Homework 3 Solution

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This is my GitHub link: Nana Ama's GitHub Link

1 Question 1

To compare the computational cost of matrix multiplication, a series of successively larger matrices, from $N = 10 \times 10$ to $N = 100 \times 100$ was applied and a plot of matrix size against time was plotted using both the for loop method and the dot method from Python.

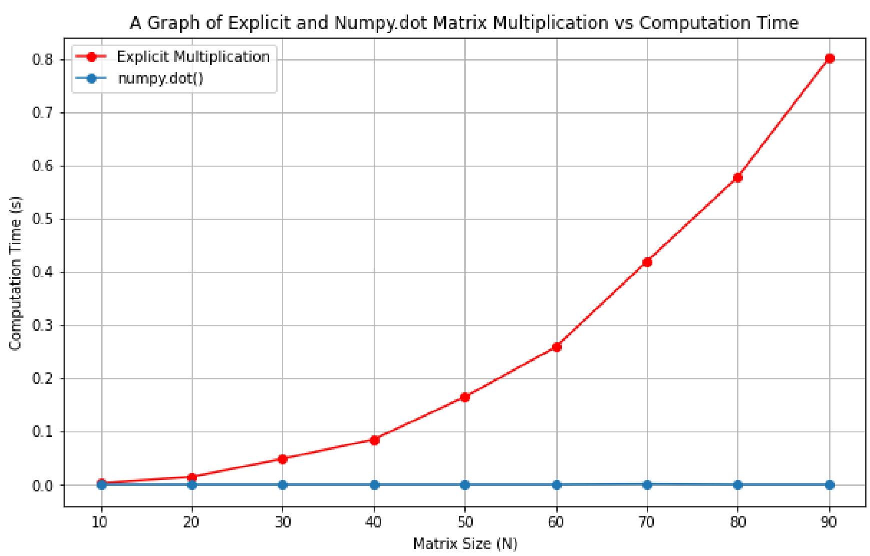

Figure 1: Comparison of Explicit Matrix Multiplication and the Dot Method

The results are seen in Figure 1. Indeed, the computational time rises as N^3 in the for loop method but stays relatively stable at 0 seconds with very little change in the dot method. This shows the difference in computational cost of both methods with the dot method being more computationally efficient.

2 Question 2

The isotope ^{213}Bi decays to stable ^{209}Bi via 2 different routes.

Route 1:$^{213}Bi \rightarrow ^{209} Tl \rightarrow ^{209} Pb \rightarrow ^{209} Bi$
Route 2: $^{213}Bi \rightarrow ^{209} Pb \rightarrow ^{209} Bi$

The half-lives of ^{213}Bi, ^{209}Tl and ^{209}Pb are $46min, 2.2min$ and $3.3min$ respectively and corresponding probabilities of ^{213}Bi decaying to ^{209}Tl and ^{209}Pb being 97.91%, 2.09%. With an initial sample of 10,000 atoms of ^{213}Bi , the decay of atoms was simulated by dividing time into slices $\delta t = 1s$

Each atom of ^{209}Pb were randomly decided to decay or not with probability

$$p(t) = 1 - 2^{-\frac{t}{\tau}} \tag{1}$$

, where τ is the half-life of the atom.

The total number of Lead atoms that decay were counted and subtracted from the number of ^{209}Pb initially and added to the number of ^{209}Bi.

Likewise, the decaying atoms are counted randomly using the probability obtained from Equation 1 They are then subtracted from the total number of Thallium atoms and added to the total of ^{209}Pb.

For ^{213}Bi, using the probability of decay from the route it takes randomly, the decayed Bismuth atom number are also counted and are added and subtracted accordingly.

