# METODE PEMULUSAN WINTER ADITIF & WINTER MULTIPLIKATIF

Pertemuan ke-4
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# OUTLINE

1. Pemulusan untuk Data Musiman
2. Pemulusan Winter Aditif
3. Pemulusan Winter Multiplikatif

# OUTLINE

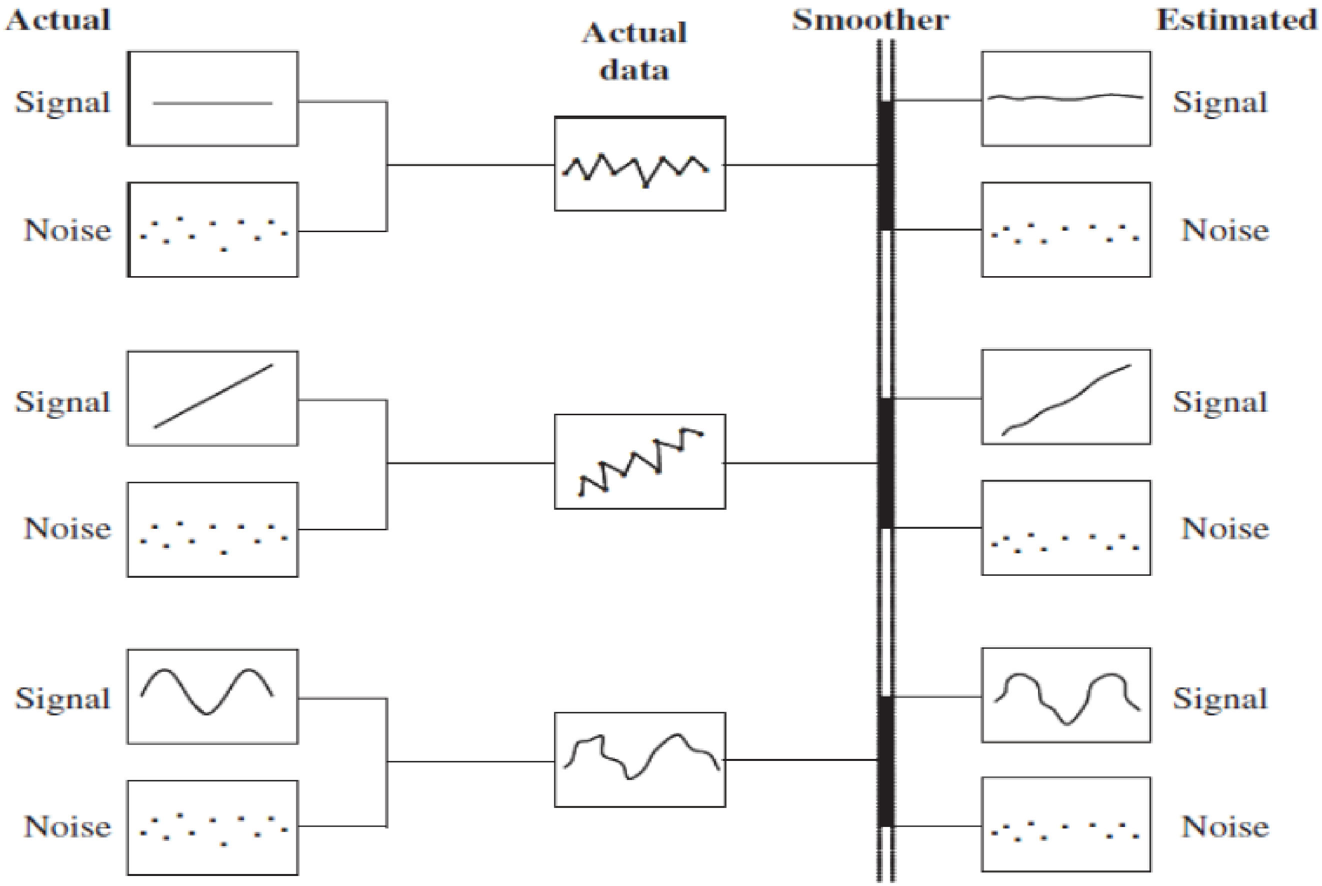
1. Pemulusan untuk Data Musiman

2. Pemulusan Winter Aditif

3. Pemulusan Winter Multiplikatif

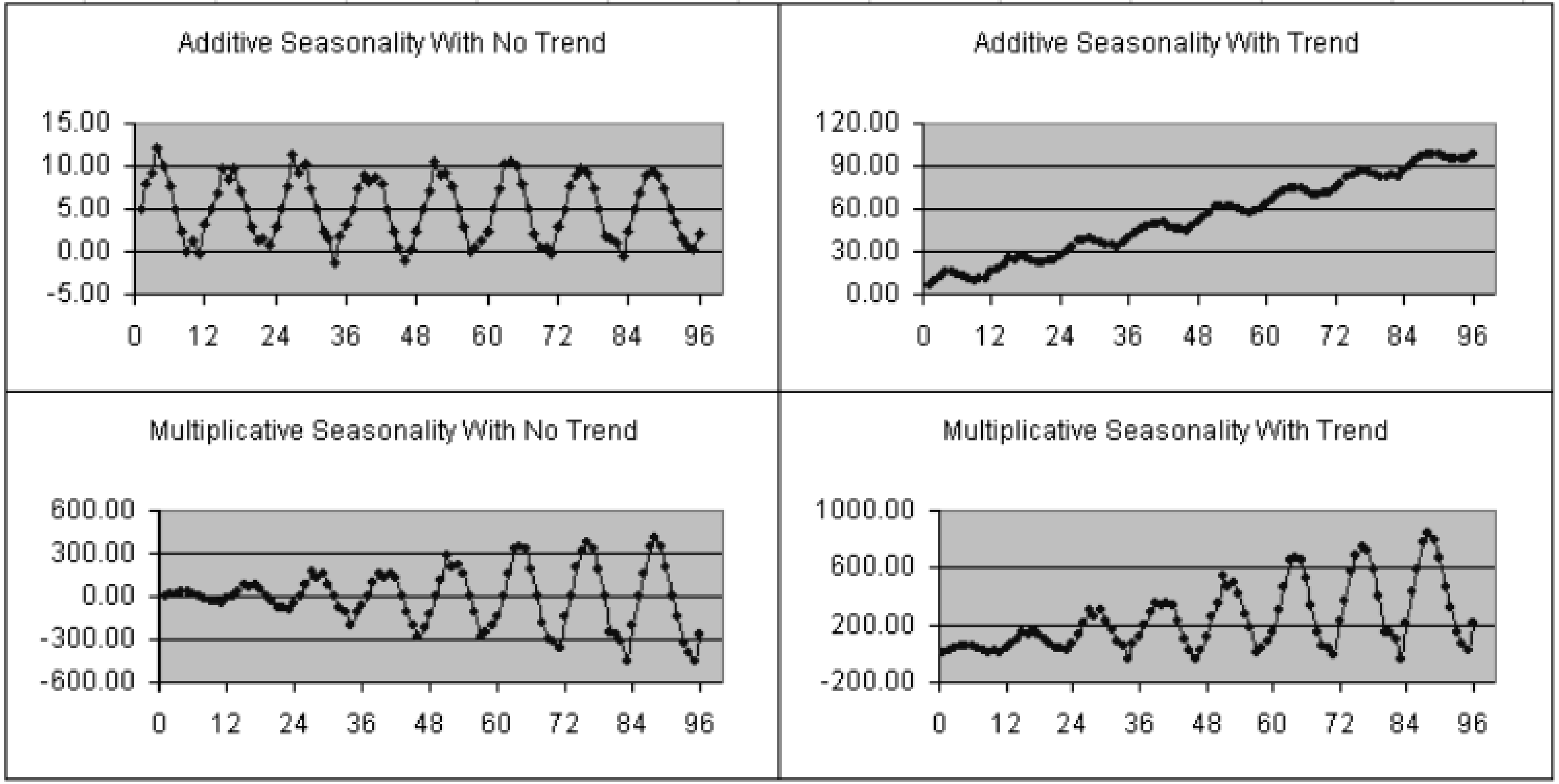
# 1. PEMULUSAN UNTUK DATA MUSIMAN

## REVIEW PEMULUSAN



# 1. PEMULUSAN UNTUK DATA MUSIMAN

## DATA MUSIMAN



# 1. PEMULUSAN UNTUK DATA MUSIMAN

## ILUSTRASI

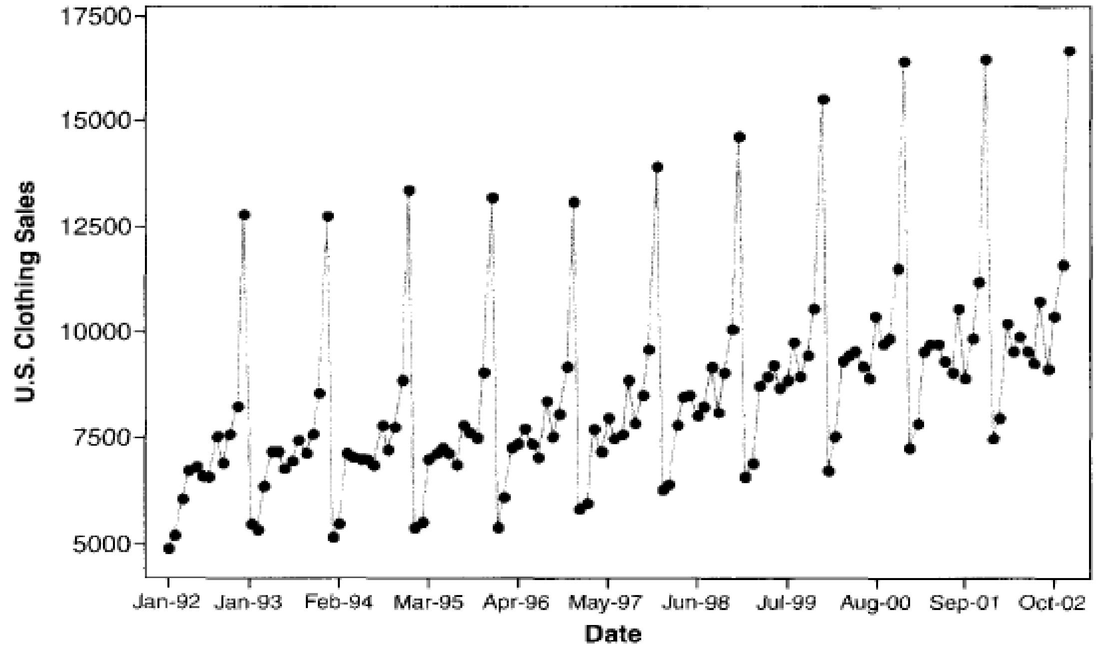



