # Modelo estructural
        homoneg~ justif + religiosidad
        justif ~~ religiosidad'

# Estimación del modelo con MLR
# varianza de los factores fijas en 1 (std.lv= T)
fit1<- sem(mod1, estimator = "MLR",
           std.lv= T, sample.mean = T,
            data=base)

# Obtención del output con:
```

```r
# índices de ajuste, resultados estandarizados y R2
summary(fit1, fit.measures = T, standardized = T, rsquare = T)
```

```
## lavaan 0.6-18 ended normally after 37 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        35
##
##                                                   Used       Total
##   Number of observations                           534         543
##
## Model Test User Model:
##                                                Standard      Scaled
##   Test Statistic                                152.247     135.794
##   Degrees of freedom                                42          42
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.121
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                              2394.616    1894.431
##   Degrees of freedom                                55          55
##   P-value                                        0.000       0.000
##   Scaling correction factor                                  1.264
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.953       0.949
##   Tucker-Lewis Index (TLI)                       0.938       0.933
##
##   Robust Comparative Fit Index (CFI)                         0.955
##   Robust Tucker-Lewis Index (TLI)                            0.941
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7995.508   -7995.508
##   Scaling correction factor                                  1.269
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -7919.385   -7919.385
##   Scaling correction factor                                  1.188
```

```
##       for the MLR correction
##
##   Akaike (AIC)                                16061.017   16061.017
##   Bayesian (BIC)                              16210.830   16210.830
##   Sample-size adjusted Bayesian (SABIC)       16099.729   16099.729
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                           0.070       0.065
##   90 Percent confidence interval - lower          0.058       0.053
##   90 Percent confidence interval - upper          0.082       0.076
##   P-value H_0: RMSEA <= 0.050                     0.003       0.017
##   P-value H_0: RMSEA >= 0.080                     0.092       0.014
##
##   Robust RMSEA                                                0.068
##   90 Percent confidence interval - lower                      0.056
##   90 Percent confidence interval - upper                      0.082
##   P-value H_0: Robust RMSEA <= 0.050                          0.009
##   P-value H_0: Robust RMSEA >= 0.080                          0.074
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                            0.040       0.040
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   justif =~
##     sj_01            0.511    0.050   10.236    0.000    0.511    0.590
##     sj_02            0.634    0.055   11.475    0.000    0.634    0.652
##     sj_03            0.529    0.058    9.179    0.000    0.529    0.566
##   homoneg =~
##     homo1_gay        0.741    0.043   17.082    0.000    0.780    0.735
##     homo2_gay        0.353    0.049    7.157    0.000    0.371    0.392
##     homo3_gay        0.973    0.044   22.270    0.000    1.024    0.792
##     homo4_gay        0.849    0.048   17.646    0.000    0.893    0.747
##     homo5_gay        1.076    0.041   25.974    0.000    1.133    0.895
```

```
##     homo6_gay         0.728    0.051   14.211    0.000    0.766    0.624
##     homo7_gay         1.039    0.039   26.613    0.000    1.094    0.871
##
## Regressions:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   homoneg ~
##     justif            0.185    0.073    2.537    0.011    0.176    0.176
##     religiosidad      0.122    0.028    4.346    0.000    0.116    0.211
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   justif ~~
##     religiosidad      0.527    0.104    5.071    0.000    0.527    0.291
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sj_01             2.150    0.037   57.399    0.000    2.150    2.484
##    .sj_02             2.221    0.042   52.728    0.000    2.221    2.282
##    .sj_03             1.904    0.040   47.105    0.000    1.904    2.038
##    .homo1_gay         1.701    0.075   22.661    0.000    1.701    1.604
##    .homo2_gay         1.372    0.050   27.321    0.000    1.372    1.447
##    .homo3_gay         2.085    0.103   20.260    0.000    2.085    1.613
##    .homo4_gay         1.770    0.087   20.274    0.000    1.770    1.481
##    .homo5_gay         1.825    0.105   17.321    0.000    1.825    1.442
##    .homo6_gay         1.882    0.080   23.593    0.000    1.882    1.533
##    .homo7_gay         1.740    0.102   17.092    0.000    1.740    1.385
##     religiosidad      3.154    0.078   40.216    0.000    3.154    1.740
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sj_01             0.488    0.046   10.670    0.000    0.488    0.652
##    .sj_02             0.545    0.063    8.604    0.000    0.545    0.575
##    .sj_03             0.593    0.060    9.827    0.000    0.593    0.680
##    .homo1_gay         0.517    0.044   11.855    0.000    0.517    0.459
##    .homo2_gay         0.760    0.101    7.520    0.000    0.760    0.847
##    .homo3_gay         0.623    0.067    9.322    0.000    0.623    0.372
##    .homo4_gay         0.631    0.069    9.141    0.000    0.631    0.442
##    .homo5_gay         0.319    0.040    7.899    0.000    0.319    0.199
##    .homo6_gay         0.920    0.070   13.119    0.000    0.920    0.611
##    .homo7_gay         0.382    0.045    8.458    0.000    0.382    0.242
##     religiosidad      3.284    0.138   23.791    0.000    3.284    1.000
##     justif            1.000                              1.000    1.000
```

```
##    .homoneg           1.000                                0.903    0.903
##
## R-Square:
##                    Estimate
##     sj_01             0.348
##     sj_02             0.425
##     sj_03             0.320
##     homo1_gay         0.541
##     homo2_gay         0.153
##     homo3_gay         0.628
##     homo4_gay         0.558
##     homo5_gay         0.801
##     homo6_gay         0.389
##     homo7_gay         0.758
##     homoneg           0.097
```

ANÁLISIS DE INVARIANZA

El análisis de invarianza supone estimar modelos anidados en pasos sucesivos variando la restricción de diferentes parámetros y comparando sus indicadores de bondad de ajuste.
Pasos:

1. Modelo de invarianza configural, estimado con todos los parámetros libres entre modelos (grupos, modelos con restricciones y diferentes tiempos).
2. Modelo de invarianza métrica débil (cargas factoriales restringidas a iguales).
3. Modelo de invarianza métrica fuerte (cargas factoriales e interceptos restringidos a iguales).
4. Modelo de invarianza métrica estricta (cargas factoriales, interceptos y residuos restringidos a iguales).
5. Modelo de invarianza de varianza de factores variables
6. Modelo de invarianza de covarianza entre variables latentes.
7. Modelo de invarianza de path de regresión.
8. Modelo de invarianza de medias de variables latentes.

Los pasos 7 y 8 pueden intercambiarse, dependiendo del objetivo de la investigación.

Análisis Multigrupo:

Invarianza del modelo de medición y estructural entre hombres (0) y mujeres (1).