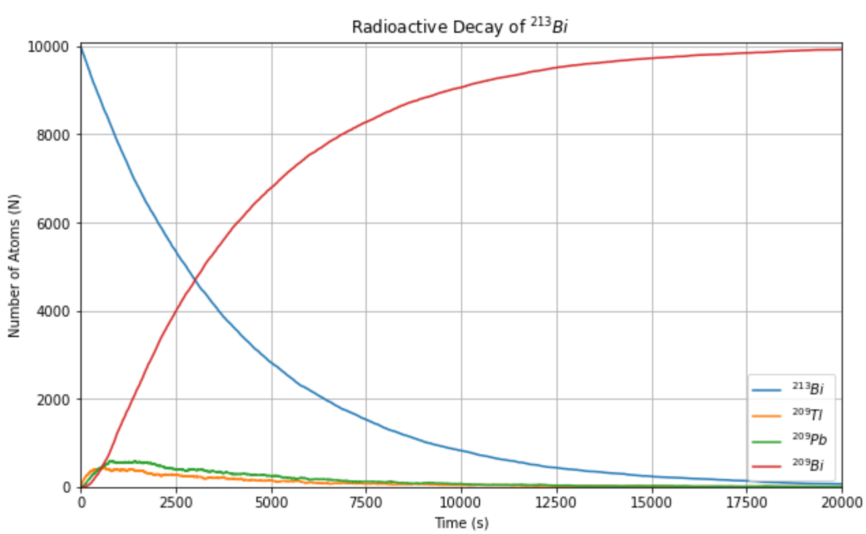

Figure 2: Radioactive Decay Series for ^{213}Bi

These were performed for $t = 20,000s$ and a graph of Number of atoms against Time was plotted on the same axes as shown in Figure 2 It can be seen that the number of ^{213}Bi atoms reduce as that of ^{209}Bi increase. Also, a few atoms of ^{209}Tl and ^{209}Pb are created but reduce as time increases as expected due to their lower decay probability values.

3 Question 3

The calculation in the decay reaction of the previous question was redone using the transformation method instead.

The probability

$$P(t)dt = 2^{-\frac{t}{\tau}} \frac{\ln 2}{\tau} dt \tag{2}$$

N random numbers are generated by drawing from the nonuniform probability distribution in Equation 2 to represent the time at which each atom decays. Using the exponential probabilty distribution

$$p(x) = \mu \exp(-\mu x) \tag{3}$$

, with $\mu = \frac{\ln 2}{\tau}$, solving for x gives

$$x = -\frac{1}{\mu} \ln(1 - z) \tag{4}$$

, where z are uniform random numbers generated in the interval $(0, 1)$

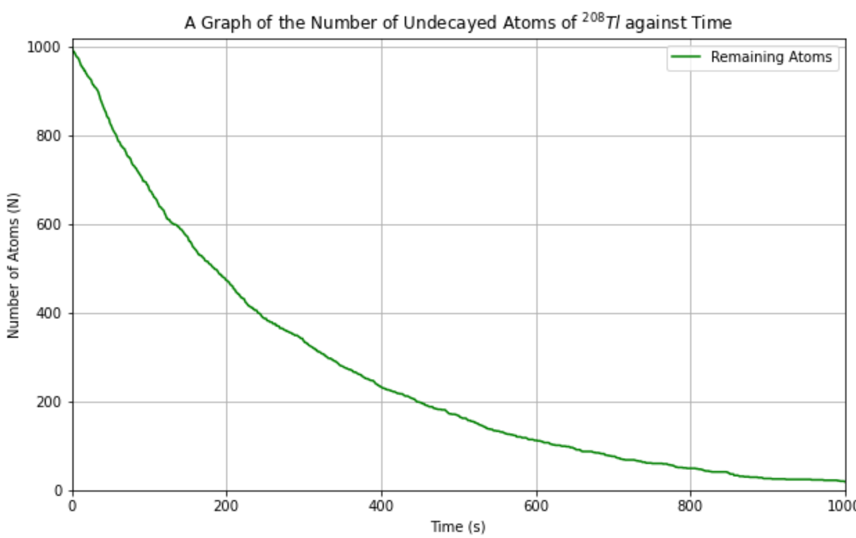

Figure 3: Number of Undecayed atoms of ^{208}Tl vs Time

1000 random numbers were generated from the nonuniform distribution of Equation 4 which represents the ties of decay of an equal number of ^{208}Tl atoms with a half-life of $3.053min$. A plot of the number of undecayed atoms was plotted as a function of time as seen in Figure 3 after being sorted in increasing order.

4 Question 4

The Central Limit Theorem states that the distribution of a normalized version of the sample mean converges to a standard normal distribution. Random variate

$$y = N^{-1} \sum_{i=0}^{N} x_i \tag{5}$$

, with x_i being a random variate distributed as $\exp(-x)$ was generated using the numpy.random package.