**FIGURE 4.26** Time series plot of U.S. clothing sales from January 1992 to December 2003.

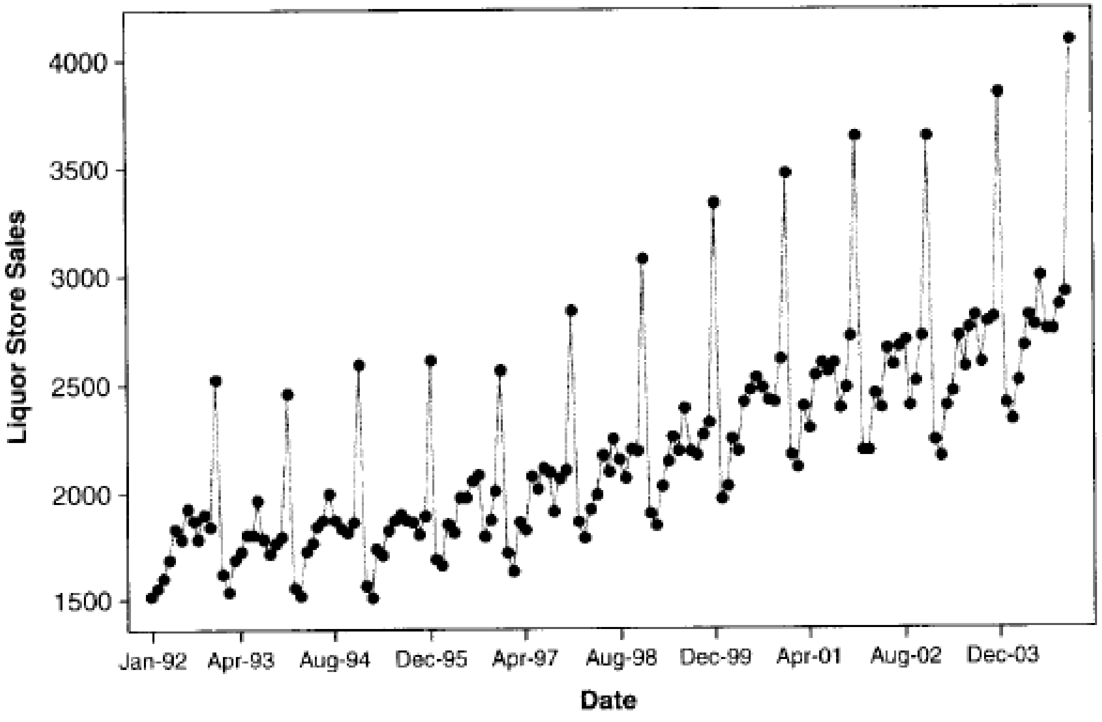



**FIGURE 4.29** Time series plot of liquor store sales data from January 1992 to December 2004.

# 1. PEMULUSAN UNTUK DATA MUSIMAN

## EXPONENTIAL SMOOTHING FOR SEASONAL DATA

- Originally introduced by Holt (1957)and Winters (1960)
- Generally known as Winters'method
- Basic idea: seasonal adjustment -> linear trend model
- Two types of adjustments are suggested:
    1. Additive
    2. Multiplicative