```
# factorizamos la Variable de agrupación
base$sexo<- as.factor(base$sexo)
str(base$sexo)

##  Factor w/ 2 levels "0","1": 2 2 2 2 2 2 1 2 2 2 ...

#### Invarianza del modelo de medición

# Estimación invarianza configural
# sobre el modelo especificado previamente "mod1"
config <- sem(mod1, data=base,
              estimator = "MLR",
              group="sexo") # Variable de agrupación

summary(config, fit.measures = T, standardized = T, rsquare = T)

## lavaan 0.6-18 ended normally after 66 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
```

```
##   Number of model parameters                         70
##
##   Number of observations per group:                Used       Total
##     1                                              347         353
##     0                                              187         190
##
## Model Test User Model:
##                                                Standard      Scaled
##   Test Statistic                                212.268     195.205
##   Degrees of freedom                                 84          84
##   P-value (Chi-square)                            0.000       0.000
##   Scaling correction factor                                   1.087
##     Yuan-Bentler correction (Mplus variant)
##   Test statistic for each group:
##     1                                           145.901     134.173
##     0                                            66.367      61.032
##
## Model Test Baseline Model:
##
##   Test statistic                               2424.359    1976.905
##   Degrees of freedom                                110         110
##   P-value                                         0.000       0.000
##   Scaling correction factor                                   1.226
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                     0.945       0.940
##   Tucker-Lewis Index (TLI)                        0.927       0.922
##
##   Robust Comparative Fit Index (CFI)                          0.947
##   Robust Tucker-Lewis Index (TLI)                             0.931
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7922.141   -7922.141
##   Scaling correction factor                                  1.249
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -7816.007   -7816.007
##   Scaling correction factor                                  1.161
##       for the MLR correction
##
##   Akaike (AIC)                               15984.282   15984.282
```

```
##   Bayesian (BIC)                                16283.910   16283.910
##   Sample-size adjusted Bayesian (SABIC)         16061.708   16061.708
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                             0.076       0.070
##   90 Percent confidence interval - lower            0.063       0.058
##   90 Percent confidence interval - upper            0.088       0.083
##   P-value H_0: RMSEA <= 0.050                       0.001       0.004
##   P-value H_0: RMSEA >= 0.080                       0.296       0.104
##
##   Robust RMSEA                                                  0.073
##   90 Percent confidence interval - lower                        0.060
##   90 Percent confidence interval - upper                        0.087
##   P-value H_0: Robust RMSEA <= 0.050                            0.003
##   P-value H_0: Robust RMSEA >= 0.080                            0.219
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                              0.045       0.045
##
## Parameter Estimates:
##
##   Standard errors                                Sandwich
##   Information bread                              Observed
##   Observed information based on                   Hessian
##
##
## Group 1 [1]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   justif =~
##     sj_01             1.000                               0.491    0.574
##     sj_02             1.436    0.229    6.283    0.000    0.704    0.719
##     sj_03             1.123    0.183    6.132    0.000    0.551    0.604
##   homoneg =~
##     homo1_gay         1.000                               0.781    0.767
##     homo2_gay         0.453    0.088    5.144    0.000    0.354    0.392
##     homo3_gay         1.213    0.083   14.664    0.000    0.948    0.761
##     homo4_gay         1.019    0.104    9.798    0.000    0.796    0.690
```

```
##     homo5_gay         1.367    0.076   17.942    0.000    1.068    0.914
##     homo6_gay         0.898    0.093    9.645    0.000    0.701    0.616
##     homo7_gay         1.289    0.096   13.442    0.000    1.007    0.853
##
## Regressions:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   homoneg ~
##     justif            0.133    0.121    1.093    0.274    0.083    0.083
##     religiosidad      0.128    0.026    4.877    0.000    0.164    0.301
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   justif ~~
##     religiosidad      0.244    0.084    2.896    0.004    0.497    0.270
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sj_01             2.210    0.046   48.160    0.000    2.210    2.585
##    .sj_02             2.219    0.053   42.173    0.000    2.219    2.264
##    .sj_03             1.839    0.049   37.532    0.000    1.839    2.015
##    .homo1_gay         1.416    0.094   15.042    0.000    1.416    1.390
##    .homo2_gay         1.241    0.062   20.154    0.000    1.241    1.373
##    .homo3_gay         1.747    0.122   14.298    0.000    1.747    1.402
##    .homo4_gay         1.529    0.100   15.233    0.000    1.529    1.325
##    .homo5_gay         1.412    0.123   11.524    0.000    1.412    1.208
##    .homo6_gay         1.616    0.093   17.397    0.000    1.616    1.419
##    .homo7_gay         1.400    0.116   12.024    0.000    1.400    1.186
##     religiosidad      3.398    0.099   34.393    0.000    3.398    1.846
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sj_01             0.490    0.051    9.568    0.000    0.490    0.671
##    .sj_02             0.464    0.083    5.570    0.000    0.464    0.483
##    .sj_03             0.529    0.058    9.147    0.000    0.529    0.635
##    .homo1_gay         0.428    0.043    9.835    0.000    0.428    0.412
##    .homo2_gay         0.691    0.125    5.550    0.000    0.691    0.847
##    .homo3_gay         0.655    0.084    7.794    0.000    0.655    0.421
##    .homo4_gay         0.697    0.094    7.404    0.000    0.697    0.524
##    .homo5_gay         0.226    0.038    5.873    0.000    0.226    0.165
##    .homo6_gay         0.805    0.083    9.745    0.000    0.805    0.621
##    .homo7_gay         0.379    0.057    6.647    0.000    0.379    0.272
##     religiosidad      3.387    0.167   20.253    0.000    3.387    1.000
```

```
##     justif            0.241    0.059    4.077    0.000    1.000    1.000
##    .homoneg           0.543    0.082    6.638    0.000    0.889    0.889
##
## R-Square:
##                    Estimate
##     sj_01             0.329
##     sj_02             0.517
##     sj_03             0.365
##     homo1_gay         0.588
##     homo2_gay         0.153
##     homo3_gay         0.579
##     homo4_gay         0.476
##     homo5_gay         0.835
##     homo6_gay         0.379
##     homo7_gay         0.728
##     homoneg           0.111
##
##
## Group 2 [0]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   justif =~
##     sj_01             1.000                               0.540    0.618
##     sj_02             0.932    0.217    4.293    0.000    0.503    0.524
##     sj_03             0.957    0.312    3.062    0.002    0.516    0.537
##   homoneg =~
##     homo1_gay         1.000                               0.720    0.661
##     homo2_gay         0.515    0.137    3.771    0.000    0.371    0.367
##     homo3_gay         1.468    0.147    9.981    0.000    1.058    0.810
##     homo4_gay         1.400    0.165    8.481    0.000    1.009    0.816
##     homo5_gay         1.566    0.158    9.910    0.000    1.129    0.853
##     homo6_gay         1.085    0.149    7.309    0.000    0.782    0.588
##     homo7_gay         1.636    0.174    9.400    0.000    1.179    0.892
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   homoneg ~
##     justif            0.416    0.179    2.322    0.020    0.312    0.312
##     religiosidad      0.094    0.034    2.764    0.006    0.130    0.217
##
## Covariances:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   justif ~~
##     religiosidad      0.301    0.088    3.426    0.001    0.557    0.334
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sj_01             2.037    0.064   31.890    0.000    2.037    2.332
##    .sj_02             2.225    0.070   31.668    0.000    2.225    2.316
##    .sj_03             2.027    0.070   28.826    0.000    2.027    2.108
##    .homo1_gay         1.987    0.115   17.239    0.000    1.987    1.823
##    .homo2_gay         1.506    0.089   16.949    0.000    1.506    1.489
##    .homo3_gay         2.436    0.167   14.625    0.000    2.436    1.865
##    .homo4_gay         1.977    0.149   13.307    0.000    1.977    1.600
##    .homo5_gay         2.277    0.165   13.822    0.000    2.277    1.722
##    .homo6_gay         2.180    0.128   17.059    0.000    2.180    1.640
##    .homo7_gay         2.062    0.168   12.251    0.000    2.062    1.560
##     religiosidad      2.701    0.122   22.162    0.000    2.701    1.621
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sj_01             0.472    0.093    5.067    0.000    0.472    0.618
##    .sj_02             0.670    0.086    7.745    0.000    0.670    0.726
##    .sj_03             0.658    0.131    5.029    0.000    0.658    0.712
##    .homo1_gay         0.669    0.089    7.522    0.000    0.669    0.563
##    .homo2_gay         0.885    0.169    5.228    0.000    0.885    0.866
##    .homo3_gay         0.588    0.117    5.044    0.000    0.588    0.344
##    .homo4_gay         0.509    0.083    6.146    0.000    0.509    0.333
##    .homo5_gay         0.476    0.089    5.315    0.000    0.476    0.272
##    .homo6_gay         1.155    0.129    8.935    0.000    1.155    0.654
##    .homo7_gay         0.357    0.071    5.066    0.000    0.357    0.205
##     religiosidad      2.777    0.241   11.531    0.000    2.777    1.000
##     justif            0.291    0.098    2.982    0.003    1.000    1.000
##    .homoneg           0.421    0.093    4.527    0.000    0.811    0.811
##
## R-Square:
##                    Estimate
##     sj_01             0.382
##     sj_02             0.274
##     sj_03             0.288
##     homo1_gay         0.437
##     homo2_gay         0.134
```

