hedging-assignment

December 19, 2018

```
In [1]: # Import modules
        import os
        import datetime
        from IPython.display import display
        import math
        import re


        import matplotlib.pyplot as plt
        import matplotlib.dates as mdates
        import numpy as np
        import pandas as pd
        import scipy
        import scipy.stats

        from utilities import *
        import hedge_functions
```

1 Master simulation source file

1.1 Read Data

```
In [2]: # Setup utilities and and data path
        datadir = os.path.join('..', 'data')

In [3]: ## Select data to analyze
        files = ['isx2008C.xls', 'isx2010C.xls']
        filename = os.path.join(datadir, files[1])

        # Read all sheets into a dictonary with sheet names as keys
        data = pd.read_excel(filename, sheet_name=None)

In [4]: sheet_names = list(data.keys())
        sheet_name = 'isx15012010C'

        sheet, (mat, T, S, Cobs, E, r) = get_sheet(data, sheet_name)
        n,m = Cobs.shape

In [5]: ## Simulation utilities

        %matplotlib inline

        def get_closest_strike(estimate, strikes):
            """
            Return the strike price closest to given estimate
            """
            return strikes[(np.abs(strikes - estimate)).argmin()]
```

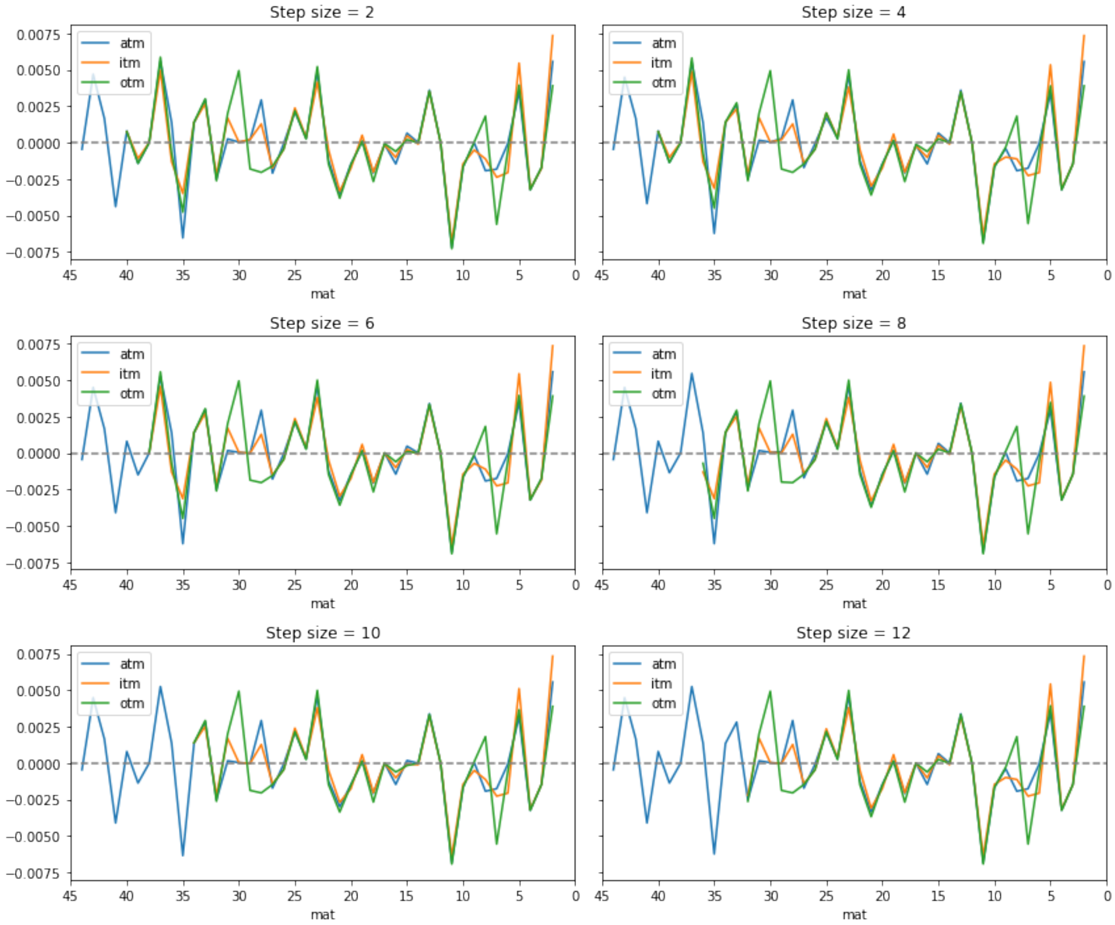
1.2 Simple Delta Simulation

```
In [6]: def simple_delta_simulation(sheet, strikes, step=1, mat=45, diff=0, sigma=0.1):
            """
            Simulate hedging for estimated costs accuracy of hedging

            Params:
                sheet (pd.DataFrame): worksheet to hedge
                strikes (pd.Series):  strike prices for convinience
                step (int):           number of days between hedges
                maturity (int):       number of days to maturity when starting hedging
                diff (int):           difference between spot and strike prices
                                      = 0 for ATM-option
                                      < 0 for out-of-the-money option
                                      > 0 for in-the-money option
            Returns:
                errors (pd.Series)
                MSE (float)
                costs (float)
            """


            spot = sheet.s_price.iloc[0]          # spot price on t0

            # Construct a single option portfolio with one option
            # C1 and hedge it with delta amount of underlying

            strike = get_closest_strike(spot + diff, strikes) # select C1 strike

            portfolio = sheet.rename(columns={
                strike: 'C1'
            })[['C1', 's_price', 'time', 'r']]

            # Include only 'mat' preceeding days
            portfolio = portfolio[portfolio.index <= mat]

            # Select days to rehedge
            hedge_rows = portfolio.iloc[::step, :]

            def call_hedge(row):
                """
                Helper function to apply on all rows row by row
                """
                # Calculate implied volatility
                sigma = hedge_functions.calculate_implied_volatility_bs(
                    row.time, strike, row.s_price, row.r, row.C1)
                # Calculate delta
                return hedge_functions.delta_hedge(row.time, strike, row.s_price, row.r, sigma)

            # Calculate deltas for given days
            delta = hedge_rows.apply(call_hedge, axis=1)

            # Keep deltas constant between rehedge days
            portfolio['delta'] = delta.reindex(portfolio.index).fillna(method='ffill')

            # Calculate change of value of single option portfolio
            portfolio['dC1'] = (portfolio.C1 - portfolio.C1.shift(1))

            # Calculate value and change of value of the replicating portfolio
            portfolio['Re'] = portfolio.delta/100 * portfolio.s_price
            portfolio['dRe'] = portfolio.Re - portfolio.Re.shift(1)

            # Calculate error as diffrence between change of value of the
            # replicating portfolio and the option portfolio
            portfolio['A'] = portfolio.dRe - portfolio.dC1

            # Calculate the change of delta. This is used for estimating
```

```python
    # turnover ratio and transaction costs
    portfolio['dDelta'] = delta - delta.shift(1)

    return portfolio

# portfolio = simple_delta_simulation(sheet, E, step=2)
# portfolio
```