# OUTLINE

1. Pemulusan untuk Data Musiman

2. Pemulusan Winter Aditif

3. Pemulusan Winter Multiplikatif

# 2. PEMULUSAN WINTER ADITIF

## Additive Model

$$y_t = \boxed{L_t} + \boxed{S_t} + \varepsilon_t$$

level or linear trend component
can in turn be represented by $\beta_0 + \beta_1 t$

the seasonal adjustment

$S_t = S_{t+m} = S_{t+2m} = \ldots$ for $t = 1, \ldots, m - 1$

length of the season (period) of the cycles

$$\sum_{t=1}^{m} S_t = 0.$$

# 2. PEMULUSAN WINTER ADITIF

## Double Exponential Vs Additive Holt-Winter's Method

Double Exponential: $\hat{y}_{t+h}(t) = \underbrace{L_t}_{\text{Level}} + \underbrace{T_t}_{\text{Trend}} h$

Holt-Winter: $\hat{y}_{t+h}(t) = \underbrace{L_t}_{\text{Level}} + \underbrace{B_t}_{\text{Trend}} h + \underbrace{S_{t+h-m}}_{\text{Seasonal}}$

Holt Winter≈Triple Exponential Smoothing

# 2. PEMULUSAN WINTER ADITIF

## Holt-Winters Additive Formulation

- Suppose the time series is denoted by $y_1, \ldots, y_n$ with $m$ seasonal period.

Estimate of the level:

$$l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$$

Estimate of the trend:

$$b_t = \gamma(l_t - l_{t-1}) + (1-\gamma)b_{t-1}$$

Estimate of the seasonal factor:

$$s_t = \delta(y_t - l_t) + (1-\delta)s_{t-m}$$

- Let $\hat{y}_{t+h}(t)$ be the $h$-step forecast made using data to time $t$

$$\hat{y}_{t+h}(t) = l_t + b_t h + s_{t+h-m}$$

# 2. PEMULUSAN WINTER ADITIF

## The Procedure

Step 1: Initialize the value of $l_t$ , $b_t$, and $s_t$

Step 2: Update the estimate of $l_t$

Step 3: Update the estimate of $b_t$

Step 4: Update the estimate of $s_t$

Step 5: Conduct the $h$-step-ahead forecast

# 2. PEMULUSAN WINTER ADITIF

## Initializing the Holt-Winters method

**Montgomery (2015):**

use the least squares estimates of the following model:

$$y_t = \underbrace{\beta_0}_{l_0} + \underbrace{\beta_1}_{b_0} t + \sum_{i=1}^{s-1} \gamma_i (I_{t,i} - I_{t,s}) + \varepsilon_t,$$

where

$$I_{t,i} = \begin{cases} 1, & t = i, i+s, i+2s, \ldots \\ 0, & \text{otherwise} \end{cases}.$$

$\hat{s}_{j-s} = \hat{y}_j$ for $1 \le j \le m-1$ , and $\hat{s}_0 = -\sum_{j=1}^{m-1} \hat{y}_j$

# 2. PEMULUSAN WINTER ADITIF

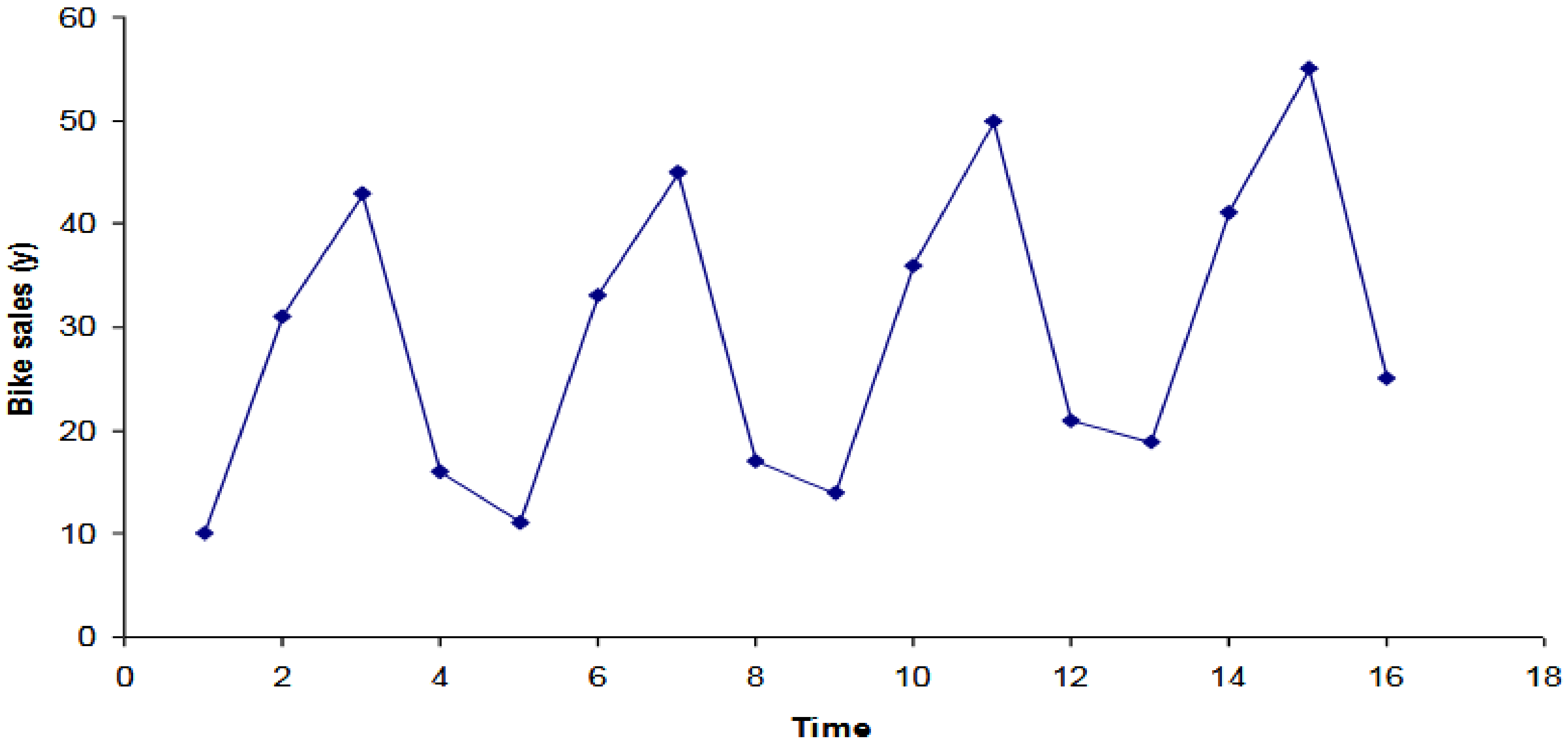
## Procedures of Additive Holt-Winters Method

