Pertemuan 3 - Regresi dengan Peubah Lag

Adelia Putri Pangestika Muhammad Rizky Nurhambali

Packages

```
## Warning: package 'dLagM' was built under R version 4.3.3

## Loading required package: nardl

## Warning: package 'nardl' was built under R version 4.3.3

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

## Loading required package: dynlm

## Warning: package 'dynlm' was built under R version 4.3.3

## Loading required package: zoo

## Warning: package 'zoo' was built under R version 4.3.3

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Warning: package 'MLmetrics' was built under R version 4.3.3

##
## Attaching package: 'MLmetrics'

## The following object is masked from 'package:dLagM':
##
##     MAPE

## The following object is masked from 'package:base':
##
##     Recall

## Warning: package 'lmtest' was built under R version 4.3.3

## Loading required package: carData
```

Impor Data

```
data <- rio::import("https://raw.githubusercontent.com/rizkynurhambali/Praktikum-MPDW-2324/main/Pertemua
str(data)
```

```
## 'data.frame':    20 obs. of  4 variables:
##  $ t     : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ Yt    : num  52.9 53.8 54.9 58.2 60 63.4 68.2 78 84.7 90.6 ...
```

```
## $ Y(t-1): num  NA 52.9 53.8 54.9 58.2 60 63.4 68.2 78 84.7 ...
## $ Xt    : num  30.3 30.9 30.9 33.4 35.1 37.3 41 44.9 46.5 50.3 ...
```

```
data
```

```
##     t    Yt Y(t-1)    Xt
## 1   1  52.9     NA  30.3
## 2   2  53.8   52.9  30.9
## 3   3  54.9   53.8  30.9
## 4   4  58.2   54.9  33.4
## 5   5  60.0   58.2  35.1
## 6   6  63.4   60.0  37.3
## 7   7  68.2   63.4  41.0
## 8   8  78.0   68.2  44.9
## 9   9  84.7   78.0  46.5
## 10 10  90.6   84.7  50.3
## 11 11  98.2   90.6  53.5
## 12 12 101.7   98.2  52.8
## 13 13 102.7  101.7  55.9
## 14 14 108.3  102.7  63.0
## 15 15 124.7  108.3  73.0
## 16 16 157.9  124.7  84.8
## 17 17 158.2  157.9  86.6
## 18 18 170.2  158.2  98.9
## 19 19 180.0  170.2 110.8
## 20 20 198.0  180.0 124.7
```

Pembagian Data

```
#SPLIT DATA
train<-data[1:15,]
test<-data[16:20,]
```

```
#data time series
train.ts<-ts(train)
test.ts<-ts(test)
data.ts<-ts(data)
```

Model Koyck

Model Koyck didasarkan pada asumsi bahwa semakin jauh jarak lag peubah independen dari periode sekarang maka semakin kecil pengaruh peubah lag terhadap peubah dependen.

Koyck mengusulkan suatu metode untuk menduga model dinamis distributed lag dengan mengasumsikan bahwa semua koefisien β mempunyai tanda sama.

Model kyock merupakan jenis paling umum dari model infinite distributed lag dan juga dikenal sebagai geometric lag

$$y_t = a(1-\lambda) + \beta_0 X_t + \beta_1 Z_t + \lambda Y_{t-1} + V_t$$

dengan

$$V_t = u_t - \lambda u_{t-1}$$

Pemodelan

Pemodelan model Koyck dengan `R` dapat menggunakan `dLagM::koyckDlm()` . Fungsi umum dari `koyckDlm` adalah sebagai berikut.

```
koyckDlm(x , y , intercept)
```

Fungsi `koyckDlm()` akan menerapkan model lag terdistribusi dengan transformasi Koyck satu prediktor. Nilai `x` dan `y` tidak perlu sebagai objek *time series* (`ts`). `intercept` dapat dibuat `TRUE` untuk memasukkan intersep ke dalam model.

```
#MODEL KOYCK
model.koyck <- koyckDlm(x = train$Xt, y = train$Yt)
summary(model.koyck)
```

```
##
## Call:
## "Y ~ (Intercept) + Y.1 + X.t"
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.75605 -1.16407  0.01599  1.17295  3.28003
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -3.7335     2.1785  -1.714   0.1146
## Y.1           0.4214     0.1158   3.639   0.0039 **
## X.t           1.1510     0.1901   6.055 8.25e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.023 on 11 degrees of freedom
## Multiple R-Squared: 0.9934,  Adjusted R-squared: 0.9922
## Wald test: 828.9 on 2 and 11 DF,  p-value: 1.01e-12
##
## Diagnostic tests:
## NULL
##
##                               alpha     beta       phi
## Geometric coefficients:  -6.452844 1.150951 0.4214181
```

```
AIC(model.koyck)
```

```
## [1] 64.08525
```

```
BIC(model.koyck)
```

```
## [1] 66.64148
```

Dari hasil tersebut, didapat bahwa peubah x_t dan y_{t-1} memiliki nilai $P-Value < 0.05$. Hal ini menunjukkan bahwa peubah x_t dan y_{t-1} berpengaruh signifikan terhadap y. Adapun model keseluruhannya adalah sebagai berikut

$$\hat{Y_t} = -3.7335 + 1.1510X_t + 0.4214Y_{t-1}$$

Peramalan dan Akurasi

Berikut adalah hasil peramalan y untuk 5 periode kedepan menggunakan model koyck

```r
fore.koyck <- forecast(model = model.koyck, x=test$Xt, h=5)
fore.koyck
```

```
## $forecasts
## [1] 146.4180 157.6420 176.5288 198.1843 223.3085
##
## $call
## forecast.koyckDlm(model = model.koyck, x = test$Xt, h = 5)
##
## attr(,"class")
## [1] "forecast.koyckDlm" "dLagM"
```

```r
mape.koyck <- MAPE(fore.koyck$forecasts, test$Yt)
#akurasi data training
GoF(model.koyck)
```

```
##              n      MAE           MPE       MAPE      sMAPE      MASE      MSE
## model.koyck 14 1.473599 -0.0004874002 0.01720083 0.01719063 0.2701945 3.216237
##                  MRAE     GMRAE
## model.koyck 0.3967354 0.3026997
```

Regression with Distributed Lag

Pemodelan model Regression with Distributed Lag dengan `R` dapat menggunakan `dLagM::dlm()` . Fungsi umum dari `dlm` adalah sebagai berikut.

```r
dlm(formula , data , x , y , q , remove )
```

Fungsi `dlm()` akan menerapkan model lag terdistribusi dengan satu atau lebih prediktor. Nilai `x` dan `y` tidak perlu sebagai objek *time series* (`ts`). q adalah integer yang mewakili panjang *lag* yang terbatas.