In [7]:
```python
# Select three options to perform simple delta hedging on.
# One at-the-money, one slighlty in-the-money and one slightly
# of-the-money option

diffs = [0, 0.006, -0.006] # how much to differiate option from spot price
labels = ['atm', 'itm', 'otm']

 # Keep these paramters fixed for all three options
hedge_freq = [2, 4, 6, 8, 10, 12]
num_sim = len(hedge_freq)
mat = 45

# Initalize matrices to store simulation results
MSE = np.zeros((num_sim, 3))
turnover = np.zeros((num_sim, 3))

# Plot results
fig, axes = plt.subplots(math.ceil(num_sim / 2),2,figsize=(12,10), sharey=True)

for i, (ax, freq) in enumerate(zip(axes.flat, hedge_freq)):
    for j, (diff, label) in enumerate(zip(diffs,labels)):

        df = simple_delta_simulation(sheet, E, step=freq, diff=diff, mat=mat)
        df.plot(y='A', ax=ax, label=label)

        MSE[i, j] = np.sqrt(df.A.pow(2).sum())
        turnover[i, j] = df.dDelta.abs().sum()

    ax.axhline(y=0, linestyle='dashed', color='grey')
    ax.set_xlim([mat, 0])
    ax.set_title('Step size = {}'.format(freq))


fig.tight_layout()

MSE_df = pd.DataFrame(MSE, columns = ["MSE_{}".format(label) for label in labels])
turnover_df = pd.DataFrame(turnover,  columns=["turnover_{}".format(label) for label in
labels])

stats = pd.concat([MSE_df, turnover_df], axis=1)

#stats['costs'] = stats.Turnover* 1e5*5e-4

display(stats)
```