Consider the Mountain Bike example,

| Quarterly sales of the TRK-50 Mountain Bike | | | | |
|---|---|---|---|---|
| | Year | | | |
| **Quarter** | **1** | **2** | **3** | **4** |
| 1 | 10 | 11 | 14 | 19 |
| 2 | 31 | 33 | 36 | 41 |
| 3 | 43 | 45 | 50 | 55 |
| 4 | 16 | 17 | 21 | 25 |

# 2. PEMULUSAN WINTER ADITIF

## Procedures of Additive Holt-Winters Method




Observations:

- Linear upward trend over the 4-year period
- Magnitude of seasonal span is almost constant as the level of the time series increases

  → *Additive Holt-Winters method can be applied to forecast future sales*

# 2. PEMULUSAN WINTER ADITIF

## Procedures of Additive Holt-Winters Method

**Step 1**: Obtain initial values for the level $\ell_0$, the growth rate $b_0$, and the seasonal factors $sn_{-3}$, $sn_{-2}$, $sn_{-1}$, and $sn_0$, by fitting a least squares trend line to at least four or five years of the historical data.

- $y$-intercept = $\ell_0$; slope = $b_0$

Example

- Fit a least squares trend line to all 16 observations
- Trend line

$$\hat{y}_t = 20.85 + 0.980882t$$

$\ell_0 = 20.85$; $b_0 = 0.9809$

| SUMMARY OUTPUT | |
|---|---|
| | |
| *Regression Statistics* | |
| Multiple R | 0.320508842 |
| R Square | 0.102725918 |
| Adjusted R Square | 0.038634912 |
| Standard Error | 14.28614022 |
| Observations | 16 |
| | |
| ANOVA | |
| | *df* |
| Regression | 1 |
| Residual | 14 |
| Total | 15 |
| | |
| | *Coefficients* |
| Intercept | 20.85 |
| Time | 0.980882353 |

# 2. PEMULUSAN WINTER ADITIF

## Procedures of Additive Holt-Winters Method

**Step 2**: Find the initial seasonal factors

1. Compute $\hat{y}_t$ for each time period that is used in finding the least squares regression equation. In this example, $t$ = 1, 2, ..., 16.

$$\hat{y}_1 = 20.85 + 0.980882(1) = 21.8309$$
$$\hat{y}_2 = 20.85 + 0.980882(2) = 22.8118$$
......
$$\hat{y}_{16} = 20.85 + 0.980882(16) = 36.5441$$

**Step 2**: Find the initial seasonal factors

2. Detrend the data by computing $S_t = y_t - \hat{y}_t$ for each observation used in the least squares fit. In this example, $t$ = 1, 2, ..., 16.

$$S_1 = y_1 - \hat{y}_1 = 10 - 21.8309 = -11.8309$$
$$S_2 = y_2 - \hat{y}_2 = 31 - 22.8112 = 8.1882$$
......
$$S_{16} = y_{16} - \hat{y}_{16} = 25 - 36.5441 = -11.5441$$

**Step 2**: Find the initial seasonal factors

3. Compute the average seasonal values for each of the $L$ seasons. The $L$ averages are found by computing the average of the detrended values for the corresponding season. For example, for quarter 1,

$$\bar{S}_{[1]} = \frac{S_1 + S_5 + S_9 + S_{13}}{4}$$
$$= \frac{(-11.8309) + (-14.7544) + (-15.6779) + (-14.6015)}{4} = -14.2162$$

**Step 2**: Find the initial seasonal factors

4. Compute the average of the $L$ seasonal factors. The average should be 0.

# 2. PEMULUSAN WINTER ADITIF

## Procedures of Additive Holt-Winters Method

**Step 3**: Calculate a point forecast of $y_1$ from time 0 using the initial values

$$\hat{y}_{T+p}(T) = \lambda_T + pb_T + sn_{T+p-L} \qquad (T = 0, p = 1)$$

$$\hat{y}_1(0) = \lambda_0 + b_0 + sn_{1-4} = \lambda_0 + b_0 + sn_{-3}$$
$$= 20.85 + 0.9809 + (-14.2162) = 7.6147$$

**Step 4**: Update the estimates $\ell_T$, $b_T$, and $sn_T$ by using some predetermined values of smoothing constants.

Example: let $\alpha = 0.2$, $\gamma = 0.1$, and $\delta = 0.1$

$$\lambda_1 = \alpha(y_1 - sn_{1-4}) + (1-\alpha)(\lambda_0 + b_0)$$
$$= 0.2(10 - (-14.2162)) + 0.8(20.85 + 0.9808) = 22.3079$$

$$b_1 = \gamma(\lambda_1 - \lambda_0) + (1-\gamma)b_0$$
$$= 0.1(22.3079 - 20.85) + 0.9(0.9809) = 1.0286$$

$$sn_1 = \delta(y_1 - \lambda_1) + (1-\delta)sn_{1-4}$$
$$= 0.1(10 - 22.3079) + 0.9(-14.2162) = -14.0254$$

$$\hat{y}_2(1) = \lambda_1 + b_1 + sn_{2-4} = \lambda_1 + b_1 + sn_{-2}$$
$$= 22.3079 + 1.0286 + 6.5529 = 29.8895$$