Pemodelan (Lag=2)

```r
model.dlm <- dlm(formula = Yt ~ Xt,
                 data = train, q = 2)
summary(model.dlm)
```

```
##
## Call:
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.4446 -0.6965 -0.2373  0.8810  1.8630
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -9.6779     1.8156  -5.330 0.000474 ***
## Xt.t          0.3179     0.1792   1.774 0.109856
## Xt.1          1.5276     0.3487   4.380 0.001770 **
## Xt.2          0.2651     0.2440   1.087 0.305388
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.322 on 9 degrees of freedom
```

```
## Multiple R-squared:  0.9974, Adjusted R-squared:  0.9965
## F-statistic:  1133 on 3 and 9 DF,  p-value: 6.471e-12
##
## AIC and BIC values for the model:
##       AIC      BIC
## 1 49.3705 52.19525
```

```
AIC(model.dlm)
```

```
## [1] 49.3705
```

```
BIC(model.dlm)
```

```
## [1] 52.19525
```

Dari hasil diatas, didapat bahwa $P - value$ dari intercept dan $x_{t-1} < 0.05$. Hal ini menunjukkan bahwa intercept dan x_{t-1} berpengaruh signifikan terhadap y. Adapun model keseluruhan yang terbentuk adalah sebagai berikut

$$\hat{Y}_t = -9.6779 + 0.3179X_t + 1.5276X_{t-1} + 0.2651X_{t-2}$$

Peramalan dan Akurasi

Berikut merupakan hasil peramalan y untuk 5 periode kedepan

```
fore.dlm <- forecast(model.dlm, x=test[,4], h=5)
fore.dlm
```

```
## $forecasts
## [1] 145.5021 166.7516 176.5405 199.5907 225.4498
##
## $call
## forecast.dlm(model = model.dlm, x = test[, 4], h = 5)
##
## attr(,"class")
## [1] "forecast.dlm" "dLagM"
```

```
mape.dlm <- MAPE(fore.dlm$forecasts, test$Yt)
#akurasi data training
GoF(model.dlm)
```

```
##            n       MAE           MPE       MAPE      sMAPE      MASE      MSE
## model.dlm 13 0.9282513 -0.0002879026 0.01227837 0.01229574 0.1595847 1.210003
##                MRAE     GMRAE
## model.dlm 0.2350414 0.1491444
```

Lag Optimum

```
#penentuan lag optimum
finiteDLMauto(formula = Yt ~ Xt,
              data = data.frame(train), q.min = 1, q.max = 6,
              model.type = "dlm", error.type = "AIC", trace = FALSE)
```

```
##   q - k    MASE       AIC       BIC   GMRAE   MBRAE R.Adj.Sq Ljung-Box
## 6     6 0.00214 -29.87368 -28.09865 0.00225 0.00125  0.99999 0.9830642
```

Berdasarkan output tersebut, lag optimum didapatkan ketika lag=6. Selanjutnya dilakukan pemodelan untuk lag=6

```r
#model dlm dengan lag optimum
model.dlm2 <- dlm(x = train$Xt,y = train$Yt , q = 6)
summary(model.dlm2)
```

```
##
## Call:
## lm(formula = model.formula, data = design)
##
## Residuals:
##         1         2         3         4         5         6         7         8
## -0.023415  0.014375  0.032641 -0.010695 -0.013096 -0.014927  0.010054  0.010809
##         9
## -0.005747
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 21.42223    1.44472  14.828   0.0429 *
## x.t          1.68749    0.04758  35.466   0.0179 *
## x.1         -1.23901    0.11688 -10.600   0.0599 .
## x.2          0.97604    0.02787  35.021   0.0182 *
## x.3         -0.23945    0.02746  -8.719   0.0727 .
## x.4          3.24431    0.14678  22.103   0.0288 *
## x.5          0.08560    0.04903   1.746   0.3312
## x.6         -3.47612    0.15157 -22.933   0.0277 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05079 on 1 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 1.277e+05 on 7 and 1 DF,  p-value: 0.002155
##
## AIC and BIC values for the model:
##         AIC       BIC
## 1 -29.87368 -28.09865
```

```r
AIC(model.dlm2)
```

```
## [1] -29.87368
```

```r
BIC(model.dlm2)
```

```
## [1] -28.09865
```

Dari hasil tersebut terdapat beberapa peubah yang berpengaruh signifikan terhadap taraf nyata 5% yaitu x_t , x_{t-2} , x_{t-4} , x_{t-6}. Adapun keseluruhan model yang terbentuk adalah

$$\hat{Y}_t = 21.42223 + 1.68749X_t + ... - 3.47612X_{t-6}$$

Adapun hasil peramalan 5 periode kedepan menggunakan model tersebut adalah sebagai berikut

```r
#peramalan dan akurasi
fore.dlm2 <- forecast(model = model.dlm2, x=test$Xt, h=5)
mape.dlm2<- MAPE(fore.dlm2$forecasts, test$Yt)
#akurasi data training
GoF(model.dlm2)
```

```
##             n        MAE           MPE         MAPE        sMAPE        MASE
## model.dlm2 9 0.01508446 -2.246354e-06 0.0001727757 0.0001727773 0.002135853
##                      MSE        MRAE       GMRAE
## model.dlm2 0.0002866713 0.003309313 0.002245928
```

Model tersebut merupakan model yang sangat baik dengan nilai MAPE yang kurang dari 10%.

