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Attribution of individual events based on data assimilation

Alexis Hannart
CNRS

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Colleagues

- **Argentina**
  - **Sammy Metref** – IFAECI, CNRS, Argentina
  - **Manuel Pulido** – Department of Physics, UNN, Argentina
  - **Juan Ruiz** – IFAECI, CONICET, Argentina

- **Europe**
  - **Marc Bocquet** – Ecole des Ponts, Paris Tech, France
  - **Alberto Carrassi** – Mohn-Sverdrup Center, NERSC, Norway
  - **Philippe Naveau** – LSCE, CNRS, France

- **US**
  - **Judea Pearl** – UCLA, USA
  - **Michael Ghil** – UCLA, USA and ENS, France

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Big Data Paris
Award Winner 2015

References

- **Hannart A., A. Carrassi, M. Bocquet, M. Ghil, P. Naveau, M. Pulido, J. Ruiz, P. Tandeo (2015) DADA: Data Assimilation for the Detection and Attribution of Weather and Climate-related Events, *Clim. Change.* (in press).**

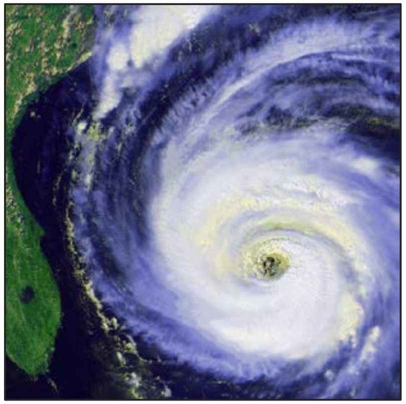
- Hannart A., M. Ghil, P. Naveau, J. Pearl (2016) Attributing an individual event or a class of events: differences and similarities (in preparation).

- **Hannart A., J. Pearl, F. E. Otto, M. Ghil (2015) Counterfactual causality theory for the attribution of weather and climate-related events, *Bull. Am. Met. Soc.* (in press).**

- Carrassi A., M. Bocquet, A. Hannart, M. Ghil (2016) Estimating model evidence using data assimilation, *Tellus* (submitted).

Starting point





Is this individual event attributable to human influence ?

Need for causal answers from scientists.

Motivations

- Conventional approach:
  - Define the event
  - Derive the probability of the event in two model ensembles:
    - factual probability: p_1
    - counterfactual probability: p_0
  - Derive the fraction of attributable risk:

$$\mathrm{FAR} = 1 - \frac{p_0}{p_1}$$

- Research questions:
  - View that "*We can't say anything for individual event, we can only say something about classes of event* "
  - What is a cause ? What is an event ? What is the FAR ? Are we doing it right ?
  - Operationalization: real-time

Motivations

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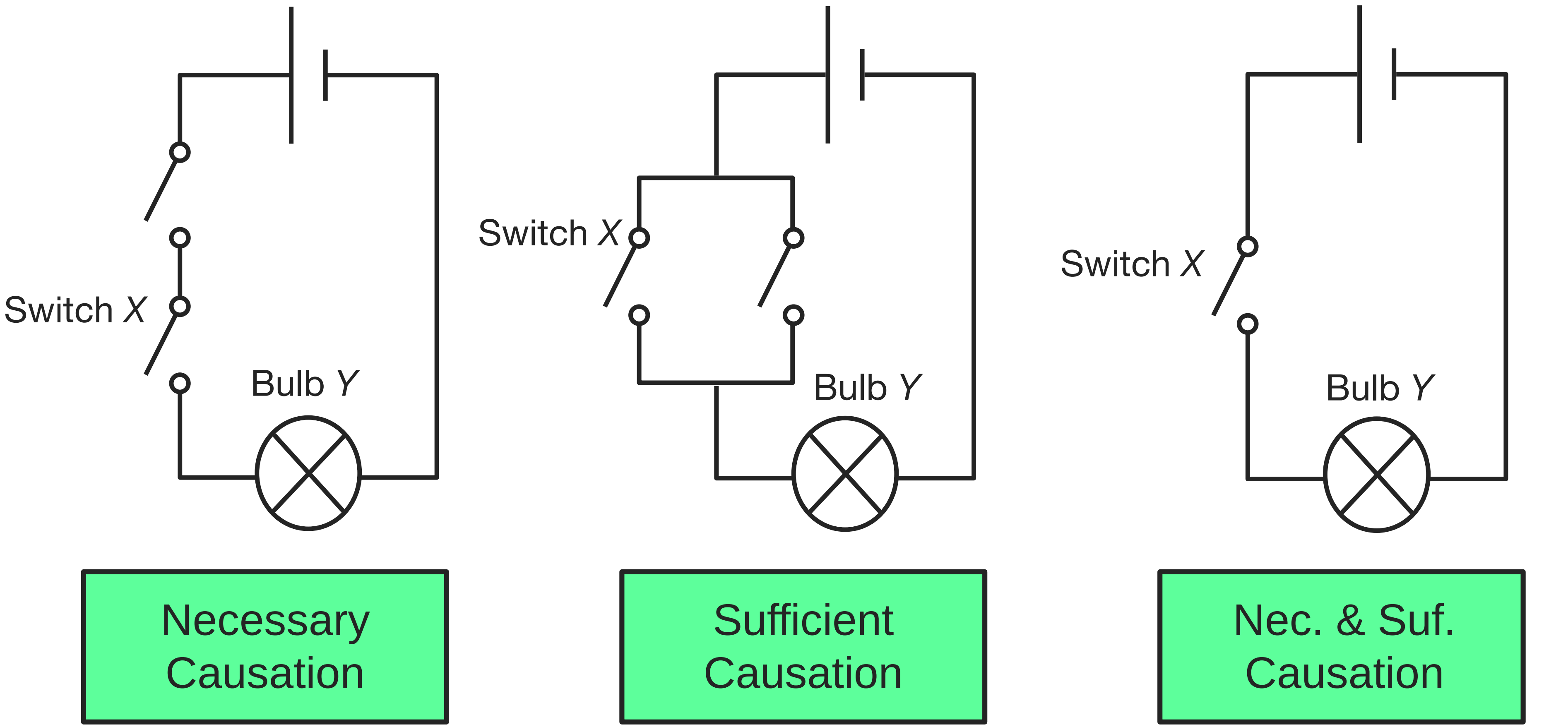
  - Operationalization: real-time

Introduce definitions : causality + events
Develop a real-time methodology

Outline

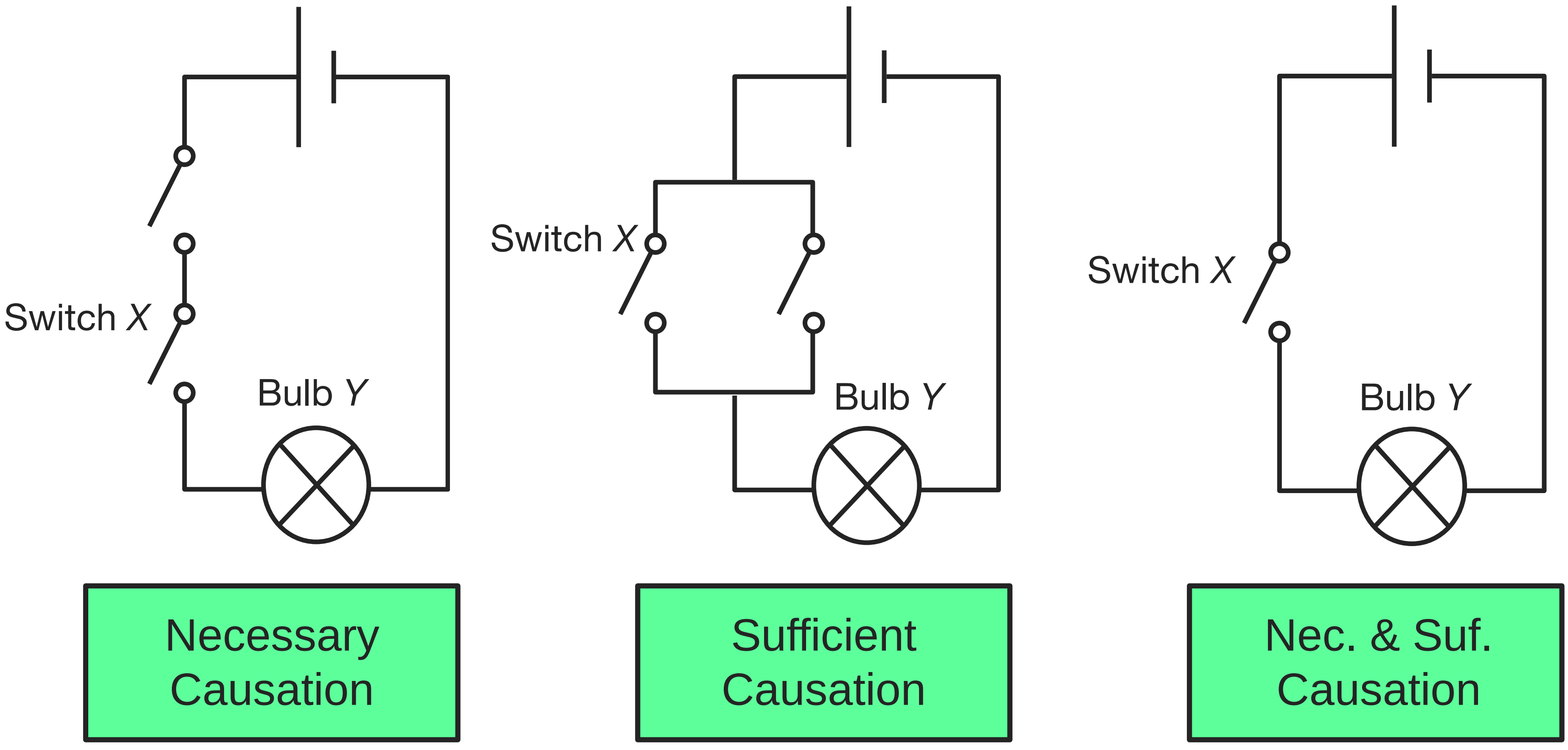
- Causal theory
- Event definition
- Attribution using data assimilation

Causal theory (Pearl): causality has two facets



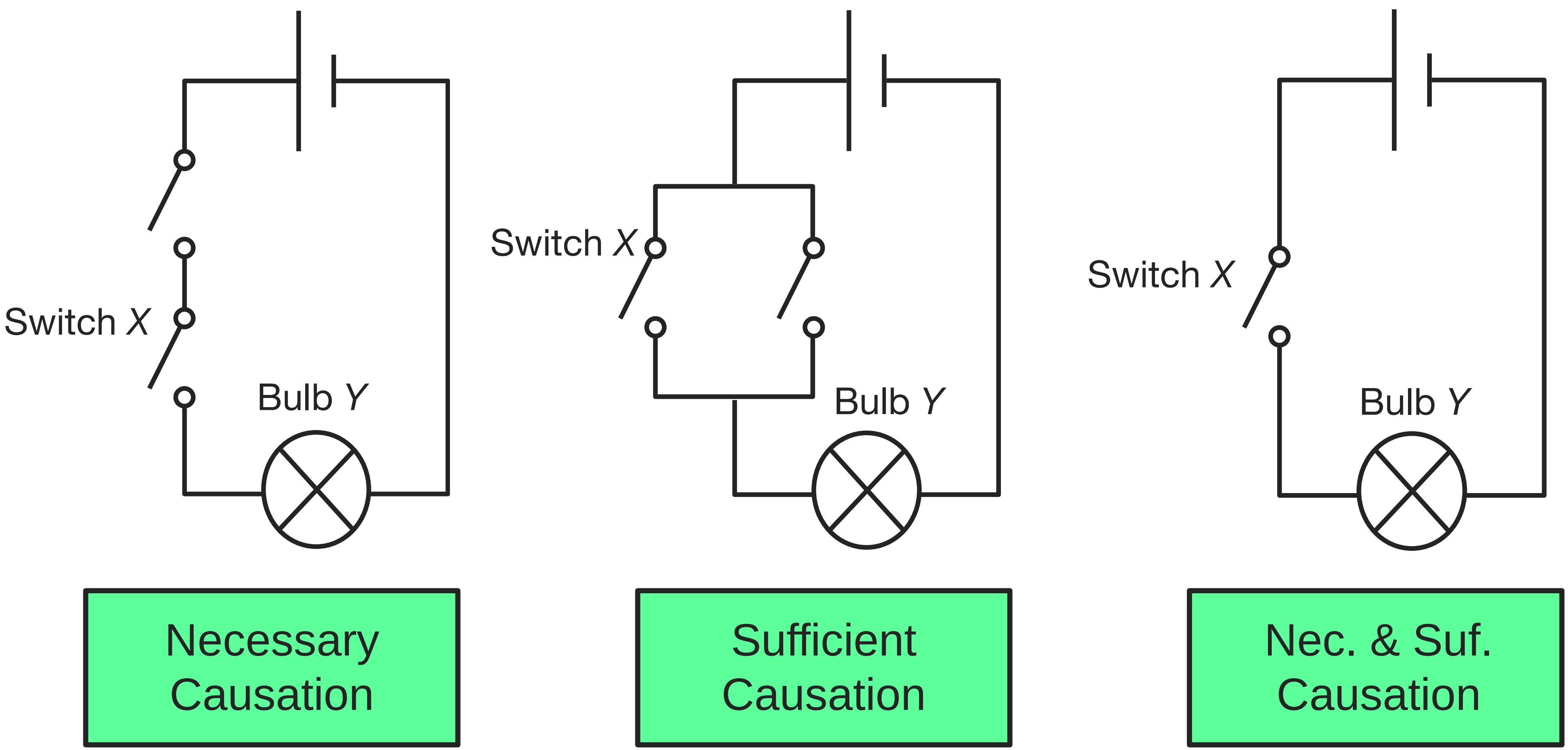
Causal theory (Pearl): Probabilities of causation

- **Probability of necessary causation** = probability that the effect is removed when the cause is turned off, conditional on the fact that the effect and the cause were initially present.



Causal theory (Pearl): Probabilities of causation

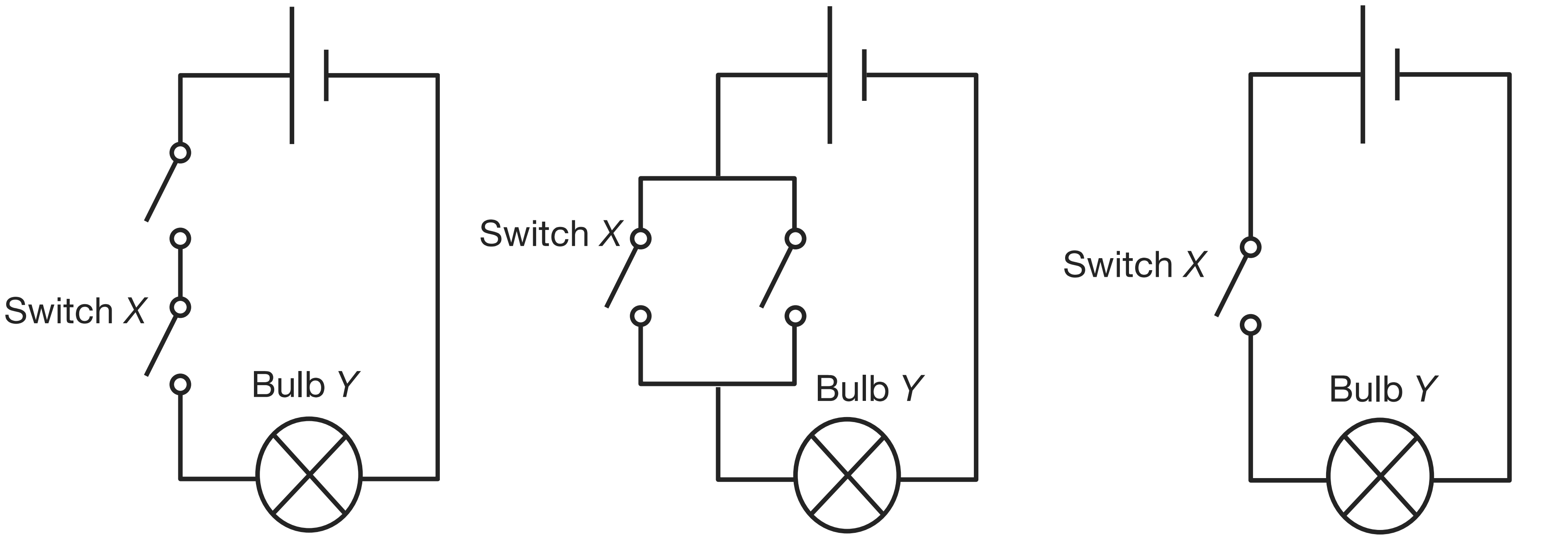
- **Probability of sufficient causation** = probability that the effect appears when the cause is turned on, conditional on the fact that the effect and the cause were initially absent.