# 2. PEMULUSAN WINTER ADITIF

## Procedures of Additive Holt-Winters Method

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | n | alpha | gamma | delta | SSE | MSE | s | |
| 2 | 16 | 0.2000 | 0.1000 | 0.1000 | 25.2166 | 1.9397 | 1.3927 | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | Forecast | | Squared |
| 6 | | | | Growth | Seasonal | Made Last | Forecast | Forecast |
| 7 | Time | y | Level | Rate | Factor | Period | Error | Error |
| 8 | -3 | | | | -14.2162 | | | |
| 9 | -2 | | | | 6.5529 | | | |
| 10 | -1 | | | | 18.5721 | | | |
| 11 | 0 | | 20.85 | 0.9809 | -10.9088 | | | |
| 12 | 1 | 10 | 22.30794 | 1.0286 | -14.0254 | 7.6147 | 2.3853 | 5.6896 |
| 13 | 2 | 31 | 23.55864 | 1.0508 | 6.6418 | 29.8895 | 1.1105 | 1.2333 |
| 14 | 3 | 43 | 24.57314 | 1.0472 | 18.5575 | 43.1815 | -0.1815 | 0.0329 |
| 15 | 4 | 16 | 25.87801 | 1.0729 | -10.8057 | 14.7115 | 1.2885 | 1.6603 |
| 16 | 5 | 11 | 26.56583 | 1.0344 | -14.1794 | 12.9256 | -1.9256 | 3.7079 |
| 17 | 6 | 33 | 27.35185 | 1.0096 | 6.5424 | 34.2420 | -1.2420 | 1.5427 |
| 18 | 7 | 45 | 27.97764 | 0.9712 | 18.4040 | 46.9190 | -1.9190 | 3.6825 |
| 19 | 8 | 17 | 28.72023 | 0.9483 | -10.8972 | 18.1431 | -1.1431 | 1.3067 |
| 20 | 9 | 14 | 29.37074 | 0.9186 | -14.2985 | 15.4892 | -1.4892 | 2.2176 |
| 21 | 10 | 36 | 30.12295 | 0.9019 | 6.4759 | 36.8317 | -0.8317 | 0.6918 |
| 22 | 11 | 50 | 31.1391 | 0.9133 | 18.4497 | 49.4289 | 0.5711 | 0.3262 |
| 23 | 12 | 21 | 32.0214 | 0.9102 | -10.9096 | 21.1553 | -0.1553 | 0.0241 |
| 24 | 13 | 19 | 33.00502 | 0.9176 | -14.2692 | 18.6331 | 0.3669 | 0.1346 |
| 25 | 14 | 41 | 34.04291 | 0.9296 | 6.5240 | 40.3985 | 0.6015 | 0.3618 |
| 26 | 15 | 55 | 35.28807 | 0.9612 | 18.5759 | 53.4222 | 1.5778 | 2.4894 |
| 27 | 16 | 25 | 36.18131 | 0.9544 | -10.9368 | 25.3396 | -0.3396 | 0.1153 |

# 2. PEMULUSAN WINTER ADITIF

## Procedures of Additive Holt-Winters Method

*p*-step-ahead forecast made at time *T*

$$\hat{y}_{T+p}(T) = \lambda_T + pb_T + sn_{T+p-L} \qquad (p = 1, 2, 3, \ldots)$$

Example

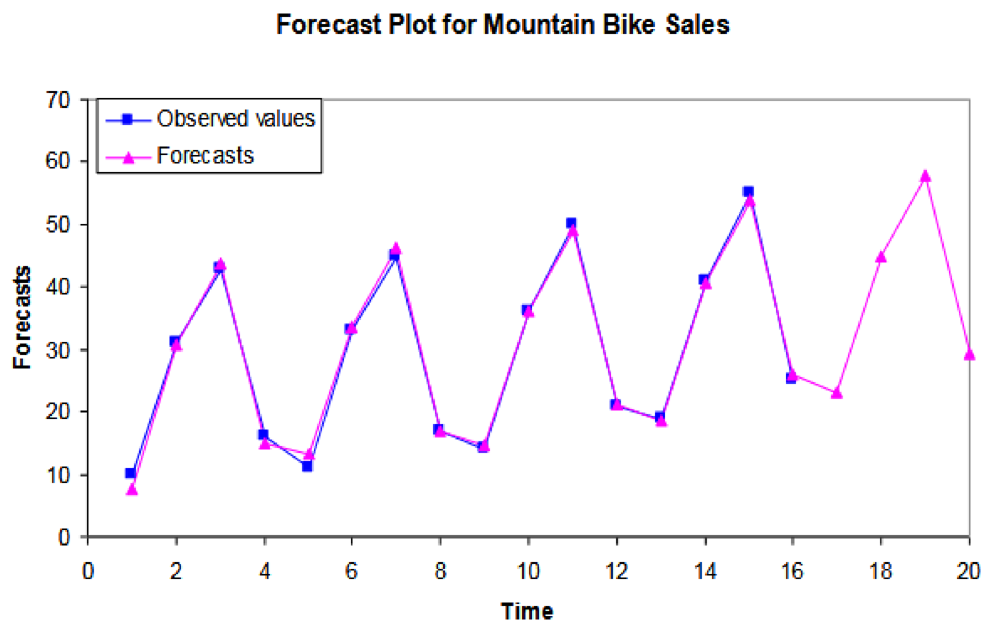
$$\hat{y}_{17}(16) = \lambda_{16} + b_{16} + sn_{17-4} = 36.3426 + 0.9809 - 14.2162 = 23.1073$$

$$\hat{y}_{18}(16) = \lambda_{16} + 2b_{16} + sn_{18-4} = 36.3426 + 2(0.9809) + 6.5529 = 44.8573$$

$$\hat{y}_{19}(16) = \lambda_{16} + 3b_{16} + sn_{19-4} = 36.3426 + 3(0.9809) + 18.5721 = 57.8573$$

$$\hat{y}_{20}(16) = \lambda_{16} + 4b_{16} + sn_{20-4} = 36.3426 + 4(0.9809) - 10.9088 = 29.3573$$

Example



# OUTLINE

1. Pemulusan untuk Data Musiman

2. Pemulusan Winter Aditif

3. Pemulusan Winter Multiplikatif

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Multiplicative ModeL

$$Y_t = (L_t)(S_t) + \varepsilon_t$$

level or linear trend component
can in turn be represented by $\beta_0 + \beta_1 t$

the seasonal adjustment

$S_t = S_{t+m} = S_{t+2m} = \ldots$ for $t = 1, \ldots, m - 1$

length of the season (period) of the cycles

$$\sum_{t=1}^{m} S_t = 0.$$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Holt-Winters Multiplicative Formulation

- Suppose the time series is denoted by $y_1, \dots, y_n$ with $m$ seasonal period.