	MSE_atm	MSE_itm	MSE_otm	turnover_atm	turnover_itm	turnover_otm
0	0.018333	0.016585	0.017966	0.778547	0.779428	0.521868
1	0.017653	0.015938	0.017488	0.475212	0.466159	0.311553
2	0.017659	0.015915	0.017426	0.270364	0.282296	0.212182
3	0.017628	0.015019	0.016402	0.289820	0.235314	0.196792
4	0.017511	0.014598	0.015698	0.172874	0.175947	0.131621
5	0.017605	0.014527	0.015501	0.154623	0.169204	0.137538



1.3 Simple Delta-Vega Simulation

```
In [8]: def simple_delta_vega_simulation(sheet, strikes, step=1, mat1=45, tdiff=15, pdiff=0,
        sigma=0.1):
            """
            Simulate hedging for estimated costs accuracy of hedging

            Params:
                sheet (pd.DataFrame): worksheet to hedge
                strikes (pd.Series):  strike prices for convinience
                step (int):           number of days between hedges
                mat1 (int):           number of days to maturity when starting hedging
                tdiff (int):          mat2 = mat1 + tdiff
                pdiff (int):          difference between spot and strike prices
                                      = 0 for ATM-option
                                      < 0 for out-of-the-money option
                                      > 0 for in-the-money option
            Returns:
                errors (pd.Series)
                MSE (float)
                costs (float)
            """

            spot = sheet.s_price.iloc[0]        # spot price on t0
```

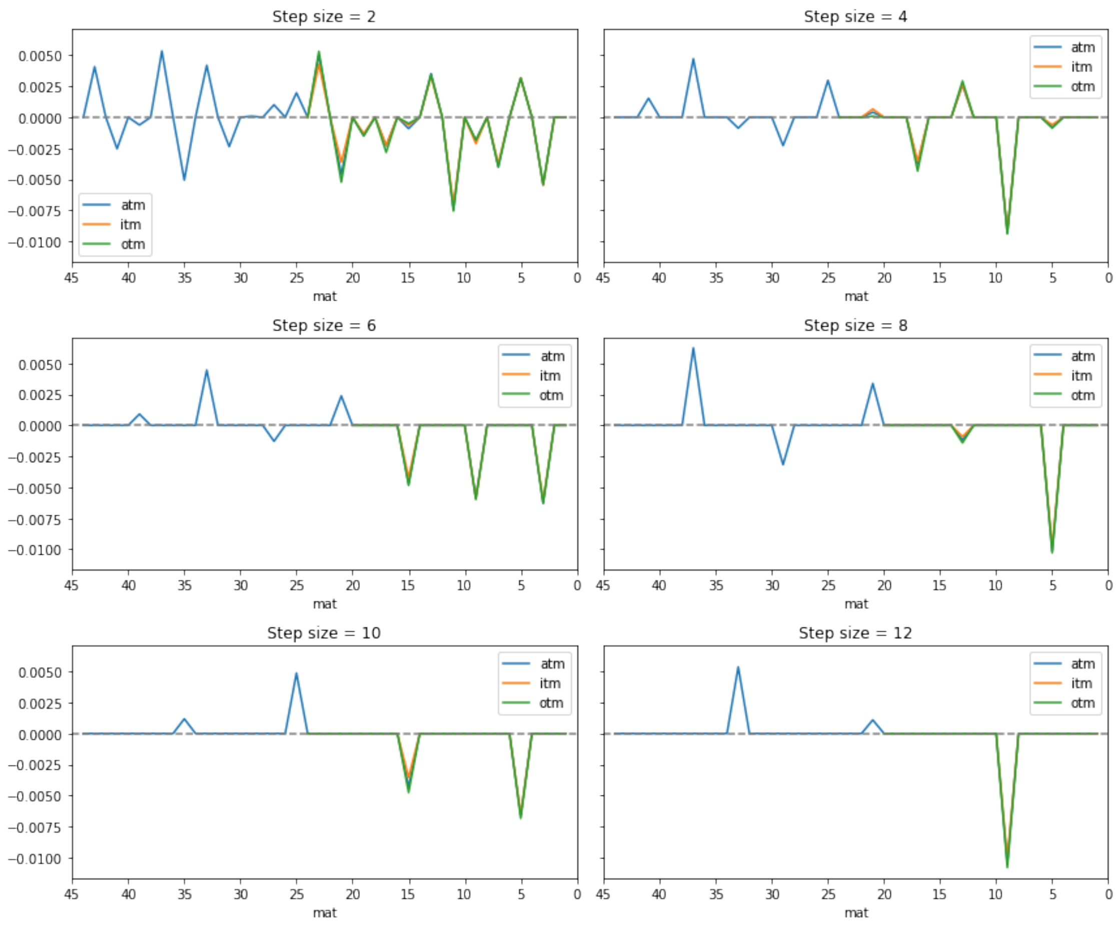
```
    # Construct a single option portfolio with one option
    # C1 and hedge it with delta amount of underlying


    # Construct a portfolio with two options on same underlying,
    # same strike but with different maturities. Call option with
    # shorter maturity C1 and the latter C2

    strike = get_closest_strike(spot + pdiff, strikes)

    portfolio = sheet.rename(columns={
        strike: 'C1',
        'time': 'mat1'
    })
    portfolio['C2'] = portfolio.C1.shift(tdiff)
    portfolio['mat2'] = portfolio.mat1.shift(tdiff)

    # For convinience select only columns we're interested in
    portfolio = portfolio[['C1', 'C2', 'mat1', 'mat2', 'r', 's_price']]

    # Select only information preceeding mat1 days from maturity of C1
    portfolio = portfolio[portfolio.index <= mat1]

    # Call rehedge on given time steps only
    hedge_rows = portfolio.iloc[::step, :]

    def call_hedge(row):
        """
        Helper function to apply on all selected rows
        """
        # Calculate implied volatility
        sigma = hedge_functions.calculate_implied_volatility_bs(
            row.mat1, strike, row.s_price, row.r, row.C1)

        # Calculate alpha and eta
        alpha, eta = hedge_functions.vega_hedge(
            row.mat1, row.mat2, strike, row.s_price, row.r, sigma)
        return pd.Series({'alpha': alpha, 'eta': eta})

    # Calculate alpha and eta parameters
    hedge_rows = pd.concat([hedge_rows, hedge_rows.apply(call_hedge, axis=1)], axis=1)

    # Fill constant alpha and eta between rehedge days
    portfolio = hedge_rows.reindex(index=portfolio.index).fillna(method='ffill')

    # Calculate change of value for portfolio consiating of only C1
    portfolio['dC1'] = portfolio.C1 - portfolio.C1.shift(1)

    # Calculate value and change of value for replicating portfolio
    portfolio['Re'] = portfolio.alpha/100 * portfolio.s_price + portfolio.eta/100 *
portfolio.C2
    portfolio['dRe'] = portfolio.Re - portfolio.Re.shift(1)

    # Calculate error as diffrence between change of value of the
    # replicating portfolio and the option portfolio
    portfolio['A'] = portfolio.dRe - portfolio.dC1

    # Estimate transaction costs by calculating turnover of C2 and underlying
    portfolio['dAlpha'] = portfolio.alpha - portfolio.alpha.shift(1)
    portfolio['dEta'] = portfolio.eta - portfolio.eta.shift(1)

    return portfolio
```