Causal theory (Pearl): Probabilities of causation

$$\mathrm{PN} = \max\{1 - \frac{p_0}{p_1},\, 0\} \qquad \mathrm{PNS} = \max\{p_1 - p_0,\, 0\}$$

$$\mathrm{PS} = \max\{1 - \frac{1-p_1}{1-p_0},\, 0\}$$



Necessary Causation	Sufficient Causation	Nec. & Suf. Causation

Causal theory (Pearl): Probabilities of causation

$$\mathrm{PN} = \max\{1 - \frac{p_0}{p_1},\, 0\}$$

$$\mathrm{PNS} = \max\{p_1 - p_0,\, 0\}$$

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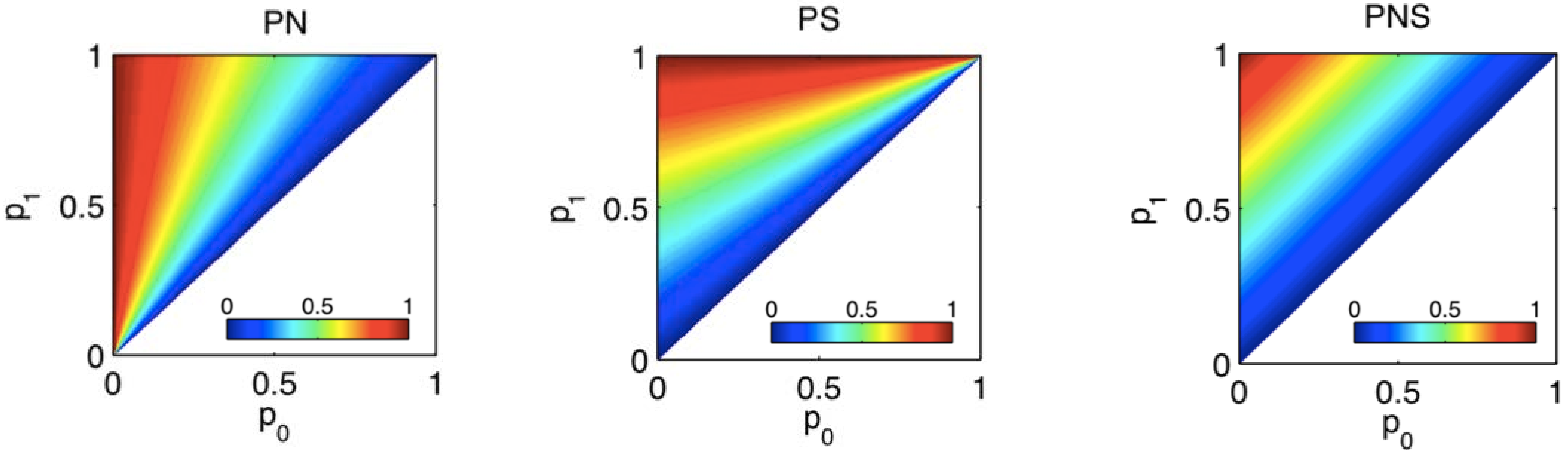

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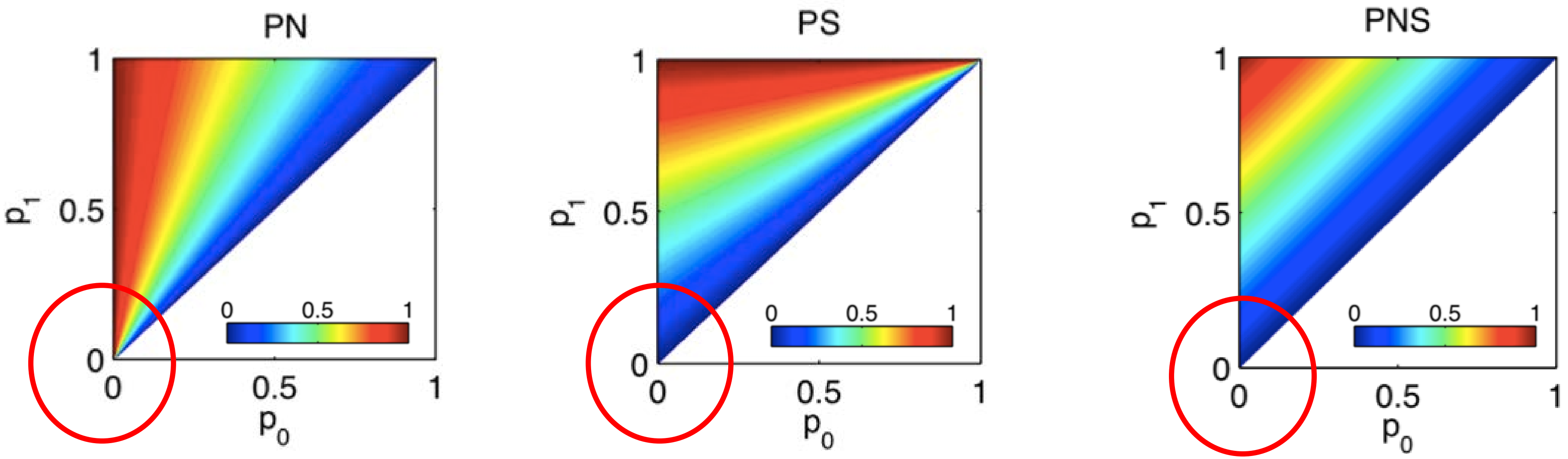
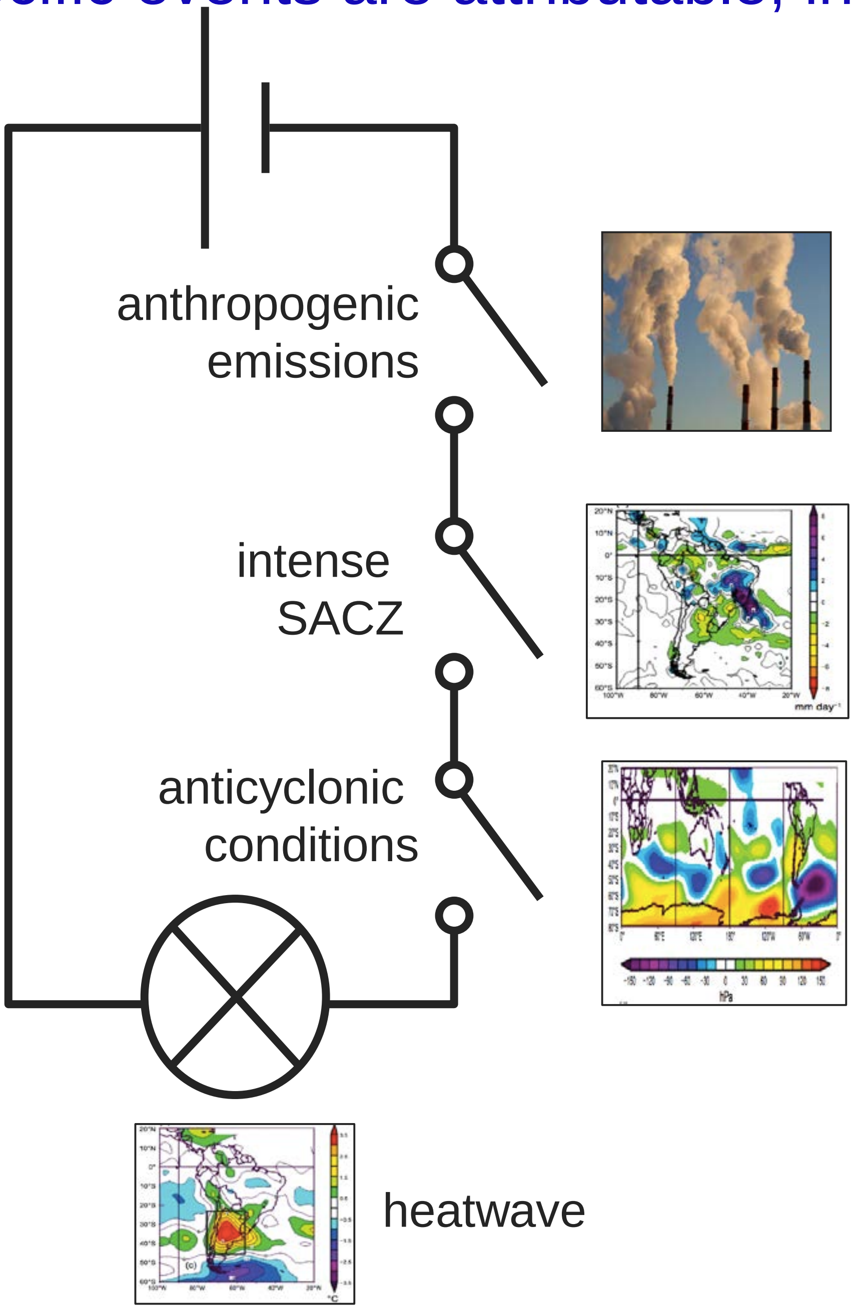


- For rare events, both p_1 and p_0 are small.

PS and PNS are always low, only PN may be high.

Specific events are attributable in a PN sense.

Specific events are attributable, in a necessary causation sense.

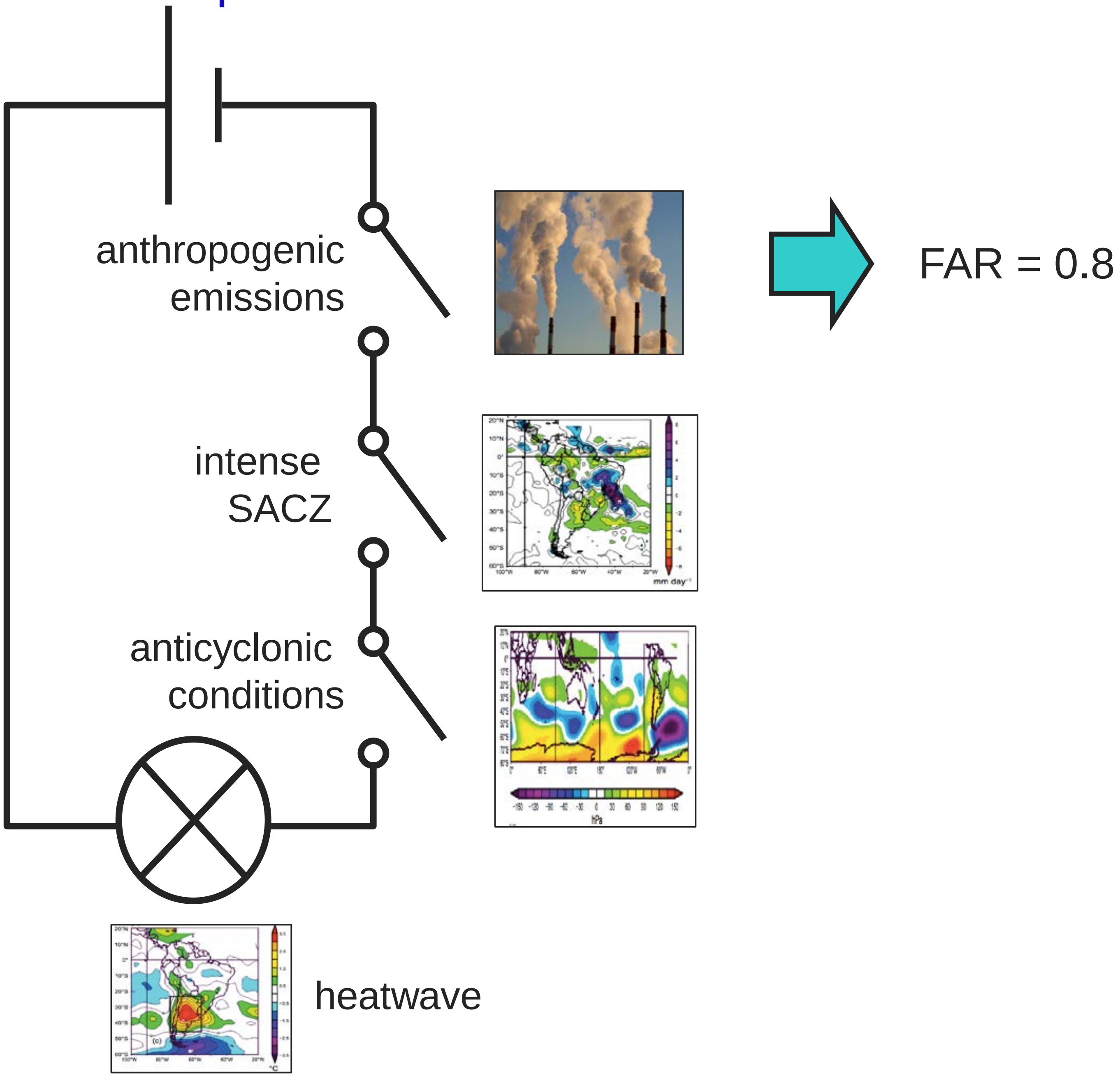


$$\mathrm{PN} = \max\{1 - \frac{p_0}{p_1}, 0\}$$

$$\Rightarrow \quad 1 - \frac{p_0}{p_1}$$

is the right quantity to look at.

Does FAR represent a fraction of attributable risk ?



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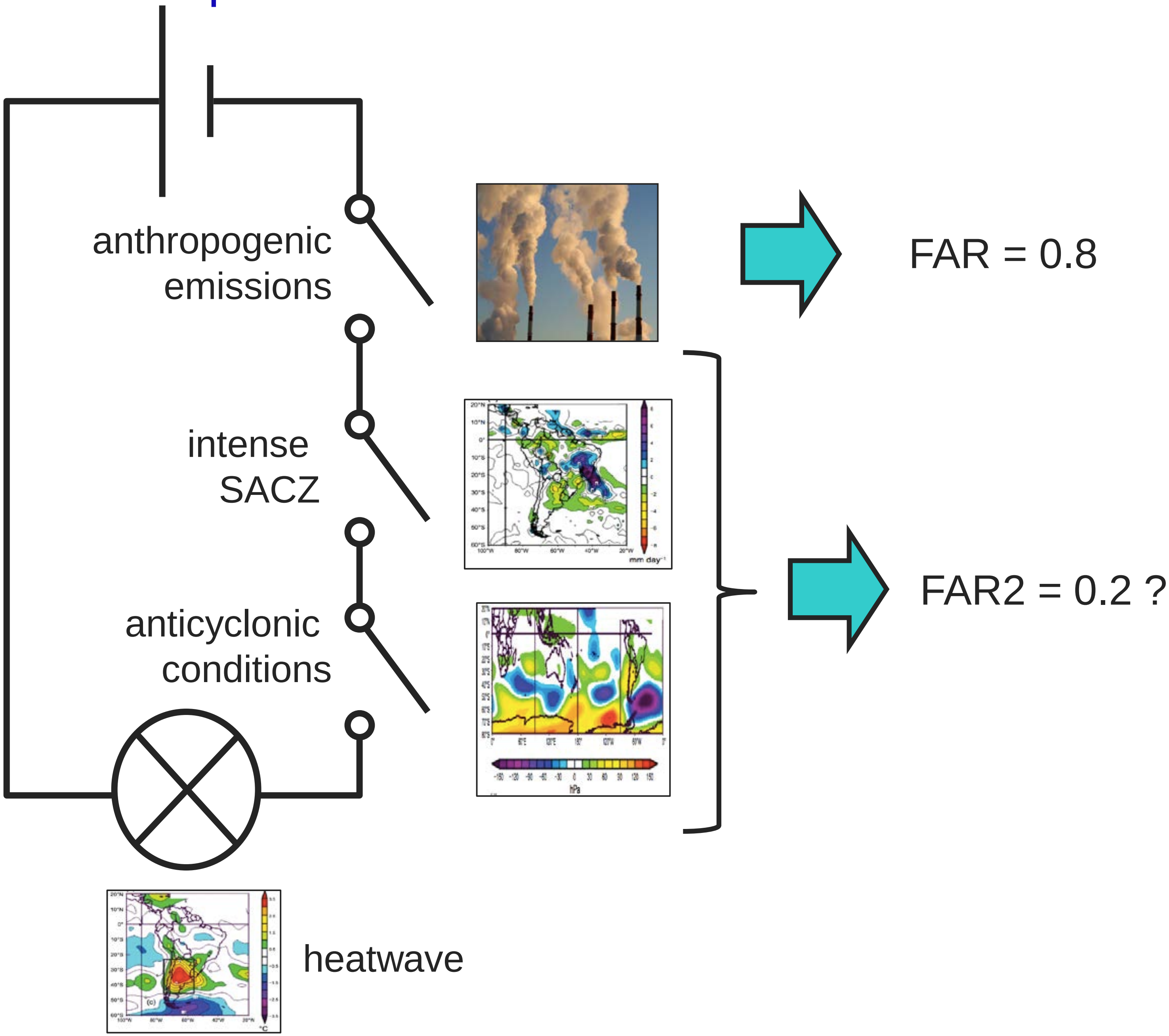