Estimate of the level:

$$L_t = \alpha \left( \frac{Y_t}{S_{t-m}} \right) + (1-\alpha)(L_{t-1} + B_{t-1})$$

Estimate of the trend:

$$B_t = \gamma(L_t - L_{t-1}) + (1-\gamma)B_{t-1}$$

Estimate of the seasonal factor:

$$S_t = \delta \left( \frac{Y_t}{L_t} \right) + (1-\delta)S_{t-m}$$

- Let $\hat{Y}_{t+h}(t)$ be the $h$-step forecast made using data to time $t$

$$\hat{Y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m}$$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## The Procedure

Step 1: Initialize the value of $L_t$ , $B_t$, and $S_t$

Step 2: Update the estimate of $L_t$

Step 3: Update the estimate of $B_t$

Step 4: Update the estimate of $S_t$

Step 5: Conduct the $h$-step-ahead forecast

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Initializing the Holt-Winters method

Montgomery (2015):

Suppose a dataset consist of $k$ seasons.

- $\hat{L}_0 = \frac{\bar{y}_k - \bar{y}_1}{(k-1)m}$ where $\bar{y}_i = \frac{1}{m}\sum_{t=(i-1)m+1}^{im} y_t$
- $\hat{B}_0 = \bar{y}_1 - \frac{m}{2}\hat{L}_0$
- $\hat{S}_{j-m} = m\left(\frac{\hat{S}_j^*}{\sum_{i=1}^{m}\hat{S}_j^*}\right)$, for $1 \le j \le s$, where $\hat{S}_j^* = \frac{1}{k}\sum_{t=1}^{k}\frac{y_{(t-1)m+j}}{\bar{y}_t - \left(\frac{s+1}{2-j}\right)\hat{\beta}_0}$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

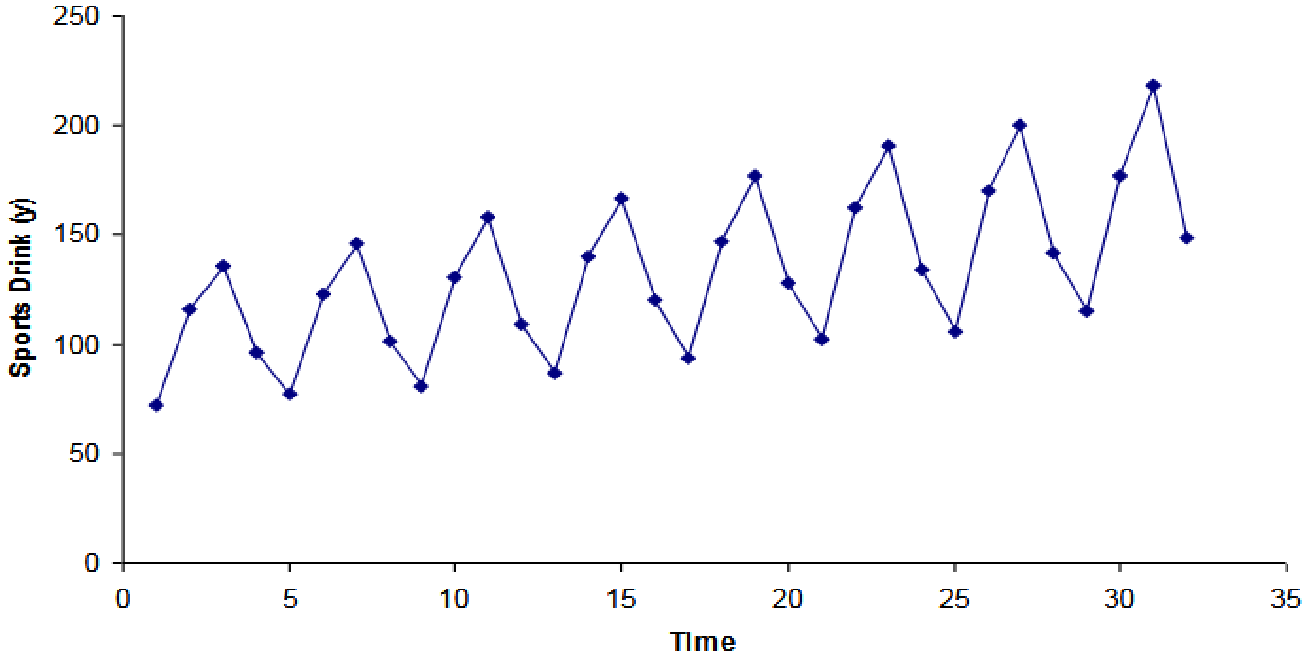
## Procedures of Multiplicative Holt-Winters Method

Use the Sports Drink example as an illustration

| Quarterly sales of Tiger Sports Drink | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Year | | | | | | | |
| **Quarter** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| 1 | 72 | 77 | 81 | 87 | 94 | 102 | 106 | 115 |
| 2 | 116 | 123 | 131 | 140 | 147 | 162 | 170 | 177 |
| 3 | 136 | 146 | 158 | 167 | 177 | 191 | 200 | 218 |
| 4 | 96 | 101 | 109 | 120 | 128 | 134 | 142 | 149 |

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method




Observations:

- Linear upward trend over the 8-year period
- Magnitude of the seasonal span increases as the level of the time series increases

⇒ Multiplicative Holt-Winters method can be applied to forecast future sales

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

**Step 1**: Obtain initial values for the level $\ell_0$, the growth rate $b_0$, and the seasonal factors $s_{-3}$, $s_{-2}$, $s_{-1}$, and $s_0$, by fitting a least squares trend line to at least four or five years of the historical data.

- $y$-intercept = $\ell_0$; slope = $b_0$

Example

- Fit a least squares trend line to the first 16 observations
- Trend line

$$\hat{y}_t = 95.2500 + 2.4706t$$

- $\ell_0 = 95.2500$; $b_0 = 2.4706$

| SUMMARY OUTPUT | |
|---|---|
| | |
| *Regression Statistics* | |
| Multiple R | 0.403809754 |
| R Square | 0.163062318 |
| Adjusted R Square | 0.103281055 |
| Standard Error | 27.58325823 |
| Observations | 16 |
| | |
| ANOVA | |
| | *df* |
| Regression | 1 |
| Residual | 14 |
| Total | 15 |
| | |
| | *Coefficients* |
| Intercept | 95.25 |
| X Variable 1 | 2.470588235 |

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

**Step 2**: Find the initial seasonal factors

1. Compute $\hat{y}_t$ for the in-sample observations used for fitting the regression. In this example, $t$ = 1, 2, ..., 16.

$$\hat{y}_1 = 95.2500 + 2.4706(1) = 97.7206$$
$$\hat{y}_2 = 95.2500 + 2.4706(2) = 100.1912$$
$$\ldots\ldots$$
$$\hat{y}_{16} = 95.2500 + 2.4706(16) = 134.7794$$

**Step 2**: Find the initial seasonal factors

2. Detrend the data by computing $S_{0,t} = y_t / \hat{y}_t$ for each time period that is used in finding the least squares regression equation. In this example, $t$ = 1, 2, ..., 16.