In [9]:
```
# Select three options to perform simple delta hedging on.
# One at-the-money, one slighlty in-the-money and one slightly
# of-the-money option
```

```python
diffs = [0, 0.006, -0.006] # how much to differiate option from spot price
labels = ['atm', 'itm', 'otm']

# Keep these paramters fixed for all three options
hedge_freq = [2, 4, 6, 8, 10, 12]
num_sim = len(hedge_freq)
mat1 = 45

# Initalize matrices to store simulation results
MSE = np.zeros((num_sim, 3))
turnover_alpha = np.zeros((num_sim, 3))
turnover_eta = np.zeros((num_sim, 3))

fig, axes = plt.subplots(math.ceil(num_sim / 2),2,figsize=(12,10), sharey=True)

for i, (ax, freq) in enumerate(zip(axes.flat, hedge_freq)):
    for j, (diff, label) in enumerate(zip(diffs,labels)):

        df = simple_delta_vega_simulation(sheet, E, step=freq, pdiff=diff, mat1=mat1)
        df.plot(y='A', ax=ax, label=label)

        MSE[i, j] = np.sqrt(df.A.pow(2).sum())
        turnover_alpha[i,j] = df.dAlpha.abs().sum()
        turnover_eta[i,j] = df.dEta.abs().sum()

    ax.axhline(y=0, linestyle='dashed', color='grey')
    ax.set_xlim([mat, 0])
    ax.set_title('Step size = {}'.format(freq))

fig.tight_layout()

MSE_df = pd.DataFrame(MSE, columns = ["MSE_{}".format(label) for label in labels])
turnover_alpha_df = pd.DataFrame(turnover_alpha,  columns=["t_alpha_{}".format(label)
for label in labels])
turnover_eta_df = pd.DataFrame(turnover_eta,  columns=["t_eta_{}".format(label) for
label in labels])

stats = pd.concat([MSE_df, turnover_alpha_df, turnover_eta_df], axis=1)
#stats['costs'] = stats.Turnover* 1e5*5e-4

display(stats)
```