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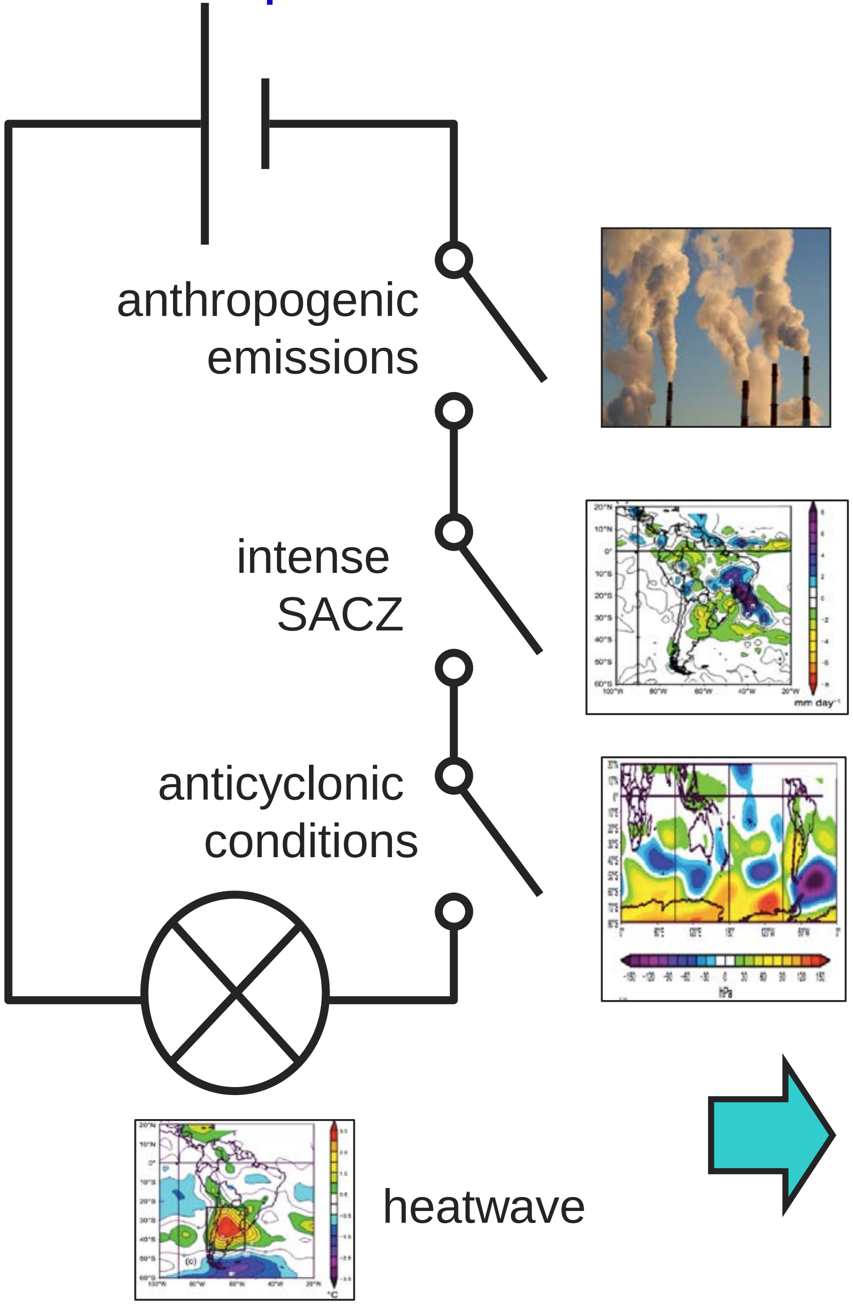


PN1 = FAR1 = 0.8

PN2 = FAR2 = 0.9*

PN3 = FAR3 = 0.9*

FAR does not represent a share of causality. FARs do not sum up to one.
PN could be used instead.

**stylized*

Outline

- Causal theory
- Event definition
- Attribution using data assimilation

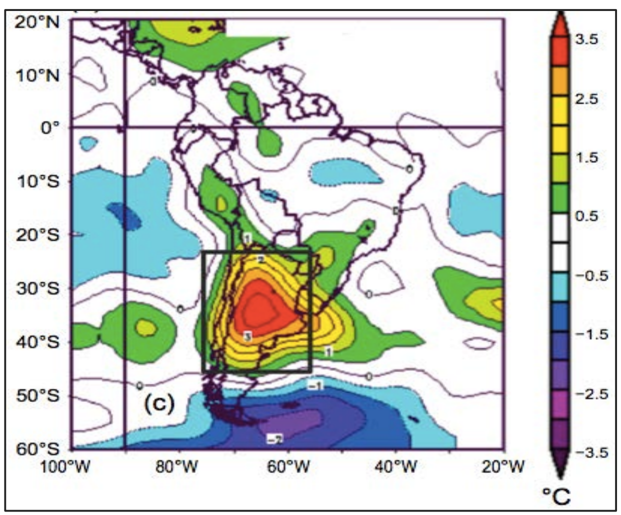
Event definition

- No matter which name is used, *1- p0/p1* is an appropriate metric: the main goal remains to derive *p0* and *p1*.

- These probabilities are affected by the chosen event definition.
  - How general / specific ?
  - Which variables ?

<u>General definition</u>

Space time-average of
a variable exceeds a threshold

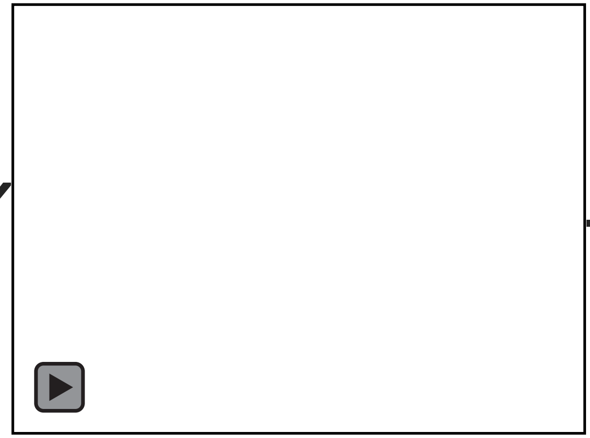


<u>Specific definition</u>

Analogue sequence of several
variables of interest over a
space-time window

Event definition: formalization

- Denote $\boldsymbol{Y} \in \mathbb{R}^n$ a high-dimensional space-time random vector concatenating all observable variables

Y  .)

- $\varphi : \mathbb{R}^n \to \mathbb{R}$ a scalar index and *E* the event:

$$E = \{\varphi(\boldsymbol{Y}) \geq u\}$$

Event definition: formalization

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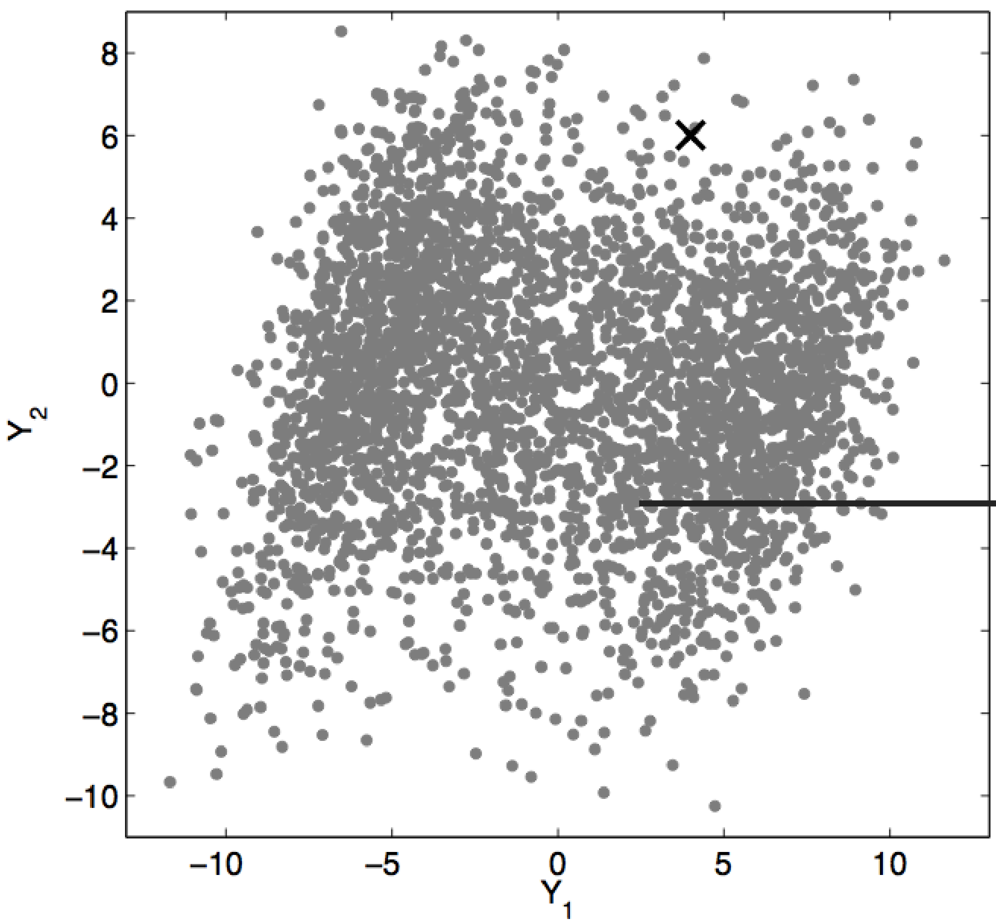
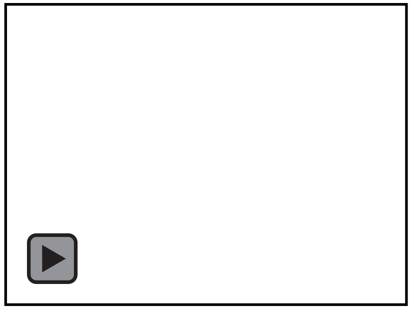


possible realizations of ***Y***



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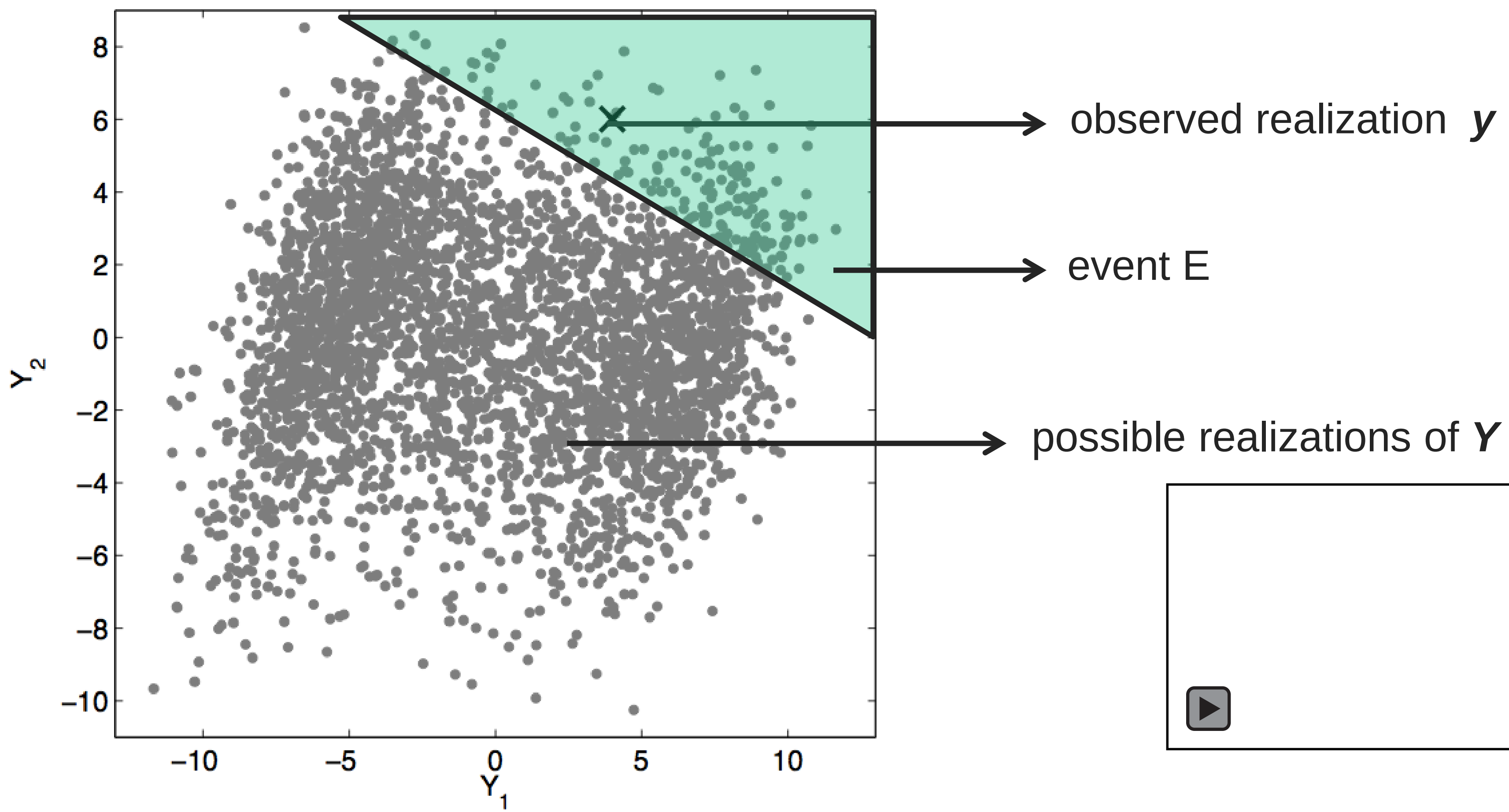


Event definition: limitations

$$E = \{\varphi(\boldsymbol{Y}) \geq u\}$$

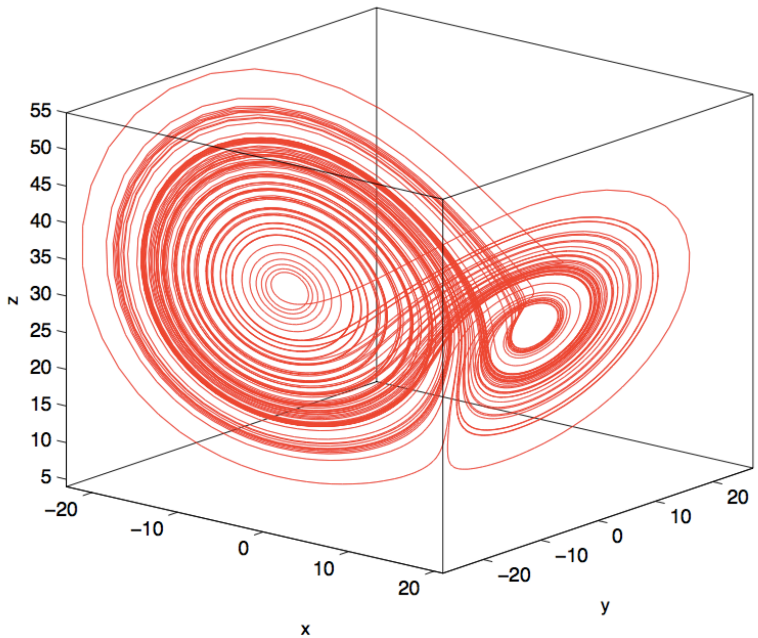
- Very sensitive to the choice of φ and ***u***.
- Does not depend on the observation ***y***
  - does not say much of the dynamic causal chain associated to the singular event ***y***.
  - table look-up approach: attribution performed beforehand for a list of pre-defined classes of events.