```
##     homo3_gay         0.656
##     homo4_gay         0.667
##     homo5_gay         0.728
##     homo6_gay         0.346
##     homo7_gay         0.795
##     homoneg           0.189
```

Los siguientes modelos están anidados en el modelo configural

```
# Invarianza débil (cargas factoriales fijas a iguales)
debil <- sem(mod1, data=base, std.lv= T, estimator = "MLR",
             group="sexo",
             group.equal="loadings")

# Invarianza fuerte (interceptos fijos a iguales)
fuerte <- sem(mod1, data=base, std.lv= T, estimator = "MLR",
              group="sexo",
              group.equal= c("loadings", "intercepts"))

# Invarianza estricta (residuos fijos a iguales)
estricta <- sem(mod1, data=base, std.lv= T, estimator = "MLR",
                group="sexo",
                group.equal= c("loadings", "intercepts", "residuals"))

## Comparación de ajuste de los modelos (chi2)
anova(config, debil, fuerte, estricta)

##
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan->lavTestLRT():
##    lavaan NOTE: The "Chisq" column contains standard test statistics, not the
##    robust test that should be reported per model. A robust difference test is
##    a function of two standard (not robust) statistics.
##           Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## config    84 15984 16284 212.27
## debil     92 15978 16244 222.52      8.758       8   0.363093
## fuerte   101 16000 16226 261.54     38.333       9  1.519e-05 ***
## estricta 112 16020 16199 303.54     27.240      11   0.004227 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
fitMeasures(config, c("chisq", "df", "pvalue", "cfi", "rmsea"))
```

```
##   chisq      df  pvalue     cfi   rmsea
## 212.268  84.000   0.000   0.945   0.076
```

```r
fitMeasures(debil, c("chisq", "df", "pvalue", "cfi", "rmsea"))
```

```
##   chisq      df  pvalue     cfi   rmsea
## 222.516  92.000   0.000   0.944   0.073
```

```r
fitMeasures(fuerte, c("chisq", "df", "pvalue", "cfi", "rmsea"))
```

```
##   chisq      df  pvalue     cfi   rmsea
## 261.535 101.000   0.000   0.931   0.077
```

```r
fitMeasures(estricta, c("chisq", "df", "pvalue", "cfi", "rmsea"))
```

```
##   chisq      df  pvalue     cfi   rmsea
## 303.543 112.000   0.000   0.917   0.080
```

Análisis de invarianza longitudinal

Para este ejercicio usaremos la base de datos ïnvarian.csv", la cual contiene 204 observaciones y 9 variables.

```
base<- read.csv("invarian.csv", header = T, sep = ";")
names(base)

## [1] "id"       "info1"    "compren1" "simil1"   "voca1"    "info2"    "compren2"
## [8] "simil2"   "voca2"

str(base)

## 'data.frame': 204 obs. of  9 variables:
##  $ id      : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ info1   : num  31.3 13.8 35 24.8 25.3 ...
##  $ compren1: num  25.6 14.8 34.7 31.4 30.3 ...
##  $ simil1  : num  22.93 7.58 28.05 8.21 15.98 ...
##  $ voca1   : num  22.2 15.4 26.8 20.2 35.4 ...
##  $ info2   : num  69.9 41.9 60.4 52.9 67.4 ...
##  $ compren2: num  44.4 44.9 50.3 42.7 86.7 ...
##  $ simil2  : num  68 33.9 35.8 45.8 72.4 ...
##  $ voca2   : num  51.2 37.7 55.5 36 60.4 ...
```

Evaluaremos si los resultados en 4 subdimensiones del wisc son invariantes en el tiempo

```
# Especificación y estimación del modelo T1
mod2<- 'habilT1 =~ info1 + compren1 + simil1 + voca1'
fit2<- cfa(mod2, std.lv=T, estimator = "MLR",meanstructure=T, data=base)
summary(fit2,fit.measures = T, standardized = T, rsquare = T)

## lavaan 0.6-18 ended normally after 17 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        12
##
##   Number of observations                           204
##
## Model Test User Model:
##                                              Standard      Scaled
##   Test Statistic                                7.147       8.576
##   Degrees of freedom                                2           2
##   P-value (Chi-square)                          0.028       0.014
```

```
##   Scaling correction factor                                  0.833
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                               261.106     230.091
##   Degrees of freedom                                 6           6
##   P-value                                        0.000       0.000
##   Scaling correction factor                                  1.135
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.980       0.971
##   Tucker-Lewis Index (TLI)                       0.939       0.912
##
##   Robust Comparative Fit Index (CFI)                         0.978
##   Robust Tucker-Lewis Index (TLI)                            0.935
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -2650.672   -2650.672
##   Scaling correction factor                                  1.082
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -2647.098   -2647.098
##   Scaling correction factor                                  1.046
##       for the MLR correction
##
##   Akaike (AIC)                                5325.343    5325.343
##   Bayesian (BIC)                              5365.161    5365.161
##   Sample-size adjusted Bayesian (SABIC)       5327.141    5327.141
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.112       0.127
##   90 Percent confidence interval - lower         0.032       0.043
##   90 Percent confidence interval - upper         0.206       0.229
##   P-value H_0: RMSEA <= 0.050                    0.088       0.063
##   P-value H_0: RMSEA >= 0.080                    0.791       0.847
##
##   Robust RMSEA                                               0.116
##   90 Percent confidence interval - lower                     0.045
##   90 Percent confidence interval - upper                     0.200
```

```
##   P-value H_0: Robust RMSEA <= 0.050                          0.062
##   P-value H_0: Robust RMSEA >= 0.080                          0.825
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                             0.025       0.025
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT1 =~
##     info1            4.582    0.448   10.232    0.000    4.582    0.751
##     compren1         7.013    0.613   11.438    0.000    7.013    0.722
##     simil1           4.843    0.569    8.510    0.000    4.843    0.642
##     voca1            4.714    0.378   12.487    0.000    4.714    0.751
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1           19.776    0.427   46.273    0.000   19.776    3.240
##    .compren1        21.797    0.680   32.036    0.000   21.797    2.243
##    .simil1          14.903    0.528   28.223    0.000   14.903    1.976
##    .voca1           20.396    0.439   46.416    0.000   20.396    3.250
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1           16.271    2.448    6.647    0.000   16.271    0.437
##    .compren1        45.257    6.787    6.668    0.000   45.257    0.479
##    .simil1          33.426    4.572    7.311    0.000   33.426    0.588
##    .voca1           17.167    3.203    5.359    0.000   17.167    0.436
##     habilT1          1.000                              1.000    1.000
##
## R-Square:
##                   Estimate
##     info1            0.563
##     compren1         0.521
##     simil1           0.412
##     voca1            0.564
```