Model Autoregressive

Peubah dependen dipengaruhi oleh peubah independen pada waktu sekarang, serta dipengaruhi juga oleh peubah dependen itu sendiri pada satu waktu yang lalu maka model tersebut disebut *autoregressive* (Gujarati 2004).

Pemodelan

Pemodelan Autoregressive dilakukan menggunakan fungsi `dLagM::ardlDlm()` . Fungsi tersebut akan menerapkan *autoregressive* berordo (p, q) dengan satu prediktor. Fungsi umum dari `ardlDlm()` adalah sebagai berikut.

```
ardlDlm(formula = NULL , data = NULL , x = NULL , y = NULL , p = 1 , q = 1 ,
         remove = NULL )
```

Dengan p adalah integer yang mewakili panjang *lag* yang terbatas dan q adalah integer yang merepresentasikan ordo dari proses *autoregressive*.

```
#model.ardl <- ardlDlm(x = train$Xt, y = train$Yt, p = 1 , q = 1)
#summary(model.ardl)
#AIC(model.ardl)
#BIC(model.ardl)
```

```
model.ardl <- ardlDlm(formula = Yt ~ Xt,
                         data = train,p = 1 , q = 1)
summary(model.ardl)
```

```
##
## Time series regression with "ts" data:
## Start = 2, End = 15
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.6274 -0.8401 -0.1767  0.8392  1.9447
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -8.3594     1.9186  -4.357  0.00143 **
## Xt.t          0.3563     0.1875   1.900  0.08661 .
## Xt.1          1.4557     0.4071   3.575  0.00505 **
## Yt.1          0.1408     0.1318   1.068  0.31055
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.269 on 10 degrees of freedom
## Multiple R-squared:  0.9976, Adjusted R-squared:  0.9969
## F-statistic:  1405 on 3 and 10 DF,  p-value: 2.006e-13
```

```r
AIC(model.ardl)
```

```
## [1] 51.69462
```

```r
BIC(model.ardl)
```

```
## [1] 54.8899
```

Hasil di atas menunjukkan bahwa selain peubah x_{t-1}, hasil uji t menunjukkan nilai-p pada peubah ≥ 0.05 Hal ini menunjukkan bahwa peubah x_{t-1} berpengaruh signifikan terhadap y_t, sementara x_t dan y_{t-1} berpengaruh signifikan terhadap y_t. Model keseluruhannya adalah sebagai berikut:

$$\hat{Y} = -8,3594 + 0,3563X_t + 1,4557X_{t-1} + 0,1408Y_{t-1}$$

Peramalan dan Akurasi

```r
fore.ardl <- forecast(model = model.ardl, x=test$Xt, h=5)
fore.ardl
```

```
## $forecasts
## [1] 145.6865 166.4608 176.3897 199.9337 225.5255
##
## $call
## forecast.ardlDlm(model = model.ardl, x = test$Xt, h = 5)
##
## attr(,"class")
## [1] "forecast.ardlDlm" "dLagM"
```

Data di atas merupakan hasil peramalan untuk 5 periode ke depan menggunakan Model Autoregressive dengan $p = 1$ dan $q = 1$.

```r
mape.ardl <- MAPE(fore.ardl$forecasts, test$Yt)
mape.ardl
```

```
## [1] 0.08313896
```

```r
#akurasi data training
GoF(model.ardl)
```

```
##             n       MAE           MPE       MAPE      sMAPE      MASE     MSE
## model.ardl 14 0.8821232 -0.0003216464 0.01183235 0.01185341 0.1617433 1.15063
##                 MRAE     GMRAE
## model.ardl 0.2332582 0.1391987
```

Berdasarkan akurasi di atas, terlihat bahwa nilai MAPE keduanya tidak jauh berbeda. Artinya, model regresi dengan distribusi lag ini tidak `overfitted` atau `underfitted`

Lag Optimum

```r
#penentuan lag optimum
model.ardl.opt <- ardlBoundOrders(data = data.frame(data), ic = "AIC",
                                  formula = Yt ~ Xt )
min_p=c()
for(i in 1:6){
  min_p[i]=min(model.ardl.opt$Stat.table[[i]])
}
q_opt=which(min_p==min(min_p, na.rm = TRUE))
```

```r
p_opt=which(model.ardl.opt$Stat.table[[q_opt]] ==
              min(model.ardl.opt$Stat.table[[q_opt]], na.rm = TRUE))
data.frame("q_optimum" = q_opt, "p_optimum" = p_opt,
           "AIC"=model.ardl.opt$min.Stat)
```

```
##   q_optimum p_optimum       AIC
## 1         1         6 -20.56587
```

Dari tabel di atas, dapat terlihat bahwa nilai AIC terendah didapat ketika $p = 6$ dan $q = 1$, yaitu sebesar `-20,56587`. Artinya, model autoregressive optimum didapat ketika $p = 6$ dan $q = 1$.

Selanjutnya dapat dilakukan pemodelan dengan nilai p dan q optimum seperti inisialisasi di langkah sebelumnya.

Pemodelan DLM & ARDL dengan Library `dynlm`

Pemodelan regresi dengan peubah *lag* tidak hanya dapat dilakukan dengan fungsi pada *packages* `dLagM` , tetapi terdapat *packages* `dynlm` yang dapat digunakan. Fungsi `dynlm` secara umum adalah sebagai berikut.

```r
dynlm(formula, data, subset, weights, na.action, method = "qr",
  model = TRUE, x = FALSE, y = FALSE, qr = TRUE, singular.ok = TRUE,
  contrasts = NULL, offset, start = NULL, end = NULL, ...)
```

Untuk menentukan `formula` model yang akan digunakan, tersedia fungsi tambahan yang memungkinkan spesifikasi dinamika (melalui `d()` dan `L()`) atau pola linier/siklus dengan mudah (melalui `trend()`, `season()`, dan `harmon()`). Semua fungsi formula baru mengharuskan argumennya berupa objek deret waktu (yaitu, `"ts"` atau `"zoo"`).

```r
#sama dengan model dlm q=1
cons_lm1 <- dynlm(Yt ~ Xt+L(Xt),data = train.ts)
#sama dengan model ardl p=1 q=0
cons_lm2 <- dynlm(Yt ~ Xt+L(Yt),data = train.ts)
#sama dengan ardl p=1 q=1
cons_lm3 <- dynlm(Yt ~ Xt+L(Xt)+L(Yt),data = train.ts)
#sama dengan dlm p=2
cons_lm4 <- dynlm(Yt ~ Xt+L(Xt)+L(Xt,2),data = train.ts)
```