	MSE_atm	MSE_itm	MSE_otm	t_alpha_atm	t_alpha_itm	t_alpha_otm	\
0	0.016978	0.012577	0.013928	0.945890	0.924626	0.841056	
1	0.012189	0.010271	0.010802	0.627705	0.576420	0.566633	
2	0.011107	0.009461	0.009937	0.699899	0.662450	0.664861	
3	0.012840	0.009860	0.010378	0.545532	0.530209	0.550200	
4	0.009354	0.007421	0.008344	0.545532	0.490427	0.509277	
5	0.011862	0.010122	0.010819	0.433663	0.378395	0.405060	

	t_eta_atm	t_eta_itm	t_eta_otm
0	0.798223	0.765087	0.762347
1	0.570996	0.528881	0.553069
2	0.692458	0.654120	0.659664
3	0.551685	0.524343	0.546274
4	0.551685	0.495926	0.512708
5	0.433939	0.371244	0.399412



1.4 Butterfly Delta Simulation

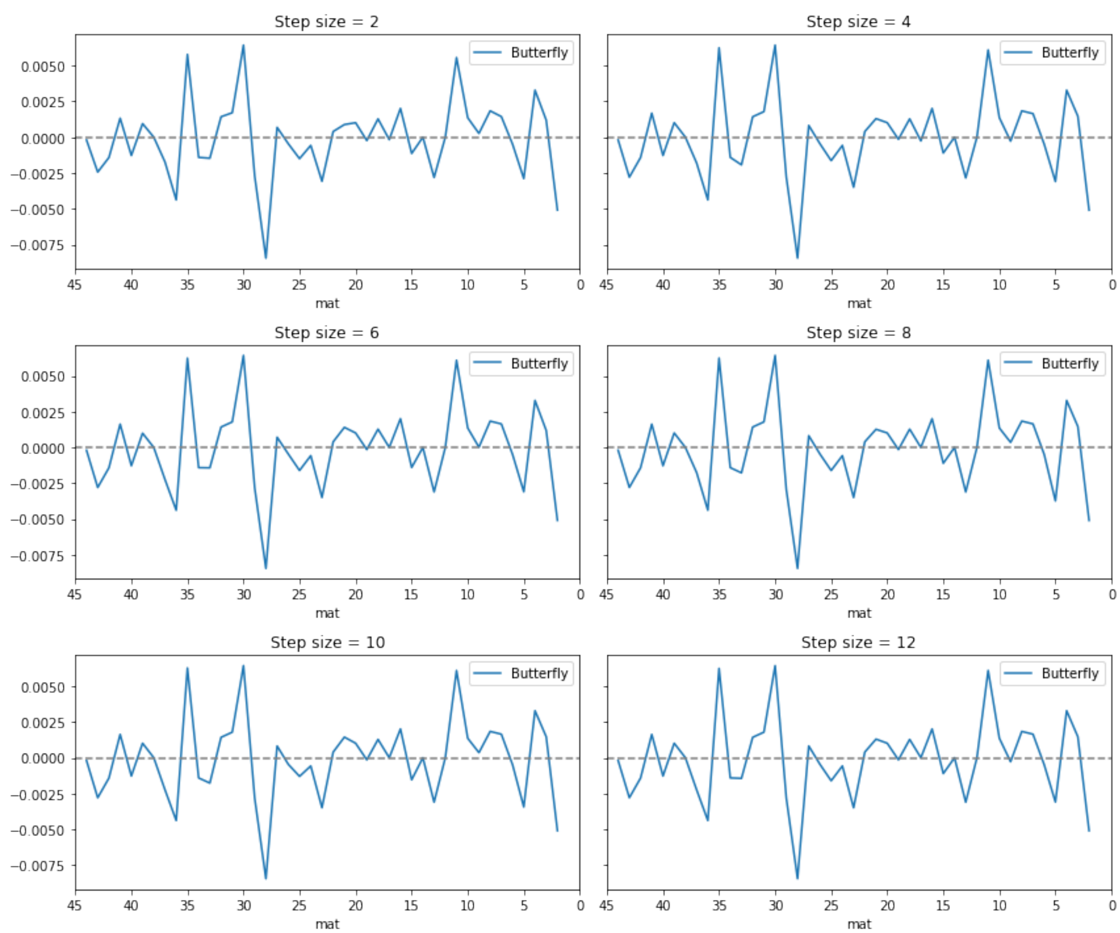
In [10]:
```
def butterfly_delta_simulation(sheet, strikes, step=1, mat=45, diff=0):
    """
    Simulate hedging for estimated costs accuracy of hedging

    Params:
        sheet (pd.DataFrame): worksheet to hedge
        strikes (pd.Series):  strike prices for convinience
        step (int):           number of days between hedges
        maturity (int):       number of days to maturity when starting hedging
        diff (int):           difference between spot and strike prices
                              = 0 for ATM-option
                              < 0 for out-of-the-money option
                              > 0 for in-the-money option
    Returns:
        errors (pd.Series)
        MSE (float)
        costs (float)
    """

    spot = sheet.s_price.iloc[0]                    # spot price at t0

    atm = get_closest_strike(spot, strikes)         # atm strike at t0
```

```
    itm = get_closest_strike(spot + 0.1, strikes)   # option with higher strike
    otm = get_closest_strike(spot - 0.1, strikes)   # option with lower strike

    # Construct portfolio of only the selected three options
    portfolio = sheet.rename(columns={
        atm: 'C_atm',
        otm: 'C_otm',
        itm: 'C_itm'
    })[['C_atm', 'C_itm', 'C_otm', 's_price', 'r', 'time']]


    # Store strike prices for calculating delta
    strikes = [itm, otm, atm]

    # Include price data only from 'mat' last days
    portfolio = portfolio[portfolio.index <= mat]

    # Select days to perform rehedge
    hedge_rows = portfolio.iloc[::step, :] # select days to rehedge

    def call_hedge(row):
        """
        Helper function to apply on all rows
        """
        # Calculate implied volatility of C_atm
        sigma = hedge_functions.calculate_implied_volatility_bs(
            row.time, atm, row.s_price, row.r, row.C_atm)
        # Calculate delta for portfolio
        return hedge_functions.delta_hedge_butterfly(row.time, strikes, row.s_price,
row.r, sigma)

    # Calculate deltas for portfolio
    delta = hedge_rows.apply(call_hedge, axis=1)

    # Fill constant deltas for portfolio between rehedge days
    portfolio['delta'] = delta.reindex(portfolio.index).fillna(method='ffill')

    # Calculate value and change of value for portfolio of options
    portfolio['V'] = portfolio.C_itm - 2*portfolio.C_atm + portfolio.C_otm
    portfolio['dV'] = portfolio.V - portfolio.V.shift(1)

    # Calculate value and change of value for replicating portfolio
    portfolio['Re'] = portfolio.delta/100 * portfolio.s_price
    portfolio['dRe'] = portfolio.Re - portfolio.Re.shift(1)

    # Calculate error as diffrence between change of value of the
    # replicating portfolio and the option portfolio
    portfolio['A'] = portfolio.dRe - portfolio.dV

    # Calculate change of delta for portfolio. This is used in estimating
    # turnover rate and transaction costs
    portfolio['dDelta'] = delta - delta.shift(1)

    return portfolio
```