```r
# Especificación y estimación del modelo T2
mod3<- 'habilT2 =~ info2 + compren2 + simil2 + voca2'

fit3<- cfa(mod3, std.lv=T, estimator = "MLR",meanstructure=T, data=base)
summary(fit3,fit.measures = T, standardized = T, rsquare = T)

## lavaan 0.6-18 ended normally after 16 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        12
##
##   Number of observations                           204
##
## Model Test User Model:
##                                              Standard      Scaled
##   Test Statistic                                 5.523       5.342
##   Degrees of freedom                                 2           2
##   P-value (Chi-square)                           0.063       0.069
##   Scaling correction factor                                  1.034
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                               461.968     357.969
##   Degrees of freedom                                 6           6
##   P-value                                        0.000       0.000
##   Scaling correction factor                                  1.291
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.992       0.991
##   Tucker-Lewis Index (TLI)                       0.977       0.972
##
##   Robust Comparative Fit Index (CFI)                         0.992
##   Robust Tucker-Lewis Index (TLI)                            0.977
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3006.127   -3006.127
##   Scaling correction factor                                  1.102
##       for the MLR correction
```

```
##   Loglikelihood unrestricted model (H1)      -3003.366   -3003.366
##   Scaling correction factor                                  1.092
##       for the MLR correction
##
##   Akaike (AIC)                                6036.255    6036.255
##   Bayesian (BIC)                              6076.072    6076.072
##   Sample-size adjusted Bayesian (SABIC)       6038.052    6038.052
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.093       0.091
##   90 Percent confidence interval - lower         0.000       0.000
##   90 Percent confidence interval - upper         0.189       0.186
##   P-value H_0: RMSEA <= 0.050                    0.159       0.168
##   P-value H_0: RMSEA >= 0.080                    0.679       0.664
##
##   Robust RMSEA                                               0.092
##   90 Percent confidence interval - lower                     0.000
##   90 Percent confidence interval - upper                     0.191
##   P-value H_0: Robust RMSEA <= 0.050                         0.166
##   P-value H_0: Robust RMSEA >= 0.080                         0.673
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.016       0.016
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT2 =~
##     info2           10.776    0.655   16.448    0.000   10.776    0.845
##     compren2         9.790    0.808   12.113    0.000    9.790    0.756
##     simil2          10.897    0.883   12.338    0.000   10.897    0.752
##     voca2            9.878    0.558   17.716    0.000    9.878    0.896
##
## Intercepts:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info2            48.510    0.893   54.319    0.000   48.510    3.803
##    .compren2         45.174    0.906   49.859    0.000   45.174    3.491
##    .simil2           41.297    1.014   40.728    0.000   41.297    2.852
##    .voca2            44.446    0.771   57.611    0.000   44.446    4.034
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info2            46.586    6.875    6.776    0.000   46.586    0.286
##    .compren2         71.628   11.131    6.435    0.000   71.628    0.428
##    .simil2           90.990   13.912    6.540    0.000   90.990    0.434
##    .voca2            23.851    4.983    4.787    0.000   23.851    0.196
##     habilT2           1.000                              1.000    1.000
##
## R-Square:
##                    Estimate
##     info2             0.714
##     compren2          0.572
##     simil2            0.566
##     voca2             0.804
```

Modelo Configural.

```
configural<- '
        habilT1 =~ info1 + compren1 + simil1 + voca1
        habilT2 =~ info2 + compren2 + simil2 + voca2

        # interceptos libremente estimados
        info1 ~1
        compren1 ~1
        simil1 ~1
        voca1 ~1
        info2 ~1
        compren2 ~1
        simil2 ~1
        voca2 ~1
        # Residuos libremente estimados
        info1 ~~ info1
        compren1 ~~ compren1
        simil1 ~~ simil1
        voca1 ~~ voca1
        info2 ~~ info2
        compren2 ~~ compren2
        simil2 ~~ simil2
        voca2 ~~ voca2
        # Varianza de los factores fijos en 1 (identificación)
        habilT1 ~~ 1*habilT1
        habilT2 ~~ 1*habilT2
        # Medias de los factores fijos en 0 (identificación)
        habilT1 ~0*1
        habilT2 ~0*1
        # Residuos de los ítems en el tiempo correlacionados
        info1 ~~ info2
        compren1 ~~ compren2
        simil1 ~~ simil2
        voca1 ~~ voca2
        # Covarianza entre los factores
        habilT1 ~~ habilT2'
```

```
fit_configural <- cfa(configural, data = base, std.lv=T,
                      estimator = "MLR", mimic = "mplus")
summary(fit_configural, fit.measures = TRUE, standardized = T)
```

```
## lavaan 0.6-18 ended normally after 54 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        29
##
##   Number of observations                           204
##   Number of missing patterns                         1
##
## Model Test User Model:
##                                              Standard      Scaled
##   Test Statistic                               24.882      21.618
##   Degrees of freedom                               15          15
##   P-value (Chi-square)                          0.052       0.118
##   Scaling correction factor                                 1.151
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                              847.740     707.947
##   Degrees of freedom                               28          28
##   P-value                                       0.000       0.000
##   Scaling correction factor                                 1.197
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                   0.988       0.990
##   Tucker-Lewis Index (TLI)                      0.977       0.982
##
##   Robust Comparative Fit Index (CFI)                        0.991
##   Robust Tucker-Lewis Index (TLI)                           0.983
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -5600.572   -5600.572
##   Scaling correction factor                                 1.091
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)     -5588.131   -5588.131
##   Scaling correction factor                                 1.112
##       for the MLR correction
##
##   Akaike (AIC)                              11259.143   11259.143
```