Ringkasan Model

```r
summary(cons_lm1)
```

```
##
## Time series regression with "ts" data:
## Start = 2, End = 15
##
## Call:
## dynlm(formula = Yt ~ Xt + L(Xt), data = train.ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.4118 -0.8790 -0.3542  0.7202  2.3047
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -9.4870     1.6123  -5.884 0.000106 ***
## Xt            0.2557     0.1632   1.567 0.145434
```

```
## L(Xt)         1.8395     0.1927   9.547 1.17e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.277 on 11 degrees of freedom
## Multiple R-squared:  0.9974, Adjusted R-squared:  0.9969
## F-statistic:  2081 on 2 and 11 DF,  p-value: 6.538e-15
```

```r
summary(cons_lm2)
```

```
##
## Time series regression with "ts" data:
## Start = 2, End = 15
##
## Call:
## dynlm(formula = Yt ~ Xt + L(Yt), data = train.ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4441 -1.1436  0.1785  1.5549  2.1584
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.54096    1.96526  -1.802    0.099 .
## Xt           0.92218    0.14482   6.368 5.31e-05 ***
## L(Yt)        0.55684    0.08922   6.241 6.34e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.827 on 11 degrees of freedom
## Multiple R-squared:  0.9946, Adjusted R-squared:  0.9936
## F-statistic:  1015 on 2 and 11 DF,  p-value: 3.344e-13
```

```r
summary(cons_lm3)
```

```
##
## Time series regression with "ts" data:
## Start = 2, End = 15
##
## Call:
## dynlm(formula = Yt ~ Xt + L(Xt) + L(Yt), data = train.ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.6274 -0.8401 -0.1767  0.8392  1.9447
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -8.3594     1.9186  -4.357  0.00143 **
## Xt            0.3563     0.1875   1.900  0.08661 .
## L(Xt)         1.4557     0.4071   3.575  0.00505 **
## L(Yt)         0.1408     0.1318   1.068  0.31055
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 1.269 on 10 degrees of freedom
## Multiple R-squared:  0.9976, Adjusted R-squared:  0.9969
## F-statistic:  1405 on 3 and 10 DF,  p-value: 2.006e-13
```

```r
summary(cons_lm4)
```

```
##
## Time series regression with "ts" data:
## Start = 3, End = 15
##
## Call:
## dynlm(formula = Yt ~ Xt + L(Xt) + L(Xt, 2), data = train.ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.4446 -0.6965 -0.2373  0.8810  1.8630
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -9.6779     1.8156  -5.330 0.000474 ***
## Xt            0.3179     0.1792   1.774 0.109856
## L(Xt)         1.5276     0.3487   4.380 0.001770 **
## L(Xt, 2)      0.2651     0.2440   1.087 0.305388
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.322 on 9 degrees of freedom
## Multiple R-squared:  0.9974, Adjusted R-squared:  0.9965
## F-statistic:  1133 on 3 and 9 DF,  p-value: 6.471e-12
```

SSE

```r
deviance(cons_lm1)
```

```
## [1] 17.94685
```

```r
deviance(cons_lm2)
```

```
## [1] 36.70155
```

```r
deviance(cons_lm3)
```

```
## [1] 16.10882
```

```r
deviance(cons_lm4)
```

```
## [1] 15.73004
```

Uji Diagnostik

```r
#uji model
if(require("lmtest")) encomptest(cons_lm1, cons_lm2)
```

```
## Encompassing test
##
## Model 1: Yt ~ Xt + L(Xt)
## Model 2: Yt ~ Xt + L(Yt)
```

```
## Model E: Yt ~ Xt + L(Xt) + L(Yt)
##           Res.Df Df      F  Pr(>F)
## M1 vs. ME     10 -1  1.141 0.31055
## M2 vs. ME     10 -1 12.784 0.00505 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
#durbin watson
dwtest(cons_lm1)
```

Autokorelasi

```
##
##  Durbin-Watson test
##
## data:  cons_lm1
## DW = 2.1065, p-value = 0.3842
## alternative hypothesis: true autocorrelation is greater than 0
```

```r
dwtest(cons_lm2)
```

```
##
##  Durbin-Watson test
##
## data:  cons_lm2
## DW = 1.441, p-value = 0.0497
## alternative hypothesis: true autocorrelation is greater than 0
```

```r
dwtest(cons_lm3)
```

```
##
##  Durbin-Watson test
##
## data:  cons_lm3
## DW = 1.9337, p-value = 0.2449
## alternative hypothesis: true autocorrelation is greater than 0
```

```r
dwtest(cons_lm4)
```

```
##
##  Durbin-Watson test
##
## data:  cons_lm4
## DW = 1.8189, p-value = 0.1911
## alternative hypothesis: true autocorrelation is greater than 0
```

```r
bptest(cons_lm1)
```

Heterogenitas

```
##
##  studentized Breusch-Pagan test
##
## data:  cons_lm1
## BP = 1.5713, df = 2, p-value = 0.4558
```

```r
bptest(cons_lm2)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  cons_lm2
## BP = 3.7022, df = 2, p-value = 0.1571
```

```r
bptest(cons_lm3)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  cons_lm3
## BP = 4.0554, df = 3, p-value = 0.2555
```

```r
bptest(cons_lm4)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  cons_lm4
## BP = 2.7921, df = 3, p-value = 0.4248
```

```r

shapiro.test(residuals(cons_lm1))
```

Kenormalan

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(cons_lm1)
## W = 0.90752, p-value = 0.145
```

```r
shapiro.test(residuals(cons_lm2))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(cons_lm2)
## W = 0.94358, p-value = 0.4661
```