In [11]:
```
hedge_freq = [2, 4, 6, 8, 10, 12]
num_sim = len(hedge_freq)
mat = 45
fig, axes = plt.subplots(math.ceil(num_sim / 2),2,figsize=(12,10), sharey=True)

MSE = np.zeros(num_sim)
turnover = np.zeros(num_sim)

for i, (ax, freq) in enumerate(zip(axes.flat, hedge_freq)):

    df = butterfly_delta_simulation(sheet, E, step=freq, diff=0, mat=mat)
    df.plot(y='A', ax=ax, label='Butterfly')
```

```
        ax.axhline(y=0, linestyle='dashed', color='grey')
        ax.set_xlim([mat, 0])
        ax.set_title('Step size = {}'.format(freq))

        MSE[i] = np.sqrt(df.A.pow(2).sum())
        turnover[i] = df.dDelta.abs().sum()

    fig.tight_layout()

    stats = pd.DataFrame({
        'MSE': MSE,
        'Turnover': turnover,
        'Hedge Freq': hedge_freq
        })

    stats['costs'] = stats.Turnover* 1e5*5e-4

    display(stats)
```

	Hedge Freq	MSE	Turnover	costs
0	2	0.017666	1.028708	51.435377
1	4	0.018344	0.386905	19.345253
2	6	0.018378	0.271855	13.592738
3	8	0.018498	0.275068	13.753392
4	10	0.018502	0.214365	10.718253
5	12	0.018383	0.212842	10.642098