```
##   Bayesian (BIC)                              11355.369   11355.369
##   Sample-size adjusted Bayesian (SABIC)       11263.488   11263.488
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.057       0.047
##   90 Percent confidence interval - lower         0.000       0.000
##   90 Percent confidence interval - upper         0.095       0.084
##   P-value H_0: RMSEA <= 0.050                    0.350       0.517
##   P-value H_0: RMSEA >= 0.080                    0.177       0.076
##
##   Robust RMSEA                                                0.049
##   90 Percent confidence interval - lower                      0.000
##   90 Percent confidence interval - upper                      0.093
##   P-value H_0: Robust RMSEA <= 0.050                          0.469
##   P-value H_0: Robust RMSEA >= 0.080                          0.138
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.030       0.030
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT1 =~
##     info1            4.470    0.411   10.865    0.000    4.470    0.732
##     compren1         6.868    0.539   12.739    0.000    6.868    0.707
##     simil1           4.636    0.559    8.297    0.000    4.636    0.614
##     voca1            5.000    0.357   13.992    0.000    5.000    0.797
##   habilT2 =~
##     info2           10.762    0.629   17.100    0.000   10.762    0.844
##     compren2         9.693    0.794   12.201    0.000    9.693    0.749
##     simil2          10.989    0.844   13.015    0.000   10.989    0.759
##     voca2            9.849    0.540   18.249    0.000    9.849    0.896
##
## Covariances:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .info1 ~~
##    .info2             0.980    2.719    0.360    0.719    0.980    0.034
##  .compren1 ~~
##    .compren2         -0.225    4.603   -0.049    0.961   -0.225   -0.004
##  .simil1 ~~
##    .simil2           -1.445    4.080   -0.354    0.723   -1.445   -0.026
##  .voca1 ~~
##    .voca2             1.741    2.258    0.771    0.441    1.741    0.094
##   habilT1 ~~
##     habilT2           0.755    0.055   13.759    0.000    0.755    0.755
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1            19.776    0.427   46.273    0.000   19.776    3.239
##    .compren1         21.797    0.680   32.036    0.000   21.797    2.243
##    .simil1           14.903    0.528   28.223    0.000   14.903    1.975
##    .voca1            20.396    0.439   46.416    0.000   20.396    3.251
##    .info2            48.510    0.893   54.319    0.000   48.510    3.802
##    .compren2         45.174    0.906   49.859    0.000   45.174    3.491
##    .simil2           41.297    1.014   40.728    0.000   41.297    2.853
##    .voca2            44.446    0.771   57.611    0.000   44.446    4.043
##     habilT1           0.000                              0.000    0.000
##     habilT2           0.000                              0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1            17.294    2.063    8.384    0.000   17.294    0.464
##    .compren1         47.245    5.791    8.158    0.000   47.245    0.500
##    .simil1           35.454    4.445    7.976    0.000   35.454    0.623
##    .voca1            14.374    2.850    5.044    0.000   14.374    0.365
##    .info2            46.958    6.556    7.163    0.000   46.958    0.288
##    .compren2         73.487   11.572    6.351    0.000   73.487    0.439
##    .simil2           88.773   13.602    6.526    0.000   88.773    0.424
##    .voca2            23.858    4.115    5.798    0.000   23.858    0.197
##     habilT1           1.000                              1.000    1.000
##     habilT2           1.000                              1.000    1.000
```

Modelo Invarianza débil

```
debil<- [1073 chars quoted with '''']

fit_debil <- cfa(debil, data = base,
                      estimator = "MLR", mimic = "mplus")
summary(fit_debil, fit.measures = TRUE, standardized = T)

## lavaan 0.6-18 ended normally after 73 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        30
##   Number of equality constraints                     4
##
##   Number of observations                           204
##   Number of missing patterns                         1
##
## Model Test User Model:
##                                              Standard      Scaled
##   Test Statistic                               40.299      35.942
##   Degrees of freedom                               18          18
##   P-value (Chi-square)                          0.002       0.007
##   Scaling correction factor                                 1.121
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                              847.740     707.947
##   Degrees of freedom                               28          28
##   P-value                                       0.000       0.000
##   Scaling correction factor                                 1.197
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                   0.973       0.974
##   Tucker-Lewis Index (TLI)                      0.958       0.959
##
##   Robust Comparative Fit Index (CFI)                        0.975
##   Robust Tucker-Lewis Index (TLI)                           0.961
##
```

```
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -5608.280   -5608.280
##   Scaling correction factor                                  0.958
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -5588.131   -5588.131
##   Scaling correction factor                                  1.112
##       for the MLR correction
##
##   Akaike (AIC)                               11268.560   11268.560
##   Bayesian (BIC)                             11354.831   11354.831
##   Sample-size adjusted Bayesian (SABIC)      11272.455   11272.455
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.078       0.070
##   90 Percent confidence interval - lower         0.046       0.038
##   90 Percent confidence interval - upper         0.110       0.101
##   P-value H_0: RMSEA <= 0.050                    0.074       0.139
##   P-value H_0: RMSEA >= 0.080                    0.490       0.323
##
##   Robust RMSEA                                               0.074
##   90 Percent confidence interval - lower                     0.038
##   90 Percent confidence interval - upper                     0.109
##   P-value H_0: Robust RMSEA <= 0.050                         0.118
##   P-value H_0: Robust RMSEA >= 0.080                         0.428
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.075       0.075
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT1 =~
##     info1     (l1)    4.955    0.336   14.767    0.000    4.955    0.780
##     compren1  (l2)    5.195    0.412   12.612    0.000    5.195    0.577
```

```
##     simil1    (l3)    5.095    0.383   13.305    0.000    5.095    0.657
##     voca1     (l4)    4.825    0.297   16.257    0.000    4.825    0.774
##   habilT2 =~
##     info2     (l1)    4.955    0.336   14.767    0.000   10.181    0.823
##     compren2  (l2)    5.195    0.412   12.612    0.000   10.676    0.783
##     simil2    (l3)    5.095    0.383   13.305    0.000   10.470    0.739
##     voca2     (l4)    4.825    0.297   16.257    0.000    9.915    0.900
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .info1 ~~
##    .info2            0.272    2.699    0.101    0.920    0.272    0.010
##  .compren1 ~~
##    .compren2        -1.585    4.692   -0.338    0.736   -1.585   -0.025
##  .simil1 ~~
##    .simil2          -2.262    4.242   -0.533    0.594   -2.262   -0.041
##  .voca1 ~~
##    .voca2            2.297    2.337    0.983    0.326    2.297    0.121
##   habilT1 ~~
##     habilT2          1.544    0.158    9.781    0.000    0.751    0.751
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1           19.776    0.427   46.273    0.000   19.776    3.114
##    .compren1        21.797    0.680   32.036    0.000   21.797    2.423
##    .simil1          14.903    0.528   28.223    0.000   14.903    1.922
##    .voca1           20.396    0.439   46.416    0.000   20.396    3.273
##    .info2           48.510    0.893   54.319    0.000   48.510    3.920
##    .compren2        45.174    0.906   49.859    0.000   45.174    3.315
##    .simil2          41.297    1.014   40.728    0.000   41.297    2.913
##    .voca2           44.446    0.771   57.611    0.000   44.446    4.035
##     habilT1          0.000                             0.000    0.000
##     habilT2          0.000                             0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1           15.780    1.990    7.929    0.000   15.780    0.391
##    .compren1        53.943    6.043    8.926    0.000   53.943    0.667
##    .simil1          34.139    4.175    8.178    0.000   34.139    0.568
##    .voca1           15.556    2.944    5.285    0.000   15.556    0.401
##    .info2           49.486    6.515    7.596    0.000   49.486    0.323
```

```
##    .compren2         71.719   11.496    6.238    0.000   71.719    0.386
##    .simil2           91.323   13.504    6.763    0.000   91.323    0.454
##    .voca2            22.996    3.906    5.887    0.000   22.996    0.190
##     habilT1           1.000                              1.000    1.000
##     habilT2           4.222    0.480    8.791    0.000    1.000    1.000
```

Comparación de modelos anidados

```
anova(fit_configural, fit_debil)

##
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan->lavTestLRT():
##    lavaan NOTE: The "Chisq" column contains standard test statistics, not the
##    robust test that should be reported per model. A robust difference test is
##    a function of two standard (not robust) statistics.
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit_configural 15 11259 11355 24.882
## fit_debil      18 11269 11355 40.299     15.856       3   0.001214 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Índices de modificación para invarianza

```
lavTestScore(fit_debil)

## Warning: lavaan->lavTestScore():
##    se is not 'standard'; not implemented yet; falling back to ordinary score
##    test

## $test
##
## total score test:
##
##    test     X2 df p.value
## 1 score 14.049  4   0.007
##
## $uni
##
## univariate score tests:
##
```

```
##    lhs op  rhs     X2 df p.value
## 1 .p1. == .p5.  4.175  1   0.041
## 2 .p2. == .p6. 11.601  1   0.001
## 3 .p3. == .p7.  1.287  1   0.257
## 4 .p4. == .p8.  0.143  1   0.705
```