```r
shapiro.test(residuals(cons_lm3))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(cons_lm3)
## W = 0.94403, p-value = 0.4723
```

```r
shapiro.test(residuals(cons_lm4))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(cons_lm4)
## W = 0.92477, p-value = 0.2907
```

Perbandingan Model

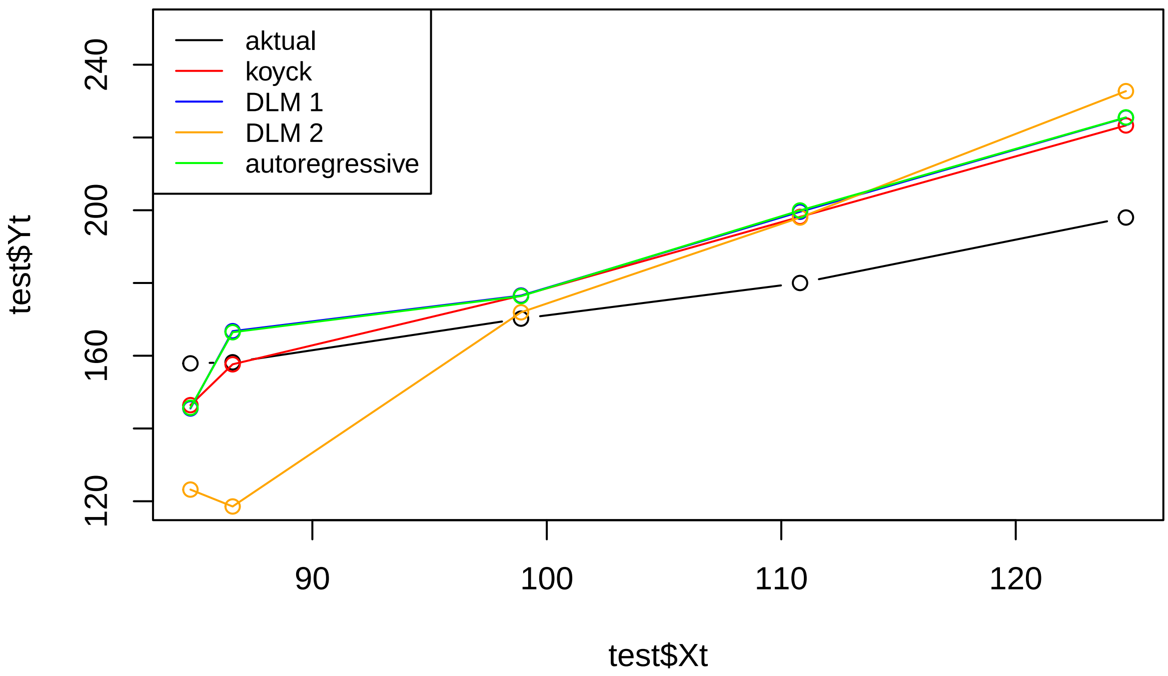
```
akurasi <- matrix(c(mape.koyck, mape.dlm, mape.dlm2, mape.ardl))
row.names(akurasi)<- c("Koyck","DLM 1","DLM 2","Autoregressive")
colnames(akurasi) <- c("MAPE")
akurasi
```

```
##                      MAPE
## Koyck          0.06845456
## DLM 1          0.08345978
## DLM 2          0.15117526
## Autoregressive 0.08313896
```

Berdasarkan nilai MAPE, model paling optimum didapat pada Model Koyck karena memiliki nilai MAPE yang terkecil.

Plot

```
par(mfrow=c(1,1))
plot(test$Xt, test$Yt, type="b", col="black", ylim=c(120,250))
points(test$Xt, fore.koyck$forecasts,col="red")
lines(test$Xt, fore.koyck$forecasts,col="red")
points(test$Xt, fore.dlm$forecasts,col="blue")
lines(test$Xt, fore.dlm$forecasts,col="blue")
points(test$Xt, fore.dlm2$forecasts,col="orange")
lines(test$Xt, fore.dlm2$forecasts,col="orange")
points(test$Xt, fore.ardl$forecasts,col="green")
lines(test$Xt, fore.ardl$forecasts,col="green")
legend("topleft",c("aktual", "koyck","DLM 1","DLM 2", "autoregressive"), lty=1, col=c("black","red","blu
```



Berdasarkan plot tersebut, terlihat bahwa plot yang paling mendekati data aktualnya adalah Model koyck, sehingga dapat disimpulkan model terbaik dalam hal ini adalah model regresi koyck

Pengayaan (Regresi Berganda)

Data

```
data(M1Germany)
data1 = M1Germany[1:144,]
```

DLM

```
#Run the search over finite DLMs according to AIC values
finiteDLMauto(formula = logprice ~ interest+logm1,
              data = data.frame(data1), q.min = 1, q.max = 5,
              model.type = "dlm", error.type = "AIC", trace = FALSE)
```

```
##   q - k    MASE       AIC       BIC   GMRAE    MBRAE R.Adj.Sq Ljung-Box
## 5     5 1.77163 -463.1393 -422.0566 1.43662 -1.60494  0.98836         0
```

```
#model dlm berganda
model.dlmberganda = dlm(formula = logprice ~ interest + logm1,
                       data = data.frame(data1) , q = 5)
summary(model.dlmberganda)
```

```
##
## Call:
```

```
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.095761 -0.028610 -0.000012  0.029496  0.102597
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -7.81759    0.11384 -68.669  < 2e-16 ***
## interest.t  -1.75616    0.80358  -2.185 0.030707 *
## interest.1   1.38935    1.22707   1.132 0.259679
## interest.2   0.40776    1.23726   0.330 0.742273
## interest.3   1.23130    1.20752   1.020 0.309830
## interest.4  -0.08718    1.20869  -0.072 0.942616
## interest.5   3.06850    0.89380   3.433 0.000808 ***
## logm1.t      0.43219    0.20876   2.070 0.040474 *
## logm1.1      0.42190    0.19807   2.130 0.035109 *
## logm1.2      0.20943    0.12883   1.626 0.106532
## logm1.3      0.22053    0.13011   1.695 0.092567 .
## logm1.4      0.05513    0.21457   0.257 0.797633
## logm1.5      0.03042    0.19192   0.159 0.874296
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04343 on 126 degrees of freedom
## Multiple R-squared:  0.9894, Adjusted R-squared:  0.9884
## F-statistic: 977.9 on 12 and 126 DF,  p-value: < 2.2e-16
##
## AIC and BIC values for the model:
##         AIC       BIC
## 1 -463.1393 -422.0566
```