An illustration: the forced Lorenz model

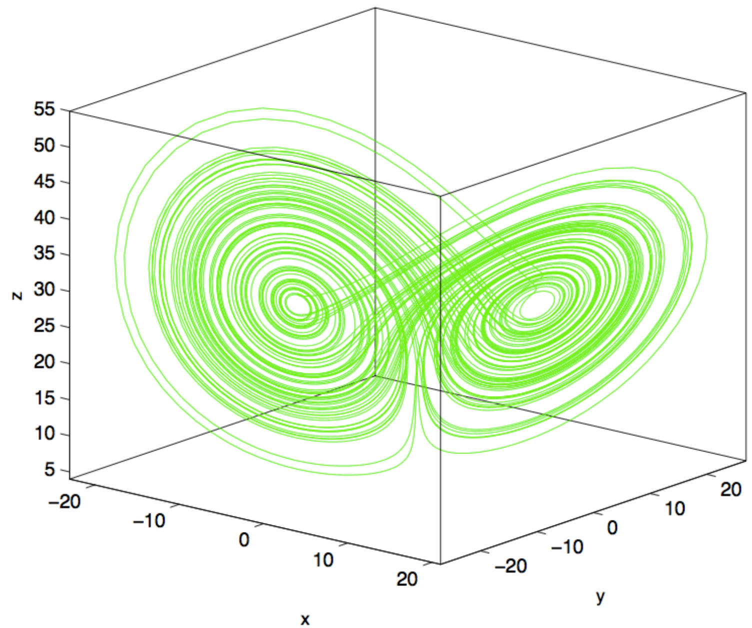
dynamic equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y-x) + \lambda_i \cos\theta_i\,, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \rho x - y - xz + \lambda_i \sin\theta_i\,, \quad \frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z\,.$$

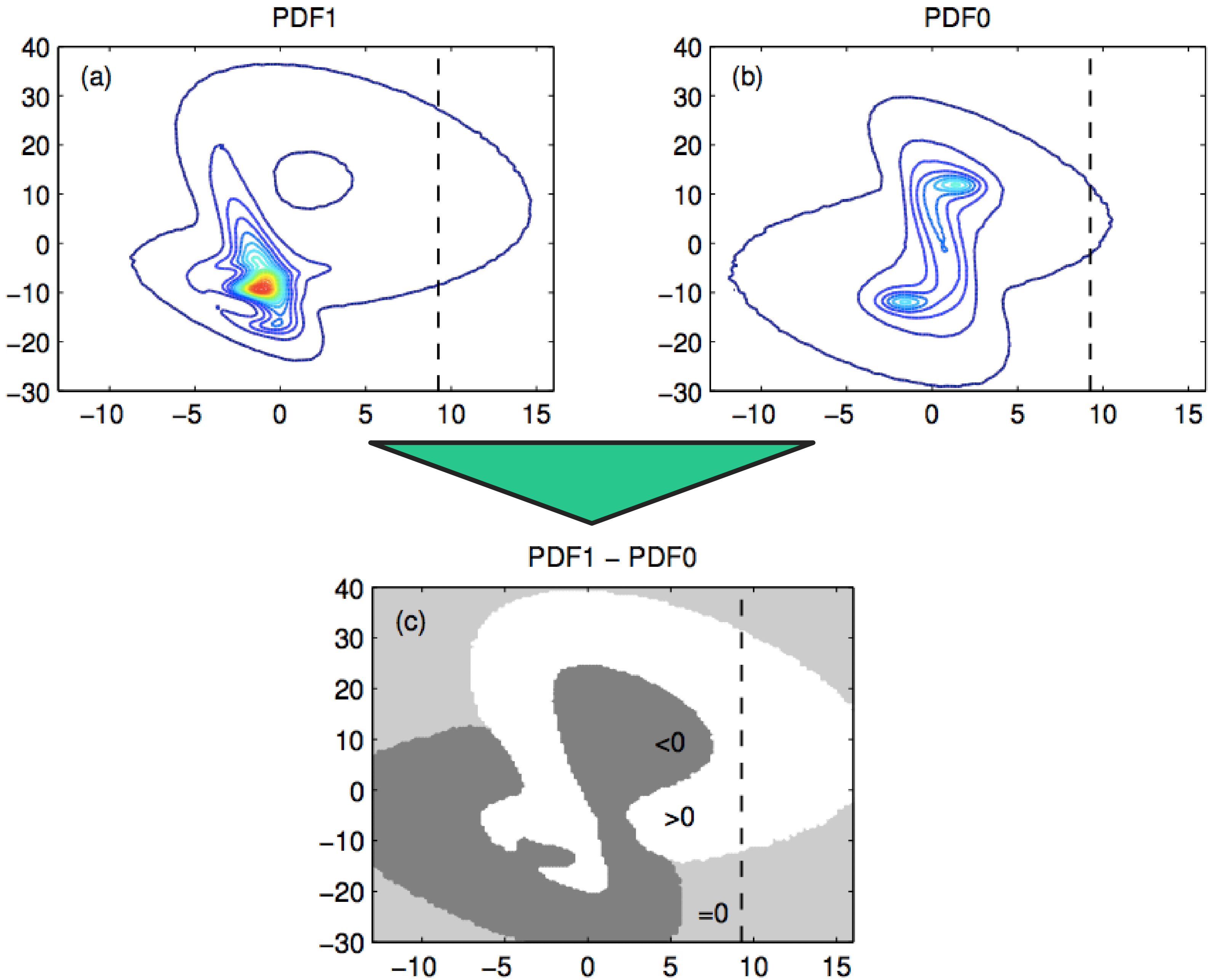
factual: $\lambda_1 = 30$



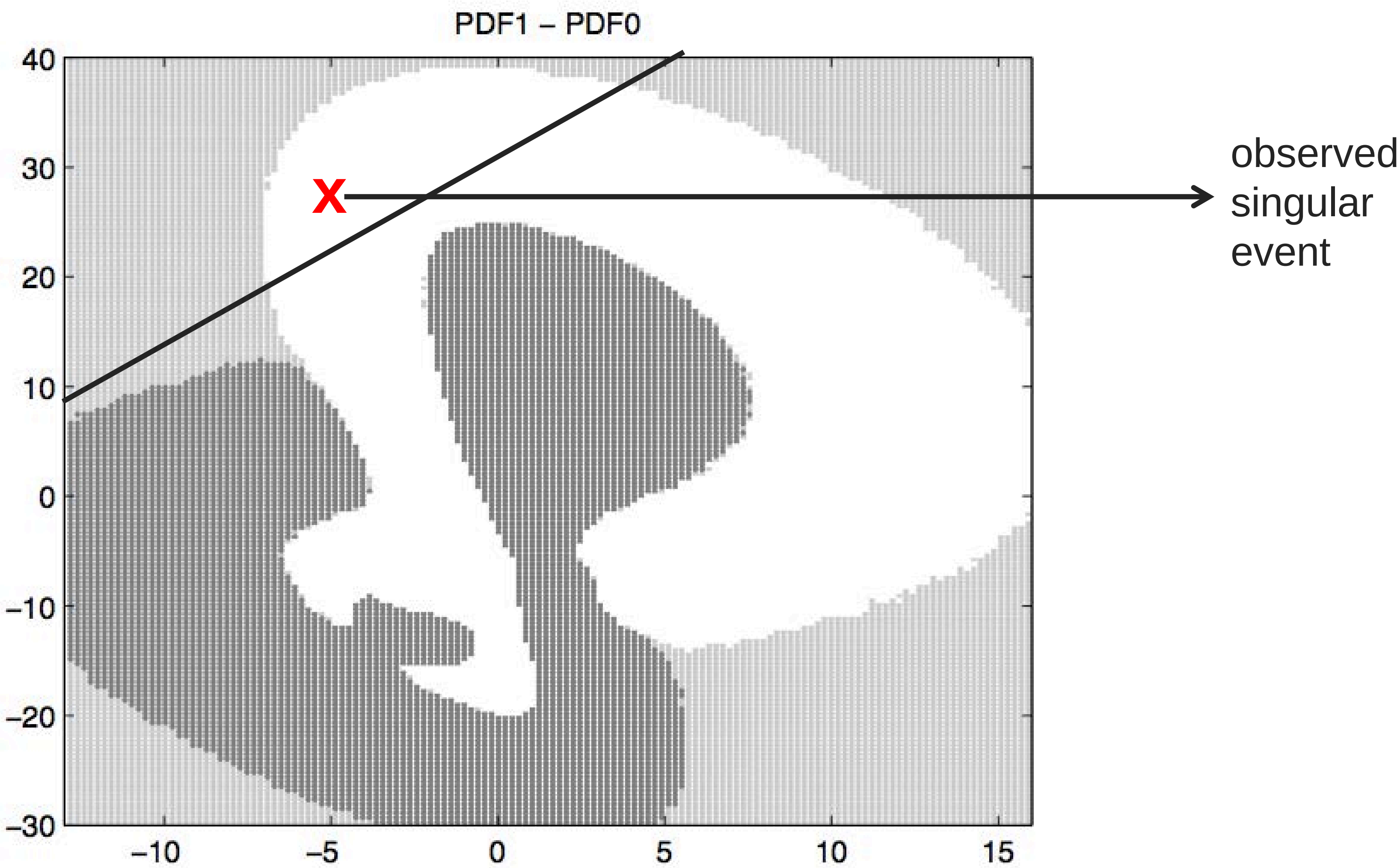
counterfactual: $\lambda_0 = 0$



Difference in attractors in the forced Lorenz model

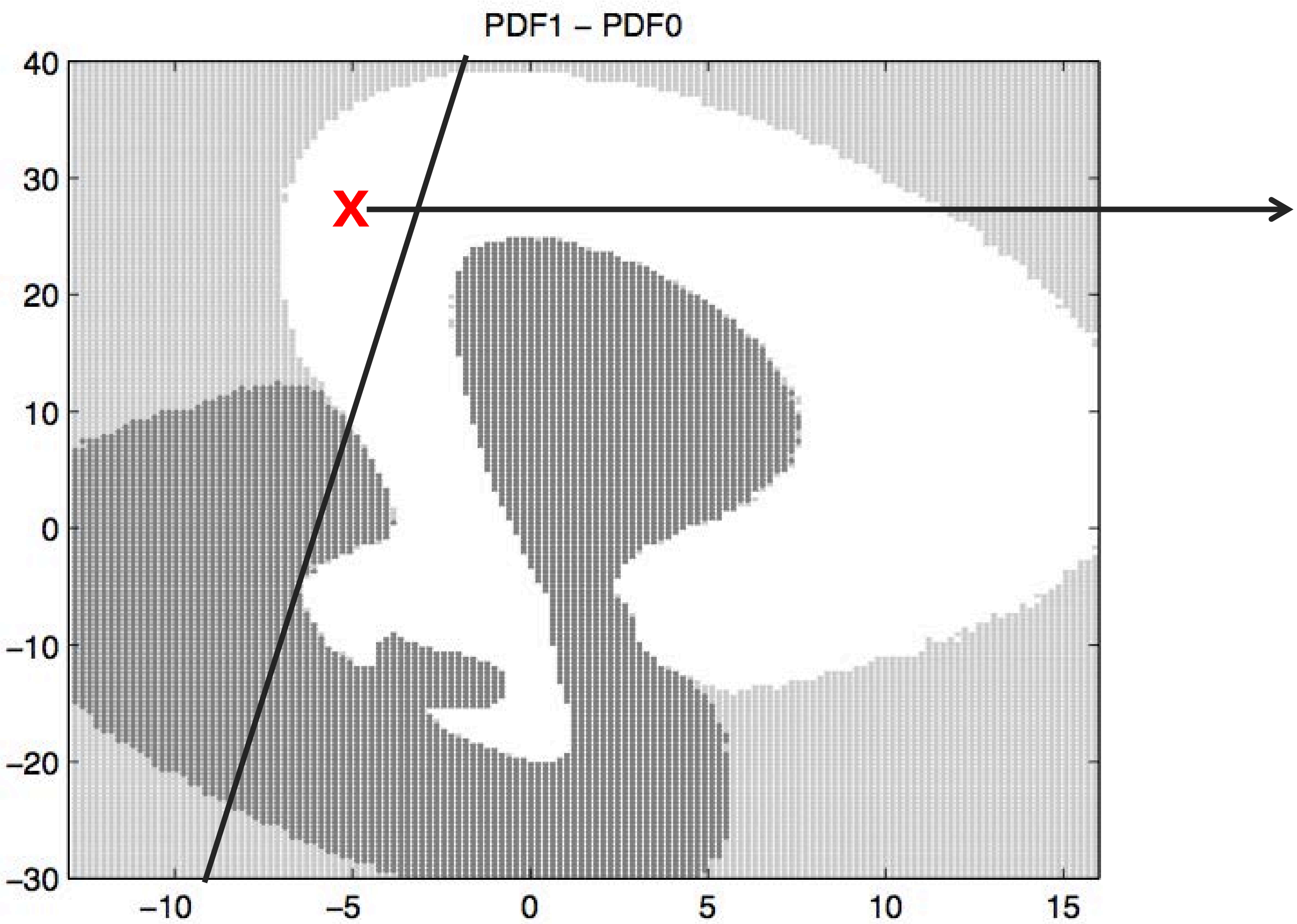


Event definition: limitations



p1>p0 = > the attribution conclusion for the chosen event definition corresponds to the position of the singular event.

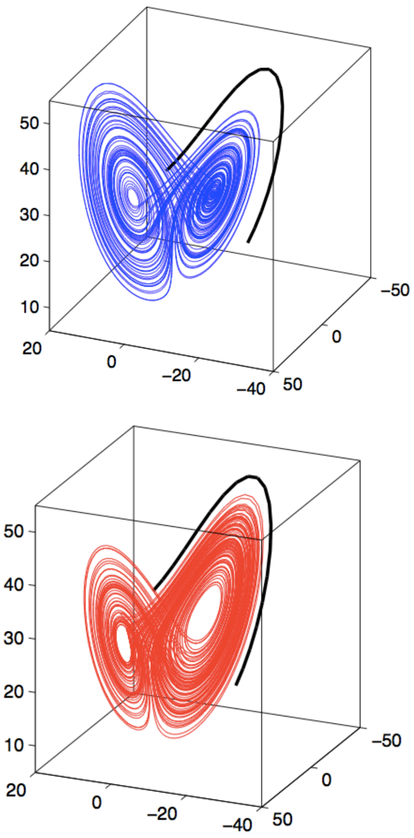
Event definition: limitations



observed singular event

p1<p0 = > the attribution conclusion for the chosen event definition does not correspond to the position of the singular event.

Event definition: limitations



$\phi 1$	$\phi 2$	$\phi 3$	p1	p0	p1/p0	PN = max{FAR,0}
-0,8	0,3	0,5	7E-05	1E-08	5E+03	0,9998
0,7	-0,6	0,4	6E-02	2E-01	4E-01	0,0000
0,1	0,8	0,5	7E-04	1E-02	5E-02	0,0000
1,0	-0,1	-0,2	5E-03	6E-03	8E-01	0,0000
0,4	-0,9	-0,2	7E-05	9E-06	8E+00	0,8755
-0,9	-0,3	0,4	9E-05	7E-08	1E+03	0,9992
0,6	-0,7	-0,3	9E-05	5E-05	2E+00	0,3768
0,3	-0,7	0,7	7E-05	2E-04	3E-01	0,0000
0,4	-0,9	-0,2	7E-05	7E-06	1E+01	0,9044
-0,9	0,5	-0,3	5E-05	3E-14	2E+09	1,0000

Event definition: limitations - illustration

- Mr. A is affected by obesity and is a Coca-Cola drinker. Is Mr. A's obesity caused by drinking Coca-Cola ?

- "Class of events" approach: $E = \{\varphi(\boldsymbol{Y}) \geq u\}$
  - "obesity" = individuals with Body Mass Index > 35
  - p1 = proba of being obese when drinking Coca-Cola
  - p0 = proba of being obese when not drinking Coca-Cola
  - p1/p0 = 1.6 (Ludwig et al. 2001)

- "Individual event" approach
  - A medical expertise shows that Mr. A's obesity is caused by an hormonal disease and is completely independent of his drinking Coca-Cola.
  - p1/p0 = 1.