Modelo Invarianza parcial débil

```
debilP<- [1086 chars quoted with '''']

fit_debilP <- cfa(debilP, data = base,
                      estimator = "MLR", mimic = "mplus")
summary(fit_debilP, fit.measures = TRUE, standardized = T)

## lavaan 0.6-18 ended normally after 86 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        30
##   Number of equality constraints                     3
##
##   Number of observations                           204
##   Number of missing patterns                         1
##
## Model Test User Model:
##                                              Standard      Scaled
##   Test Statistic                               28.092      24.592
##   Degrees of freedom                               17          17
##   P-value (Chi-square)                          0.044       0.104
##   Scaling correction factor                                 1.142
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                              847.740     707.947
##   Degrees of freedom                               28          28
##   P-value                                       0.000       0.000
##   Scaling correction factor                                 1.197
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                   0.986       0.989
##   Tucker-Lewis Index (TLI)                      0.978       0.982
##
##   Robust Comparative Fit Index (CFI)                        0.989
##   Robust Tucker-Lewis Index (TLI)                           0.983
##
```

```
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -5602.177   -5602.177
##   Scaling correction factor                                  0.983
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -5588.131   -5588.131
##   Scaling correction factor                                  1.112
##       for the MLR correction
##
##   Akaike (AIC)                               11258.353   11258.353
##   Bayesian (BIC)                             11347.942   11347.942
##   Sample-size adjusted Bayesian (SABIC)      11262.398   11262.398
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.057       0.047
##   90 Percent confidence interval - lower         0.009       0.000
##   90 Percent confidence interval - upper         0.093       0.082
##   P-value H_0: RMSEA <= 0.050                    0.351       0.518
##   P-value H_0: RMSEA >= 0.080                    0.156       0.064
##
##   Robust RMSEA                                               0.050
##   90 Percent confidence interval - lower                     0.000
##   90 Percent confidence interval - upper                     0.091
##   P-value H_0: Robust RMSEA <= 0.050                         0.459
##   P-value H_0: Robust RMSEA >= 0.080                         0.123
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.045       0.045
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT1 =~
##     info1     (l1)    4.771    0.339   14.067    0.000    4.771    0.763
##     compren1          6.856    0.539   12.724    0.000    6.856    0.706
```

```
##     simil1    (l3)    4.907    0.379   12.954    0.000    4.907    0.642
##     voca1     (l4)    4.602    0.302   15.236    0.000    4.602    0.757
##   habilT2 =~
##     info2     (l1)    4.771    0.339   14.067    0.000   10.426    0.831
##     compren2          4.446    0.421   10.560    0.000    9.714    0.751
##     simil2    (l3)    4.907    0.379   12.954    0.000   10.723    0.748
##     voca2     (l4)    4.602    0.302   15.236    0.000   10.056    0.904
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .info1 ~~
##    .info2            0.887    2.708    0.328    0.743    0.887    0.031
##  .compren1 ~~
##    .compren2        -0.178    4.596   -0.039    0.969   -0.178   -0.003
##  .simil1 ~~
##    .simil2          -1.688    4.117   -0.410    0.682   -1.688   -0.030
##  .voca1 ~~
##    .voca2            1.933    2.288    0.845    0.398    1.933    0.102
##   habilT1 ~~
##     habilT2          1.633    0.161   10.162    0.000    0.747    0.747
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1           19.776    0.427   46.273    0.000   19.776    3.163
##    .compren1        21.797    0.680   32.036    0.000   21.797    2.243
##    .simil1          14.903    0.528   28.223    0.000   14.903    1.949
##    .voca1           20.396    0.439   46.416    0.000   20.396    3.353
##    .info2           48.510    0.893   54.319    0.000   48.510    3.866
##    .compren2        45.174    0.906   49.859    0.000   45.174    3.491
##    .simil2          41.297    1.014   40.728    0.000   41.297    2.882
##    .voca2           44.446    0.771   57.611    0.000   44.446    3.997
##     habilT1          0.000                             0.000    0.000
##     habilT2          0.000                             0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1           16.325    1.973    8.273    0.000   16.325    0.418
##    .compren1        47.411    5.851    8.104    0.000   47.411    0.502
##    .simil1          34.411    4.180    8.232    0.000   34.411    0.588
##    .voca1           15.824    2.868    5.517    0.000   15.824    0.428
##    .info2           48.717    6.542    7.447    0.000   48.717    0.309
```

```
##    .compren2         73.081   11.469    6.372    0.000   73.081    0.436
##    .simil2           90.407   13.489    6.702    0.000   90.407    0.440
##    .voca2            22.492    3.973    5.661    0.000   22.492    0.182
##     habilT1           1.000                              1.000    1.000
##     habilT2           4.775    0.560    8.533    0.000    1.000    1.000
```

```
anova(fit_configural, fit_debilP)

##
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan->lavTestLRT():
##    lavaan NOTE: The "Chisq" column contains standard test statistics, not the
##    robust test that should be reported per model. A robust difference test is
##    a function of two standard (not robust) statistics.
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit_configural 15 11259 11355 24.882
## fit_debilP     17 11258 11348 28.092     2.9797       2     0.2254
```