$$S_{0,1} = y_1 / \hat{y}_1 = 72 / 97.7206 = 0.7368$$
$$S_{0,2} = y_2 / \hat{y}_2 = 116 / 100.1912 = 1.1578$$
$$\ldots\ldots$$
$$S_{0,16} = y_{16} / \hat{y}_{16} = 120 / 134.7794 = 0.8903$$

**Step 2**: Find the initial seasonal factors

3. Compute the average seasonal values for each of the $k$ seasons. The $k$ averages are found by computing the average of the detrended values for the corresponding season. For example, for quarter 1,

$$\bar{S}_{[1]} = \frac{S_1 + S_5 + S_9 + S_{13}}{4} = \frac{0.7368 + 0.7156 + 0.6894 + 0.6831}{4} = 0.7062$$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

**Step 2**: Find the initial seasonal factors

4. Multiply the average seasonal values by the normalizing constant

$$CF = \frac{m}{\sum_{i=1}^{m} \bar{S}_{[i]}}$$

such that the average of the seasonal factors is 1. The initial seasonal factors are

$$S_{i-m} = \bar{S}_{[i]}(CF) \qquad (i = 1, 2, \ldots, L)$$

**Step 2**: Find the initial seasonal factors

4. Multiply the average seasonal values by the normalizing constant such that the average of the seasonal factors is 1.
   - Example

     *CF* = 4/3.9999 = 1.0000

$$S_{-3} = S_{1-4} = \bar{S}_{[1]}(CF) = 0.7062(1) = 0.7062$$

$$S_{-2} = S_{2-4} = \bar{S}_{[2]}(CF) = 1.1114(1) = 1.1114$$

$$S_{-1} = S_{3-4} = \bar{S}_{[3]}(CF) = 1.2937(1) = 1.2937$$

$$S_{0} = S_{4-4} = \bar{S}_{[1]}(CF) = 0.8886(1) = 0.8886$$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

**Step 3**: Calculate a point forecast of $y_1$ from time 0 using the initial values

$$\hat{y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m} \qquad (t = 1\,, h = 0\,)$$

$$\hat{y}_1(0) = (L_0 + B_0)S_{1-4} = (L_0 + B_0)S_{-3}$$

$$= (95.25 + 2.4706)0.7062$$

$$= 69.0103$$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

**Step 4**: Update the estimates $\ell_T$, $b_T$, and $sn_T$ by using some predetermined values of smoothing constants.

Example: let $\alpha = 0.2$, $\gamma = 0.1$, and $\delta = 0.1$

$$l_1 = \alpha(y_1 / s_{1-4}) + (1-\alpha)(l_0 + b_0)$$
$$= 0.2(72/0.7062) + 0.8(95.2500 + 2.4706) = 98.5673$$

$$b_1 = \gamma(l_1 - l_0) + (1-\gamma)b_0$$
$$= 0.1(98.5673 - 95.2500) + 0.9(2.4706) = 2.5553$$

$$sn_1 = \delta(y_1 / l_1) + (1-\delta)s_{1-4}$$
$$= 0.1(72/98.5673) + 0.9(0.7062) = 0.7086$$

$$\hat{y}_2(1) = (l_1 + b_1)s_{2-4}$$
$$= (98.5673 + 2.5553)(1.1114) = 112.3876$$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

$$\begin{aligned}\lambda_2 &= \alpha(y_2/s_{2-4}) + (1-\alpha)(\lambda_1 + b_1)\\ &= 0.2(116/1.1114) + 0.8(98.5673 + 2.5553)\\ &= 101.7727\end{aligned}$$

$$\begin{aligned}b_2 &= \gamma(\lambda_2 - \lambda_1) + (1-\gamma)b_1\\ &= 0.1(101.7727 - 98.5673) + 0.9(2.5553)\\ &= 2.62031\end{aligned}$$

$$\begin{aligned}sn_2 &= \delta(y_2/\lambda_2) + (1-\delta)s_{2-4}\\ &= 0.1(116/101.7727) + 0.9(1.1114)\\ &= 1.114239\end{aligned}$$

$$\begin{aligned}\hat{y}_3(2) &= (\lambda_2 + b_2)s_{3-4}\\ &= (101.7727 + 2.62031)(1.2937)\\ &= 135.053\end{aligned}$$

$$\begin{aligned}\lambda_4 &= \alpha(y_4/s_{4-4}) + (1-\alpha)(\lambda_3 + b_3)\\ &= 0.2(96/0.8886) + 0.8(104.5393 + 2.6349)\\ &= 107.3464\end{aligned}$$

$$\begin{aligned}b_4 &= \gamma(\lambda_4 - \lambda_3) + (1-\gamma)b_3\\ &= 0.1(107.3464 - 104.5393) + 0.9(2.6349)\\ &= 2.65212\end{aligned}$$

$$\begin{aligned}sn_4 &= \delta(y_4/\lambda_4) + (1-\delta)s_{4-4}\\ &= 0.1(96/107.3464) + 0.9(0.8886)\\ &= 0.889170\end{aligned}$$

$$\begin{aligned}\hat{y}_5(4) &= (\lambda_4 + b_4)s_{5-4}\\ &= (107.3464 + 2.65212)(0.7086)\\ &= 77.945\end{aligned}$$