The attribution conclusion for the chosen event definition (obesity) does not correspond to the situation of the singular event (Mr. A).

b
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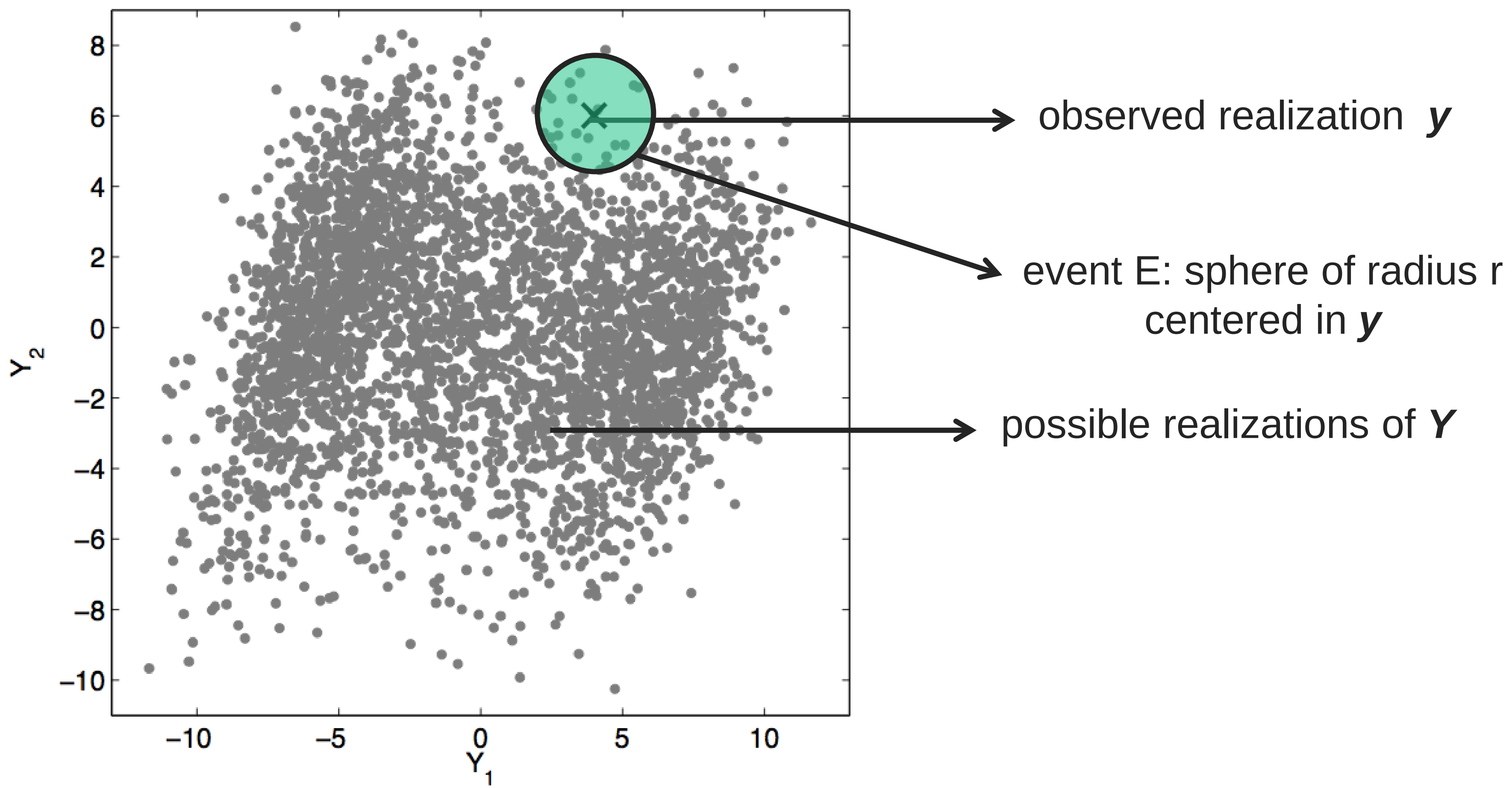
t

Event definition: analogues

$$E = \{\|\boldsymbol{Y} - \boldsymbol{y}\| \leq r\}$$

- same problem for r too large (mismatch),
- sampling becomes too costly for r too small

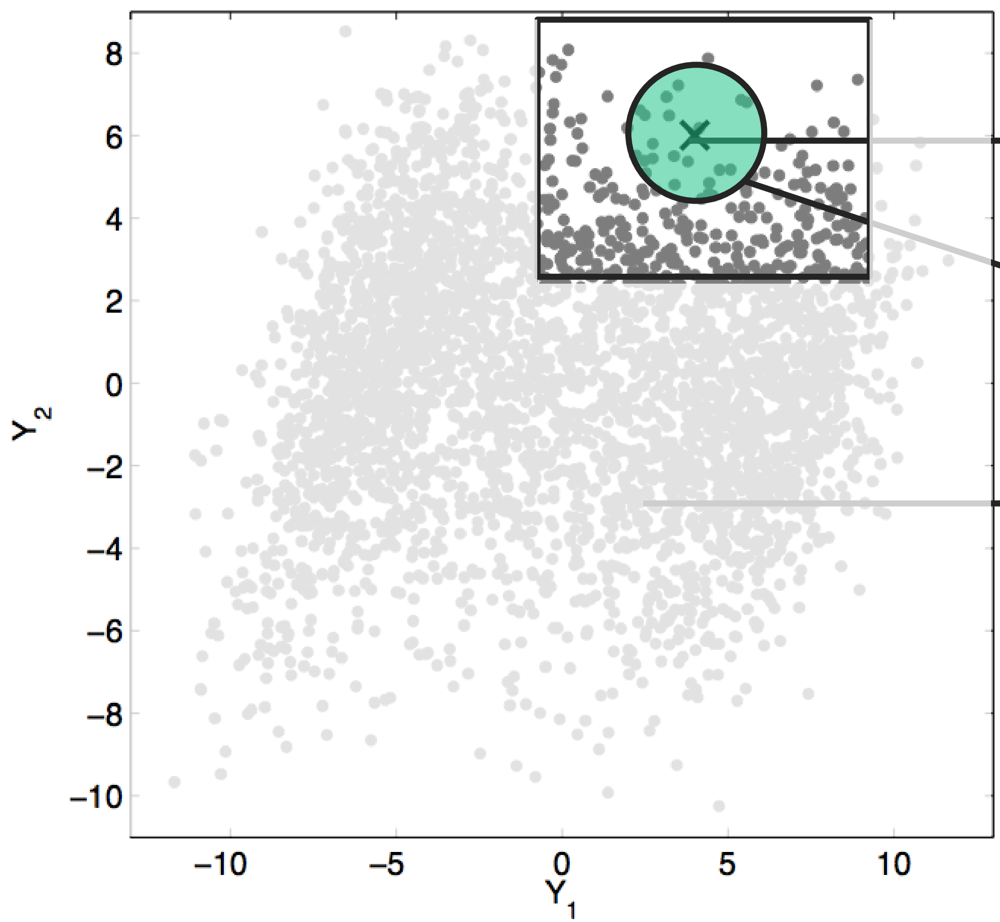


Event definition: conditional analogues

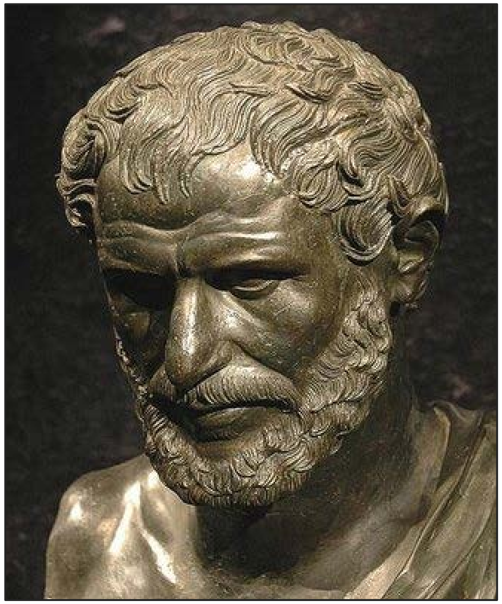
$$E = \{\|\boldsymbol{Y} - \boldsymbol{y}\| \leq r\}$$

with small r and conditioning to make sampling easier

but does not mean the same thing



A quote on the unicity of events



Heraclitus of Ephesus
535-475 BC

"*You cannot step into the same river twice, for other waters are continually flowing in.*"

Every event is unique.

"*We suggest that a different framing is desirable which asks why such extremes unfold the way they do.*"
(Trenberth et al. 2015)

Proposed definition: a singular event is a **realization**

$$E = \{\boldsymbol{Y} = \boldsymbol{y}\}$$

- We want to identify all the features of this unique event that reveal the causal influence of a forcing, with no a priori.
- These may be different every time and should be observation-dependent.

Causal implications

$$E = \{\boldsymbol{Y} = \boldsymbol{y}\}$$

The event is unique and has probability zero in both worlds:

$$p_1 = p_0 = 0$$

Hence the probability of sufficient causation is always zero:

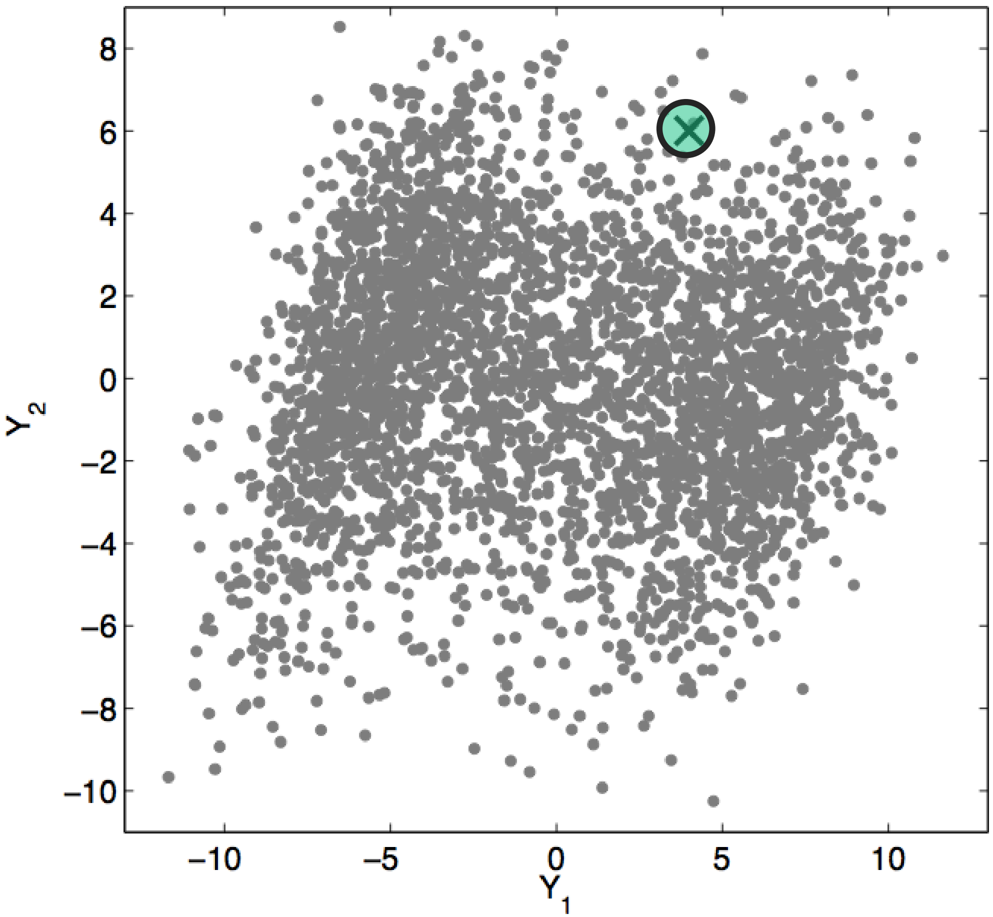
$$\mathrm{PS} = \max\{1 - \frac{1 - p_1}{1 - p_0}, 0\} = 0$$

What happens to the probability of necessary causation?

$$\mathrm{PN} = \max\{1 - \frac{p_0}{p_1}, 0\}$$

Causal implications

- Let us take the radius *r* to zero. We have:



$$p_0 \simeq f_0(\boldsymbol{y})\,\mathrm{d}V_r$$
$$p_1 \simeq f_1(\boldsymbol{y})\,\mathrm{d}V_r$$

where $f_1(\boldsymbol{y})$ is the PDF of the random variable $\boldsymbol{Y}$ in point $\boldsymbol{y}$ in the factual world and $f_0(\boldsymbol{y})$ is the same quantity in the counterfactual one.

$$\mathrm{PN} \to \max\{1 - \frac{f_0(\boldsymbol{y})}{f_1(\boldsymbol{y})}, 0\}$$

Deriving the likelihood of the event $\{\boldsymbol{Y}=\boldsymbol{y}\}$ in both worlds

Objectives:

- comparing the PDF (or **likelihood**) of $\boldsymbol{y}$ in both worlds.
- identifying the features creating a gap.

Difficulties:

- $\boldsymbol{y}$ is a very high-dimensional vector ($N \sim 10^8$).
- the same event will never be randomly simulated.

→ **Proposed solution: Data Assimilation.**

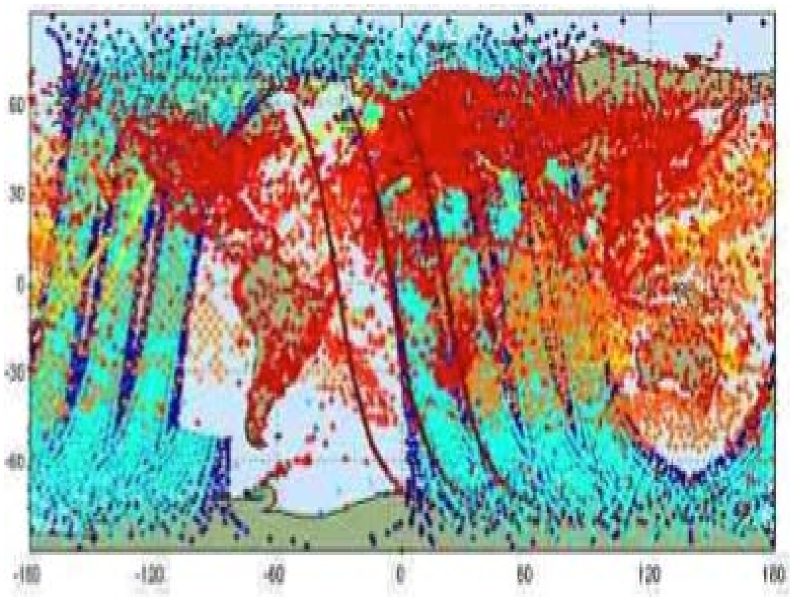
Outline

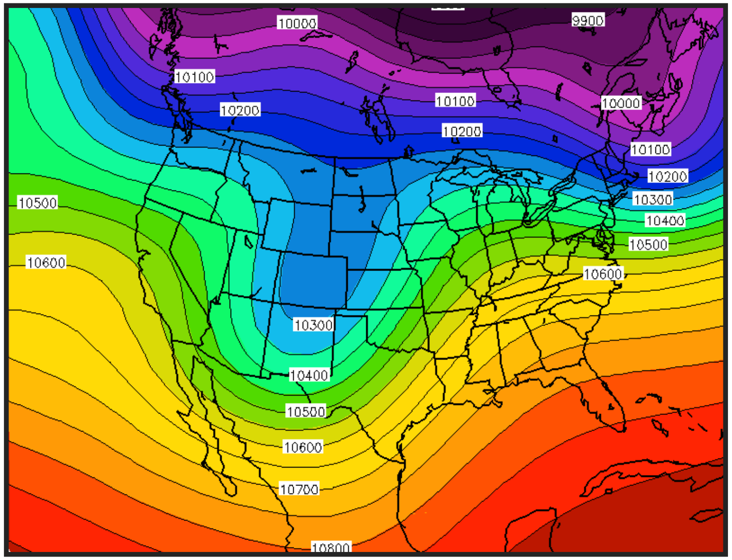
- Causal theory
- Event definition
- Attribution using data assimilation

Outlook of Data Assimilation: origins

<u>Observations:</u>
multiple
sensors

<u>State vector:</u>
atmospheric
model





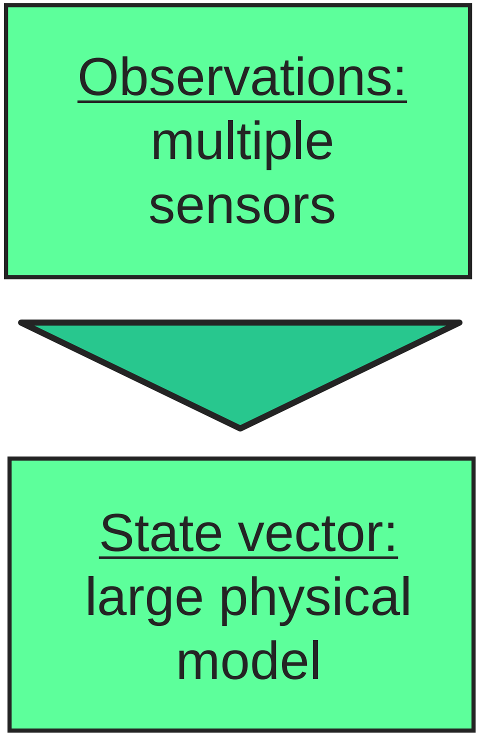
Numerical Weather Prediction requires to **initialize** the model every six hours with **new observations**.