```
model.dlmberganda2 = dlm(formula = logprice ~ interest + logm1,
                         data = data.frame(data1) , q = 1)
summary(model.dlmberganda2)
```

```
##
## Call:
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.134002 -0.044697  0.006407  0.036962  0.113063
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -7.77917    0.13299 -58.492  < 2e-16 ***
## interest.t  -3.22103    0.94184  -3.420 0.000824 ***
## interest.1   6.52775    0.94501   6.908 1.66e-10 ***
## logm1.t      0.73918    0.08419   8.780 5.61e-15 ***
## logm1.1      0.63330    0.08429   7.513 6.55e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05443 on 138 degrees of freedom
```

```
## Multiple R-squared:  0.9832, Adjusted R-squared:  0.9828
## F-statistic:  2025 on 4 and 138 DF,  p-value: < 2.2e-16
##
## AIC and BIC values for the model:
##         AIC       BIC
## 1 -419.7575 -401.9805
```

ARDL

```
#Mencari orde lag optimum model ARDL
ardlBoundOrders(data = data1 , formula = logprice ~ interest + logm1,
                ic="AIC")
```

```
## $p
##    interest logm1
## 65        0     4
##
## $q
## [1] 4
##
## $Stat.table
##             q = 1     q = 2     q = 3     q = 4     q = 5     q = 6     q = 7
## p = 1  -760.1786 -757.9195 -846.8342 -975.2079 -965.7536 -958.9072 -956.7315
## p = 2  -760.0433 -759.3090 -843.6247 -971.2514 -961.7929 -955.2809 -953.4890
## p = 3  -753.7746 -753.7746 -841.2485 -970.4543 -961.4343 -953.7173 -950.0412
## p = 4  -829.8076 -832.6436 -832.6436 -971.0837 -962.1804 -955.0429 -953.4667
## p = 5  -749.4144 -753.2292 -962.9290 -962.9290 -961.7063 -954.3406 -951.7660
## p = 6  -742.2103 -742.9945 -891.6195 -952.3771 -952.3771 -952.2461 -950.1105
## p = 7  -728.9374 -733.0286 -851.2943 -945.7445 -944.6879 -944.6879 -949.3720
## p = 8  -747.9277 -746.2948 -812.4289 -937.9446 -938.9491 -937.3393 -937.3393
## p = 9  -722.6891 -724.5786 -863.2734 -928.9215 -927.2914 -926.8716 -936.6432
## p = 10 -714.8175 -714.5658 -816.3319 -918.5218 -918.6350 -916.9076 -921.1246
## p = 11 -703.1807 -705.3383 -794.0772 -909.6457 -908.8225 -906.9542 -912.9605
## p = 12 -716.7111 -714.7403 -774.0127 -910.0315 -910.6834 -908.7146 -909.6612
## p = 13 -697.7175 -698.1931 -793.4602 -895.5927 -894.9273 -893.5995 -897.7589
## p = 14 -686.5600 -685.7967 -766.5292 -886.0709 -885.4341 -885.2283 -890.1638
## p = 15 -676.7280 -678.3689 -753.2854 -875.6392 -874.1257 -874.3117 -879.2727
##             q = 8     q = 9    q = 10    q = 11    q = 12    q = 13    q = 14
## p = 1  -954.3375 -946.6293 -936.5328 -927.7728 -920.6435 -917.5463 -918.3110
## p = 2  -951.1470 -943.9360 -933.7047 -924.7949 -917.5334 -913.6213 -914.4063
## p = 3  -948.4683 -941.1039 -930.8509 -922.0563 -914.5728 -910.5351 -913.4996
## p = 4  -948.2330 -941.8238 -931.5689 -923.2663 -916.2063 -911.6023 -913.9345
## p = 5  -947.5994 -939.3767 -929.0155 -920.4475 -913.5968 -909.0781 -911.6312
## p = 6  -945.5758 -937.4076 -927.2439 -919.3949 -911.9537 -907.7394 -910.2890
## p = 7  -945.5181 -937.1826 -926.9640 -917.9619 -910.2774 -905.9449 -907.8712
## p = 8  -941.9617 -933.5959 -923.3691 -914.6251 -907.0608 -902.2187 -903.9255
## p = 9  -936.6432 -935.7172 -925.2881 -917.0877 -911.6973 -903.9027 -904.6405
## p = 10 -926.6891 -926.6891 -924.6986 -917.0904 -911.4197 -903.4313 -903.0612
## p = 11 -917.9145 -918.2328 -918.2328 -919.2867 -913.3674 -904.8733 -903.6541
## p = 12 -916.1321 -914.4362 -914.4610 -914.4610 -912.5159 -904.2394 -901.6216
## p = 13 -905.4744 -903.7559 -902.4406 -902.2530 -902.2530 -902.9434 -901.2363
## p = 14 -896.2370 -896.2620 -894.2896 -897.5711 -899.1407 -899.1407 -902.2350
## p = 15 -884.5637 -886.8221 -884.9832 -890.5665 -893.2335 -891.6220 -891.6220
##            q = 15
```