1.5 Butterfly Delta-Vega Simulation

```
In [12]: def butterfly_delta_vega_simulation(sheet, strikes, step=1, mat1=45, pdiff=0, tdiff=15):
             """
             Simulate hedging for estimated costs accuracy of hedging

             Params:
                 sheet (pd.DataFrame): worksheet to hedge
                 strikes (pd.Series):  strike prices for convinience
                 step (int):           number of days between hedges
                 mat1 (int):           number of days to maturity when starting hedging
                 tdiff (int):          mat2 = mat1 + tdiff

             Returns:
                 errors (pd.Series)
                 MSE (float)
                 costs (float)
             """

             spot = sheet.s_price.iloc[0]                    # spot price at t0

             atm = get_closest_strike(spot, strikes)         # atm strike at t0
             itm = get_closest_strike(spot + 0.1, strikes)   # option with higher strike
             otm = get_closest_strike(spot - 0.1, strikes)   # option with lower strike

             # Construct portfolio of three options with different strikes
             portfolio = sheet.rename(columns={
                 atm: 'C1_atm',
                 otm: 'C1_otm',
                 itm: 'C1_itm',
                 'time': 'T1'
             })[['C1_atm', 'C1_itm', 'C1_otm', 's_price', 'r', 'T1']]

             # Hedge with underlying and three options with selected strikes but
             # with longer maturity. We'll refer to these as C2_name

             portfolio['C2_atm'] = portfolio.C1_atm.shift(tdiff)
             portfolio['C2_itm'] = portfolio.C1_itm.shift(tdiff)
             portfolio['C2_otm'] = portfolio.C1_otm.shift(tdiff)

             # Store maturity of longer options
             portfolio['T2'] = portfolio.T1.shift(tdiff)

             # Store strike prices of options for calculating delta
             strikes = [itm, otm, atm]

             # Include price data only from 'mat1' last days
             portfolio = portfolio[portfolio.index <= mat1]

             # Select days to perform rehedge
             hedge_rows = portfolio.iloc[::step, :] # select days to rehedge

             def call_hedge(row):
                 """
                 Helper function to apply on all rows
                 """
                 # Calculate implied volatility of C1_atm
                 sigma = hedge_functions.calculate_implied_volatility_bs(
                     row.T1, atm, row.s_price, row.r, row.C1_atm)

                 # Calculate delta for portfolio
                 alpha, eta_itm, eta_atm, eta_otm =  hedge_functions.vega_hedge_butterfly(
```

```
                row.T1, row.T2, strikes, row.s_price, row.r, sigma)

            return pd.Series({
                'alpha': alpha,
                'eta_itm': eta_itm,
                'eta_atm': eta_atm,
                'eta_otm': eta_otm
            })

        # Calculate alphas and etas for portfolio
        alpha_eta = hedge_rows.apply(call_hedge, axis=1)

        # Fill constant alphas and etas for portfolio between rehedge days
        alpha_eta = alpha_eta.reindex(portfolio.index).fillna(method='ffill')
        portfolio = pd.concat([portfolio, alpha_eta], axis=1)

        # Calculate value and change of value for portfolio of options
        portfolio['V'] = portfolio.C1_itm - 2*portfolio.C1_atm + portfolio.C1_otm
        portfolio['dV'] = portfolio.V - portfolio.V.shift(1)

        # Calculate value and change of value for replicating portfolio
        portfolio['Re'] = (portfolio.alpha/100 * portfolio.s_price +
                           portfolio.eta_itm/100 * portfolio.C2_itm +
                           portfolio.eta_atm/100 * portfolio.C2_atm +
                           portfolio.eta_otm/100 * portfolio.C2_otm)

        portfolio['dRe'] = portfolio.Re - portfolio.Re.shift(1)

        # Calculate error as diffrence between change of value of the
        # replicating portfolio and the option portfolio
        portfolio['A'] = portfolio.dRe - portfolio.dV

        # Estimate transaction costs by calculating turnover of C2 and underlying
        portfolio['dAlpha'] = portfolio.alpha - portfolio.alpha.shift(1)
        portfolio['dEta_itm'] = portfolio.eta_itm - portfolio.eta_itm.shift(1)
        portfolio['dEta_atm'] = portfolio.eta_atm - portfolio.eta_atm.shift(1)
        portfolio['dEta_otm'] = portfolio.eta_otm - portfolio.eta_otm.shift(1)

        return portfolio
```