Outlook of Data Assimilation: evolution

Trend: expansion towards new applications, general framework for interfacing large models and observations.

Examples:

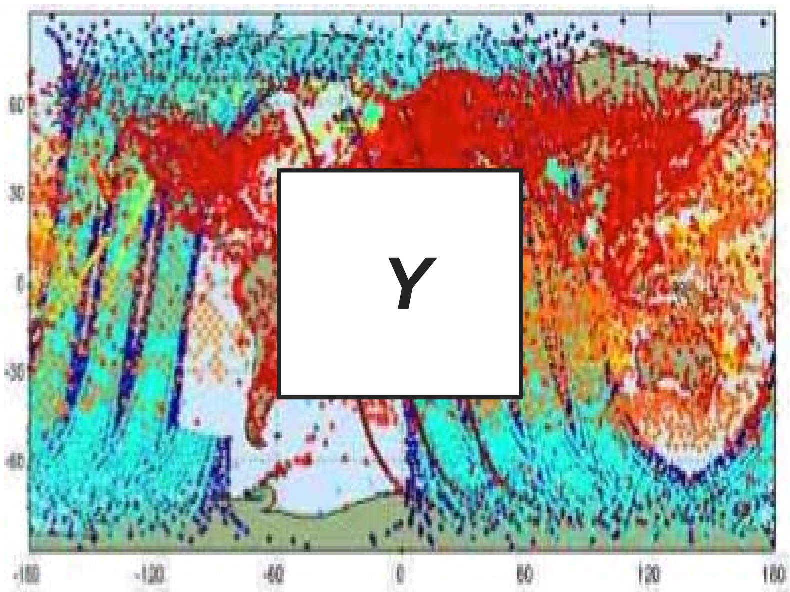
- initialization:
  - weather forecast
  - climate prediction (seasonal to decadal)
  - nowcasting (storm, tornadoes)
- reconstruction:
  - reanalysis
  - paleoclimate
  - carbon cycle
  - Earth’s core
  - Oil reservoir
- estimation: model parameters
- DADA: real time causal analysis of weather events

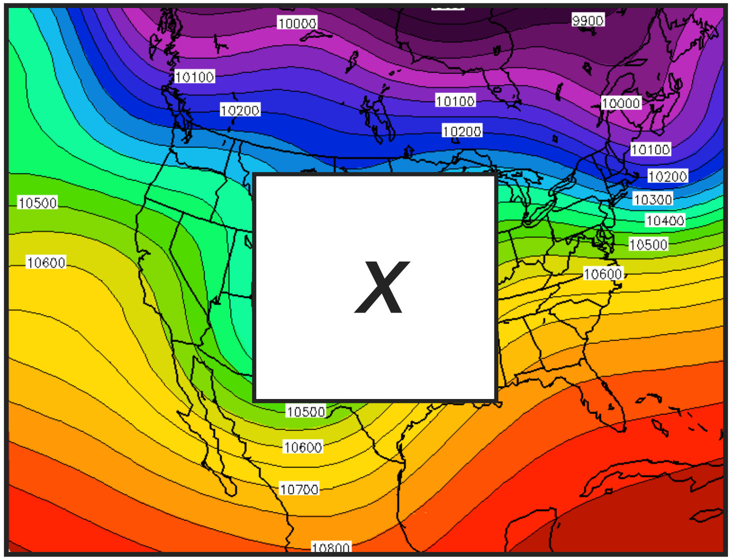


Outlook of Data Assimilation: algorithms

<u>Observations:</u>
multiple
sensors

<u>State vector:</u>
atmospheric
model





<u>Goal</u>: deriving the PDF of ***X*** conditional on ***Y*** = ***y***

high dimensional Bayesian update.

The “primitive equations” of data assimilation

<u>Assumptions:</u>
Hidden Markov model

- Dynamic equation:

$$\mathbf{X}_{t+1} = \mathrm{M}(\mathbf{X}_t, \mathbf{F}_t) + \mathbf{v}_t$$

- Observational equation:

$$\mathbf{Y}_t = \mathrm{H}(\mathbf{X}_t) + \mathbf{w}_t$$

- $\boldsymbol{v_t}$ and $\boldsymbol{w}_t$ Gaussian error terms with covariance $\boldsymbol{Q}$ and $\boldsymbol{R}$;
- M is the model with $\boldsymbol{F_t}$ external forcing;
- H is the observation operator.

The "primitive equations" of data assimilation

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- $\boldsymbol{v}_t$ and $\boldsymbol{w}_t$ Gaussian error terms with covariance $\boldsymbol{Q}$ and $\boldsymbol{R}$;
- M is the model with $\boldsymbol{F}_t$ external forcing;
- H is the observation operator.

Solution:
Gaussian linear approximation

- Propagation equation:

$$\mathbf{x}^f_{t+1} = \mathbf{M}\mathbf{x}^a_t$$

$$\mathbf{P}^f_{t+1} = \mathbf{M}\mathbf{P}^a_t\mathbf{M}' + \mathbf{Q}$$

- Update equation:

$$\boxed{\mathbf{x}^a_t = \mathbf{x}^f_t + \mathbf{K}(\mathbf{y}_t - \mathbf{H}\mathbf{x}^f_t)}$$

$$\mathbf{P}^a_t = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^f_t$$

$$\mathbf{K} = \mathbf{P}^f_t\mathbf{H}'(\mathbf{H}\mathbf{P}^f_t\mathbf{H}' + \mathbf{R})^{-1}$$

The likelihood *f*(***y***) is a by-product of data assimilation

<u>Solution:</u>
Gaussian linear approximation

<u>By-product:</u>
PDF of observation ***y***

- Propagation equation:

$$\mathbf{x}_{t+1}^f = \mathbf{M}\mathbf{x}_t^a$$
$$\mathbf{P}_{t+1}^f = \mathbf{M}\mathbf{P}_t^a\mathbf{M}' + \mathbf{Q}$$

- Update equation:

$$\mathbf{x}_t^a = \mathbf{x}_t^f + \mathbf{K}(\mathbf{y}_t - \mathbf{H}\mathbf{x}_t^f)$$
$$\mathbf{P}_t^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_t^f$$
$$\mathbf{K} = \mathbf{P}_t^f\mathbf{H}'(\mathbf{H}\mathbf{P}_t^f\mathbf{H}' + \mathbf{R})^{-1}$$

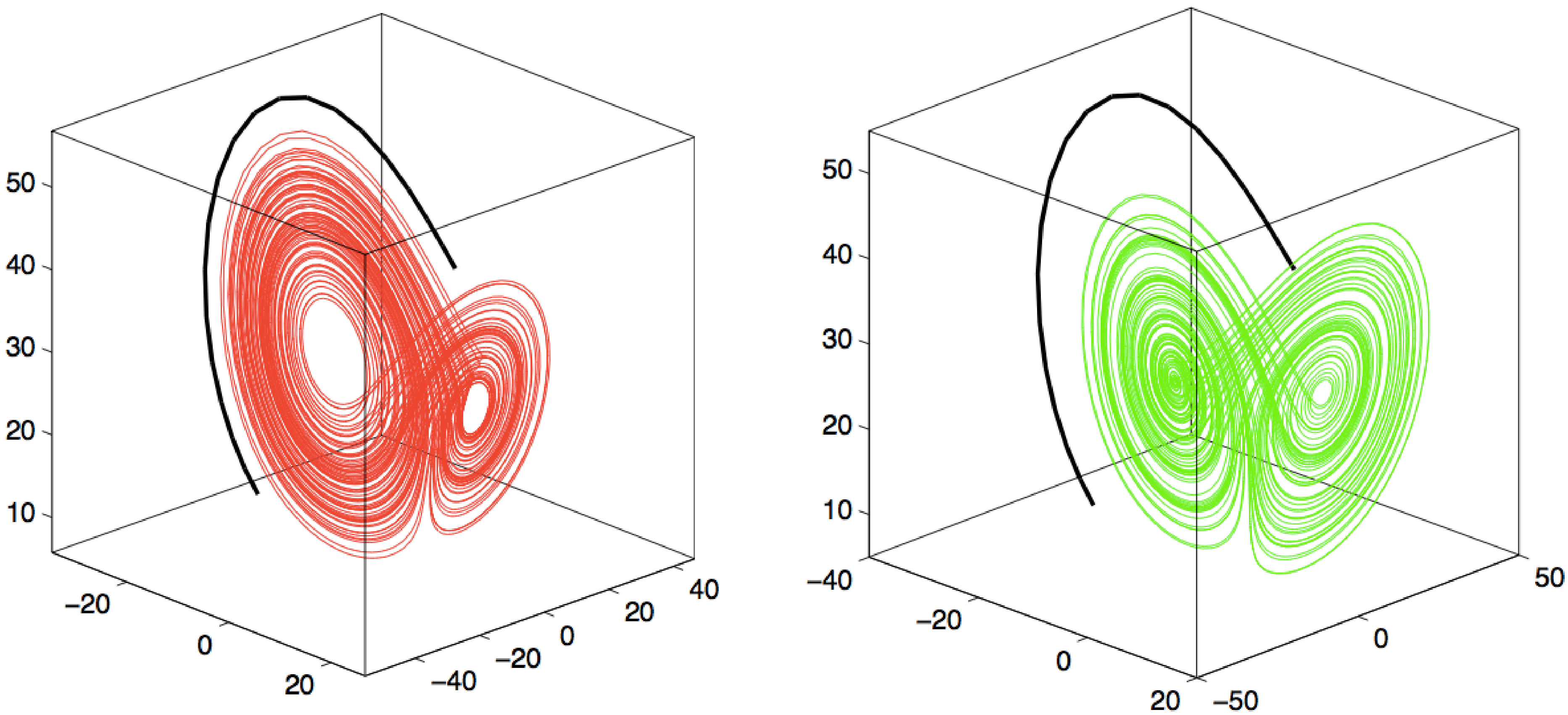
- Likelihood equation:

$$\boxed{-\log f(\mathbf{y}) = \sum_{t=0}^{T} \frac{1}{2}\log|\mathbf{\Sigma}_t| + \frac{1}{2}\mathbf{d}_t'\mathbf{\Sigma}_t^{-1}\mathbf{d}_t}$$

with:

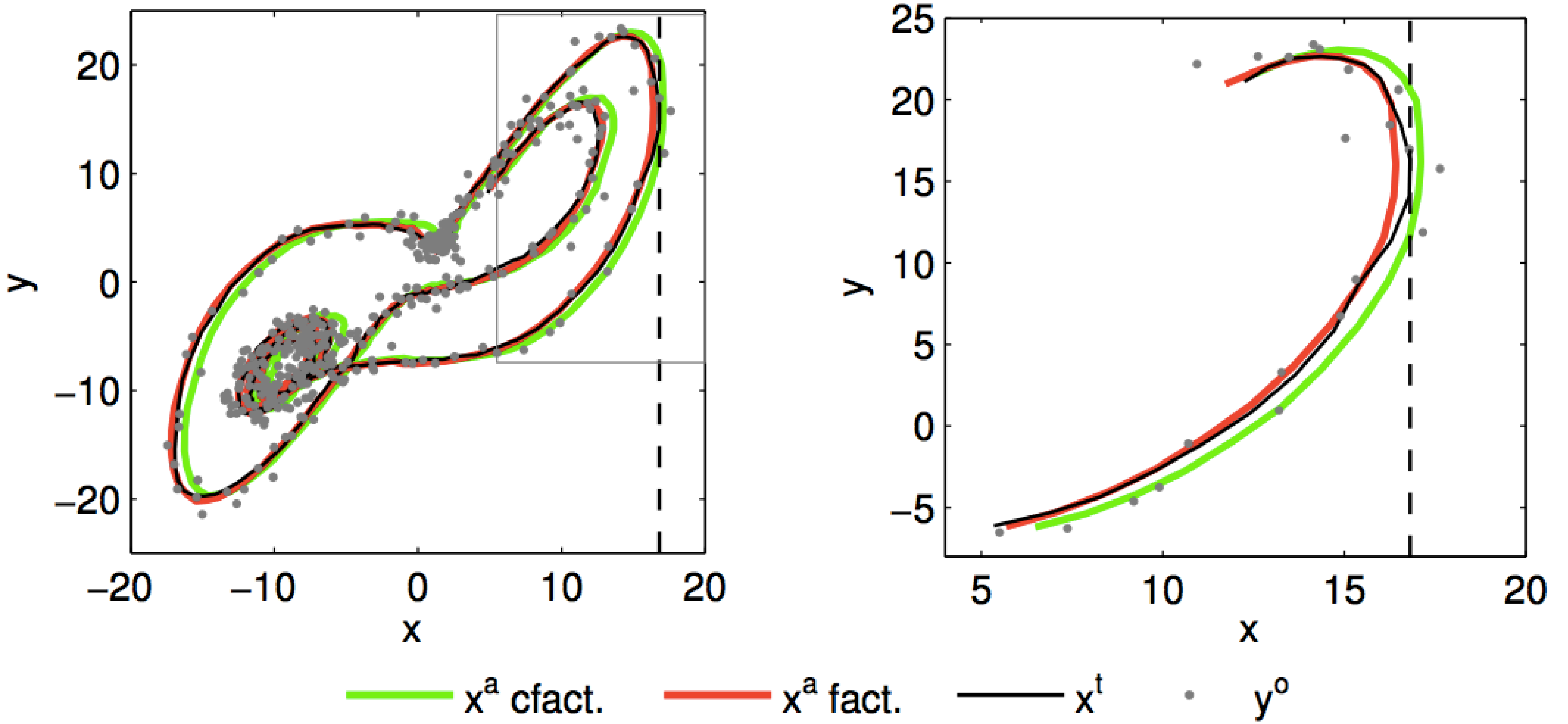
$$\mathbf{d}_t = \mathbf{y}_t - \mathbf{H}\mathbf{x}_t^f$$
$$\mathbf{\Sigma}_t = \mathbf{H}\mathbf{P}_t^f\mathbf{H}' + \mathbf{R}$$

Experiments in the forced Lorenz model



Experiments in the forced Lorenz model

- DADA likelihoods of each event's trajectory are derived:



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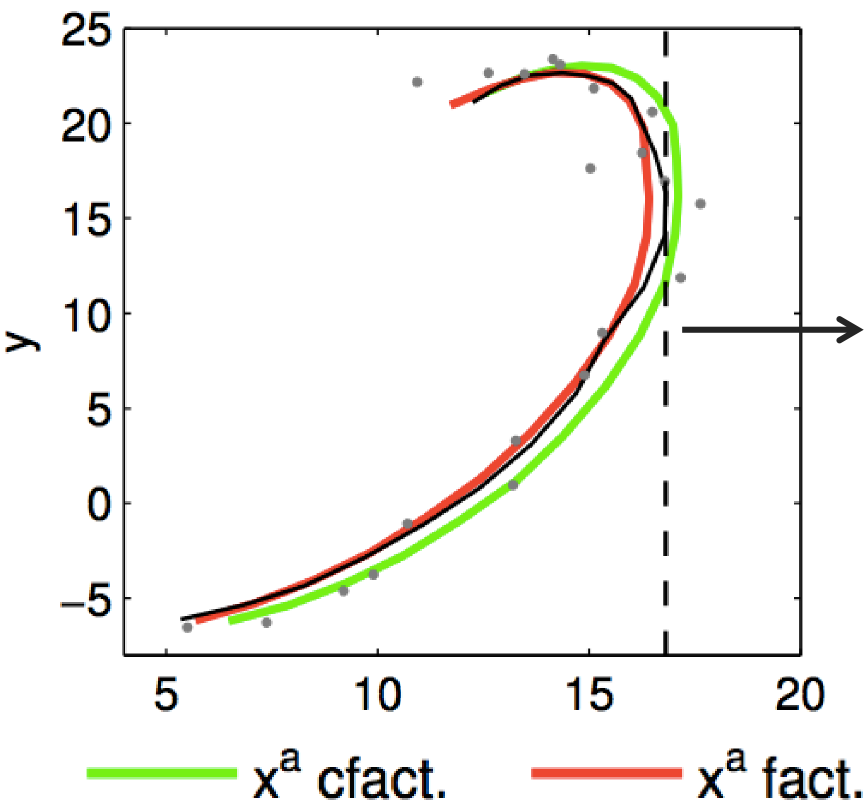
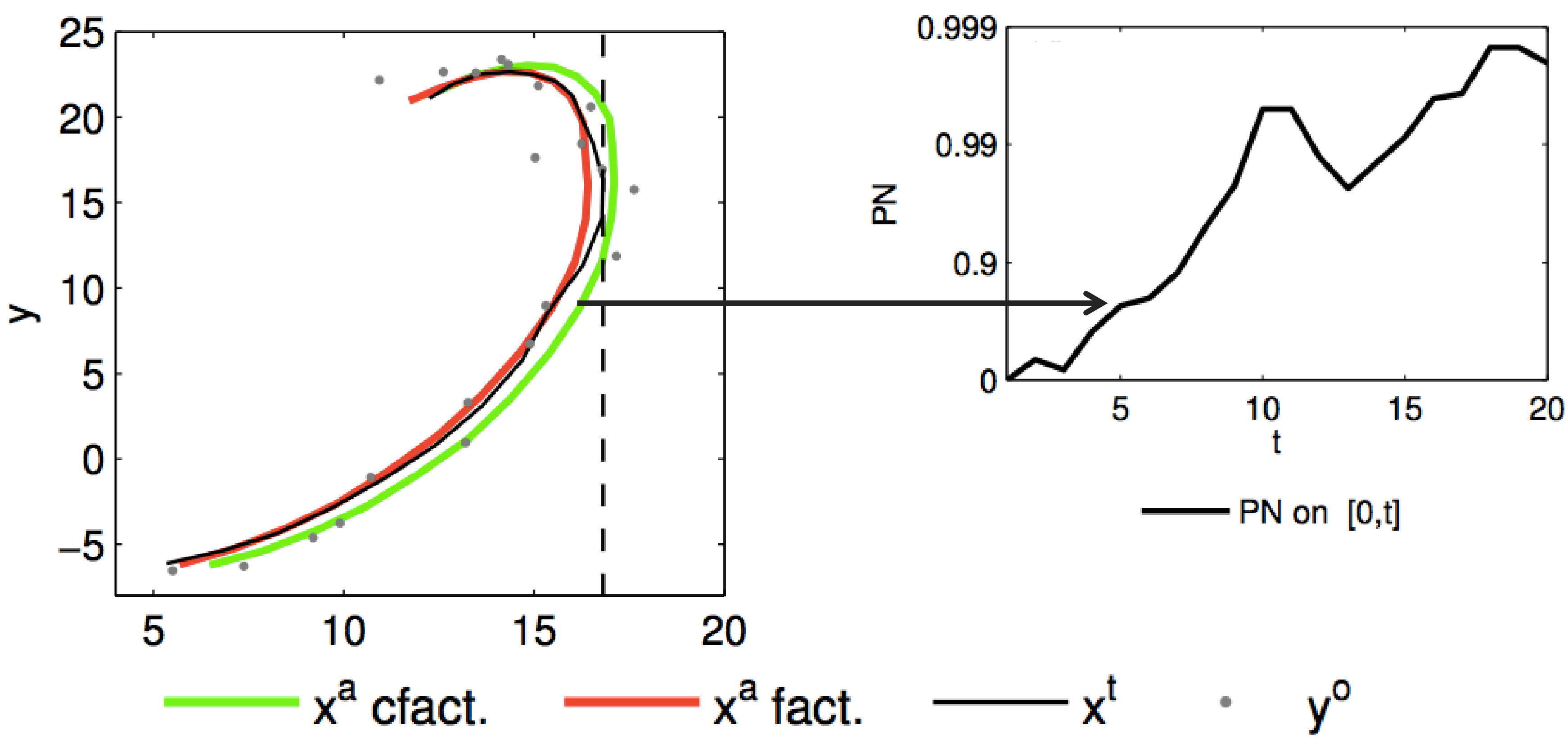


The factual reconstruction is usually slightly better than the counterfactual one.

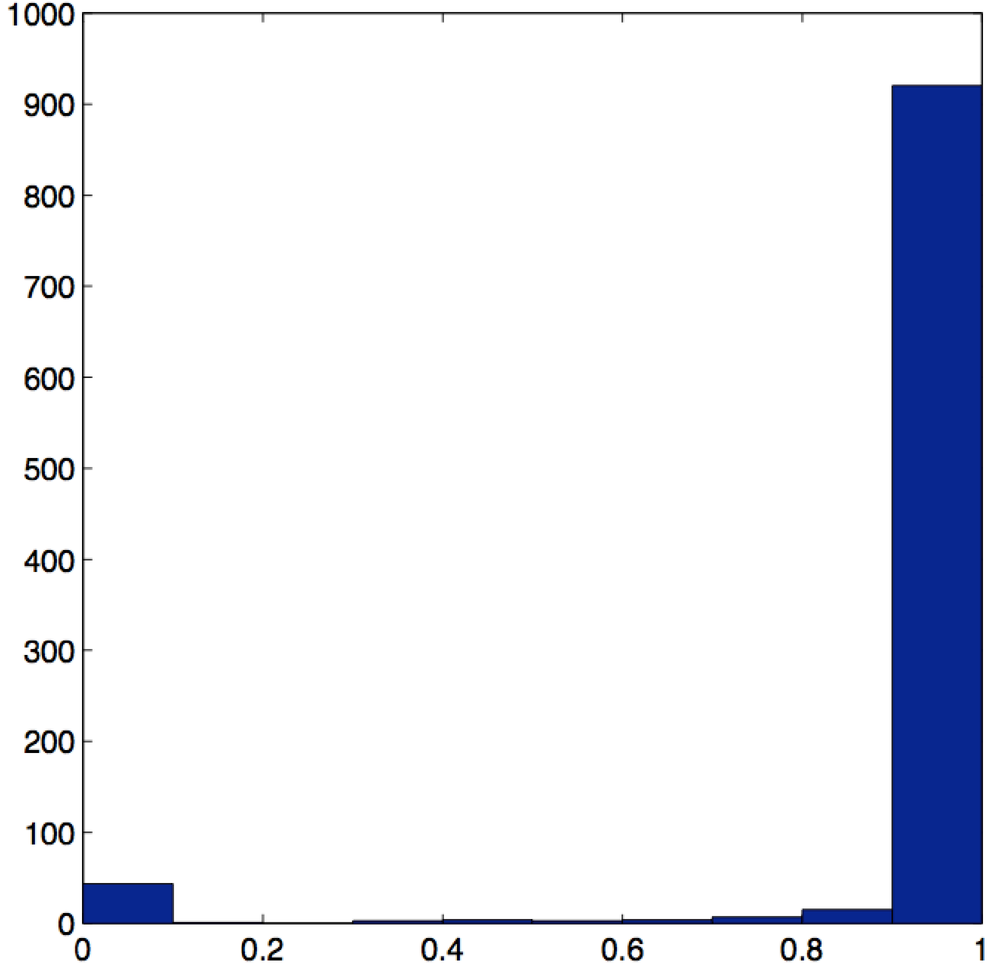
Experiments in the forced Lorenz model

- The factual reconstruction is usually slightly better than the counterfactual one.
- Small local differences pile up into a large amount of causal evidence overall.



Experiments in the forced Lorenz model

Distribution of PN for several individual events
sampled within a fixed class of events having PN = 0.7



Performance of an event attribution method: a proposal

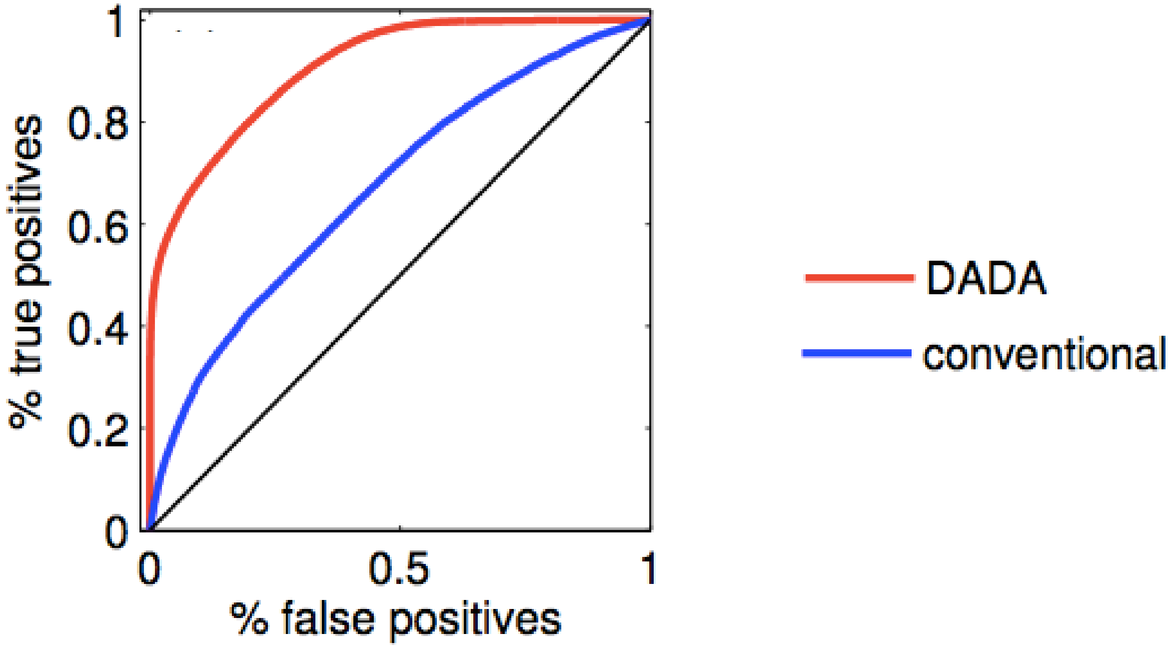
- For a given class of events, simulate singular events ***y*** using the factual model and then using the counterfactual model.

- Determine which model was used to simulate the observed event ***y*** based on p1 and p0.

- Measure the discriminative performance (ROC curve and Gini index) of this procedure.

Experiments in the forced Lorenz model

- The performance of DADA is better overall (ROC curve, Gini index).



Experiments in the ICTP AGCM model

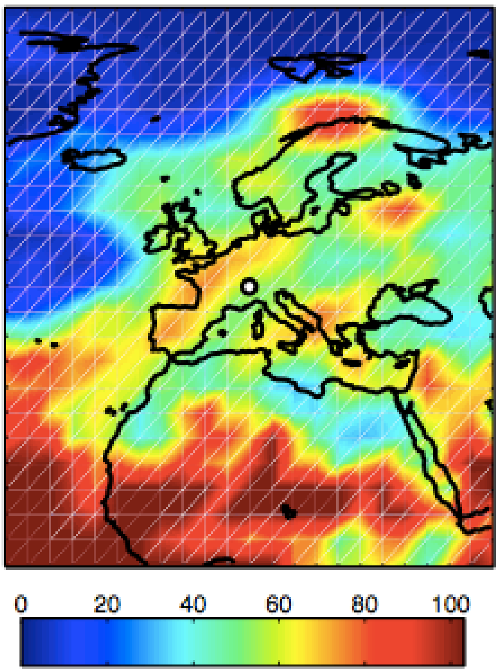
factual

counterfactual

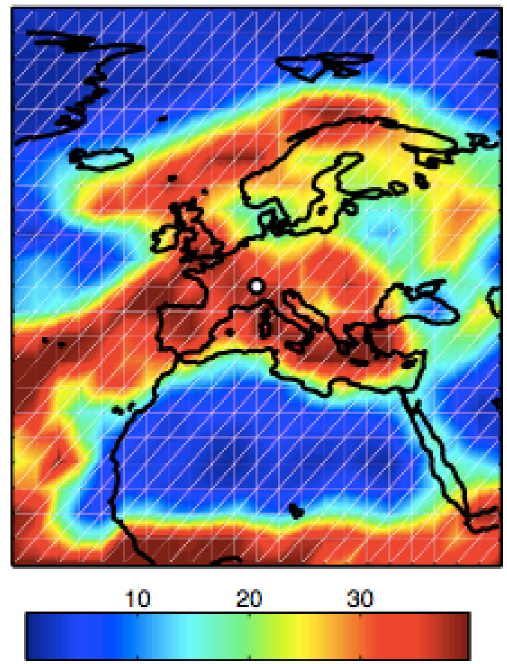
Experiments in the ICTP AGCM model

<u>mse of analysis:</u>

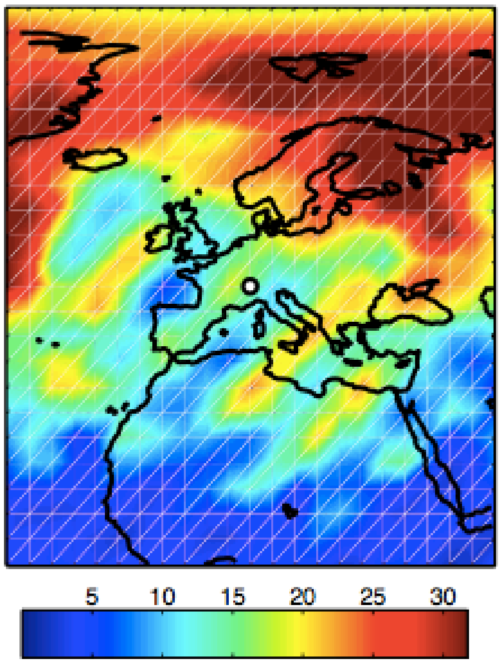
<u>surface temperature</u>



<u>wind</u>



<u>geopotential heigth</u>



Summary

- Causal theory can help.
- An individual event can be defined as a zero probability realization.
- Causal diagnostic of singular events may be implemented in an automated, real time and systematic way using DA.
- Synergy with existing infrastructure at NWP centers.
- The likelihood may also be used for the purpose of model evaluation.
- Research under way:
  - Experiments using larger models (ICTP AGCM, WRF)
  - Implementation on real case studies.
  - Theoretical developments for computing the PDF. (Carrassi et al. 2016)

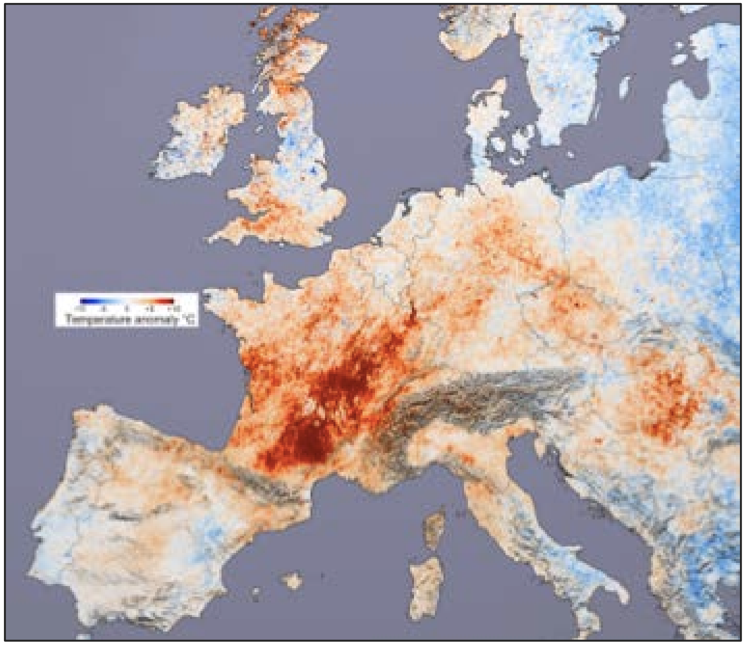
Thank you

Founders of Dadaïsm, Zürich, 1915

Inference from numerical simulations

- very rare events may never occur even in a large ensemble.