```
## p = 1  -908.0863
## p = 2  -904.1665
## p = 3  -903.3006
## p = 4  -903.9256
## p = 5  -901.6220
## p = 6  -900.1824
## p = 7  -897.9867
## p = 8  -894.1031
## p = 9  -894.7387
## p = 10 -893.6199
## p = 11 -893.6060
## p = 12 -892.4805
## p = 13 -892.5115
## p = 14 -893.6214
## p = 15 -891.3741
##
## $min.Stat
## [1] -977.2745
##
## $Stat.p
##     interest logm1      Stat
## 65         0     4 -977.2745
## 1          0     0 -976.5191
## 2          1     0 -976.2558
## 17         0     1 -975.9606
## 66         1     4 -975.6027
## 18         1     1 -975.2079
## 49         0     3 -974.4859
## 3          2     0 -974.4275
## 33         0     2 -974.0166
## 50         1     3 -973.7500
## 67         2     4 -973.6028
## 34         1     2 -973.2324
## 19         2     1 -973.2188
## 68         3     4 -972.5992
## 4          3     0 -972.4875
## 51         2     3 -971.7743
## 20         3     1 -971.3872
## 35         2     2 -971.2514
## 69         4     4 -971.0837
## 5          4     0 -970.5114
## 52         3     3 -970.4543
## 81         0     5 -969.9284
## 53         4     3 -969.5311
## 21         4     1 -969.4756
## 36         3     2 -969.3907
## 82         1     5 -968.6783
## 37         4     2 -967.4756
## 83         2     5 -966.8835
## 84         3     5 -965.6393
## 85         4     5 -963.9662
## 86         5     5 -962.9290
## 70         5     4 -961.2547
## 54         5     3 -960.9580
```

```
## 97          0     6 -960.7402
## 6           5     0 -960.6858
## 22          5     1 -959.8419
## 98          1     6 -959.6604
## 38          5     2 -957.8547
## 99          2     6 -957.7528
## 100         3     6 -956.7875
## 101         4     6 -955.2416
## 71          6     4 -954.8953
## 87          6     5 -954.6855
## 102         5     6 -954.3662
## 103         6     6 -954.0973
## 7           6     0 -954.0615
## 113         0     7 -953.9160
## 55          6     3 -953.2860
## 23          6     1 -953.1080
## 114         1     7 -952.6540
## 39          6     2 -951.1356
## 115         2     7 -950.6562
## 116         3     7 -949.6038
## 88          7     5 -949.2090
## 72          7     4 -948.5194
## 117         4     7 -947.7999
## 104         7     6 -947.7424
## 56          7     3 -947.6915
## 8           7     0 -947.5092
## 120         7     7 -947.3660
## 24          7     1 -947.0094
## 118         5     7 -946.9631
## 119         6     7 -946.8080
## 40          7     2 -945.0123
## 129         0     8 -943.9035
## 130         1     8 -942.6627
## 131         2     8 -940.6818
## 145         0     9 -940.0114
## 132         3     8 -939.6913
## 89          8     5 -939.1878
## 73          8     4 -938.5330
## 146         1     9 -938.2680
## 133         4     8 -937.8368
## 105         8     6 -937.6834
## 57          8     3 -937.6370
## 9           8     0 -937.5705
## 121         8     7 -937.5351
## 136         7     8 -937.3948
## 25          8     1 -937.0088
## 134         5     8 -936.9393
## 135         6     8 -936.8904
## 147         2     9 -936.3875
## 148         3     9 -936.3159
## 137         8     8 -935.5389
## 41          8     2 -935.0088
## 149         4     9 -934.3458
## 150         5     9 -934.1858
```

```
## 152         7     9 -934.0733
## 151         6     9 -932.9538
## 153         8     9 -932.3338
## 154         9     9 -930.9065
## 161         0    10 -929.8056
## 90          9     5 -929.2731
## 74          9     4 -928.5254
## 162         1    10 -928.1257
## 10          9     0 -927.9853
## 58          9     3 -927.9744
## 122         9     7 -927.9061
## 106         9     6 -927.6344
## 26          9     1 -927.4482
## 164         3    10 -926.5271
## 163         2    10 -926.2965
## 138         9     8 -926.1307
## 42          9     2 -925.4484
## 165         4    10 -924.5287
## 168         7    10 -924.2716
## 166         5    10 -924.0521
## 167         6    10 -922.7596
## 169         8    10 -922.5928
## 155        10     9 -921.2169
## 170         9    10 -921.1777
## 177         0    11 -920.2608
## 171        10    10 -920.0124
## 91         10     5 -919.0182
## 178         1    11 -918.7342
## 75         10     4 -918.4135
## 11         10     0 -917.8597
## 59         10     3 -917.7711
## 123        10     7 -917.6569
## 107        10     6 -917.3861
## 27         10     1 -917.2925
## 179         2    11 -916.9417
## 180         3    11 -916.8682
## 193         0    12 -916.1477
## 139        10     8 -915.9643
## 92         11     5 -915.3201
## 43         10     2 -915.2941
## 156        11     9 -915.0851
## 181         4    11 -914.8854
## 194         1    12 -914.4423
## 124        11     7 -914.3141
## 184         7    11 -914.1880
## 76         11     4 -914.1395
## 182         5    11 -914.0440
## 108        11     6 -913.4052
## 140        11     8 -913.3026
## 195         2    12 -913.1680
## 172        11    10 -913.0914
## 60         11     3 -912.7714
## 183         6    11 -912.7548
## 196         3    12 -912.5820
```

```
## 185          8    11 -912.5636
## 12          11     0 -912.2009
## 28          11     1 -912.0389
## 186          9    11 -911.1737
## 157         12     9 -911.1513
## 188         11    11 -911.1189
## 93          12     5 -910.7693
## 198          5    12 -910.7434
## 197          4    12 -910.6154
## 125         12     7 -910.5873
## 141         12     8 -910.0719
## 44          11     2 -910.0439
## 187         10    11 -909.9928
## 200          7    12 -909.4197
## 173         12    10 -909.2473
## 77          12     4 -909.1913
## 109         12     6 -908.7753
## 199          6    12 -908.7635
## 201          8    12 -908.1609
## 61          12     3 -908.0357
## 29          12     1 -907.8613
## 209          0    13 -907.6473
## 13          12     0 -907.6158
## 205         12    12 -907.5931
## 204         11    12 -907.5525
## 202          9    12 -907.3633
## 189         12    11 -907.3200
## 210          1    13 -906.1005
## 45          12     2 -905.9070
## 203         10    12 -905.7653
## 211          2    13 -904.7293
## 212          3    13 -903.9077
## 214          5    13 -902.0824
## 158         13     9 -901.9574
## 213          4    13 -901.9144
## 94          13     5 -901.6338
## 126         13     7 -901.3766
## 142         13     8 -900.9367
## 216          7    13 -900.5676
## 225          0    14 -900.5066
## 174         13    10 -900.1413
## 215          6    13 -900.1102
## 78          13     4 -900.0282
## 110         13     6 -899.6703
## 226          1    14 -899.0967
## 217          8    13 -899.0866
## 62          13     3 -898.8589
## 30          13     1 -898.7940
## 190         13    11 -898.4409
## 221         12    13 -898.4110
## 220         11    13 -898.3058
## 218          9    13 -898.2568
## 14          13     0 -898.2039
## 206         13    12 -897.9014
```