In [13]:

```
# Perform delta-vega hedging on Butterfly spread portfolio
# consisting of 1 itm, -2 atm and 1 otm option

# Keep these paramters fixed
hedge_freq = [2, 4, 6, 8, 10, 12, 16, 20]
num_sim = len(hedge_freq)
mat1 = 45

# Initalize matrices to store simulation results
MSE = np.zeros((num_sim, 3))
turnover_alpha = np.zeros((num_sim, 3))

turnover_eta_itm = np.zeros((num_sim, 3))
turnover_eta_atm = np.zeros((num_sim, 3))
turnover_eta_otm = np.zeros((num_sim, 3))

fig, axes = plt.subplots(math.ceil(num_sim / 2),2,figsize=(12,10), sharey=True)

for i, (ax, freq) in enumerate(zip(axes.flat, hedge_freq)):
    for j, (diff, label) in enumerate(zip(diffs,labels)):

        df = butterfly_delta_vega_simulation(sheet, E, step=freq, mat1=mat1)
        df.plot(y='A', ax=ax, label=label)

        MSE[i, j] = np.sqrt(df.A.pow(2).sum())
        turnover_alpha[i,j] = df.dAlpha.abs().sum()
        turnover_eta_itm[i,j] = df.dEta_itm.abs().sum()
        turnover_eta_atm[i,j] = df.dEta_atm.abs().sum()
```

```python
            turnover_eta_otm[i,j] = df.dEta_otm.abs().sum()

        ax.axhline(y=0, linestyle='dashed', color='grey')
        ax.set_xlim([mat, 0])
        ax.set_title('Step size = {}'.format(freq))

    fig.tight_layout()

    # Calculate statistics for each rehedge frequency simulations

    # MSE denotes means-squared-error of A values
    # turnover_alpha denotes the turnover of underlying
    # turnover_eta_itm denotes the turnover of itm option (longer maturity)
    # turnover_eta_atm denotes the turnover of atm option (longer maturity)
    # turnover_eta_otm denotes the turnover of otm option (longer maturity)

    MSE_df = pd.DataFrame(MSE, columns = ["MSE_{}".format(label) for label in labels])
    MSE_df.insert(0, 'Freq', hedge_freq)

    turnover_alpha_df = pd.DataFrame(turnover_alpha,  columns=["t_alpha_{}".format(label)
    for label in labels])

    turnover_eta_itm_df = pd.DataFrame(
        turnover_eta_itm,  columns=["t_eta_itm_{}".format(label) for label in labels])

    turnover_eta_atm_df = pd.DataFrame(
        turnover_eta_atm,  columns=["t_eta_atm_{}".format(label) for label in labels])

    turnover_eta_otm_df = pd.DataFrame(
        turnover_eta_otm,  columns=["t_eta_otm_{}".format(label) for label in labels])

    stats = pd.concat([MSE_df,
                       turnover_alpha_df,
                       turnover_eta_itm_df,
                       turnover_eta_atm_df,
                       turnover_eta_otm_df], axis=1)

    #stats['costs'] = stats.Turnover* 1e5*5e-4

    display(stats)
```

```
   Freq   MSE_atm   MSE_itm   MSE_otm  t_alpha_atm  t_alpha_itm  t_alpha_otm  \
0     2  0.018287  0.018287  0.018287     1.495361     1.495361     1.495361
1     4  0.018648  0.018648  0.018648     1.161471     1.161471     1.161471
2     6  0.018462  0.018462  0.018462     0.895924     0.895924     0.895924
3     8  0.018936  0.018936  0.018936     0.906258     0.906258     0.906258
4    10  0.019013  0.019013  0.019013     0.741529     0.741529     0.741529
5    12  0.018415  0.018415  0.018415     0.621562     0.621562     0.621562
6    16  0.018328  0.018328  0.018328     0.284527     0.284527     0.284527
7    20  0.018843  0.018843  0.018843     0.741529     0.741529     0.741529

   t_eta_itm_atm  t_eta_itm_itm  t_eta_itm_otm  t_eta_atm_atm  t_eta_atm_itm  \
0       0.809536       0.809536       0.809536       0.798223       0.798223
1       0.649266       0.649266       0.649266       0.570996       0.570996
2       0.590173       0.590173       0.590173       0.692458       0.692458
3       0.542259       0.542259       0.542259       0.551685       0.551685
4       0.539832       0.539832       0.539832       0.551685       0.551685
5       0.428841       0.428841       0.428841       0.433939       0.433939
6       0.444011       0.444011       0.444011       0.259989       0.259989
7       0.539832       0.539832       0.539832       0.551685       0.551685
```

	t_eta_atm_otm	t_eta_otm_atm	t_eta_otm_itm	t_eta_otm_otm
0	0.798223	1.088701	1.088701	1.088701
1	0.570996	0.976009	0.976009	0.976009
2	0.692458	0.527708	0.527708	0.527708
3	0.551685	0.656668	0.656668	0.656668
4	0.551685	0.527693	0.527693	0.527693
5	0.433939	0.527392	0.527392	0.527392
6	0.259989	0.523704	0.523704	0.523704
7	0.551685	0.527693	0.527693	0.527693