- **<u>example</u>**: estimates of the return time of the 2003 European heatwave range from 500 years to 40,000 years.



Inference from numerical simulations – the issue

- Estimating a small probability through brute force direct simulation:

- Generate a large number of samples and count:

$$\widehat{p}_E = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_E(\boldsymbol{Y}^{(i)})$$

- Variance and relative error:

$$\begin{cases} \mathbb{V}(\widehat{p}_E) = \dfrac{\widehat{p}_E(1-\widehat{p}_E)}{N} \simeq \dfrac{\widehat{p}_E}{N} \\ \mathrm{RE} = \dfrac{1}{\sqrt{\widehat{p}_E N}} \end{cases}$$

⇒ To estimate a return time of ~5000 years with a 10% error, one needs a 500,000 years of simulation.

Rare event simulation is a thematical field in statistics

- Main underlying idea: **importance sampling**

- Assume we can sample $\widetilde{\boldsymbol{Y}}$ from the PDF:

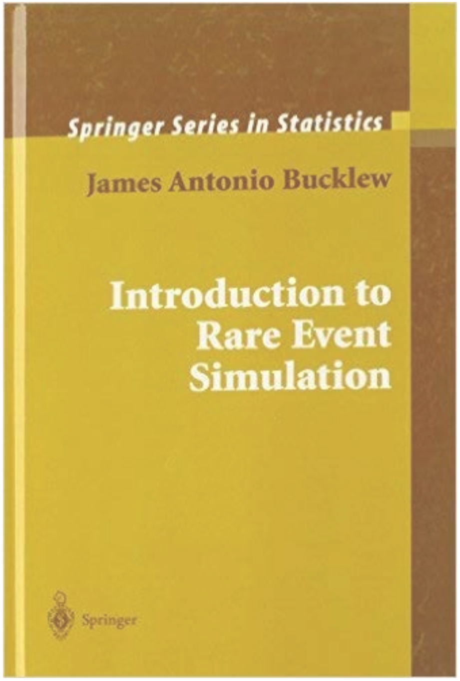
$$\widetilde{f}(\widetilde{\boldsymbol{Y}}) = f(\widetilde{\boldsymbol{Y}})\,.\,L(\widetilde{\boldsymbol{Y}})$$

- Then we can derive a new estimator:

$$\widetilde{p}_E = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_E(\widetilde{\boldsymbol{Y}}^{(i)})\,.\,L^{-1}(\widetilde{\boldsymbol{Y}}^{(i)})$$

- The variance may be substantially improved with an adequate *L*:

$$\mathbb{V}(\widetilde{p}_E) \ll \mathbb{V}(\widehat{p}_E)$$

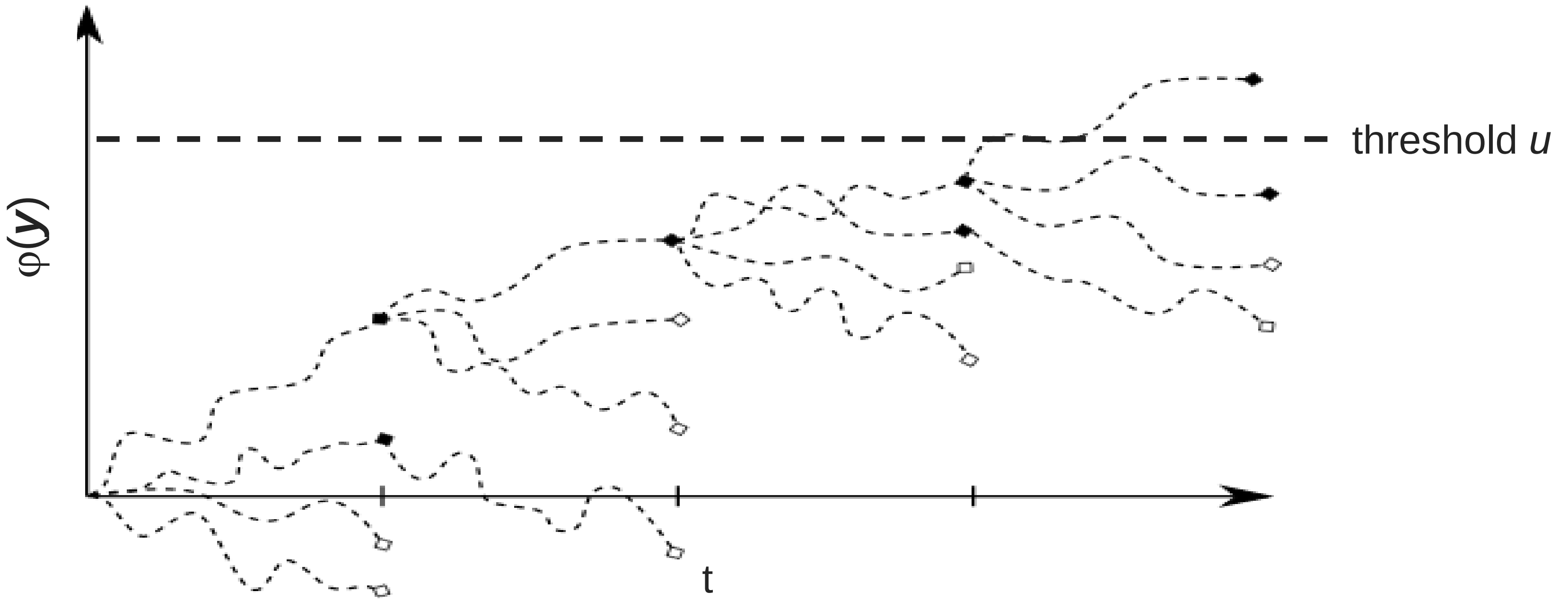


Rare event simulation is a thematical field in statistics

- Main underlying idea: **importance sampling**

$$\widetilde{f}(\widetilde{\boldsymbol{Y}}) = f(\widetilde{\boldsymbol{Y}})\,.\,L(\widetilde{\boldsymbol{Y}})$$

- For a stochastic process: interacting particle algorithm

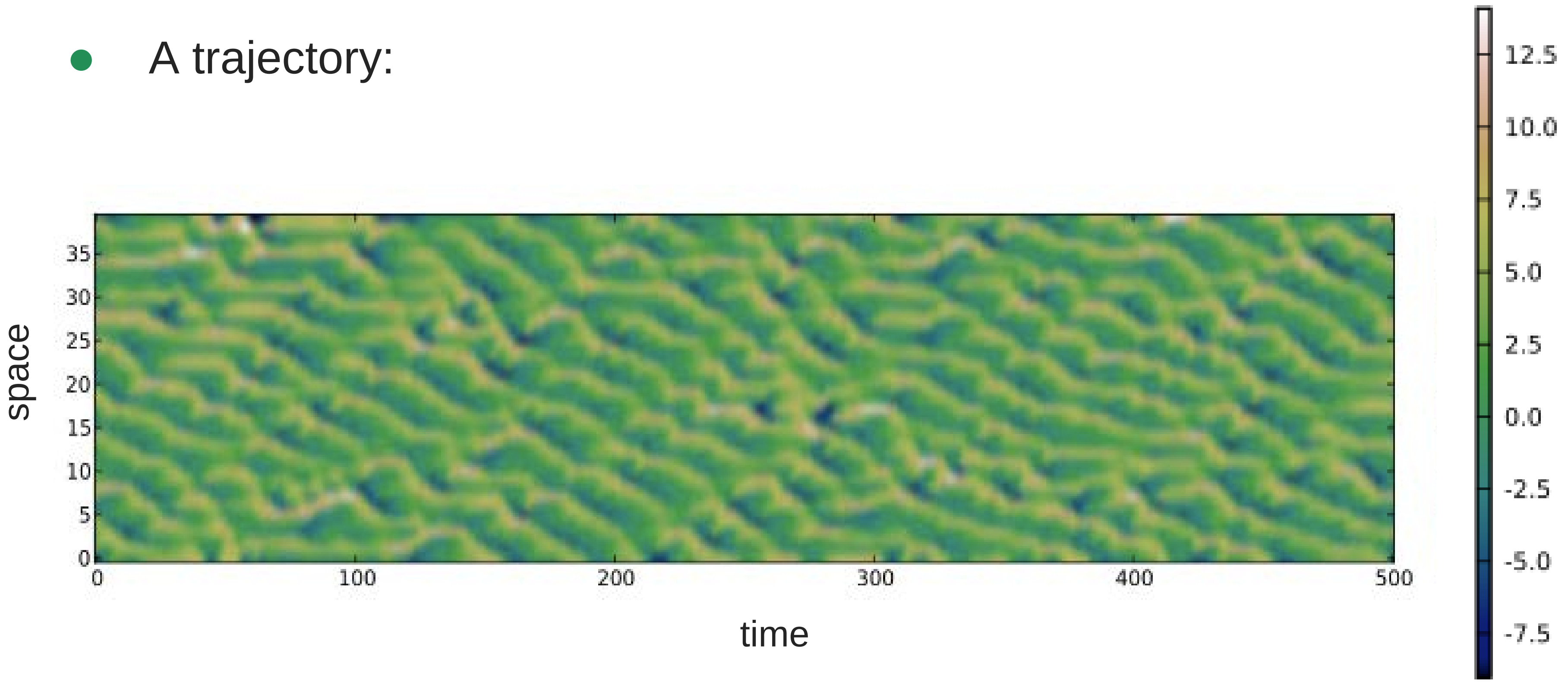


Rare event simulation – illustration in the L95 model

- Dynamic equation:

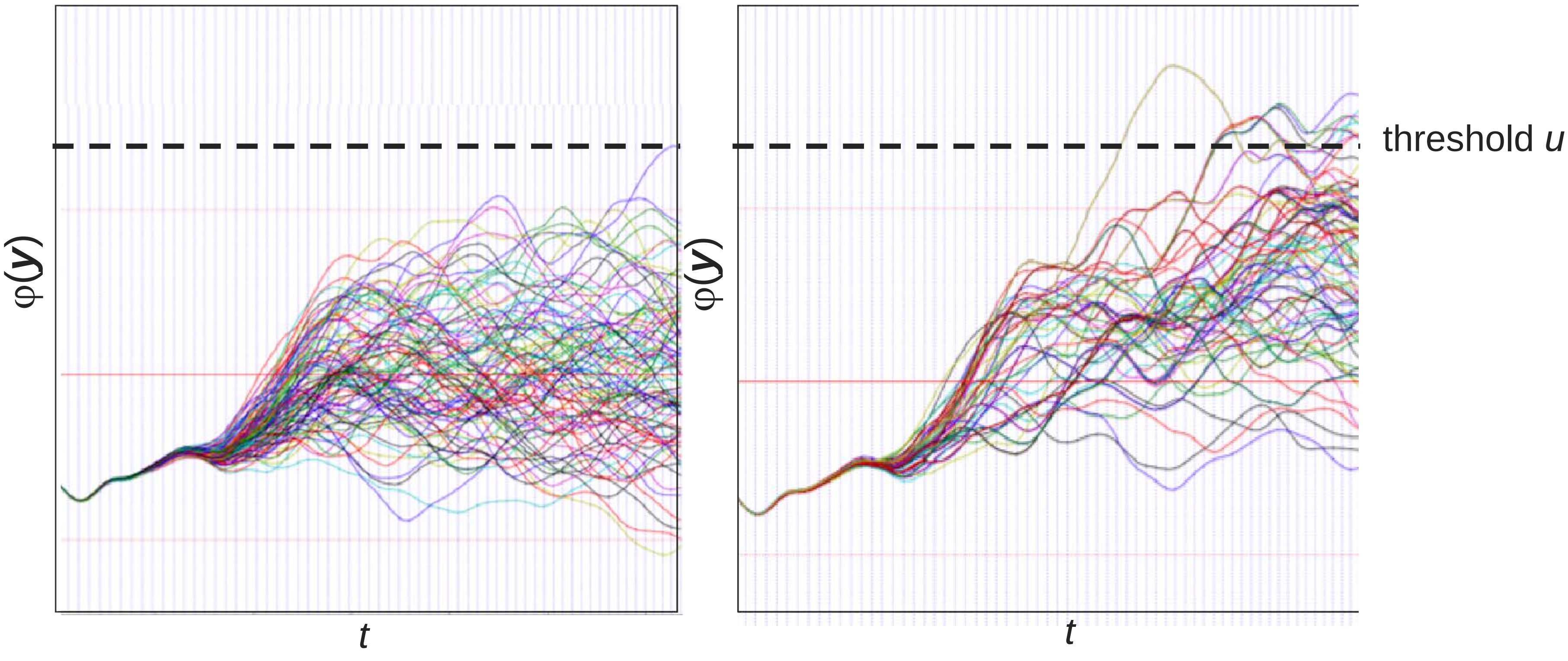
$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = (x_{i+1} - x_{i-2})x_{i-1} - x_i + F$$

- A trajectory:



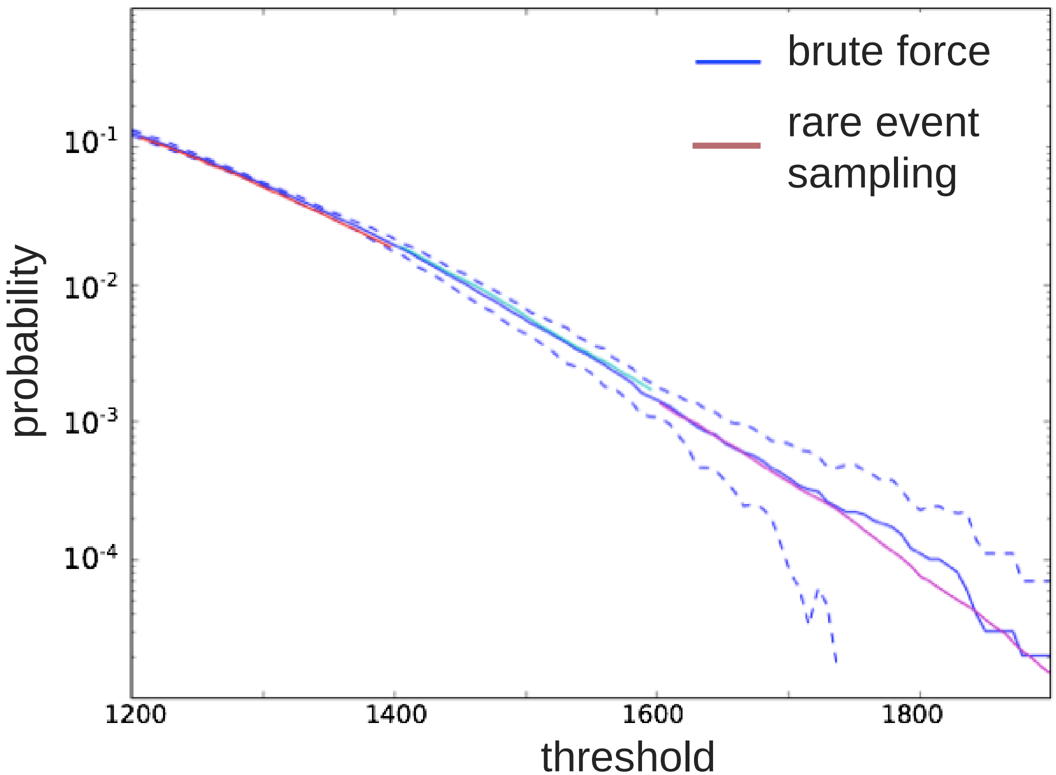
Rare event simulation – illustration in the L95 model

- The function φ is the total energy.
- The interacting particle algorithm is applied.



Rare event simulation – illustration in the L95 model

- The function φ is the total energy.
- The interacting particle algorithm is applied.
- Probabilities of exceedances are accurately estimated at a much lower computational expense.



Take aways

- It is possible to use **importance sampling** in dynamical systems (e.g. interacting particle algorithm).

- For the deterministic L95 model, probabilities of order 10^{-16} can be sampled as easily as probabilities of order 10^{-3}.

- This technique is promising for realistic climate models.

- Theoretical research and numerical experiments are needed.