Modelo Invarianza parcial fuerte

```
fuerte<- [1119 chars quoted with '''']
```

```
fit_fuerte <- cfa(fuerte, data = base,
                      estimator = "MLR", mimic = "mplus")
summary(fit_fuerte, fit.measures = TRUE, standardized = T)

## lavaan 0.6-18 ended normally after 104 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        31
##   Number of equality constraints                     6
##
##   Number of observations                           204
##   Number of missing patterns                         1
##
## Model Test User Model:
##                                                Standard      Scaled
##   Test Statistic                                 33.524      30.465
##   Degrees of freedom                                 19          19
##   P-value (Chi-square)                            0.021       0.046
##   Scaling correction factor                                   1.100
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                                847.740     707.947
##   Degrees of freedom                                 28          28
##   P-value                                         0.000       0.000
##   Scaling correction factor                                   1.197
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                     0.982       0.983
##   Tucker-Lewis Index (TLI)                        0.974       0.975
##
##   Robust Comparative Fit Index (CFI)                          0.985
##   Robust Tucker-Lewis Index (TLI)                             0.977
##
## Loglikelihood and Information Criteria:
```

```
##
##   Loglikelihood user model (H0)              -5604.892   -5604.892
##   Scaling correction factor                                  0.903
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -5588.131   -5588.131
##   Scaling correction factor                                  1.112
##       for the MLR correction
##
##   Akaike (AIC)                               11259.785   11259.785
##   Bayesian (BIC)                             11342.738   11342.738
##   Sample-size adjusted Bayesian (SABIC)      11263.530   11263.530
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.061       0.054
##   90 Percent confidence interval - lower         0.024       0.012
##   90 Percent confidence interval - upper         0.095       0.087
##   P-value H_0: RMSEA <= 0.050                    0.268       0.383
##   P-value H_0: RMSEA >= 0.080                    0.194       0.107
##
##   Robust RMSEA                                               0.057
##   90 Percent confidence interval - lower                     0.008
##   90 Percent confidence interval - upper                     0.093
##   P-value H_0: Robust RMSEA <= 0.050                         0.346
##   P-value H_0: Robust RMSEA >= 0.080                         0.163
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.061       0.061
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT1 =~
##     info1     (l1)    5.035    0.332   15.165    0.000    5.035    0.787
##     compren1          6.823    0.539   12.646    0.000    6.823    0.702
##     simil1    (l3)    4.739    0.325   14.578    0.000    4.739    0.629
```

```
##     voca1     (l4)    4.317    0.279   15.472    0.000    4.317    0.724
##   habilT2 =~
##     info2     (l1)    5.035    0.332   15.165    0.000   11.194    0.857
##     compren2          4.362    0.412   10.597    0.000    9.697    0.749
##     simil2    (l3)    4.739    0.325   14.578    0.000   10.534    0.743
##     voca2     (l4)    4.317    0.279   15.472    0.000    9.596    0.886
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .info1 ~~
##    .info2             0.573    2.697    0.212    0.832    0.573    0.022
##  .compren1 ~~
##    .compren2          0.259    4.687    0.055    0.956    0.259    0.004
##  .simil1 ~~
##    .simil2           -1.695    4.083   -0.415    0.678   -1.695   -0.031
##  .voca1 ~~
##    .voca2             2.896    2.247    1.289    0.197    2.896    0.140
##   habilT1 ~~
##     habilT2           1.648    0.166    9.926    0.000    0.742    0.742
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1     (i1)   19.882    0.417   47.632    0.000   19.882    3.107
##    .compren1         21.797    0.680   32.036    0.000   21.797    2.243
##    .simil1    (i3)   14.833    0.516   28.768    0.000   14.833    1.969
##    .voca1     (i4)   20.311    0.438   46.389    0.000   20.311    3.409
##    .info2     (i1)   19.882    0.417   47.632    0.000   19.882    1.522
##    .compren2         20.650    1.815   11.377    0.000   20.650    1.596
##    .simil2    (i3)   14.833    0.516   28.768    0.000   14.833    1.046
##    .voca2     (i4)   20.311    0.438   46.389    0.000   20.311    1.875
##     habilT1           0.000                              0.000    0.000
##     habilT2           5.623    0.363   15.497    0.000    2.529    2.529
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1            15.599    1.990    7.839    0.000   15.599    0.381
##    .compren1         47.906    5.912    8.104    0.000   47.906    0.507
##    .simil1           34.284    4.122    8.318    0.000   34.284    0.604
##    .voca1            16.866    2.871    5.875    0.000   16.866    0.475
##    .info2            45.334    6.476    7.000    0.000   45.334    0.266
##    .compren2         73.461   11.568    6.350    0.000   73.461    0.439
```

```
##    .simil2           89.942   12.999    6.919    0.000   89.942    0.448
##    .voca2            25.213    3.786    6.660    0.000   25.213    0.215
##     habilT1           1.000                              1.000    1.000
##     habilT2           4.942    0.595    8.307    0.000    1.000    1.000
```

```
anova(fit_debilP, fit_fuerte)

##
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan->lavTestLRT():
##    lavaan NOTE: The "Chisq" column contains standard test statistics, not the
##    robust test that should be reported per model. A robust difference test is
##    a function of two standard (not robust) statistics.
##            Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit_debilP 17 11258 11348 28.092
## fit_fuerte 19 11260 11343 33.524     7.2994       2      0.026 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
lavTestScore(fit_fuerte)

## Warning: lavaan->lavTestScore():
##    se is not 'standard'; not implemented yet; falling back to ordinary score
##    test

## $test
##
## total score test:
##
##    test    X2 df p.value
## 1 score 8.067  6   0.233
##
## $uni
##
## univariate score tests:
##
##     lhs op   rhs    X2 df p.value
## 1  .p1. ==  .p5. 4.854  1   0.028
## 2  .p3. ==  .p7. 0.009  1   0.923
## 3  .p4. ==  .p8. 4.877  1   0.027
## 4  .p9. == .p13. 5.296  1   0.021
```

```
## 5 .p11. == .p15. 0.541  1   0.462
## 6 .p12. == .p16. 2.881  1   0.090
```

Modelo Invarianza parcial fuerte

```
fuerteP<- [1113 chars quoted with '''']
```

```
fit_fuerteP <- cfa(fuerteP, data = base,
                       estimator = "MLR", mimic = "mplus")
summary(fit_fuerteP, fit.measures = TRUE, standardized = T)

## lavaan 0.6-18 ended normally after 106 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        31
##   Number of equality constraints                     5
##
##   Number of observations                           204
##   Number of missing patterns                         1
##
## Model Test User Model:
##                                                Standard      Scaled
##   Test Statistic                                 28.238      25.169
##   Degrees of freedom                                 18          18
##   P-value (Chi-square)                            0.059       0.120
##   Scaling correction factor                                   1.122
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                                847.740     707.947
##   Degrees of freedom                                 28          28
##   P-value                                         0.000       0.000
##   Scaling correction factor                                   1.197
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                     0.988       0.989
##   Tucker-Lewis Index (TLI)                        0.981       0.984
##
##   Robust Comparative Fit Index (CFI)                          0.990
##   Robust Tucker-Lewis Index (TLI)                             0.985
##
## Loglikelihood and Information Criteria:
```

```
##
##   Loglikelihood user model (H0)              -5602.249   -5602.249
##   Scaling correction factor                                  0.926
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -5588.131   -5588.131
##   Scaling correction factor                                  1.112
##       for the MLR correction
##
##   Akaike (AIC)                               11256.499   11256.499
##   Bayesian (BIC)                             11342.770   11342.770
##   Sample-size adjusted Bayesian (SABIC)      11260.394   11260.394
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.053       0.044
##   90 Percent confidence interval - lower         0.000       0.000
##   90 Percent confidence interval - upper         0.089       0.080
##   P-value H_0: RMSEA <= 0.050                    0.413       0.564
##   P-value H_0: RMSEA >= 0.080                    0.113       0.049
##
##   Robust RMSEA                                               0.047
##   90 Percent confidence interval - lower                     0.000
##   90 Percent confidence interval - upper                     0.087
##   P-value H_0: Robust RMSEA <= 0.050                         0.506
##   P-value H_0: Robust RMSEA >= 0.080                         0.092
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.045       0.045
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT1 =~
##     info1     (l1)    4.771    0.340   14.036    0.000    4.771    0.764
##     compren1          6.854    0.539   12.708    0.000    6.854    0.705
##     simil1    (l3)    4.974    0.332   14.988    0.000    4.974    0.648
```

```
##     voca1     (l4)    4.560    0.291   15.695    0.000    4.560    0.752
##   habilT2 =~
##     info2     (l1)    4.771    0.340   14.036    0.000   10.450    0.832
##     compren2          4.433    0.417   10.621    0.000    9.711    0.750
##     simil2    (l3)    4.974    0.332   14.988    0.000   10.895    0.755
##     voca2     (l4)    4.560    0.291   15.695    0.000    9.988    0.902
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .info1 ~~
##    .info2             0.848    2.701    0.314    0.754    0.848    0.030
##  .compren1 ~~
##    .compren2         -0.132    4.607   -0.029    0.977   -0.132   -0.002
##  .simil1 ~~
##    .simil2           -1.755    4.119   -0.426    0.670   -1.755   -0.032
##  .voca1 ~~
##    .voca2             2.071    2.243    0.923    0.356    2.071    0.108
##   habilT1 ~~
##     habilT2           1.635    0.161   10.125    0.000    0.746    0.746
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1            19.776    0.427   46.273    0.000   19.776    3.166
##    .compren1         21.797    0.680   32.036    0.000   21.797    2.243
##    .simil1    (i3)   14.937    0.515   28.998    0.000   14.937    1.945
##    .voca1     (i4)   20.382    0.435   46.881    0.000   20.382    3.361
##    .info2            23.311    1.205   19.351    0.000   23.311    1.857
##    .compren2         21.756    1.720   12.646    0.000   21.756    1.681
##    .simil2    (i3)   14.937    0.515   28.998    0.000   14.937    1.035
##    .voca2     (i4)   20.382    0.435   46.881    0.000   20.382    1.840
##     habilT1           0.000                              0.000    0.000
##     habilT2           5.282    0.349   15.136    0.000    2.411    2.411
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1            16.265    1.971    8.250    0.000   16.265    0.417
##    .compren1         47.444    5.862    8.094    0.000   47.444    0.502
##    .simil1           34.255    4.181    8.194    0.000   34.255    0.581
##    .voca1            15.982    2.860    5.587    0.000   15.982    0.435
##    .info2            48.427    6.479    7.475    0.000   48.427    0.307
##    .compren2         73.148   11.494    6.364    0.000   73.148    0.437
```

```
##    .simil2           89.745   13.280    6.758    0.000   89.745    0.431
##    .voca2            22.906    3.791    6.042    0.000   22.906    0.187
##     habilT1           1.000                              1.000    1.000
##     habilT2           4.798    0.564    8.509    0.000    1.000    1.000
```

```
anova(fit_debilP, fit_fuerteP)

##
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan->lavTestLRT():
##    lavaan NOTE: The "Chisq" column contains standard test statistics, not the
##    robust test that should be reported per model. A robust difference test is
##    a function of two standard (not robust) statistics.
##             Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit_debilP  17 11258 11348 28.092
## fit_fuerteP 18 11256 11343 28.238    0.18776       1     0.6648
```

