Homework 9 Solution

Nana Ama Nyamekye Darpaah

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This is my GitHub link: Nana Ama's GitHub Link

1 Question 1

The harmonic and anharmonic oscillator equations are expressed as

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x \tag{1}$$

and

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x^3 \tag{2}$$

respectively.

The fourth-order Runge-Kutta method was employed to solve all the differential equations in this report.

For the harmonic oscillator, the initial conditions of x = 1 and $\frac{\mathrm{d}x}{\mathrm{d}t}$ had a value of 0, where x represents the amplitude. The initial and final times were $t = 0$ and $t = 50$ respectively. ω was set equal to 1 in this case. The amplitudes were made to vary at x = 1, 2, and 3 and a plot of position against time was made as shown in Figure 1. It can be seen that the period has no significant changes as the amplitudes are varied.

In the case of the anharmonic oscillator, a plot of time and varying amplitudes was also plotted with the result as shown in Figure 2. The amplitudes were varied with small differences as against that in Figure 3. It can be seen from the 2 plots that the oscillator oscillates faster at higher amplitudes.

A phase space plot of velocity against position of the anharmonic oscillator can be seen in Figure 4.

The equation of the Van der Pol oscillator is given by

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \mu(1 - x^2)\frac{\mathrm{d}x}{\mathrm{d}t} - \omega^2 x \tag{3}$$

The constant μ was made to vary as $\mu = 1, 2, 4$ and the phase space diagrams to each value of mu were plotted on the same graph as shown in Figure 5. Here, the initial and final times were