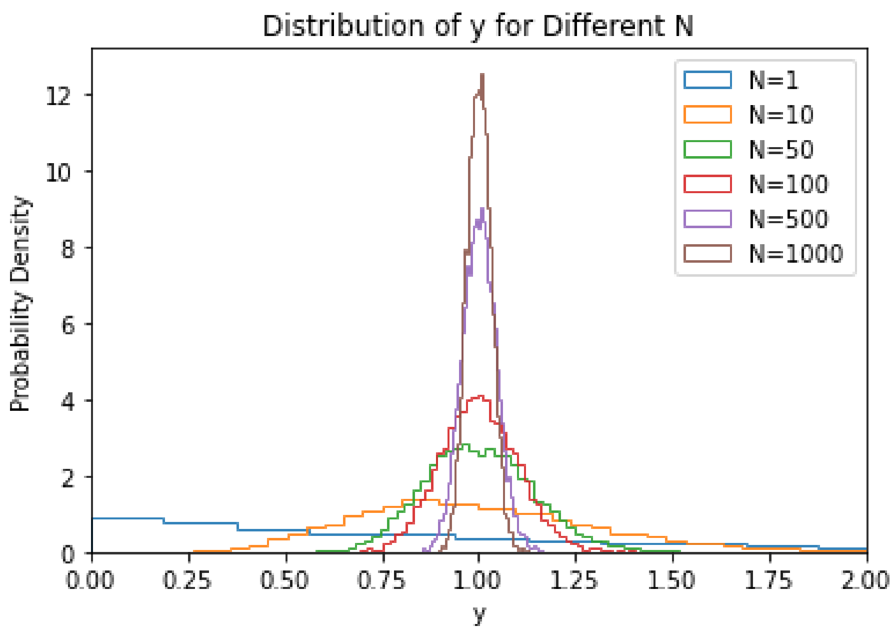

Figure 4: Shape of the Distribution as N is increased

As seen in Figure 4, when N is large, the distribution of y tends towards a Gaussian.

The mean, variance, skewness and kurtosis of the distribution changes as follows from Figure 5. The Mean seems to converge at 1, which is the expected value of the mean of an exponential distribution. The variance also converges at 0 which agree with expected analytical results.

It can be estimated from the graph in Figure 5 that the kurtosis seemed to have reached 1% of its value at about N = 50. The value of the skewness could not be determined form the graph, even when N was increased past 1000.

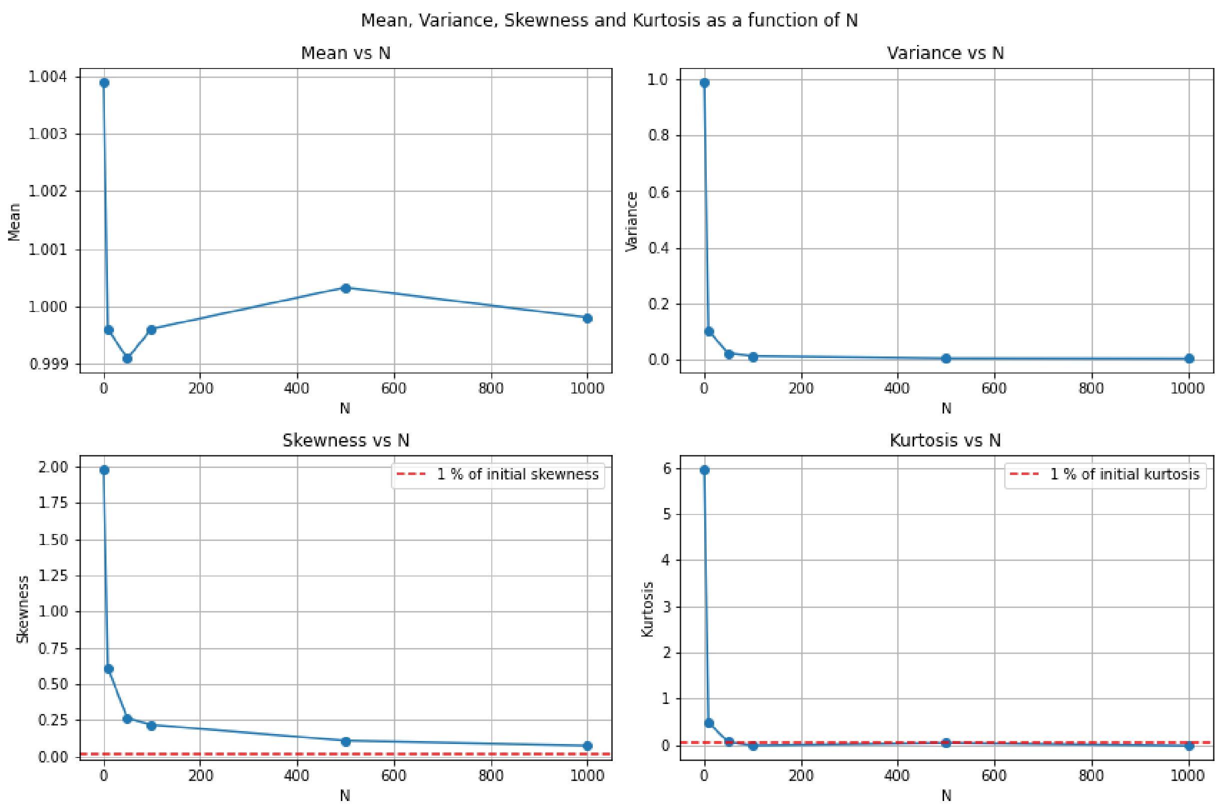

Figure 5: Plots showing the Mean, Vaiance, Skewness and Kurtosis of a Random Distribution