Modelo Invarianza estricta

```
fuerteP<- [1125 chars quoted with '''']
```

```
fit_fuerteP <- cfa(fuerteP, data = base,
                       estimator = "MLR", mimic = "mplus")
summary(fit_fuerteP, fit.measures = TRUE, standardized = T)

## lavaan 0.6-18 ended normally after 92 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        31
##   Number of equality constraints                     7
##
##   Number of observations                           204
##   Number of missing patterns                         1
##
## Model Test User Model:
##                                                Standard      Scaled
##   Test Statistic                                 69.471      61.472
##   Degrees of freedom                                 20          20
##   P-value (Chi-square)                            0.000       0.000
##   Scaling correction factor                                   1.130
##     Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##   Test statistic                                847.740     707.947
##   Degrees of freedom                                 28          28
##   P-value                                         0.000       0.000
##   Scaling correction factor                                   1.197
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                     0.940       0.939
##   Tucker-Lewis Index (TLI)                        0.916       0.915
##
##   Robust Comparative Fit Index (CFI)                          0.943
##   Robust Tucker-Lewis Index (TLI)                             0.920
##
## Loglikelihood and Information Criteria:
```

```
##
##   Loglikelihood user model (H0)              -5622.866   -5622.866
##   Scaling correction factor                                  0.849
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -5588.131   -5588.131
##   Scaling correction factor                                  1.112
##       for the MLR correction
##
##   Akaike (AIC)                               11293.732   11293.732
##   Bayesian (BIC)                             11373.367   11373.367
##   Sample-size adjusted Bayesian (SABIC)      11297.328   11297.328
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.110       0.101
##   90 Percent confidence interval - lower         0.083       0.074
##   90 Percent confidence interval - upper         0.139       0.128
##   P-value H_0: RMSEA <= 0.050                    0.000       0.001
##   P-value H_0: RMSEA >= 0.080                    0.963       0.906
##
##   Robust RMSEA                                               0.107
##   90 Percent confidence interval - lower                     0.077
##   90 Percent confidence interval - upper                     0.138
##   P-value H_0: Robust RMSEA <= 0.050                         0.002
##   P-value H_0: Robust RMSEA >= 0.080                         0.931
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.107       0.107
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   habilT1 =~
##     info1     (l1)    4.655    0.339   13.749    0.000    4.655    0.752
##     compren1          6.893    0.540   12.756    0.000    6.893    0.709
##     simil1    (l3)    4.931    0.335   14.703    0.000    4.931    0.538
```

```
##     voca1     (l4)    4.499    0.290   15.522    0.000    4.499    0.722
##   habilT2 =~
##     info2     (l1)    4.655    0.339   13.749    0.000   10.475    0.829
##     compren2          4.272    0.410   10.415    0.000    9.612    0.743
##     simil2    (l3)    4.931    0.335   14.703    0.000   11.096    0.820
##     voca2     (l4)    4.499    0.290   15.522    0.000   10.123    0.920
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .info1 ~~
##    .info2             1.301    2.731    0.476    0.634    1.301    0.045
##  .compren1 ~~
##    .compren2         -0.241    4.602   -0.052    0.958   -0.241   -0.004
##  .simil1 ~~
##    .simil2           -1.153    3.927   -0.294    0.769   -1.153   -0.019
##  .voca1 ~~
##    .voca2             1.326    2.095    0.633    0.527    1.326    0.071
##   habilT1 ~~
##     habilT2           1.699    0.158   10.780    0.000    0.755    0.755
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1             19.776    0.427   46.273    0.000   19.776    3.194
##    .compren1          21.797    0.680   32.036    0.000   21.797    2.243
##    .simil1    (i3)    14.915    0.518   28.787    0.000   14.915    1.626
##    .voca1     (i4)    20.393    0.437   46.707    0.000   20.393    3.274
##    .info2             23.614    1.211   19.499    0.000   23.614    1.868
##    .compren2          22.331    1.701   13.127    0.000   22.331    1.726
##    .simil2    (i3)    14.915    0.518   28.787    0.000   14.915    1.103
##    .voca2     (i4)    20.393    0.437   46.707    0.000   20.393    1.854
##     habilT1            0.000                             0.000    0.000
##     habilT2            5.348    0.361   14.802    0.000    2.377    2.377
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info1             16.670    2.041    8.169    0.000   16.670    0.435
##    .compren1          46.900    5.866    7.995    0.000   46.900    0.497
##    .simil1    (r3)    59.823    6.643    9.006    0.000   59.823    0.711
##    .voca1     (r4)    18.558    2.655    6.991    0.000   18.558    0.478
##    .info2             49.996    6.242    8.009    0.000   49.996    0.313
##    .compren2          75.053   11.466    6.546    0.000   75.053    0.448
```

```
##    .simil2    (r3)   59.823    6.643    9.006    0.000   59.823    0.327
##    .voca2     (r4)   18.558    2.655    6.991    0.000   18.558    0.153
##     habilT1            1.000                              1.000    1.000
##     habilT2            5.063    0.591    8.571    0.000    1.000    1.000
```

```
anova(fit_debilP, fit_fuerteP)

##
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan->lavTestLRT():
##    lavaan NOTE: The "Chisq" column contains standard test statistics, not the
##    robust test that should be reported per model. A robust difference test is
##    a function of two standard (not robust) statistics.
##             Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit_debilP  17 11258 11348 28.092
## fit_fuerteP 20 11294 11373 69.471     39.001       3  1.735e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```