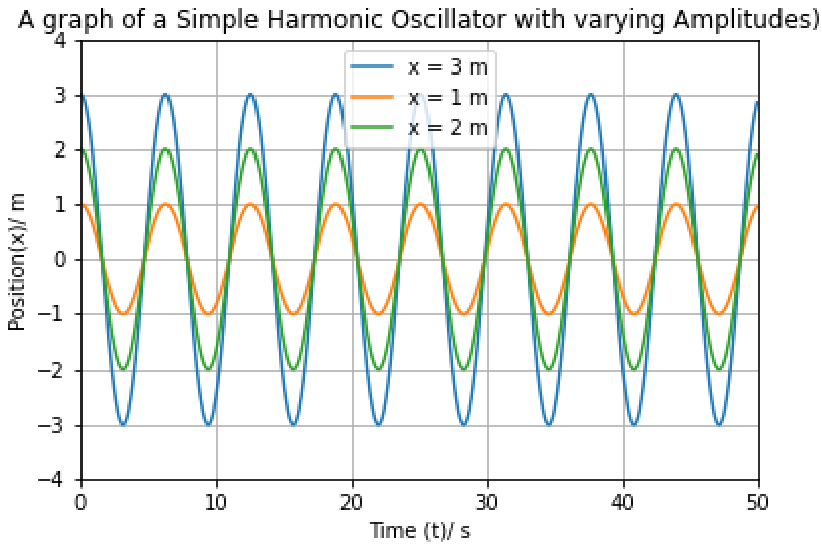

Figure 1: A graph of a simple harmonic oscillator with varying amplitudes

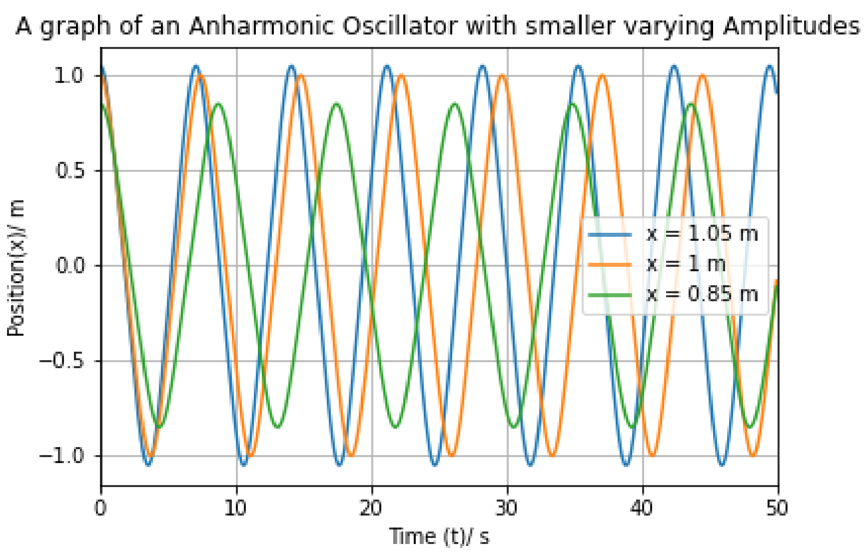

Figure 2: A graph of an anharmonic oscillator with small varying amplitudes

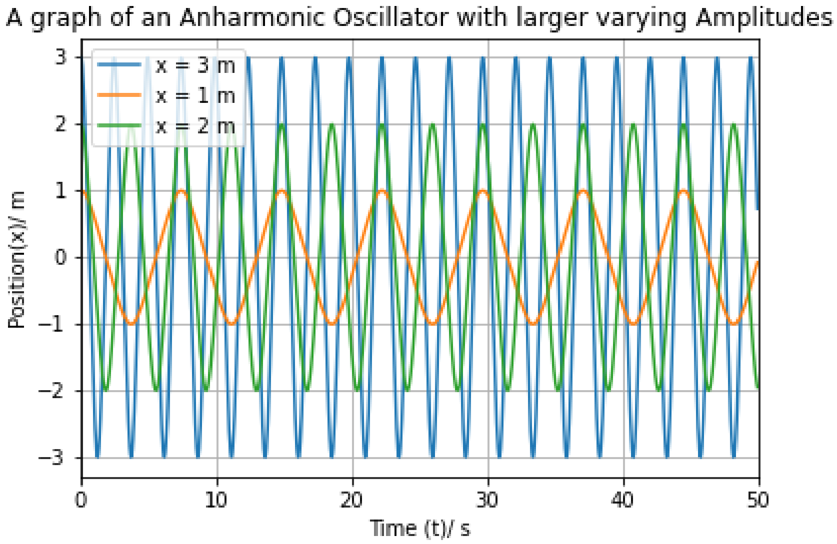

Figure 3: A graph of an anharmonic oscillator with large varying amplitudes

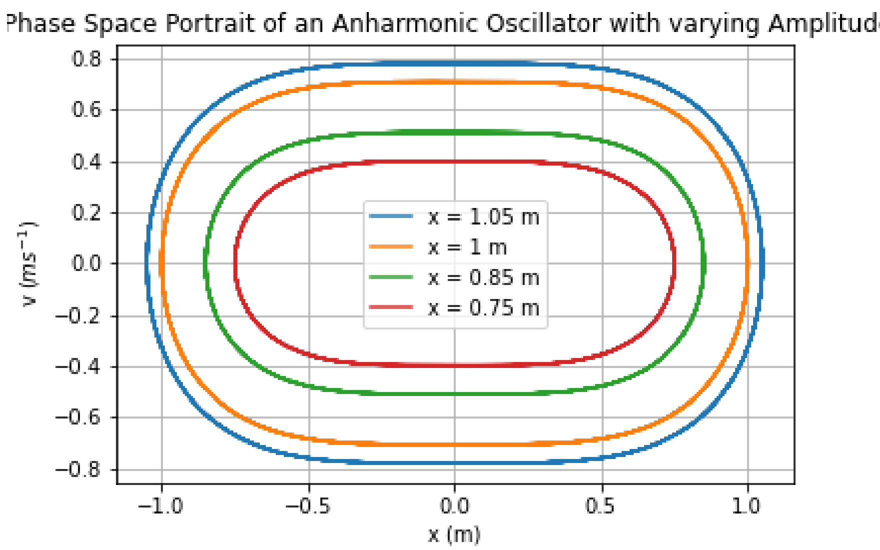

Figure 4: Phase Space plot of an anharmonic oscillator at varying amplitudes

given as $t = 0$ and $t = 20$ respectively and h was made small enough to ensure the smoothness of the plot.

