

PS-4 Solution

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1 Question 1

The heat capacity of a solid at temperature T , as expressed by Debye's theory, is given by

$$C_v = 9V\rho k_B(T/\theta_D^3) \int_0^{\theta_D/T} (x^4 \exp(x))/(\exp(x) - 1)^2 dx \quad (1)$$

where V is the volume, ρ is the number density of atoms, k_B is Boltzmann's constant and θ_D is the Debye temperature.

At $V = 1000\text{cm}^3$, $\rho = 6.022 \times 10^{28}\text{m}^{-3}$ and $\theta_D = 428\text{K}$ at $N = 50$ sample points, the function $c_v(T)$ was created using Gaussian quadrature which yielded the heat capacity at a specific temperature.

A plot of heat capacity against temperature ranging from $T = 5\text{K}$ and $T = 500\text{K}$ was made using the function above. The result is as shown in Figure 1.

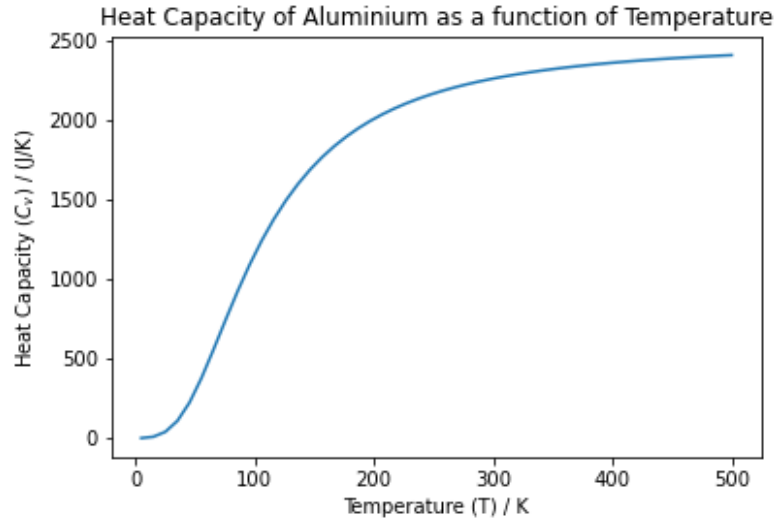


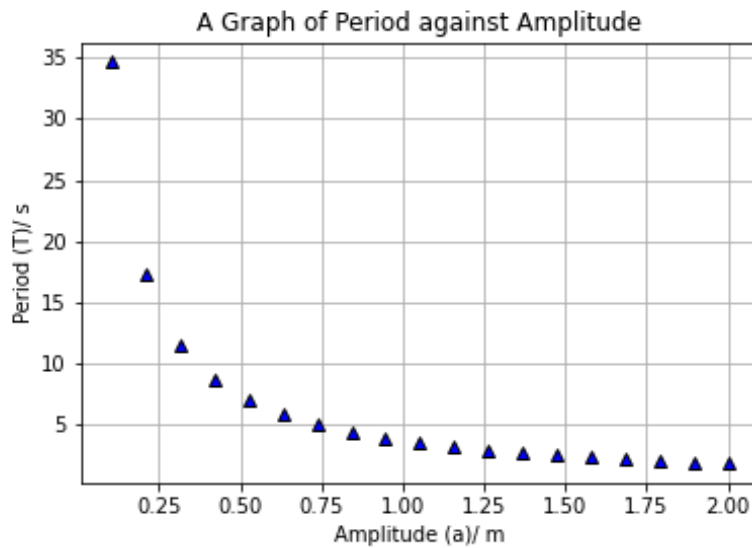
Figure 1: A graph of the heat capacity of aluminium against temperature

2 Question 2

Using

$$T = \sqrt{8m} \int_0^a dx / \sqrt{V(a) - V(x)} \quad (2)$$

Gaussian quadrature was used to evaluate the integral within the function using $N = 20$ points and a graph of the period against amplitude was plotted from $a = 0$ to $a = 2$. The graph is shown below.



It can be seen from the graph in Figure 2 that as amplitudes increase, the oscillator gets faster.

3 Question 3

For $n = 0, 1, 2, 3$ at $x = -4$ to $x = 4$, the graph of ψ_n against x yields the result in Figure 2

For $n = 30$ at $x = -10$ to $x = 10$, the graph of ψ_n against x yields The uncertainty was found to be 2.3452078737858177

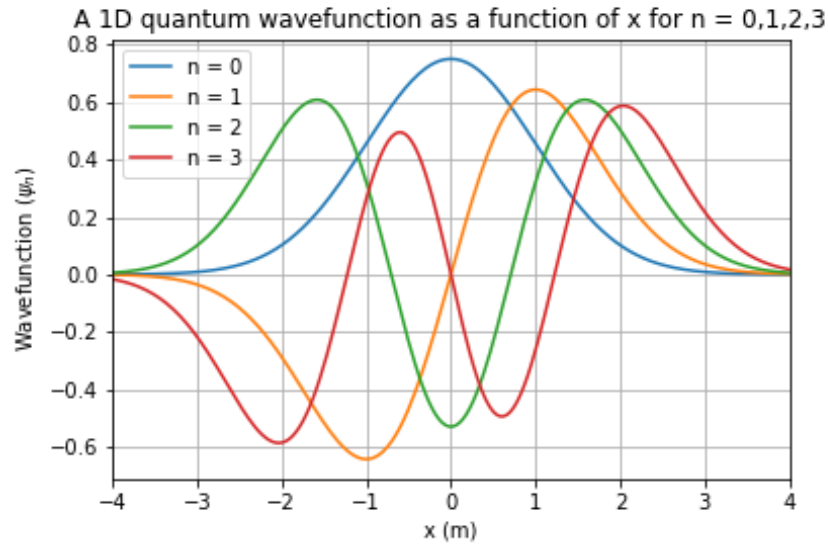


Figure 2: A graph of first 4 wavefunction plots of n against x

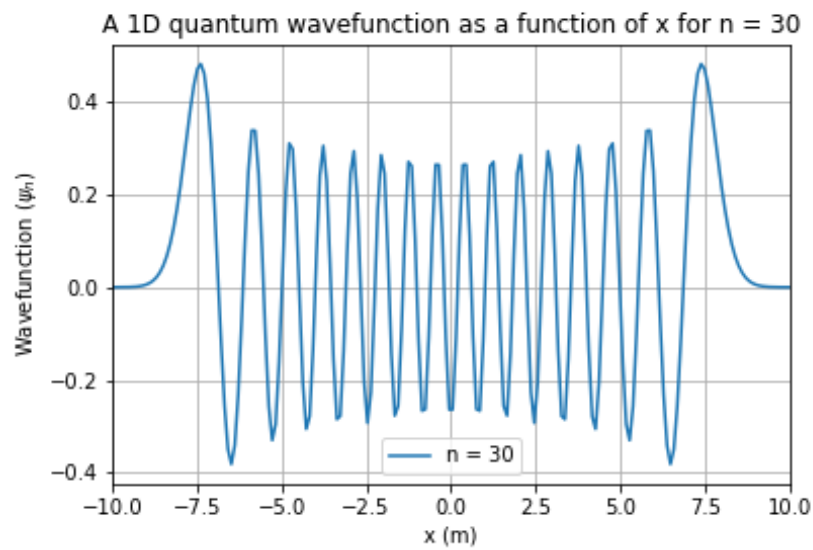


Figure 3: A plot of the wavefunction at $n = 30$ against x