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | n | alpha | gamma | delta | SSE | MSE | s | |
| 2 | 32 | 0.2 | 0.1 | 0.1 | 177.3223 | 6.1146 | 2.4728 | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | Forecast | | Squared |
| 6 | | | | Growth | Seasonal | Made Last | Forecast | Forecast |
| 7 | Time | y | Level | Rate | Factor | Period | Error | Error |
| 8 | -3 | | | | 0.7062 | | | |
| 9 | -2 | | | | 1.1114 | | | |
| 10 | -1 | | | | 1.2937 | | | |
| 11 | 0 | | 95.25 | 2.4706 | 0.8886 | | | |
| 12 | 1 | 72 | 98.56729 | 2.5553 | 0.7086 | 69.0103 | 2.9897 | 8.9384 |
| 13 | 2 | 116 | 101.7726 | 2.6203 | 1.1142 | 112.3876 | 3.6124 | 13.0494 |
| 14 | 3 | 136 | 104.5393 | 2.6349 | 1.2944 | 135.0531 | 0.9469 | 0.8967 |
| 15 | 4 | 96 | 107.3464 | 2.6521 | 0.8892 | 95.2350 | 0.7650 | 0.5853 |
| 16 | 5 | 77 | 109.731 | 2.6254 | 0.7079 | 77.9478 | -0.9478 | 0.8984 |
| 17 | 6 | 123 | 111.9629 | 2.5860 | 1.1127 | 125.1919 | -2.1919 | 4.8043 |
| 18 | 7 | 146 | 114.1974 | 2.5509 | 1.2928 | 148.2750 | -2.2750 | 5.1755 |
| 19 | 8 | 101 | 116.1165 | 2.4877 | 0.8872 | 103.8091 | -2.8091 | 7.8911 |
| 20 | 9 | 81 | 117.7668 | 2.4040 | 0.7059 | 83.9641 | -2.9641 | 8.7858 |
| 21 | 10 | 131 | 119.6835 | 2.3552 | 1.1109 | 133.7108 | -2.7108 | 7.3482 |
| 22 | 11 | 158 | 122.0734 | 2.3587 | 1.2930 | 157.7754 | 0.2246 | 0.0504 |
| 23 | 12 | 109 | 124.1164 | 2.3271 | 0.8863 | 110.4005 | -1.4005 | 1.9615 |
| 24 | 13 | 87 | 125.8035 | 2.2631 | 0.7045 | 89.2593 | -2.2593 | 5.1044 |
| 25 | 14 | 140 | 127.6589 | 2.2224 | 1.1094 | 142.2642 | -2.2642 | 5.1268 |
| 26 | 15 | 167 | 129.7369 | 2.2079 | 1.2924 | 167.9337 | -0.9337 | 0.8718 |
| | ...... | | | | | ...... | | |
| 38 | 27 | 200 | 156.1396 | 2.1752 | 1.2903 | 202.0396 | -2.0396 | 4.1601 |
| 39 | 28 | 142 | 158.5505 | 2.1988 | 0.8908 | 140.9508 | 1.0492 | 1.1008 |
| 40 | 29 | 115 | 161.2803 | 2.2519 | 0.7047 | 113.1314 | 1.8686 | 3.4918 |
| 41 | 30 | 177 | 162.8178 | 2.1804 | 1.1046 | 180.9529 | -3.9529 | 15.6252 |
| 42 | 31 | 218 | 165.7889 | 2.2595 | 1.2928 | 212.8988 | 5.1012 | 26.0220 |
| 43 | 32 | 149 | 167.8899 | 2.2437 | 0.8905 | 149.7057 | -0.7057 | 0.4981 |

# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Procedures of Multiplicative Holt-Winters Method

*p*-step-ahead forecast made at time *T*

$$\hat{y}_{t+h}(t) = (L_t + B_t h) S_{t+h-m} \qquad (h = 1, 2, 3, \dots)$$

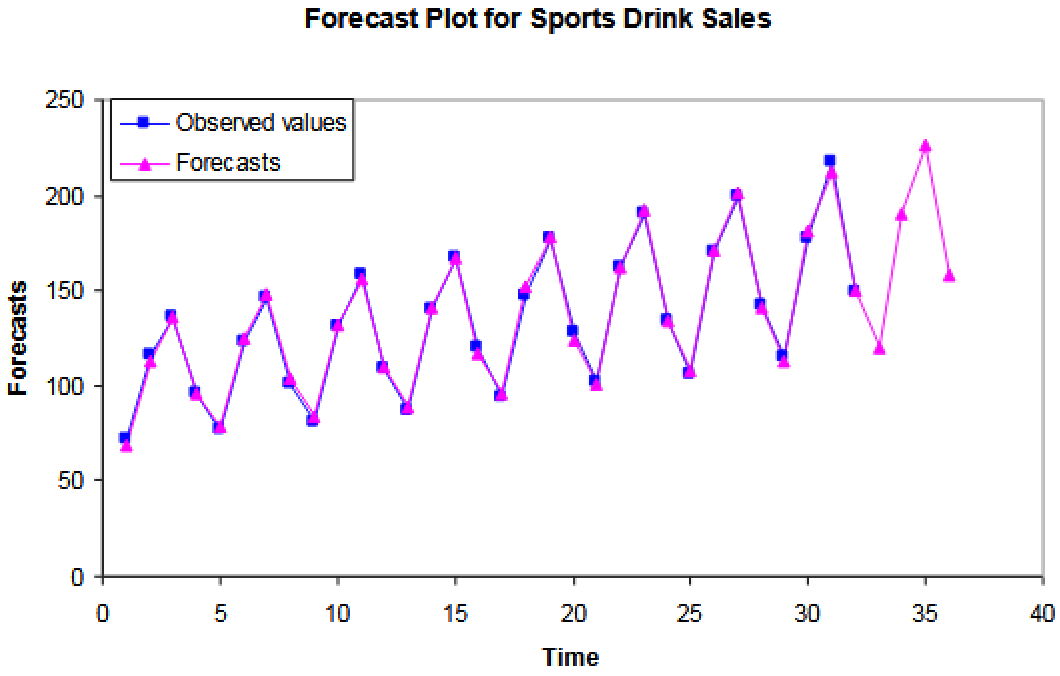
Example

$$\hat{y}_{33}(32) = (l_{32} + b_{32}) s_{33-4} = (168.1213 + 2.3028)(0.7044) = 120.0467$$

$$\hat{y}_{34}(32) = (l_{32} + 2b_{32}) s_{34-4} = [168.1213 + 2(2.3028)](1.1038) = 190.6560$$

$$\hat{y}_{35}(32) = (l_{32} + 3b_{32}) s_{35-4} = [(168.1213 + 3(2.3028)](1.2934) = 226.3834$$

$$\hat{y}_{36}(32) = (l_{32} + 4b_{32}) s_{36-4} = [(168.1213 + 4(2.3028)](0.8908) = 157.9678$$



# 3. PEMULUSAN WINTER MULTIPLIKATIF

## Holt-Winters Additive Vs Multiplicative Formulation

Suppose the time series is denoted by $y_1, \dots, y_n$ with $m$ seasonal period.

| | Additive | Multiplicative |
|---|---|---|
| Est. of level | $L_t = \alpha(Y_t - S_{t-m}) + (1-\alpha)(L_{t-1} + B_{t-1})$ | $L_t = \alpha\left(\frac{Y_t}{S_{t-m}}\right) + (1-\alpha)(L_{t-1} + B_{t-1})$ |
| Est. of trend | $B_t = \gamma(L_t - L_{t-1}) + (1-\gamma)B_{t-1}$ | $B_t = \gamma(L_t - L_{t-1}) + (1-\gamma)B_{t-1}$ |
| Est. of seasonal | $S_t = \delta(Y_t - L_t) + (1-\delta)S_{t-m}$ | $S_t = \delta\left(\frac{Y_t}{L_t}\right) + (1-\delta)S_{t-m}$ |
| Forecast | $\hat{Y}_{t+h}(t) = L_t + B_t h + S_{t+h-m}$ | $\hat{Y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m}$ |

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# TERIMAKASIH