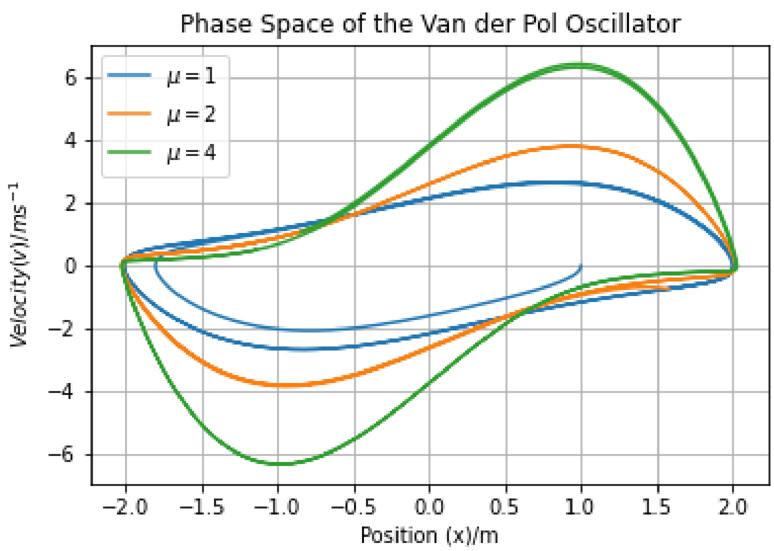

Figure 5: Phase Space plot of a Van der Pol oscillator at varying amplitudes

2 Question 2

Air resistance is expressed as

$$F = \frac{1}{2}\pi R^2 \rho C v^2 \tag{4}$$

where R is the radius of the spherical cannonball , ρ is the density of air, C is the drag coefficient and v is the velocity, $v = \sqrt{\dot{x}^2 + \dot{y}^2}$.

The total forces acting on the cannonball are expressed as

$$F = ma = F_{grav} + F_{drag} \tag{5}$$

where a is the acceleration, F_{grav} is the gravitational force and F_{drag} is the air resistance force. This implies

$$\ddot{r} = -mg\hat{y} - \frac{1}{2}\pi R^2 \rho C v^2 \hat{v} \tag{6}$$

and $\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

For the x-direction,

$$
\begin{aligned}
m\ddot{x} &= -\frac{1}{2}\pi R^2 \rho C v^2 \frac{\dot{x}}{v} && (7)\\
&= -\frac{1}{2}\pi R^2 \rho C \dot{x}\sqrt{\dot{x}^2+\dot{y}^2} && (8)
\end{aligned}
$$

Likewise, for the y-direction,

$$
\begin{aligned}
m\ddot{y} &= -mg - \frac{1}{2}\pi R^2 \rho C v^2 \frac{\dot{y}}{v} && (9)\\
&= -g - \frac{1}{2}\pi R^2 \rho C \dot{y}\sqrt{\dot{x}^2+\dot{y}^2} && (10)
\end{aligned}
$$

Hence, the 2 differential equations are

$$
\begin{aligned}
\ddot{x} &= -\frac{1}{2}\pi R^2 \rho C \dot{x}\sqrt{\dot{x}^2+\dot{y}^2} && (11)\\
\ddot{y} &= -g - \frac{1}{2}\pi R^2 \rho C \dot{y}\sqrt{\dot{x}^2+\dot{y}^2} && (12)
\end{aligned}
$$

Rescaling the variables to produce a unitless equation was made through the $t = t'/T$ subsitution as well as the $x = gT^2x'$ and $y = gT^2y'$.

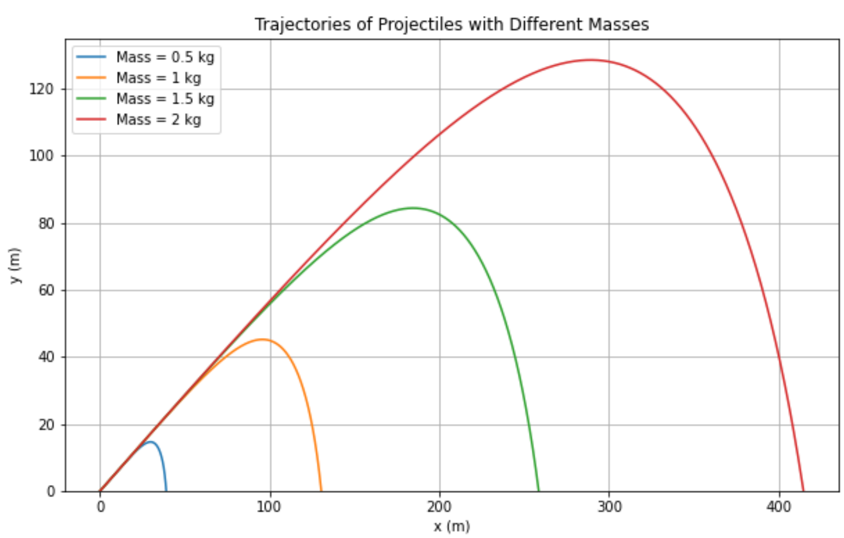

Figure 6: Trajectories of Projectile Motion with Air Resistance at varying Masses

A series of trajectories for different masses of the cannonball was plotted with the initial velocity being $100ms^{-1}$ at $30°$ to the horizontal as shown in Figure 6. As shown, the total horizontal distance does indeed depend on the mass of the projectile. From the plot, it is seen that heavier masses travel further than lighter masses.