```
## 227          2     14 -897.3889
## 46          13      2 -896.8637
## 219         10     13 -896.6244
## 222         13     13 -896.4458
## 228          3     14 -896.2512
## 230          5     14 -895.1320
## 95          14      5 -894.6021
## 229          4     14 -894.3023
## 159         14      9 -894.2497
## 127         14      7 -893.9663
## 143         14      8 -893.6932
## 231          6     14 -893.4037
## 79          14      4 -893.1343
## 232          7     14 -893.1064
## 111         14      6 -892.6253
## 175         14     10 -892.5085
## 63          14      3 -891.9131
## 191         14     11 -891.1895
## 233          8     14 -891.1877
## 234          9     14 -891.1729
## 31          14      1 -890.7573
## 236         11     14 -890.5576
## 241          0     15 -890.5500
## 15          14      0 -890.3449
## 237         12     14 -890.1854
## 235         10     14 -889.8957
## 207         14     12 -889.7107
## 242          1     15 -889.0419
## 47          14      2 -888.9410
## 238         13     14 -888.1867
## 223         14     13 -887.7488
## 239         14     14 -887.6659
## 243          2     15 -887.3088
## 244          3     15 -886.0691
## 246          5     15 -884.7479
## 96          15      5 -884.2869
## 245          4     15 -884.1417
## 160         15      9 -883.9364
## 128         15      7 -883.6409
## 144         15      8 -883.4503
## 247          6     15 -883.0158
## 80          15      4 -882.8148
## 248          7     15 -882.7881
## 112         15      6 -882.3106
## 176         15     10 -882.2093
## 64          15      3 -881.6497
## 253         12     15 -881.4274
## 252         11     15 -881.3077
## 250          9     15 -881.1831
## 192         15     11 -880.9028
## 249          8     15 -880.8964
## 32          15      1 -880.5983
## 251         10     15 -880.2736
## 16          15      0 -880.2468
```

```
## 254       13    15 -879.4467
## 208       15    12 -879.4364
## 255       14    15 -879.2846
## 48        15     2 -878.8432
## 224       15    13 -877.4985
## 240       15    14 -877.4570
```

```
model.ardlDlmberganda = ardlDlm(formula = logprice ~ interest + logm1,
                        data = data.frame(data1) , p = 4 , q = 4)
summary(model.ardlDlmberganda)
```

```
##
## Time series regression with "ts" data:
## Start = 5, End = 144
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0290527 -0.0075965  0.0005726  0.0072745  0.0304486
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0145022  0.1822785   0.080  0.93671
## interest.t   0.0067985  0.2135315   0.032  0.97465
## interest.1   0.6093502  0.3240545   1.880  0.06238 .
## interest.2   0.0798544  0.3221168   0.248  0.80461
## interest.3  -0.3638172  0.3238873  -1.123  0.26347
## interest.4   0.2084240  0.2447331   0.852  0.39604
## logm1.t      0.0828689  0.0457486   1.811  0.07248 .
## logm1.1     -0.0092841  0.0399079  -0.233  0.81642
## logm1.2     -0.1166129  0.0390732  -2.984  0.00342 **
## logm1.3      0.0007016  0.0389297   0.018  0.98565
## logm1.4      0.0447857  0.0425474   1.053  0.29455
## logprice.1   0.3274245  0.0651574   5.025  1.7e-06 ***
## logprice.2   0.1323801  0.0684485   1.934  0.05537 .
## logprice.3  -0.1448245  0.0674268  -2.148  0.03365 *
## logprice.4   0.6730871  0.0636443  10.576  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01132 on 125 degrees of freedom
## Multiple R-squared:  0.9993, Adjusted R-squared:  0.9992
## F-statistic: 1.273e+04 on 14 and 125 DF,  p-value: < 2.2e-16
```

```
#model p interest 0 p logm1 4
rem.p = list(interest = c(1,2,3,4))
remove = list(p = rem.p)
model.ardlDlmberganda2 = ardlDlm(formula = logprice ~ interest + logm1,
                         data = data.frame(data1) , p = 4 , q = 4 ,
                         remove = remove)
summary(model.ardlDlmberganda2)
```

```
##
```

```
## Time series regression with "ts" data:
## Start = 5, End = 144
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0290369 -0.0083445  0.0009024  0.0079199  0.0303652
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.174838   0.133708   1.308  0.19333
## interest.t   0.448826   0.098736   4.546 1.24e-05 ***
## logm1.t      0.056659   0.043836   1.293  0.19849
## logm1.1     -0.017025   0.039159  -0.435  0.66446
## logm1.2     -0.118413   0.037399  -3.166  0.00193 **
## logm1.3     -0.006454   0.038112  -0.169  0.86580
## logm1.4      0.060220   0.040337   1.493  0.13789
## logprice.1   0.319059   0.062107   5.137 1.00e-06 ***
## logprice.2   0.111794   0.066101   1.691  0.09320 .
## logprice.3  -0.122129   0.065114  -1.876  0.06297 .
## logprice.4   0.699061   0.062611  11.165  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01149 on 129 degrees of freedom
## Multiple R-squared:  0.9993, Adjusted R-squared:  0.9992
## F-statistic: 1.73e+04 on 10 and 129 DF,  p-value: < 2.2e-16
```

Proses selanjutnya sama dengan pemodelan menggunakan peubah tunggal.